Landau-Khalatnikov hydrodynamics
&
phenomenology of dark energy

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1. Quantum vacuum & matter: two fluid hydrodynamics

2. Quantum vacuum as Lorentz invariant medium

3. Hydrodynamics and thermodynamics of quantum vacuum

4. Relaxation of vacuum energy

5. Problem of remnant cosmological constant

6. Osmotic pressure of matter in quantum vacuum

dark energy problem

recipe to cook Universe

Dark Energy	70%
Dark Matter	30%
Baryonic Matter	4%
Visible Matter	0,4%

Dark and Dark!
What is the difference?

dark matter forms clusters
like ordinary matter

cosmological constant Λ is possible candidate for dark energy

$$\Lambda = \varepsilon_{\text{Dark Energy}}$$

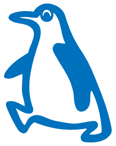
Cosmological constant paradox

$$\Lambda_{\text{observation}} = \varepsilon_{\text{Dark Energy}} \sim 2\text{–}3\ \varepsilon_{\text{DM}} \sim 10^{-47}\ \text{GeV}^4$$

$$\Lambda_{\text{theory}} = \varepsilon_{\text{zero point energy}} \sim E^{4}_{\text{Planck}} \sim 10^{76}\ \text{GeV}^4$$

$$\Lambda_{\text{observation}} \sim 10^{-123}\,\Lambda_{\text{Theory}}$$

too bad for theory



problems:

- **Why is vacuum not extremely heavy?**
- **Why is vacuum gravitating? Why is Λ non-zero?**
- **Why is vacuum as heavy as (dark) matter ?**

it is easier to accept that $\Lambda=0$
than 123 *orders of magnitude smaller*

$$\boxed{\Lambda_{\text{exp}} \sim 2\text{-}3\ \varepsilon_{\text{Dark Matter}} \sim 10^{-123}\Lambda_{\text{bare}}}$$

$$\Lambda_{\text{bare}} \sim \varepsilon_{\text{zero point}}$$

*it is easier to accept that $\Lambda = 0$ than 123 orders smaller

*magic word: *regularization*

wisdom of particle physicist:

$$\boxed{\frac{1}{0} = 0}$$

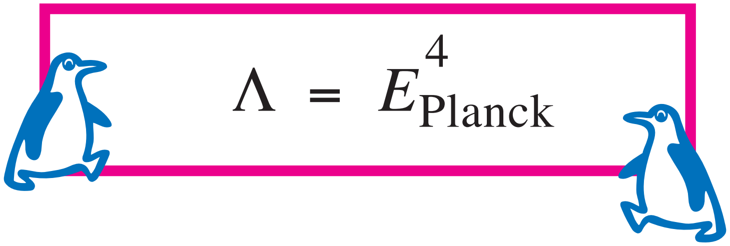
*Polyakov conjecture: dynamical screeneng of Λ by infrared fluctuations of metric

A.M. Polyakov
Phase transitions and the Universe, UFN **136,** 538 (1982)
De Sitter space and eternity, Nucl. Phys. **B 797**, 199 (2008)

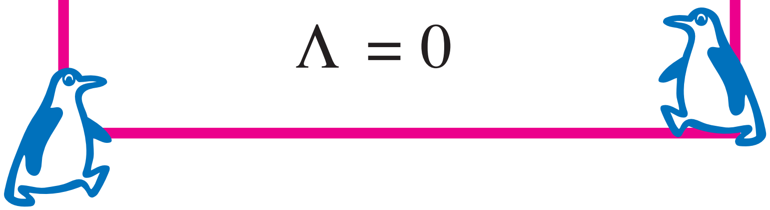
*Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)
Barankov & Levitov, ...

what is natural value of cosmological constant ?

