Solitonic quasi-particles in electronic systems : 1979 - 2009

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Доложено на юбилее основателя и директора Института,
Ученого и организатора и азартного игрока -
Исаака Марковича Халатникова

"На этих выборах, Институт Ландау ведет нечестную игру:
он пользуется интеллектуальным превосходством своих сотрудников."

Протест одного академика на выборных баталиях в АН СССР.

Услышано в комнате #5 ИФП от самого Халатникова,

Solitons as elementary excitations in electronic systems with *conventional* spontaneous symmetry breakings

standard symmetries: Superconductors, Antiferromagnetic Mott Insulator, Spin/ Charge Density Waves

Secure starting level: one - dimensional systems.

Solitons in the ground state and as elementary excitations.
Conversion of electrons to various solitons;
Separated or even anomalous charge, spin, currents;
Mid-gap states (zero fermion modes)

Quasi one- dimensional route:

Recent experimental confirmations.
Confinement of solitons and dimensional crossover.
Spin-charge recombination due to 3D confinement.

Extrapolation to arbitrary correlated systems:

Combined symmetry and spin-charge reconfinement;
Spin- or Charge- roton-like excitations with charge- or spin- kinks localized in the core;
Inverse rout: from regular stripe- or FFLO solitonic lattices to combined solitons.

Singlet ground state gapful systems:
SuperConductors SCs and
Charge Density Waves CDWs.

Standard BCS-Bogolubov view:

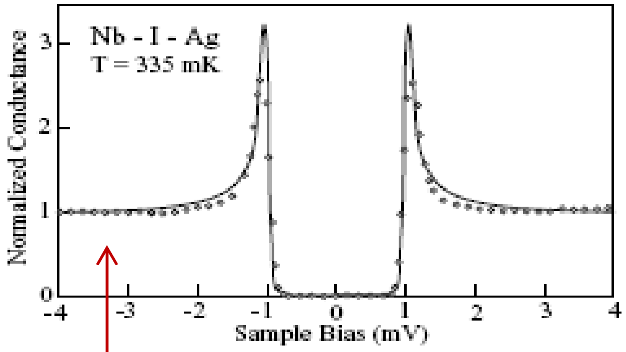
Spectra : $\mathbf{E(k)= \pm(\Delta^2+(v_f k)^2)^{1/2}}$

States = linear combinations of :

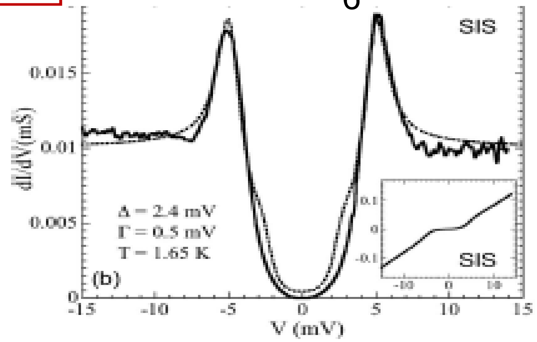
electrons and holes at $\pm$**p** for SC

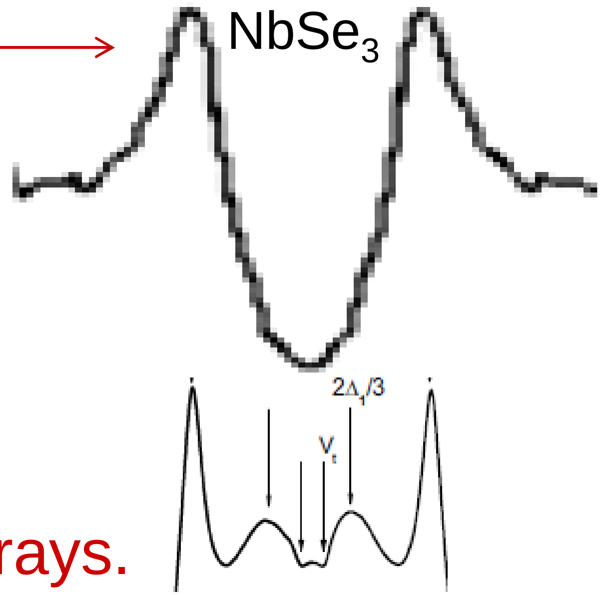
electrons at **–p** and **p+2p$_f$** for CDW

Figures:
pair-breaking gaps from tunneling experiments.



CaC_6





Is it always true?

Proved "**yes**" for typical SCs.

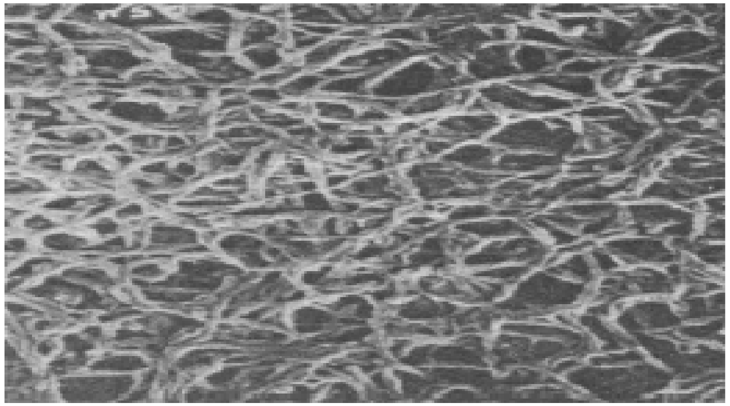
Questionable for strong coupling : *High-T_c, real space pairs, cold atoms, bi-polarons.*

Certainly incomplete for CDWs
as proved by many modern experiments.

Certainly inconsistent for 1D and even
quasi 1D systems as proved theoretically.

Guilty and Most Wanted : solitons and their arrays.

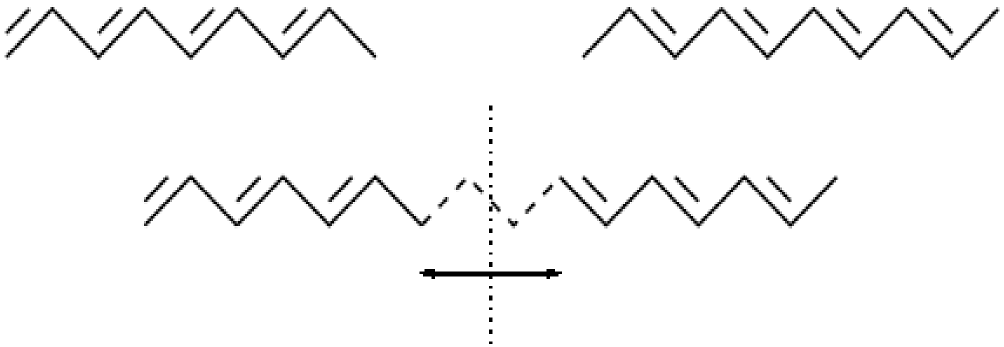
Inside the noodles: trans-$(CH)_x$ chains with one excess electron per site. Chains should be metallic (gapless 1D electrons) but the spontaneous atomic dimerization opens gap 2Δ.
Peierls – BCS effect, equivalence to Gross-Neveu model.
Basis for exact solutions for solitons and their superstructures:
Relations to nonlinear equations (Schroedinger, KdV, MKdV) -
Fateev, Novikov and Dubrovin, Its, Krichever;
S.B. with Kirova, Dzyaloshinskii, Matveenko



$$Tr\begin{vmatrix} -i\partial_x & \Delta^* \\ \Delta & i\partial_x \end{vmatrix} + K\overline{|\Delta|^2}$$

Ground state double degeneracy allows for topological solitons = kinks = trajectories connecting equivalent vacuums (+/-1).

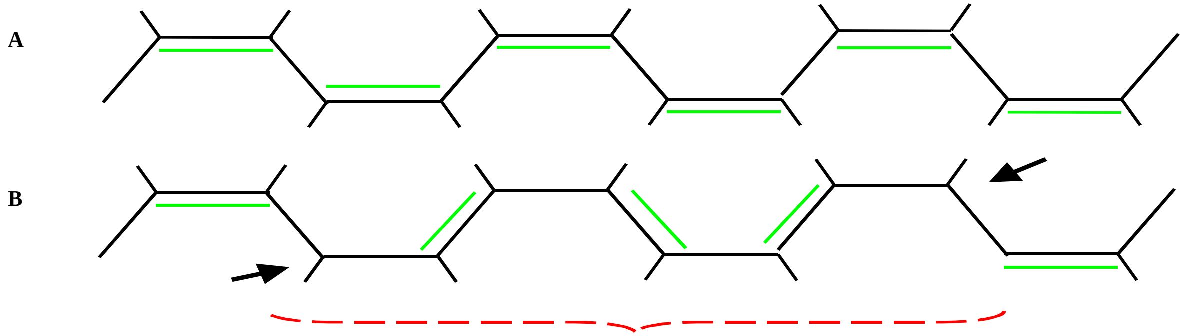
Phase of Δ is fixed, Z_2



Major properties of kinks:

1. Their energy < Δ: selftrapping of electrons into kinks (2→2)
2. They bear mid-gap states = zero fermionic mode
3. They carry either charge or spin

Fatal effect upon kinks: lifting of degeneracy, hence confinement.
Trivial but spectacular: global lifting.



