

Incoherent magnetoresistance in layered metals

Pavel D. Grigoriev

L. D. Landau Institute for Theoretical Physics,
Chernogolovka, Russia



Plan of the talk

- Introduction. Layered compounds, background magnetoresistance, angular oscillations and magnetic quantum oscillations.
- Coherence – incoherence crossover in quasi-2D metallic interlayer magnetoresistance. Role of disorder. Experimental observations and the puzzles.
- The model of incoherent interlayer electron transport and its experimental test [M. V. Kartsovnik, P. D. Grigoriev, W. Biberacher, and N. D. Kushch, PRB 79, 165120 (2009).]

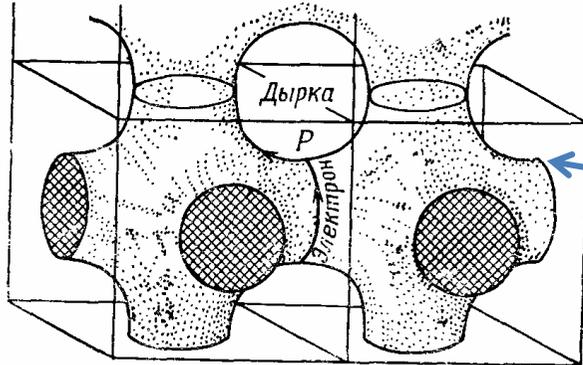
Introduction

Background magnetoresistance in normal 3D metals (strong fields)

In strong magnetic field the magnetoresistance depends on the shape and topology of the Fermi surface (FS), because now the electrons can encircle the FS before being scattered ($\omega_c\tau \gg 1$).

The conductivity tensor for closed trajectories

$$\sigma = \begin{pmatrix} \frac{A_{xx}}{H^2} & -\frac{A_{yx}}{H} & -\frac{A_{zx}}{H} \\ \frac{A_{yx}}{H} & \frac{A_{yy}}{H^2} & -\frac{A_{zy}}{H} \\ \frac{A_{zx}}{H} & \frac{A_{zy}}{H} & A_{zz} \end{pmatrix}$$



FS, containing open and closed trajectories

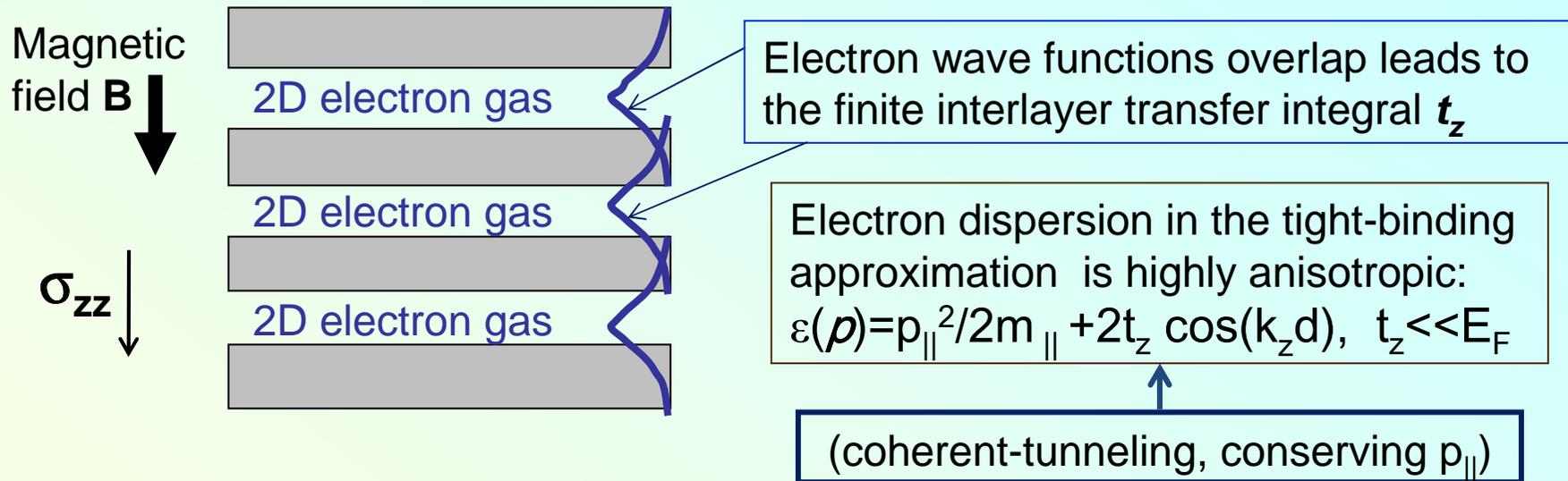
For open trajectories (open orbit along x-axis) the conductivity tensor is

$$\sigma = \begin{pmatrix} B_{xx} & -\frac{A_{yx}}{H} & -B_{zx} \\ \frac{A_{yx}}{H} & \frac{A_{yy}}{H^2} & -\frac{A_{zy}}{H} \\ B_{zx} & \frac{A_{zy}}{H} & A_{zz} \end{pmatrix}$$