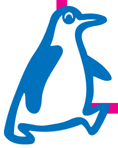


$$\Lambda = E_{\text{Planck}}^4$$



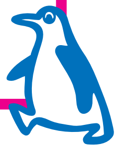
$$\Lambda = 0$$

time dependent cosmological constant



$$\boxed{\Lambda \sim E_{\text{Planck}}^{4}}$$

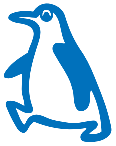
could be in early Universe



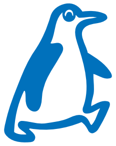
$$\boxed{\Lambda \sim 0}$$

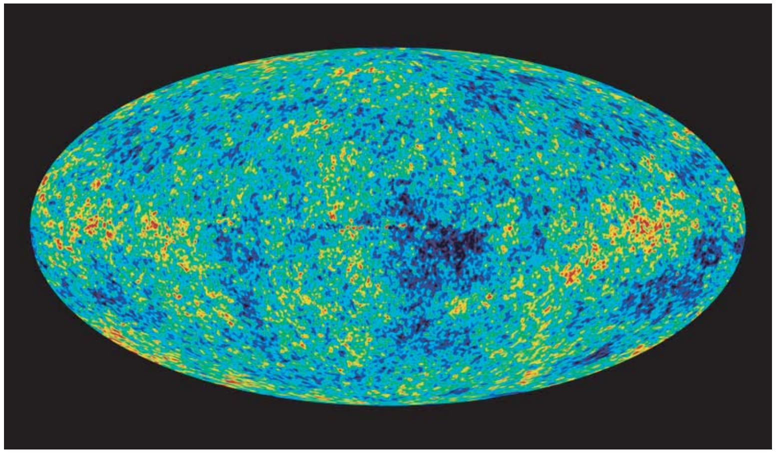
should be in old Universe

Superfluids



Universe



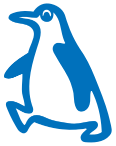


acoustic gravity **metric theories of gravity** **general relativity**

geometry of effective space time
for matter (phonons)

$g_{\mu\upsilon}$

geometry of space time
for matter



geodesics for phonons $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$ geodesics for photons

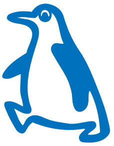
equation
for phonons

$$\boxed{T^{\mu\nu}_{;\nu\ \mathrm{Matter}} = 0}$$

equation
for matter

Khalatninkov
"Theory of superfluidity"
Nauka, 1971

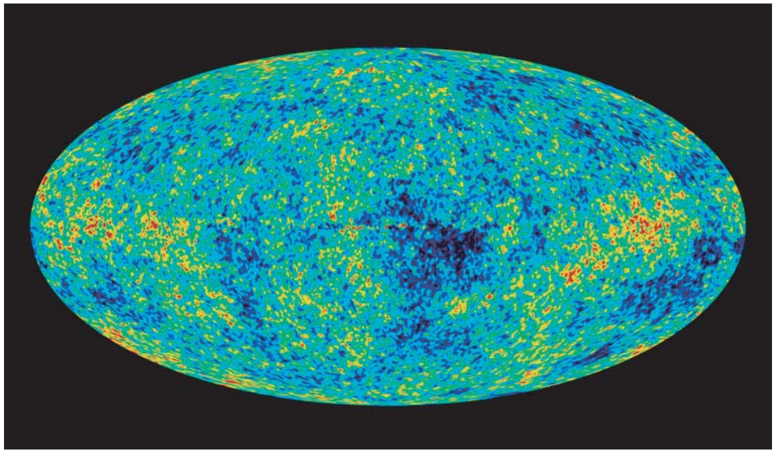
1/2 of GR



where is another 1/2 of GR ?

superfluid component

quantum vacuum



equation for superfluid velocity

$$\boxed{\dot{\mathbf{v}}_s + \nabla\left(\mu + \mathbf{v}_s{}^2/2\right) = 0}$$

Einstein equations for metric field

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = \Lambda g_{\mu\nu} + T^{\text{Matter}}_{\mu\nu}}$$

equation for mass density

$$\boxed{\dot{\rho} + \nabla\cdot\left(\rho\mathbf{v}_s + \mathbf{P}^{\text{Matter}}\right) = 0}$$

equation for quantum vacuum

what is ρ in quantum vacuum?

equation for phonons

$$\boxed{T^{\mu\nu}_{;\nu\,\text{Matter}} = 0}$$

equation for matter

how to describe quantum vacuum & vacuum energy Λ ?