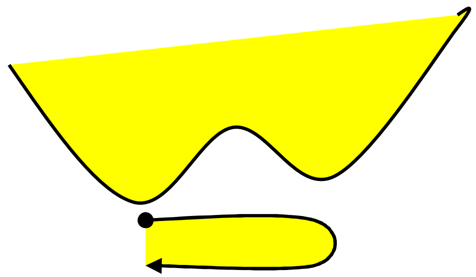
Nature present:

cis-isomer of $(CH)_x$

Build-in slight inequivalence of bonds.

Confinement of kinks pairs into 2e charged (bipolaron) or neutral (exciton) formation.
Symmetry determined picture of optical differences for trans- and cis (CH)x. Photoconductivity versus photoluminescence, new optical features due to hybridization of midgap states.

$$Tr\begin{vmatrix} -i\partial_x & \Delta^* \\ \Delta & i\partial_x \end{vmatrix} + K\overline{\left|\Delta - \Delta_{build-in}\right|^2}$$



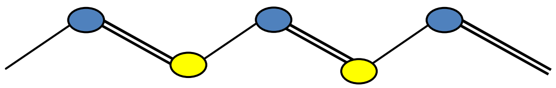
S. B. and N. Kirova, 1981: Interpretation of experiments and « accidental » exact solution for the Peierls-Gross-Neveu model with confinement.

SOILITONS WITH NONINTEGER VARIABLE CHARGES:

Orthogonal mixing of static and dynamic mass generations.

Realisation: modified polyacetylene $(CRCR')_x$.

Theories for solitons with variable charges: S.B. & N.K. , E.Mele and M.Rice



$$\Delta = \sqrt{\Delta_{ex}^2 + \Delta_{in}^2}$$

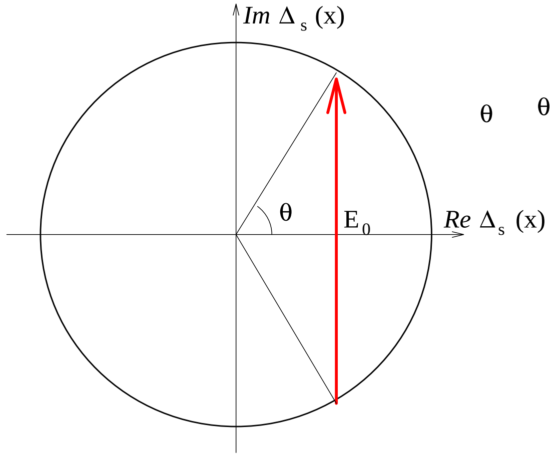
Joint effect of extrinsic Δ_{ex} and intrinsic Δ_{in} contributions to dimerization gap Δ.

Δ_{ex} comes from the build-in site dimerization – inequivalence of sites A and B.

Δ_{in} - from spontaneous dimerization of bonds $\Delta_{in}=\Delta_b$ - generic Peierls effect.

$$Tr\begin{vmatrix} -i\partial_x & \Delta_1 + i\Delta_2 \\ \Delta_1 - i\Delta_2 & i\partial_x \end{vmatrix} + K\overline{|\Delta_2|^2}$$

$$\Delta_1 = cnst, \quad \max \Delta_2 = \pm\sqrt{\Delta_0^2 - \Delta_1^2}$$



Nontrivial chiral angle 0<2θ<π of the soliton trajectory corresponds to the noninteger electric charge q= eθ/π

Continuous symmetries, still exact solutions**.**

Solitons are stable energetically but not topologically.

Special significance: allowance for a direct transformation of one electron into one soliton. *(Only 2→2 were allowed for kinks in discrete symmetries)*

Peierls-Fröhlich model for incommensurate CDWs

= chiral Gross-Neveu model in field theory

$$Tr\begin{vmatrix} -i\partial_x & \Delta^* \\ \Delta & i\partial_x \end{vmatrix} + K\overline{|\Delta|^2} + Ai\left(\Delta^*\partial_x\Delta - \Delta\partial_x\Delta^*\right)$$

Order Parameter $\mathbf{O_{cdw} \sim A(x)\ COS[Qx+\varphi]}$

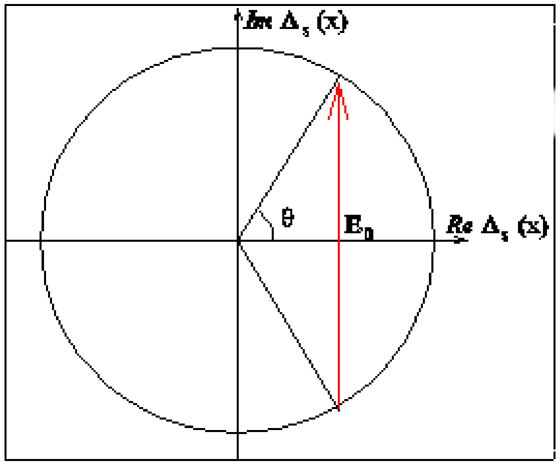
$\mathbf{\Delta= A\ exp[i\varphi]}$; **A** - amplitude , φ - phase

Ground State with an odd number of particles:

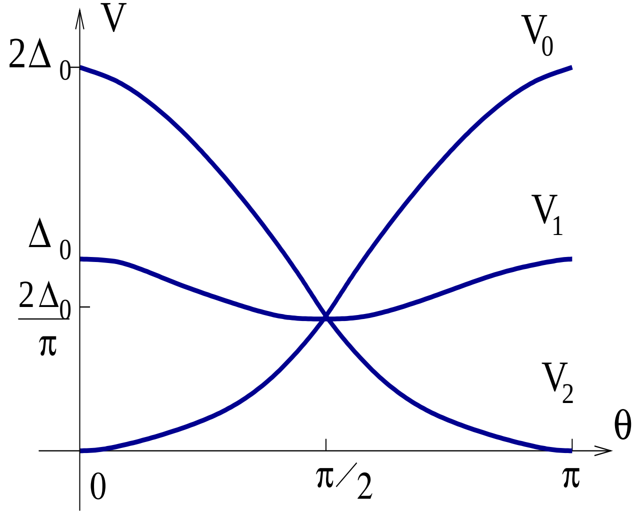
In 1D - *Amplitude Soliton* AS $\mathbf{\Delta(x=-\infty) \Leftrightarrow -\Delta(x=\infty)}$

via **A ⇔-A** at arbitrary **φ=cnst**.

Spin **½** and Charge **0** (*fractional variable charge at circumstances: S.B. with Dzyaloshinskii, Kirova, Matveenko*)



Sequence of chordus solitons develops from bare **θ=0** through AS **2θ=π** to the full phase slip **2θ=2π**.
Midgap state evolves from Δ_0 to $-\Delta_0$ providing spectral flow across the gap.

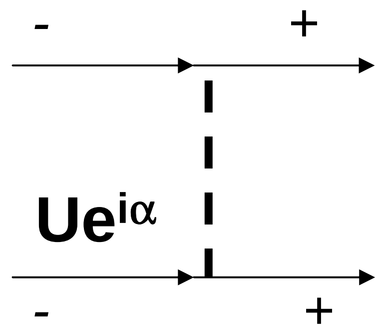


Solitons' workshop in quasi 1D organic conductors

Charge ordering and related ferroelectricity in 2000-01

Felix Nad, Pierre Monceau and S.B.