Introduction

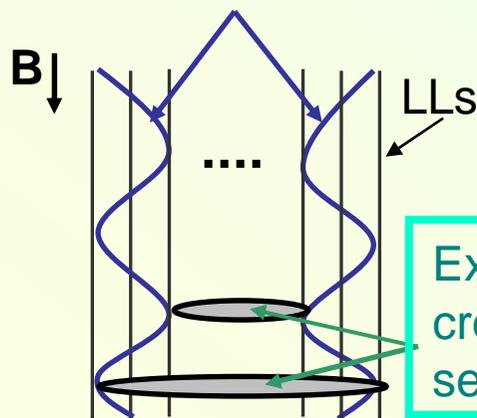
Layered quasi-2D metals

(Examples: heterostructures, organic metals, high-Tc superconductors)

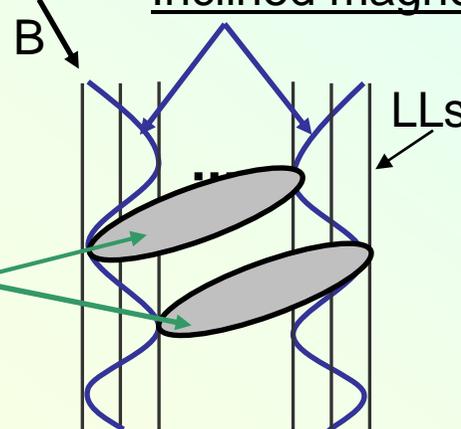


Fermi surface

$B \parallel$ conducting layers



Inclined magnetic field

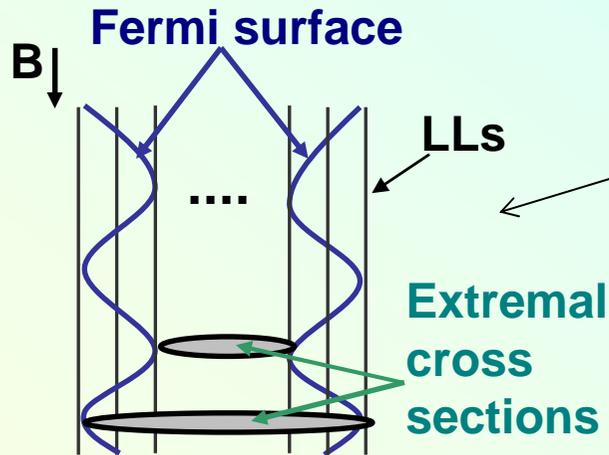


Fermi surface in quasi-2D metals is a warped cylinder

Introduction

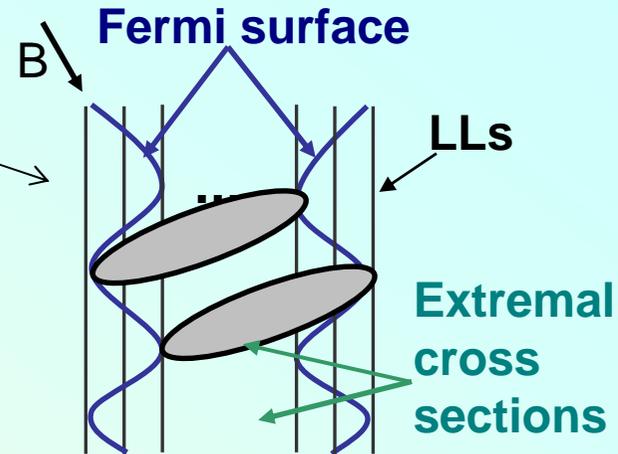
Angular magnetoresistance oscillations in q2D

$B \parallel$ conducting layers



Geometrical interpretation of the Yamaji angles in quasi-2D metals

Inclined magnetic field



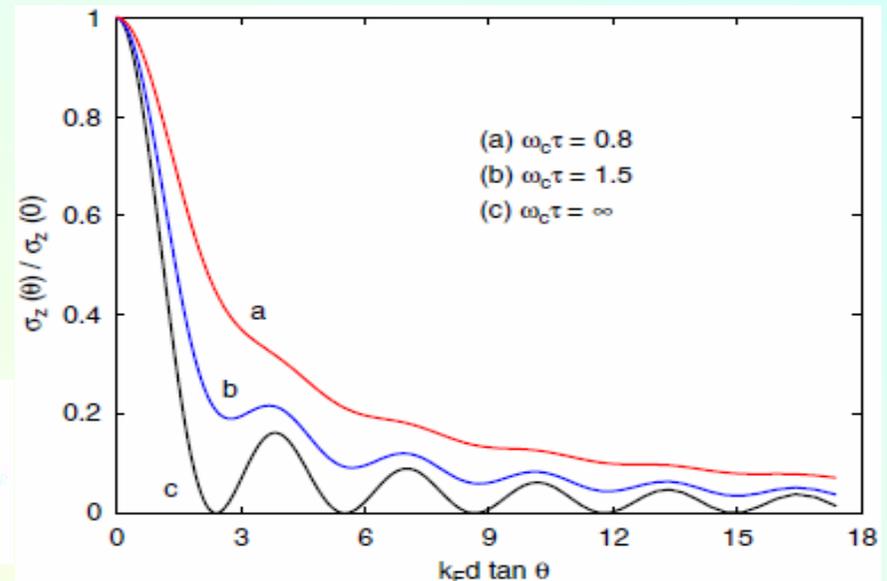
Cross section area and the electron dispersion have strong k_z -dependence

Cross section area and the electron dispersion are almost k_z -independent

Electron dispersion in the tight-binding approximation $\varepsilon(\mathbf{p}) = p_{\parallel}^2 / 2m_{\parallel} + 2t_z \cos(k_z d)$.