* quantum vacuum has equation of state **w=–1**

$$\Lambda = \boxed{\varepsilon_{\rm vac} = w_{\rm vac} P_{\rm vac}}$$

* quantum vacuum is Lorentz-invariant

$$\boxed{w_{\rm vac} = -1}$$

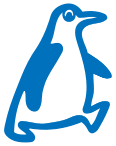
* quantum vacuum is a self-sustained medium,
which may exist in the absence of environment

* for that, vacuum must be described by conserved charge q

q is analog of particle density n in liquids

q must be Lorentz invariant

$$\boxed{L\, q = q}$$

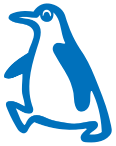


charge density n
is not Lorentz invariant

$$\boxed{L\, n = \gamma(n + \mathbf{v}\cdot\mathbf{j})}$$

Hawking suggested to introduce special
field which describes the vacuum only
Hawking, Phys. Lett. B **134**, 403 (1984)

does such q exist ?



relativistic invariant conserved charges q

possible

$$\boxed{\nabla_\alpha q^{\alpha\beta} = 0}$$

$$\boxed{\nabla_\alpha q^{\alpha\beta\mu\nu} = 0}$$

$$\boxed{q^{\alpha\beta} = q\, g^{\alpha\beta}}$$

$$\boxed{q^{\alpha\beta\mu\nu} = q\, e^{\alpha\beta\mu\nu}}$$

Duff & van Nieuwenhuizen
Phys. Lett. **B 94,** 179 (1980)

impossible

$$\boxed{\nabla_\alpha q^{\alpha} = 0}$$

$$\boxed{q^{\alpha} = ?}$$

examples of vacuum variable q

4-form field

$$\boxed{F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}}$$

$$\boxed{F_{\kappa\lambda\mu\nu} = q\,(\text{-g})^{1/2}\, e_{\kappa\lambda\mu\nu}}$$

$$\boxed{q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}}$$

gluon condensates in QCD

$$\boxed{<\mathbf{G}_{\alpha\beta}\mathbf{G}_{\mu\nu}>= \frac{\mathrm{q}}{12}(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\nu}g_{\beta\mu})}$$

$$\boxed{q = <\mathbf{G}_{\alpha\beta}\mathbf{G}^{\alpha\beta}>}$$

$$<\mathbf{G}_{\alpha\beta}> = 0$$

$$\boxed{<\mathbf{G}_{\alpha\beta}\mathbf{G}_{\mu\nu}>= \frac{\mathrm{q}}{24}(\text{-g})^{1/2}\; e_{\alpha\beta\mu\nu}}$$

$$\boxed{q = <\tilde{\mathbf{G}}_{\alpha\beta}\mathbf{G}^{\alpha\beta}>}$$

topological
charge density

Einstein-aether theory (T. Jacobson, A. Dolgov)

$$\boxed{\nabla_\mu u_\nu = q\; g_{\mu\nu}}$$

thermodynamics in flat space

the same as in cond-mat

conserved charge Q

$$\boxed{Q = \int dV\, q}$$

thermodynamic potential

$$\boxed{\Omega = E - \mu Q = \int dV\, (\varepsilon\,(q) - \mu q)}$$

Lagrange multiplier or chemical potential μ

pressure

$$\boxed{\begin{array}{c} P = -\, dE/dV = -\varepsilon + q\, d\varepsilon/dq \\ E = V\, \varepsilon(Q/V) \end{array}}$$

$$d\Omega/dq = 0$$

equilibrium vacuum

$$\boxed{d\varepsilon/dq = \mu}$$

equilibrium self-sustained vacuum

$$\boxed{\begin{array}{l} d\varepsilon/dq = \mu \\ \varepsilon - q\, d\varepsilon/dq = -P = 0 \end{array}}$$

vacuum energy & cosmological constant

equilibrium self-sustained vacuum

$$d\varepsilon/dq = \mu$$

$$\varepsilon - q\, d\varepsilon/dq = -P = 0$$

$$q \sim \mu \sim E^2_{\rm Planck}$$

vacuum variable in equilibrium self-sustained vacuum

$$\varepsilon\,(q) \sim E^4_{\rm Planck}$$

energy of equilibrium self-sustained vacuum

pressure

$$P = -\varepsilon + q\, d\varepsilon/dq = -\Omega$$

$$\Lambda = \Omega = \varepsilon - \mu\, q$$

cosmological constant

$$P = -\Omega$$

equation of state

cosmological constant in equilibrium self-sustained vacuum

$$\Lambda = \underset{\substack{\downarrow \\ E^4_{\rm Planck}}}{\varepsilon} - \underset{\substack{\downarrow \\ E^4_{\rm Planck}}}{\mu q} = 0$$