Acess to switching on/off of the Mott state and to the Zoo of solitons.



$$H\sim [v\,(\partial_x\varphi\,)^2 + (\partial_t\varphi\,)^2\,/v] - U\cos\,(2\varphi\,-2\alpha)$$

U = Umklapp scattering amplitude, leading to the Mott state
(*Dzyaloshinskii & Larkin, Luther & Emery; recall Finkelstein, Wiegmann, Zamolodchikov*)

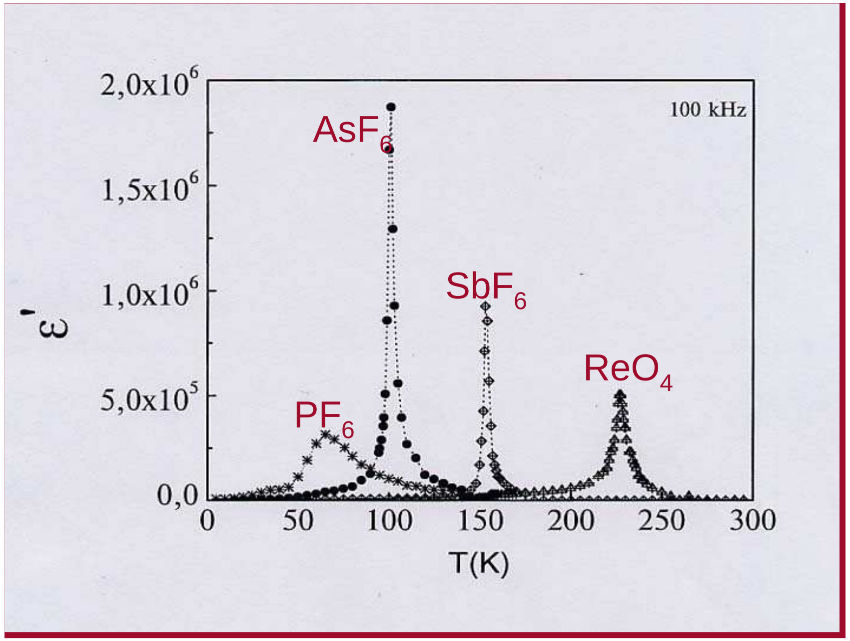
News from 2000's:

1. U may appear spontaneously at the Charge Ordering transition T_{CO}
2. Phase centre shift α - may appear at the ferroelectric transition T_{FE}

In our case $T_{FE}=T_{CO}$!

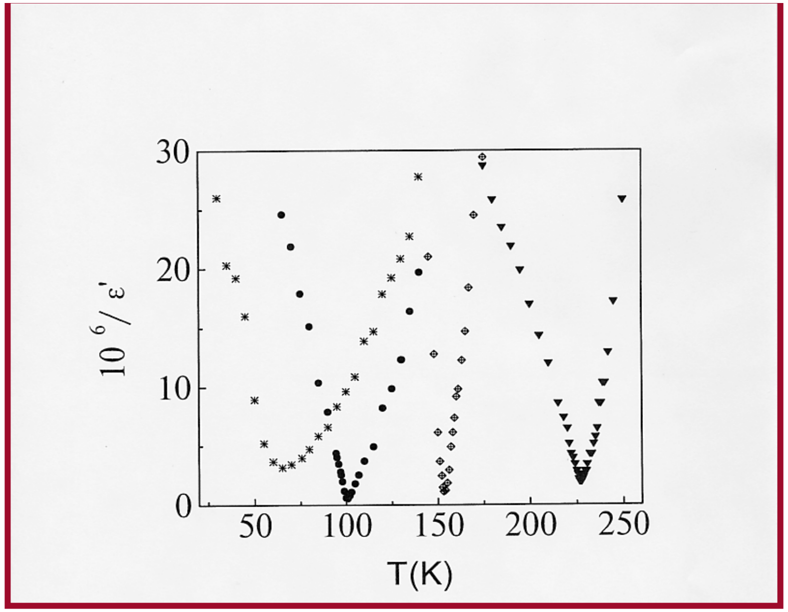
Degeneracy **φ→φ+π** originates spinless charge current carriers: ±π solitons with charges ±e, as well proved experimentally in our interpretation of conductivity and optics.

Spontaneous α itself can change sign between different FE domains α and -α.
Hence the phase soliton with the non-*integer* charge **$q=-2\alpha/\pi$** per chain.
It is the wall between domains of opposite ferroelectric polarizations



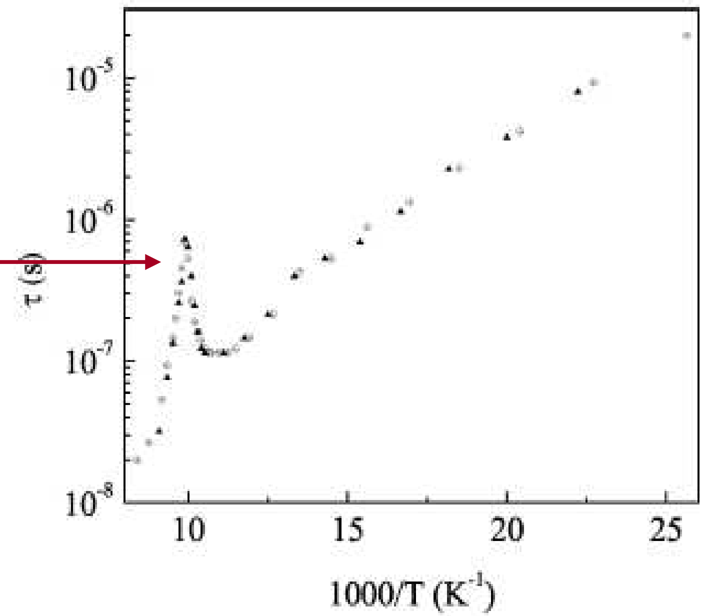
Real part of dielectric constant
P.Monceau, F. N and S.B. PRL

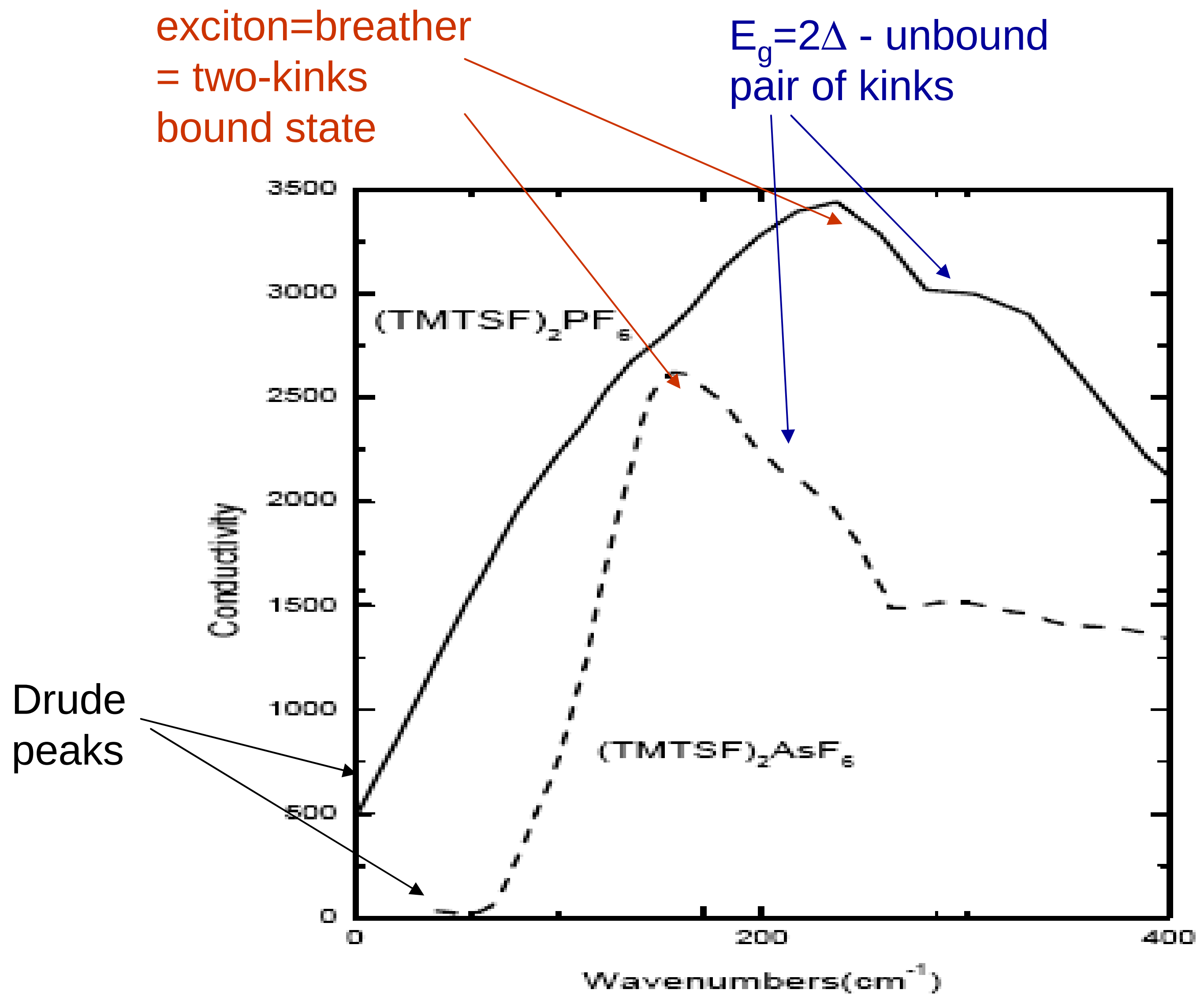
(2001)