***self-tuning*:**
two Planck-scale quantities
cancel each other
in equilibrium self-sustained vacuum

dynamics of q in flat space

whatever is the origin of q the motion equation for q is the same

action

$$S = \int dV\, dt\, \varepsilon(q)$$

motion equation

$$\nabla_\kappa \left(d\varepsilon/dq \right) = 0$$

solution

$$d\varepsilon/dq = \mu$$

integration constant μ in dynamics
becomes chemical potential in thermodynamics

4-form field $F_{\kappa\lambda\mu\nu}$ as an example of conserved charge q in relativistic vacuum

$$q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}$$

$$F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}$$

$$F^{\kappa\lambda\mu\nu} = q\, e^{\kappa\lambda\mu\nu}$$

→

Maxwell equation

$$\nabla_\kappa \left(F^{\kappa\lambda\mu\nu} q^{-1} d\varepsilon/dq \right) = 0$$

→

$$\nabla_\kappa \left(d\varepsilon/dq \right) = 0$$

general dynamics of q in curved space

action

$$\boxed{S = \int d^{\,4}x\,(-g)^{1/2}\left[\,\varepsilon\,(q) + K(q)R\,\right] + S_{\rm matter}}$$

gravitational coupling $K(q)$ is determined by vacuum and thus depends on vacuum variable q

motion equation

$$d\varepsilon/dq + R\,dK/dq = \mu$$

integration constant

Einstein equations

$$K\big(Rg_{\mu\nu} - 2R_{\mu\nu}\big) + \big(\varepsilon - \mu q\big)g_{\mu\nu} - 2\big(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\big)K = T_{\mu\nu}$$

Einstein tensor

cosmological term $\neq \varepsilon\, g_{\mu\nu}$

$$\boxed{\nabla_\mu T^{\mu\nu} = 0}$$

matter

case of $K=const$ restores original Einstein equations

$$\boxed{K = \frac{1}{16\pi G}}$$

G - Newton constant

motion equation

$d\varepsilon/dq = \mu \qquad q = \text{const}$

original Einstein equations

$$\frac{1}{16\pi G}\left(R g_{\mu\nu} - 2R_{\mu\nu}\right) + \Lambda\, g_{\mu\nu} = T_{\mu\nu} \qquad \Lambda = \varepsilon - \mu q$$

Λ - cosmological constant

Minkowski solution

Maxwell equations

$$d\varepsilon/dq + R\, dK/dq = \mu$$

Einstein equations

$$K(Rg_{\mu\nu} - 2R_{\mu\nu}) + (\varepsilon - \mu q)g_{\mu\nu} - 2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda)K = T_{\mu\nu}$$