2nd order phase transition –
ideal Landau theory

Landau-
Khalatnikov
critical relaxation



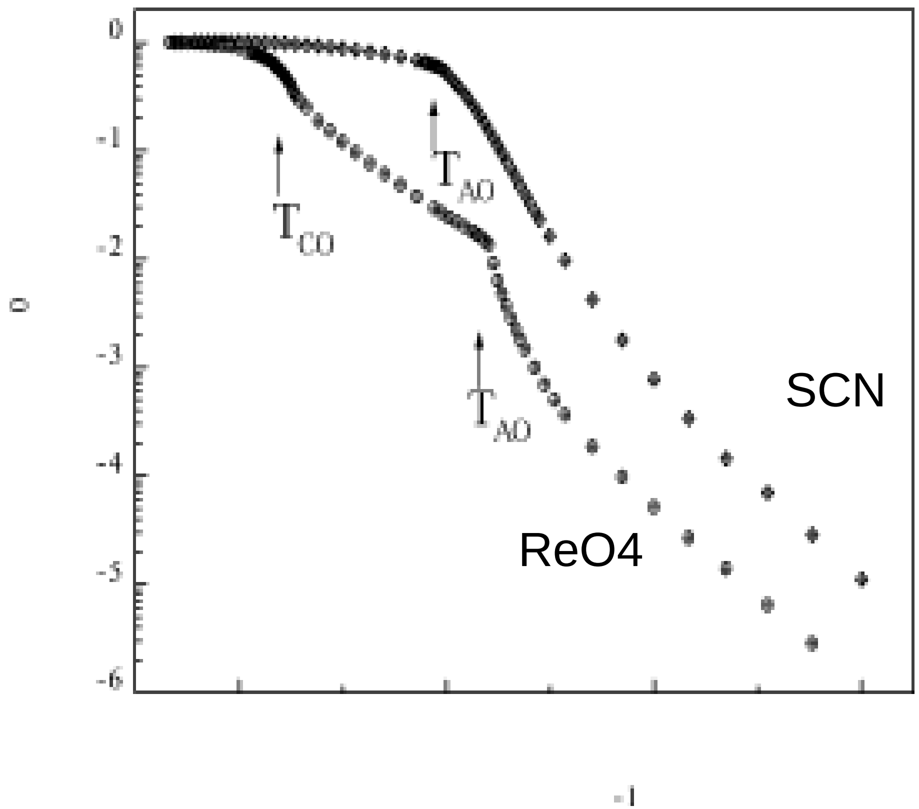


Optical excitation of sin-Gordon solitons' pairs within the coexisting metal – Mott insulator state. (Two – fluids of ^{4}He ?)

Effects of subsequent transitions.

Combined solitons. Spin-Charge reconfinement.

Another present from the Nature:
tetramerization in $(TMTTF)_2ReO_4$ at $\mathbf{T_{AO}<T_0}$



Spin-charge reconfinement below $\mathbf{T_{AO}}$ of tetramerisation. Enhanced gap Δ comes from topologically coupled **π**- solitons in both sectors of the charge and the spin. The last is weakly localized.

What does it mean ?

Spin degrees of freedom enter the game:

$$\Psi_{\pm\sigma} \sim \exp[\pm i(\varphi + \sigma\theta / 2]$$

θ - spin phase, θ´/ π = smooth spin density

Further symmetry lifting of lattice tetramerization or of spin-Peierls order mixes charge and spin: additional energy

$V\cos(\varphi-\beta)\cos\theta$ - on top of **$U\cos(2\varphi-2\alpha)$**

φ and θ -- chiral phases counting the charge and the spin

φ'/π and θ'/π = smooth charge and spin densities

$\cos\theta$ sign instructs the CDW to make spin singlets over sorter bonds

Major effects of the small V - term :

Opens spin gap $2\Delta_\sigma$:

triplet pair of $\delta\theta=\pi$ solitons at $\varphi=cnst$

- Prohibits $\delta\varphi=\pi$ solitons – now bound in pairs by spin strings
- Allows for combined spin-charge topologically bound solitons:

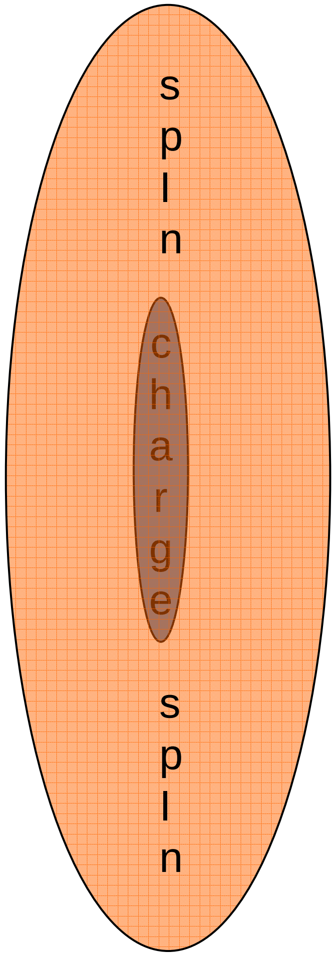
$\{\delta\varphi=\pi, \delta\theta=\pi\}$ – leaves the **V** term invariant

Quantum numbers of the compound particle --

charge **e** , spin ½ but differently localized:

charge **e** , $\delta\varphi=\pi$ sharply within $\hbar v_F/\Delta_\rho$

spin ½ , $\delta\theta=\pi$ loosely within $\hbar v_F/\Delta_\sigma$



CASES OF CONTINUOUS DEGENERACY

Limitation of the above cases of discrete symmetry breaking :
Solitons can be created only in pairs, even in 1D.
In D>1 they are either permanently confined in pairs
or aggregated into macroscopic domain walls –
topologically stable objects. (*Recall Mineev , Volovik and others*)

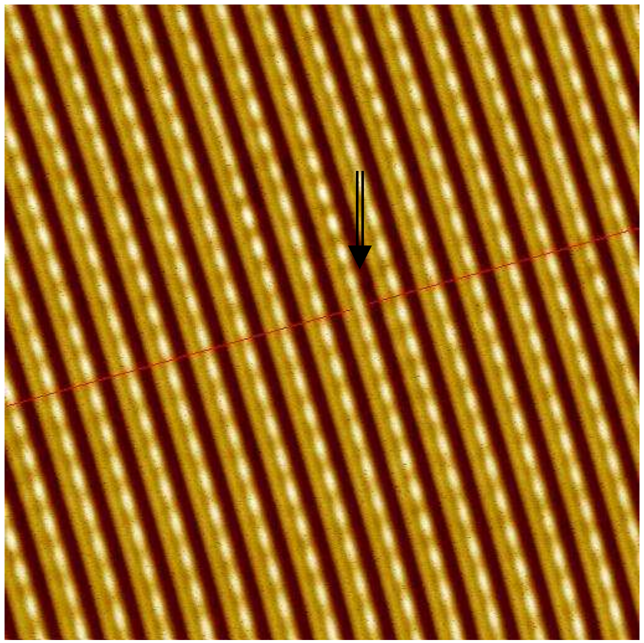
A higher continuous degeneracy -
superconductors, SDWs, incommensurate CDWs -
allows for existence and creation of isolated solitons,
of direct conversion of a single electron into a single soliton,
These are the topologically unstable objects – stabilized only
energetically by absorbing the electrons from the level E≈Δ to the E=0
midgap state.

In 1D, a single electron becomes unstable with respect to its
selftrapping - transformation into a soliton.
In D>1 an undressed electron is metastable,
hence both particles may be observed simultaneously

Incommensurate Charge Density Wave – **ICDW ~cos(Qx + ϕ)**

Charge Ordering in organic Mott state was a crystal of electrons.
Conventional CDW is a crystal of electron pairs.
Its lowest energy current carrier may be a charge **2e** defect of adding/missing one period at a defected chain.
It is the $\pm$**2π** soliton of the **ICDW order O= Acos(2K$_F$x + ϕ)**

Visualization of the **2π** soliton = **2e** prefabricated electrons' pair
C. Brun and Z.Z. Wang STM scan of $NbSe_3$

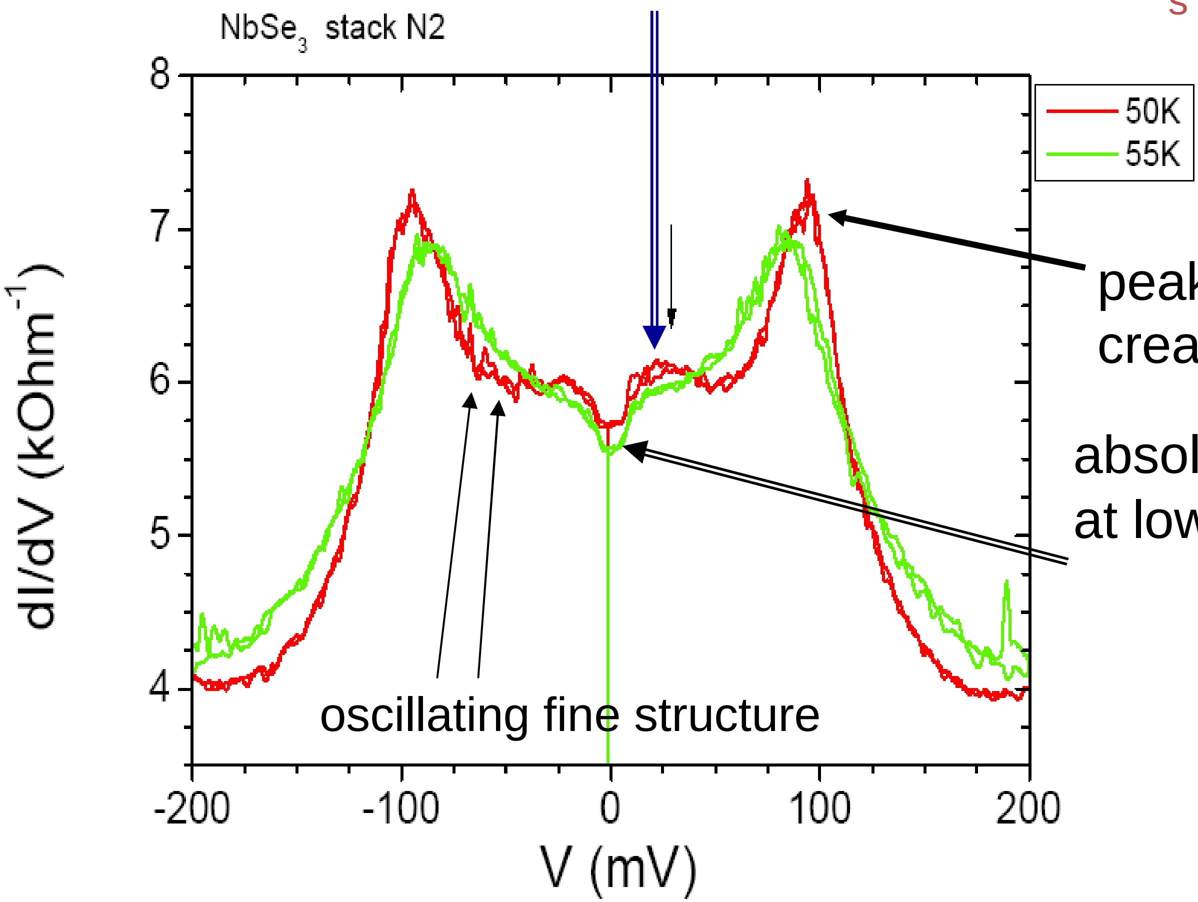


At the (red) front line the defected chain is displaced by half of the period.
Along the defected chain the whole period $\pm$**2π** is missed or gained – a pair of electrons is accommodated to the ground state.

Direct observation of solitons and their arrays in tunneling on $NbSe_3$

Latyshev, Monceau, SB, et al 2004-2006

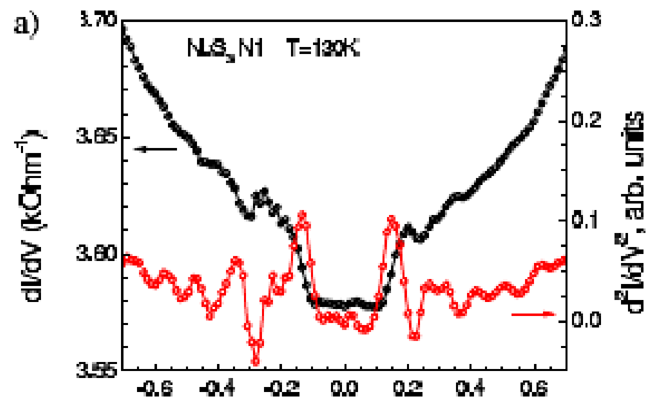
creation of soliton at $E_s = 2\Delta/\pi$



All features scale with $\Delta(T)$!

peak 2Δ for inter-gap creation of e-h pairs

absolute threshold at low $V_t \approx 0.2\Delta$



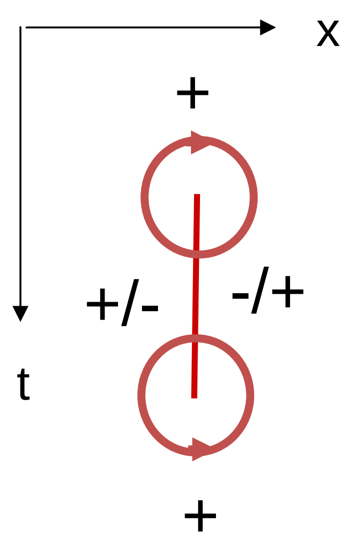
What does tunnel at the subgap voltages ? $\pm 2\pi$ phase solitons, already seen by the STM, - elementary particles with the charge $\pm 2e$ and the energy $E_s \sim$ 3D ordering temperature T_p.

Probabilities to create combined topological defects in 1D:

AS creates the π- discontinuity **δφ=π** along its world line: (0<t<T, x=0)
compensating for the sign change of the amplitude :

To be topologically allowed = to have a finite action S, the line must terminate with half integer space-time vortices located at (**0,0**) , (**0,T**) :

their circulation provides the jump A→-A combined with **φ→φ+π**
which leaves invariant the order parameter **Aexp(iφ)**.
The phase action as a function of time **T**:



$$S_{phase} \sim \frac{v_F}{u} \ln\left(\frac{uT}{\xi_0}\right)$$

Contrary to usual 2π- vortices, the connecting line is the physical singularity which string tension gives

$$S_{core} \; [illegible] \; W_s \; [illegible] \; T$$

Total action **S=S$_{core}$+S$_{phase}$** , **min{S}~ln(T)**

hence the power law for **I(Ω)~exp(-S)**

$$I(\Omega) \propto \left(\frac{\Omega - W_s}{W_s}\right)^{\beta}, \; \beta = \frac{v_F}{2u}$$

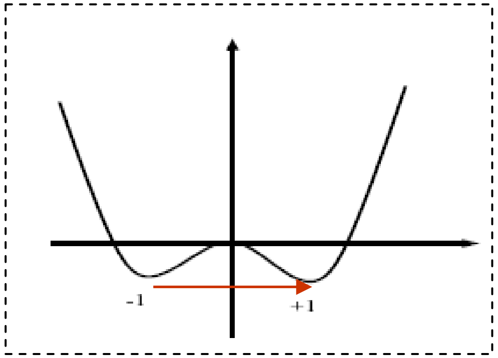
Puzzle and inspiration of the above described experiments on CDWs:

amplitude solitons has been observed within the long range ordered phase at $T<T_c$

Obstacle : *confinement.*

Changing the minima on one chain would lead to a loss of interchain ordering energy ~ total length.

Need to activate other modes to cure the defect !