Einstein tensor

cosmological term

matter

$$\nabla_\mu T^{\mu\nu} = 0$$

Minkowski vacuum solution

$$R = 0 \qquad d\varepsilon/dq = \mu$$

$$\Lambda = \varepsilon(q) - \mu q = 0$$

vacuum energy in action: $\varepsilon(q) \sim E^4_{\mathrm{Planck}}$

thermodynamic vacuum energy: $\varepsilon - \mu q = 0$

Model vacuum energy

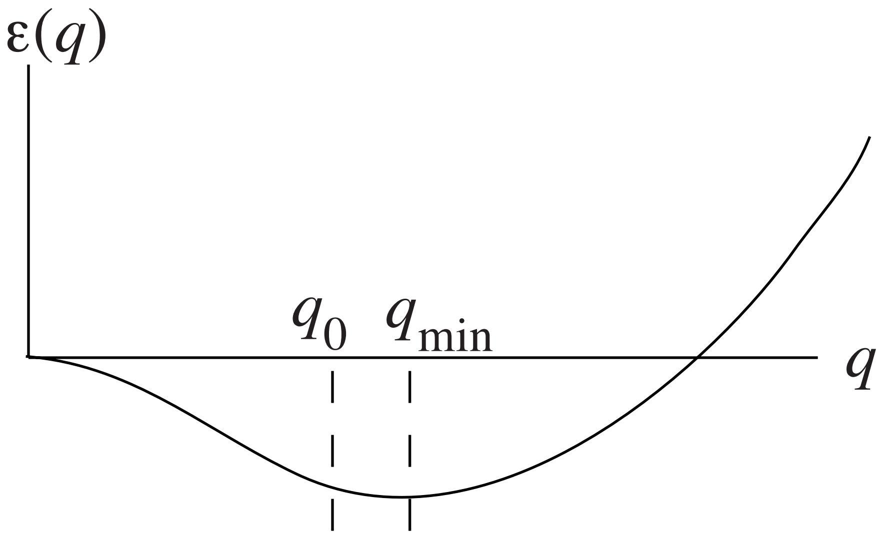
$$\varepsilon\,(q) = \frac{1}{2\chi}\left(-\frac{q^2}{q_0{}^2} + \frac{q^4}{3q_0{}^4}\right)$$

Minkowski vacuum solution

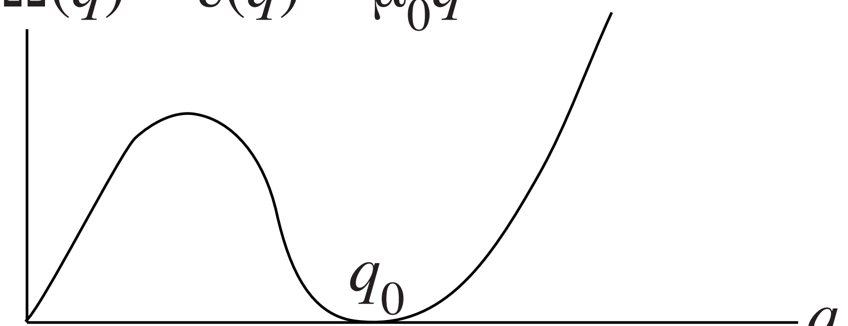
$$\boxed{\begin{aligned} d\varepsilon/dq &= \mu \\ \varepsilon - \mu q &= 0 \end{aligned}}$$

→

$$\boxed{\begin{aligned} q &= q_0 \\ \mu = \mu_0 &= -\frac{1}{3\chi q_0} \end{aligned}}$$



$$\Omega(q) = \varepsilon(q) - \mu_0 q$$



empty space

$q = 0$

self-sustained vacuum

$q = q_0$

vacuum compressibility

$$\boxed{\chi = -\frac{1}{V}\frac{dV}{dP}}$$

$$\boxed{\frac{1}{\chi} = \left(q^2\, d^2\varepsilon/dq^2\right)_{q=q_0} > 0}$$

vacuum stability

Minkowski vacuum (q-independent properties)

$$\Lambda = \Omega_{\mathrm{vac}} = -P_{\mathrm{vac}}$$

energy density of vacuum

pressure of vacuum

$$P_{\mathrm{vac}} = -\ dE/dV = -\ \Omega_{\mathrm{vac}}$$

$$\chi_{\mathrm{vac}} = -(1/V)\ dV/dP$$

compressibility of vacuum

$$<(\Delta P_{\mathrm{vac}})^2> = T/(V\chi_{\mathrm{vac}})$$

$$<(\Delta\Lambda)^2> = <(\Delta P)^2>$$

pressure fluctuations

natural value of Λ *determined by macroscopic physics*

$$\boxed{\Lambda = 0}$$

natural value of χ_{vac} *determined by microscopic physics*

$$\boxed{\chi_{\mathrm{vac}} \sim E^{-4}_{\mathbf{Planck}}}$$

volume of Universe is large:

$$\boxed{V > T_{\mathrm{CMB}}/(\Lambda^2\chi_{\mathrm{vac}})}$$

$$V > 10^{28}\ V_{\mathrm{hor}}$$

dynamics of q in curved space: relaxation of Λ

motion equation

$$\boxed{d\varepsilon/dq + R\, dK/dq = \mu}$$

Einstein equations

$$\boxed{K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + g_{\mu\nu}\,\Lambda(q) - 2\left(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\right) K = T_{\mu\nu}^{\text{matter}}}$$