Unifying observation :

combination of a discrete and continuous symmetries

Complex Order Parameter **O= A exp[iφ]** ; **A** - amplitude , **φ** - phase

Ground State with an odd number of particles:

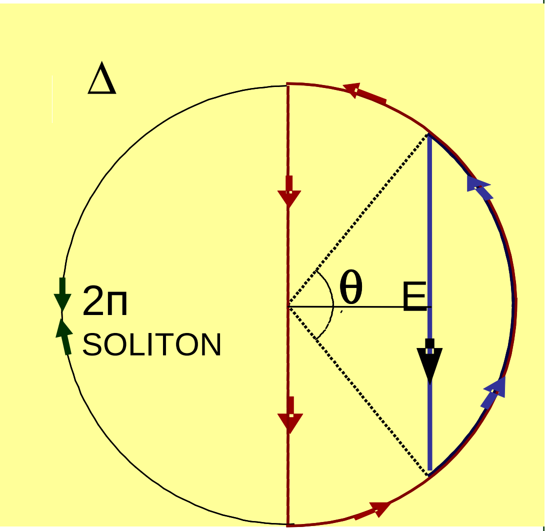
In 1D - *Amplitude Soliton* **O(x=-∞) ⇔ - O(x=∞)**

performed via **A ⇔-A** at arbitrary φ=cnst

Favorable in energy in comparison with an electron, but:

Prohibited to be created dynamically even in 1D

Prohibited to exist even stationary at D>1



RESOLUTION – Combined Symmetry :

A ⇔-A combined with φ→φ+π – phase wings = semi-vortex of phase rotation compensates for the amplitude sign change

SPIN-GAP phases with a long range order - superconductivity or incommensurate CDW. Bosonisation language :

$H_{1D} \sim \{(\partial\theta)^2 - V\cos(2\theta)\} + (\partial\varphi)^2$

$V \sim g_1$ - from the backward exchange scattering of electrons

In **1D** : Spinon as a soliton $\theta \rightarrow \theta+\pi$ hence **s=1/2**

+ Gapless charge sound in φ.

CDW order parameter $\sim \psi^{\dagger}_{+\uparrow}\psi_{-\uparrow} + \psi^{\dagger}_{+\downarrow}\psi_{-\downarrow} \sim \exp[i\varphi]\cos\theta$

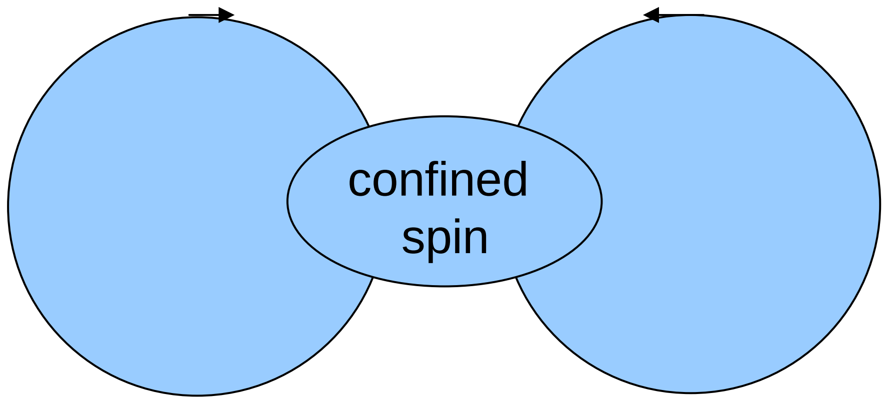
- Its amplitude **$\cos\theta$** changes the sign along the allowed π soliton

At higher D : allowed mixed configuration

$\theta \rightarrow \theta+\pi$, s=1/2 ↑spin soliton ↑

$\varphi \rightarrow \varphi+\pi$, e=1 ↑charged wings↑

Spinon as a soliton + semi-integer dislocation loop = π - vortex of φ $\equiv$ confined spin + semi dislocation loop



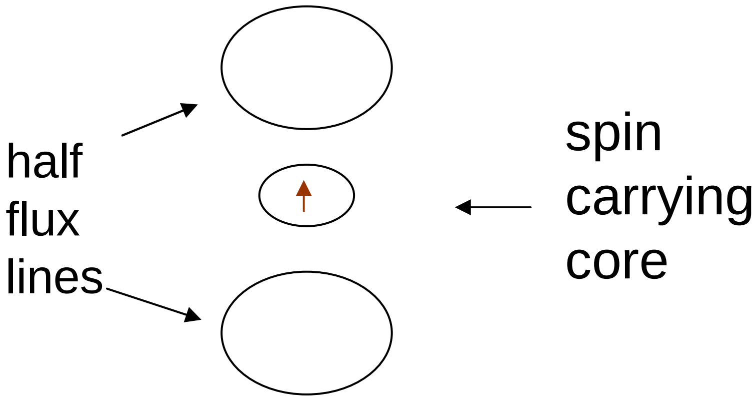
Here, we have broken with the language of 1D systems, keeping the overlap for quasi1D

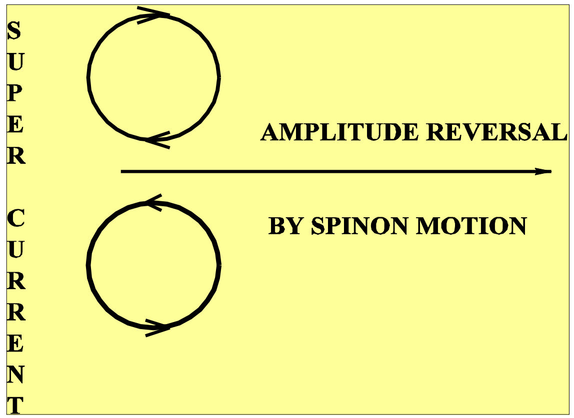
Singlet Superconductivity

$$O_{SC} \sim \psi_{+\uparrow}\, \psi_{-\downarrow} + \psi_{+\downarrow}\, \psi_{-\uparrow} \sim \exp[i\chi] \cos\theta$$

$\theta \rightarrow \theta+\pi$ s=1/2 $\chi \rightarrow \chi+\pi$

↑spin soliton ↑ ↑wings of supercurents↑





Quasi 1d view : spinon as a π- Josephson junction in the superconducting wire (*applications: Yakovenko et al*).

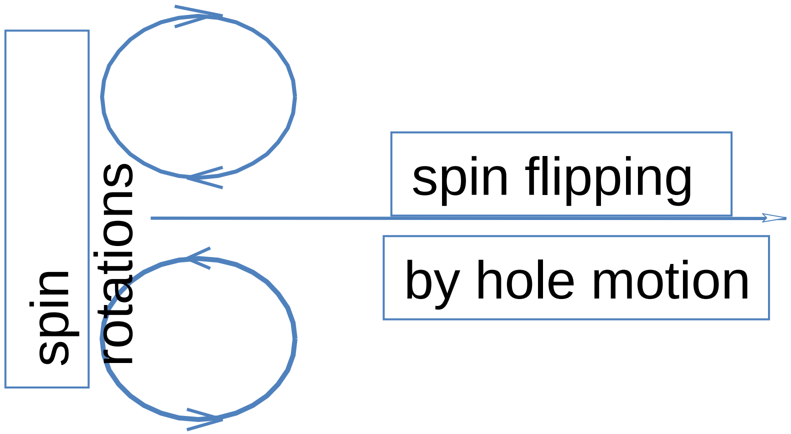
2D view : pair of π- vortices shares the common core bearing unpaired spin.

3D view : half-flux vortex stabilized by the confined spin.

Updown view: nucleus of melted FFLO phase in spin-polarized SC

Half filled band with repulsion.
SDW rout to the doped Mott-Hubbard insulator.

$$H_{1D} \sim (\partial\varphi)^2 - U\cos(2\varphi)\} + (\partial\theta)^2$$



U - Umklapp amplitude
(*Dzyaloshinskii & Larkin ; Luther & Emery*).
φ - chiral phase of charge displacements
θ - chiral phase of spin rotations.
Degeneracy of the ground state:
$\varphi \rightarrow \varphi+\pi$ = translation by one site

Excitations in 1D :

holon as a π soliton in φ, spin sound in θ

Higher D : A hole in the AFM environment.

Staggered magnetization ≡ AFM=SDW order parameter:

$O_{SDW} \sim \cos(\varphi) \exp\{\pm i(Qx+\theta)\}$, amplitude **$A = \cos(\varphi)$** changes the sign

To survive in D>1 :
The π soliton in φ : $\cos\varphi \rightarrow -\cos\varphi$
enforces a π rotation in θ to preserve O_{SDW}

Paradox of the "spinless hole" spin:

the central chain, **y=0**: $\delta\theta=\pi \rightarrow$ **s=1/2**

like an added electron of spin **s=1/2**, charge **e=1**.

But integrally over cross section:

$\int\delta\theta d^2r_\perp=0$ $\Rightarrow$ net spin **s=0**

3D quantum numbers are like for normal electron spin

s=1/2 (while in wings) charge **e=1** (while in the core).

But integrally over a perpendicular cross-section:

net AFM spin $S_{afm}=0$;

integrally over any cross section:

net magnetization $S_{fer}=0$

The LRO in D>1 reconfines the charge and the spin but only in a core.

But integrally one of the two is transferred to the collective mode.

Locally we restore single electronic quantum numbers

but with different scales of localization.

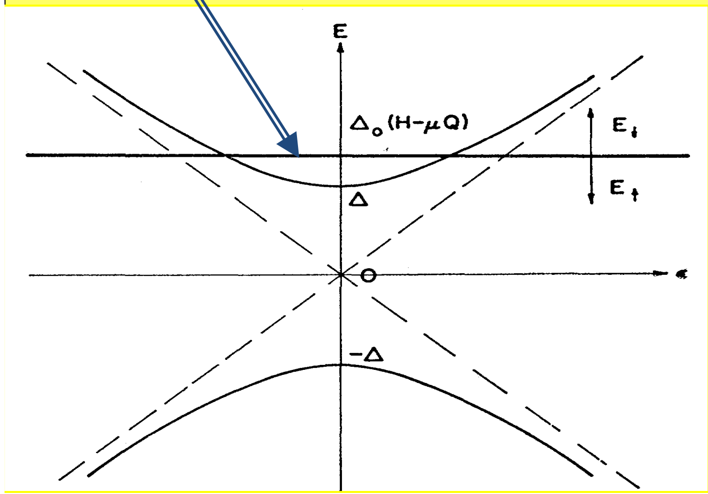
Integrally – still the spin without the charge.

Solitonic lattices in CDWs or stripes in doped AFM or FFLO in SC

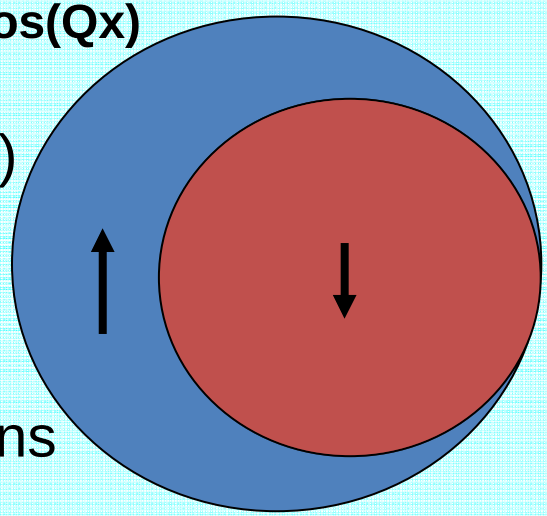
FFLO in superconductors SC with imbalanced spin population :
FF=Fulde&Ferrell 1964, LO=Larkin&Ovchinnikov 1964

Even if the single excitations were of the BCS-Bogolubov type,
their finite concentration
(CDW/SDW incommensurability, Mott/AFM/SDW states dopping,
CDW or SC Zeeman breakdown, cold atom imbalance)
gives rise **not** to pockets occupying E(k) above Δ, but to
solitonic lattices in CDW = stripes in AFM = FFLO in SC

1. Homogeneous phase:
Fill excess spins to
states above the gap



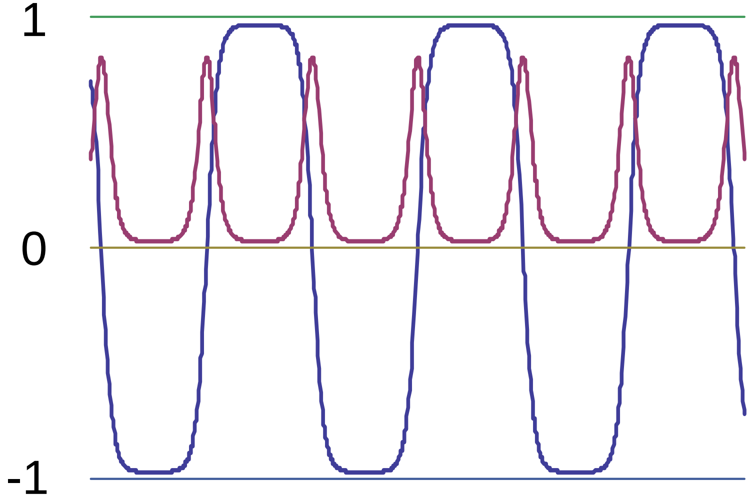
2. Modulated phase: wave number Q≠0
FF: **Δ~exp(iQx)** & LO: **Δ~ cos(Qx)**
erases a mismatching
at some (all in quasi-1D)
parts of the FS.



Valid for both suggestions
FF and LO

3. Build a structure of local walls so strong as to create intra-gap states which are able to accommodate access spins.

Able to evolve into the LO phase **Δ~ cos(Qx)**
(*not to FF – gap passes through zeros*),
Proved by theory in quasi-1D,
by recent experiments on CDW in high magnetic fields.



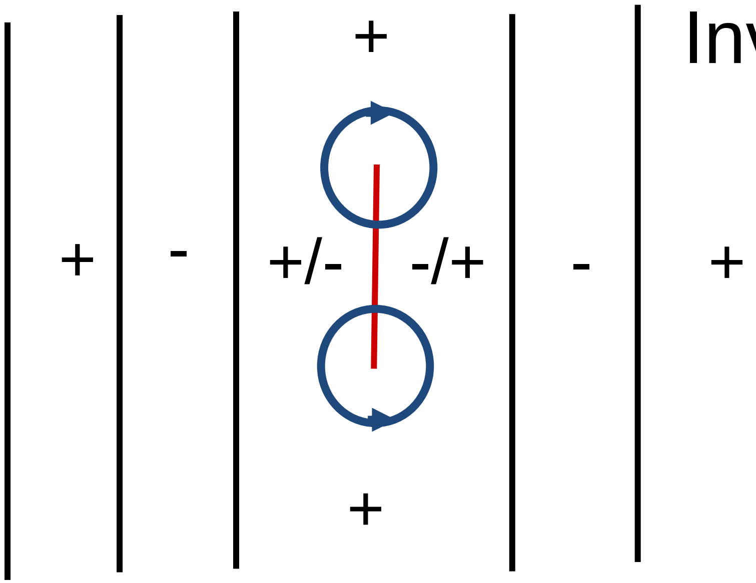
CDW or SC under slightly supercritical Zeeman splitting. *plotted*:

Solitonic lattice of the order parameter,

Unpaired spins = mid-gap states density distributed near the gap zeros.

If melted, each element becomes a particle - Amplitude Soliton = Spinon

Inverse rout: from stripes to solitons



Kink-roton complexes as nucleuses of melted lattices: FFLO phase for superconductors or strips for doped AFMs.

Defect is embedded into the regular stripe structure (black lines).
+/- are the alternating signs of the order parameter amplitude.
Termination points of a finite segment L (red color) of the zero line must be encircled by semi-vortices of the π rotation (blue circles) to resolve the signs conflict.
The minimal segment would correspond to the spin carrying kink.

Vortices cost $\sim \mathbf{E_{phase} \log L}$ is less than the gain $\sim -\Delta \mathbf{L}$ for the string formation at long L. Can we shrink to the atomic scale?

For smallest **L**~"unit cell", it is still valid in quasi 1D : $\mathbf{E_{phase} \sim T_c < \Delta}$
For isotropic SCs, $\mathbf{E_{phase} \sim E_F}$ – strong coupling is necessary.

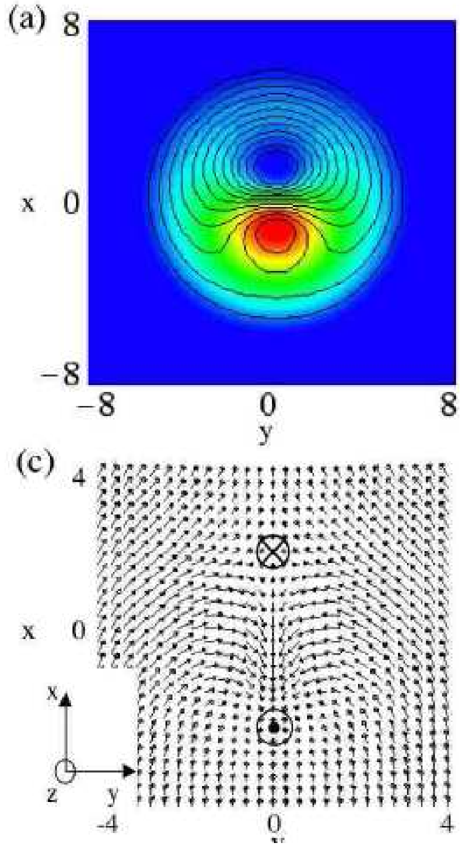
In absence of microscopic theory for a strong coupling vortex (with a single intra-gap state), we search the literature for numeric, while still phenomenological, models. And it works.