$$\Lambda(q) = \varepsilon(q) - \mu_0 q$$

dynamic solution: approach to equilibrium vacuum

$$\boxed{q(t) - q_0 \sim q_0 \frac{\sin \omega t}{t}} \qquad \boxed{\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}} \qquad \boxed{H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)}$$

$$\omega \sim E_{\text{Planck}}$$

similar to scalar field with mass $M \sim E_{\text{Planck}}$

A.A. Starobinsky, Phys. Lett. **B 91**, 99 (1980)

Relaxation of Λ (generic q-independent properties)

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

cosmological "constant"

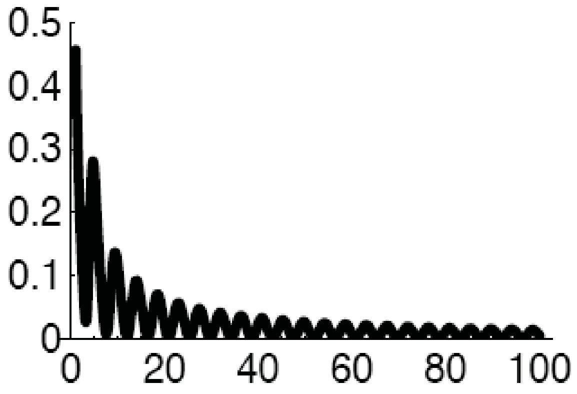
$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

Hubble parameter

$$G(t) = G_N \left(1 + \frac{\sin \omega t}{\omega t}\right)$$

Newton "constant"

$$\omega \sim E_{\mathrm{Planck}}$$

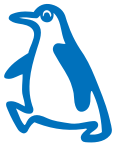


$$<\Lambda(t_{\mathrm{Planck}})> \sim E_{\mathrm{Planck}}^4$$

$$\Lambda(t = \infty) = 0$$

natural solution of the main cosmological problem ?

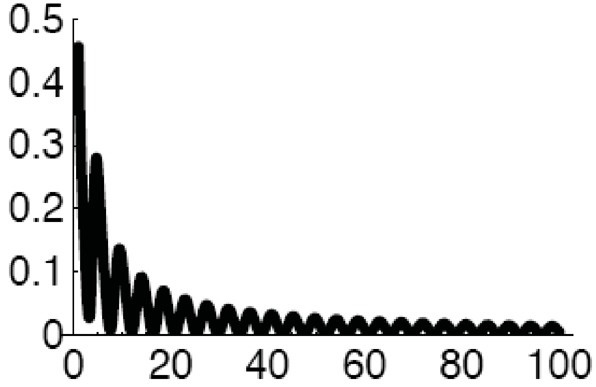
Λ relaxes from natural Planck scale value
to natural zero value



present value of Λ

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$\omega \sim E_{\mathrm{Planck}}$$



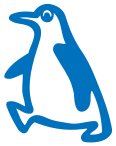
dynamics of Λ: from Planck to present value

$$<\Lambda(t_{\mathrm{Planck}})> \sim E^4_{\mathrm{Planck}}$$

$$<\Lambda(t_{\mathrm{present}})> \sim E^2_{\mathrm{Planck}} \,/\, t^2_{\mathrm{present}} \sim 10^{-120}\, E^4_{\mathrm{Planck}}$$

coincides with present value of dark energy

something to do with coincidence problem ?



Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

dynamics of Λ in cosmology

$$\boxed{\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}}$$

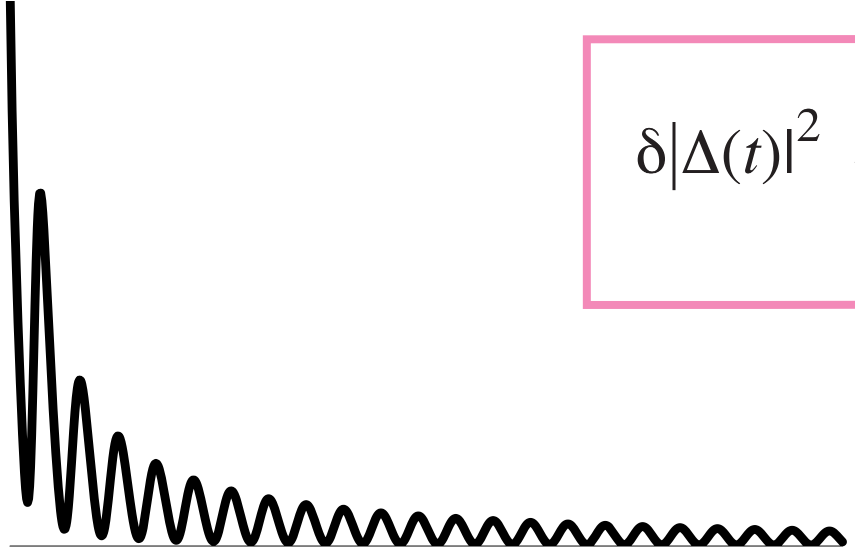
$\omega \sim E_{\text{Planck}}$

F.R. Klinkhamer & G.E. Volovik
Dynamic vacuum variable &
equilibrium approach in cosmology
PRD **78**, 063528 (2008)
Self-tuning vacuum variable &
cosmological constant,
PRD **77**, 085015 (2008)

dynamics of Δ in superconductor

$$\boxed{\delta|\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}}$$

$\omega = 2\Delta$



intial states:

nonequilibrium vacuum with $\Lambda \sim E_{\text{Planck}}^4$

superconductor with nonequilibrium gap Δ

final states:

equilibrium vacuum with $\Lambda = 0$

ground state of superconductor

$$\varepsilon(t) - \varepsilon_{\text{vac}} \sim \omega \frac{\sin^2 \omega t}{t}$$

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)
Barankov & Levitov, ...

properties of relativistic quantum vacuum as a self-sustained system

* **quantum vacuum is characterized by conserved charge** q

q has Planck scale value in equilibrium

$$\boxed{\varepsilon(q) \sim E^{4}_{\rm Planck}}$$

* **vacuum energy has Planck scale value in equilibrium**

but this energy is not gravitating

$$\boxed{T_{\mu\nu} = \Lambda g_{\mu\nu} \neq \varepsilon(q) g_{\mu\nu}}$$

* **gravitating energy**
is thermodynamic vacuum energy

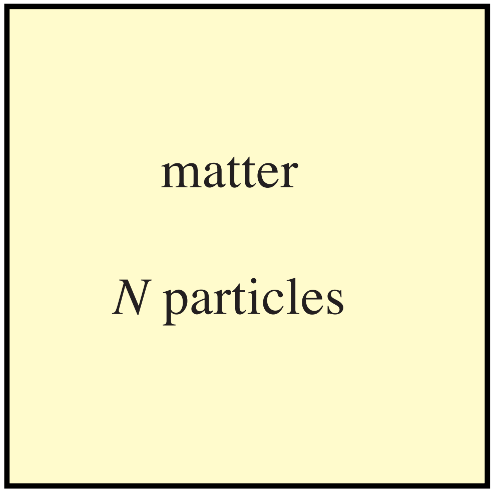
$$\boxed{T_{\mu\nu} = \Lambda g_{\mu\nu} = \Omega(q) g_{\mu\nu}}$$

$$\boxed{\Omega(q) = \varepsilon - q\, d\varepsilon/dq}$$

* **thermodynamic energy**
of equilibrium vacuum

$$\boxed{\Omega(q_0) = \varepsilon(q_0) - q_0\, d\varepsilon/dq_0 = 0}$$

matter in the box



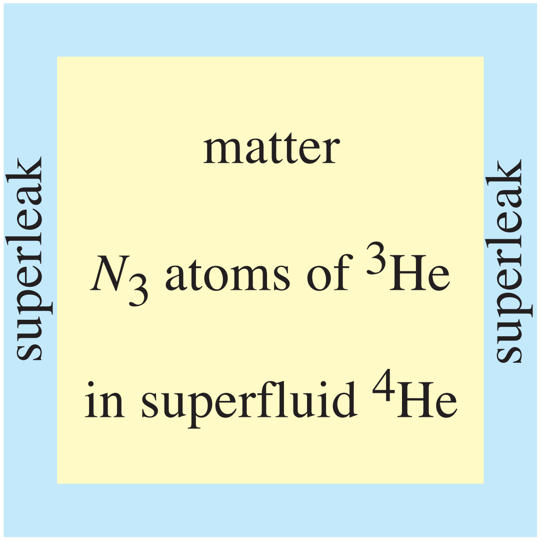
matter

N particles

vacuum is outside and inside too:

**vacuum may penetrate the wall of the box,
but not matter**

analog: dilute ^{3}He in superfluid ^{4}He



superleak

matter

N_3 atoms of ^{3}He

in superfluid ^{4}He

superleak

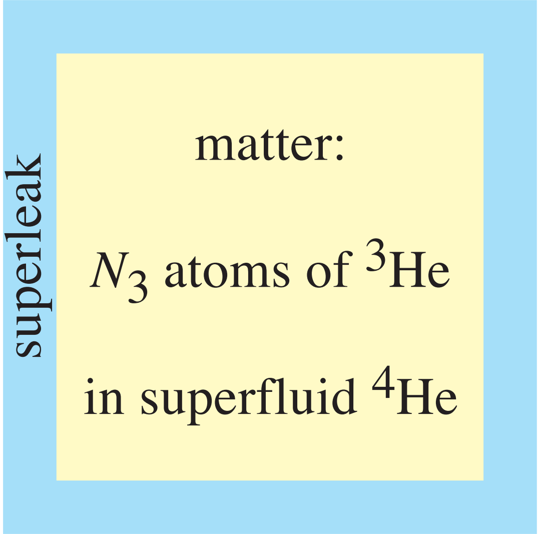
superleak is porous material

superfluid ^{4}He is outside and inside too:

vacuum (superfluid ^{4}He) **may penetrate superleak,
but not matter** (^{3}He atoms)

Khalatninkov
"Theory of superfluidity" Chapter XVI
Theory of Fermi-Bose Quantum Liquids

chemical potential of superfluid ^{4}He **is the same across the superleak**



matter:

N_3 atoms of ^{3}He

in superfluid ^{4}He

superfluid ^{4}He

$P_4 = 0$

$n_3 = 0$

$n_4 = n_{40}$

$P_{osm} = P_3 + P_4$

n_3 , n_4

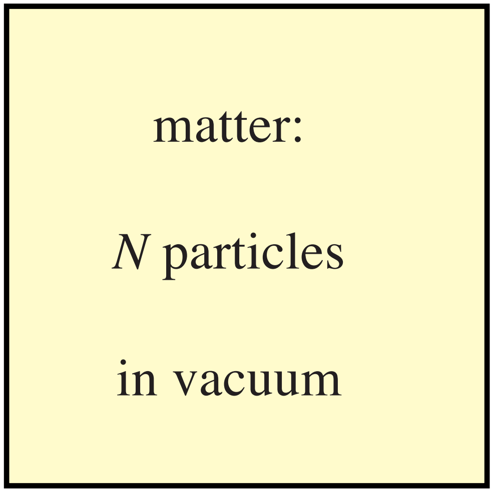
chemical potential of vacuum is the same across the wall

vacuum

$P_{vac} = 0$

$n = 0$

$q = q_0$



matter:

N particles

in vacuum

$P_{osm} = P_{mat} + P_{vac}$

n , q

pressure of superfluid ^{4}He **inside the box**

$$\boxed{P_4 = -\,(1/2)\chi_4\,[n_4\, d\varepsilon_3\,/\,dn_4]^2}$$

pressure of vacuum inside the box

$$\boxed{P_{vac} = -\,(1/2)\chi_{vac}\,[q\, d\varepsilon_{mat}\,/\,dq]^2}$$

$$\chi = -(1/V)\, dV/dP$$

compressibility

$$\boxed{1/\chi_{vac} = q^2\, d^2\varepsilon_{vac}\,/dq^2}$$

conclusion

* quantum vacuum is relativistic invariant self-sustained medium

* quantum vacuum of early Universe
belongs to nontrivial topological class of chiral Fermi systems with Fermi points

* quantum vacuum of present Universe
belongs to nontrivial class of topological insulators

* Universe experienced metal-insulator topological transition