At presence of unpaired spins, the vortex created by rotation (magnetic field) splits into two semi-vortices.

Spatial Line Nodes and Fractional Vortex Pairs in the FFLO Vortex State of Superconductors
Agterberg, et al 2008

Vortex molecules in coherently coupled two-component Bose-Einstein condensates
Kasamatsu et al 2004

Last step: reformulate these results inversely – unpaired spin creates the vortex pair at NO rotation/MF.

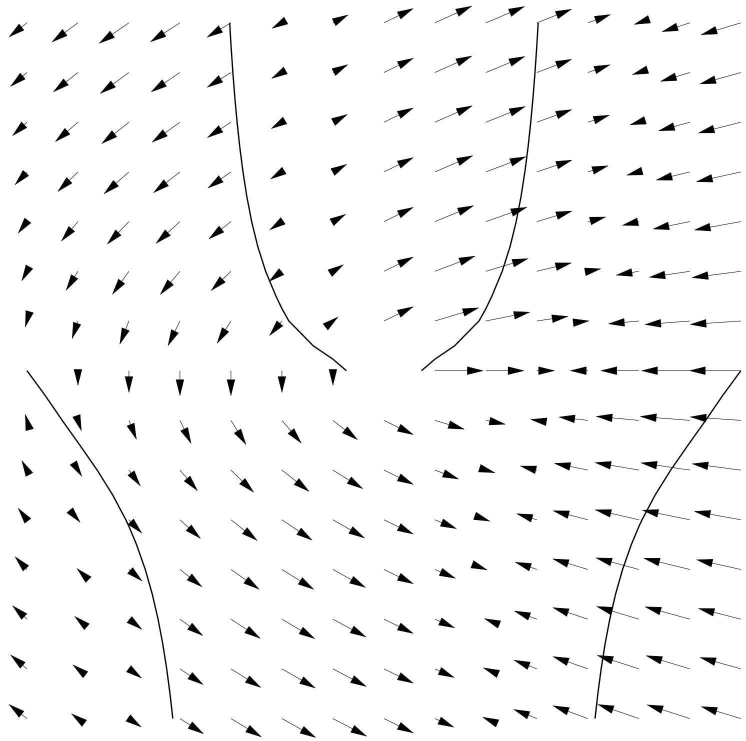


TOPOLOGICAL COUPLING OF DISLOCATIONS AND VORTICES
IN INCOMMENSURATE Spin DENSITY WAVES

N. Kirova, S. Brazovskii, 2000

An attempt to rehabilitate the Density Waves against more fascinating symmetries:
He^3 (Volovik et al), skyrmions in QHE (Yu.Bychkov et al)

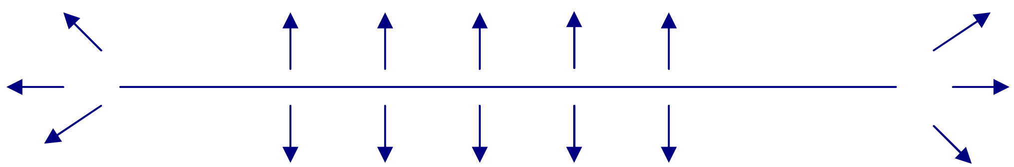
ISDW order parameter: $O_{SDW} \sim \mathbf{m}\cos(Qx+\varphi)$
m – staggered magnetization vector



Three types of self mapping for the O_{SDW} :

1. normal dislocation, 2π translation:
 $\varphi\to\varphi+2\pi$, $\mathbf{m}\to\mathbf{m}$
2. normal **m** - vortex, 2π rotation:
 $\mathbf{m}\to R_{2\pi}\mathbf{m}$, $\varphi\to\varphi$
3. combined object :
 $\varphi\to\varphi+\pi$, $m\to R_{\pi}\, m = -m$

Coulomb energy favors splitting the phase dislocation at a smaller cost of creating spin semi-vortices.



Effect of rotational anisotropy:
String tension binds semi-vortices

FINITE TEMPERATURE, ENSEMBLES OF SOLITONS,
PHASE TRANSITIONS OF CONFINEMENT AND AGGREGATION.
DISCRETE SYMMETRY only.

Fatal effect upon kinks: global lifting of degeneracy, hence confinement.

Nontrivial but still spectacular:

local lifting in the state with long range order.

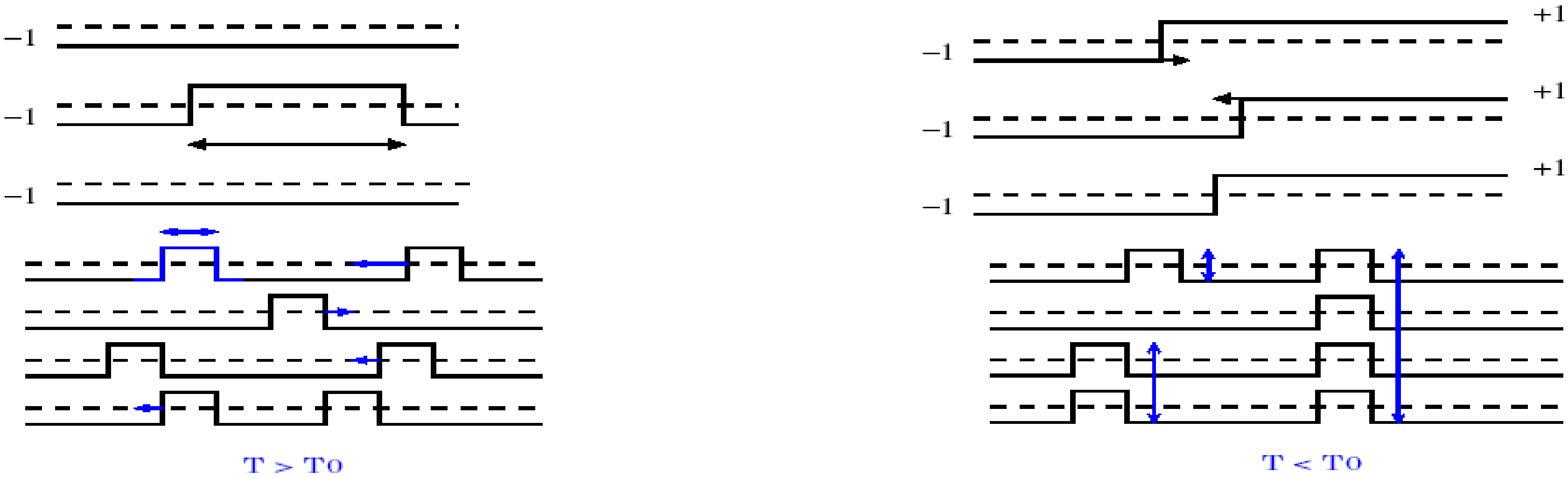
Interchain coupling of the order parameter.

$$H_I = -\sum_{<\alpha,\beta>} \int dx V_\perp \Delta_\alpha(x) \Delta_\beta(x)$$

Two competing effects:

Binding of kinks into pairs at $T<T_c$;

Aggregation into macroscopic domain walls at $T<T_0<T_c$.



Solution for a statistical model *T.Bohr and S.B. 1983, S.Teber et al 2000's*
Recall N. Nagaosa on superionic conductors

SUMMARY

- Existence of solitons is proved experimentally in single- or bi-electronic processes of 1D regimes in quasi 1D materials.
- They feature self-traping of electrons into mid-gap states and separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales.
- Topologically unstable configurations are of particular importance allowing for direct transformation of electrons into solitons.
- Continuously broken symmetries allow for solitons to enter D>1 world of long range ordered states: SC, ICDW, SDW.
- They take forms of amplitude kinks topologically bound to semi-vortices of gapless modes – half integer rotons
- These combined particles substitute for electrons certainly in quasi-1D systems – valid for both charge- and spin- gaped cases
- The description is extrapolatable to strongly correlated isotropic cases. Here it meets the picture of fragmented stripe phases