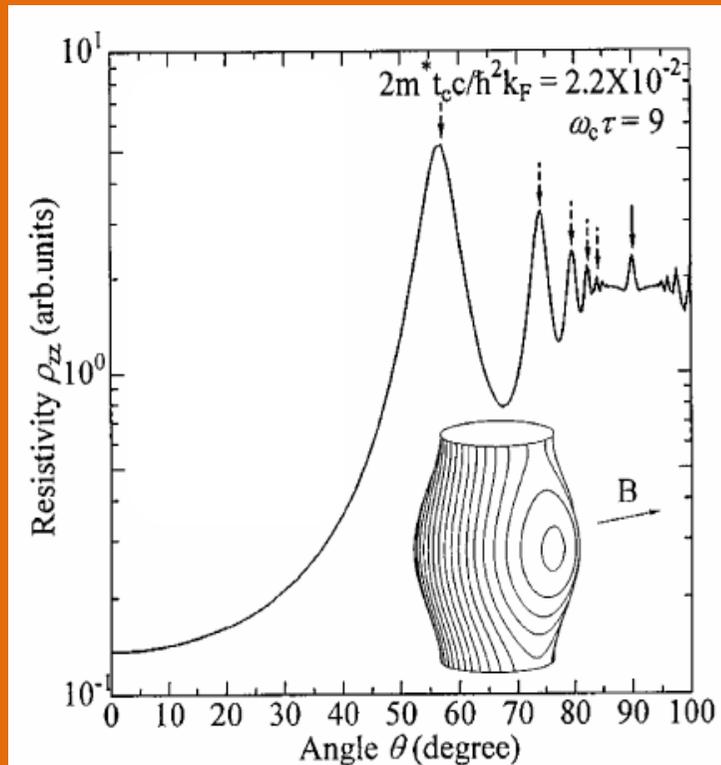
Theoretical prediction for AMRO of interlayer conductivity in quasi-2D metals in magnetic field:

$$\frac{\sigma_z(\mathbf{B})}{\sigma_z(0)} = J_0^2(k_F d \tan \theta) + 2 \sum_{j=1}^{\infty} \frac{J_j^2(k_F d \tan \theta)}{1 + (j\omega_c \tau)^2}$$



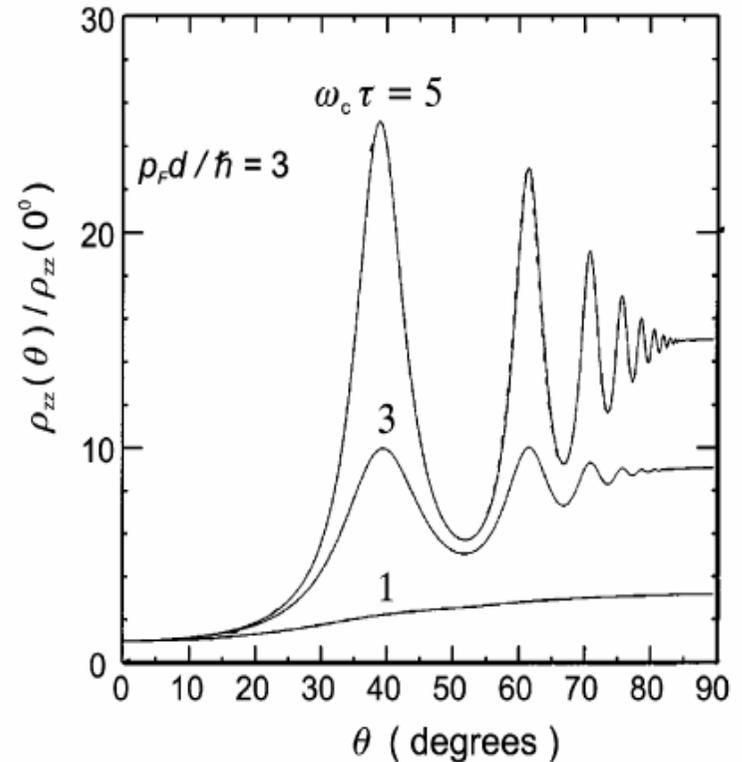
Theory: coherent vs. incoherent regimes (effect of impurities)

Coherent regime



D. Andres et al, PRB **72**, 174513 (2005)
N. Hanasaki et al., PRB **57**, 1336 (1998)

Weakly incoherent regime

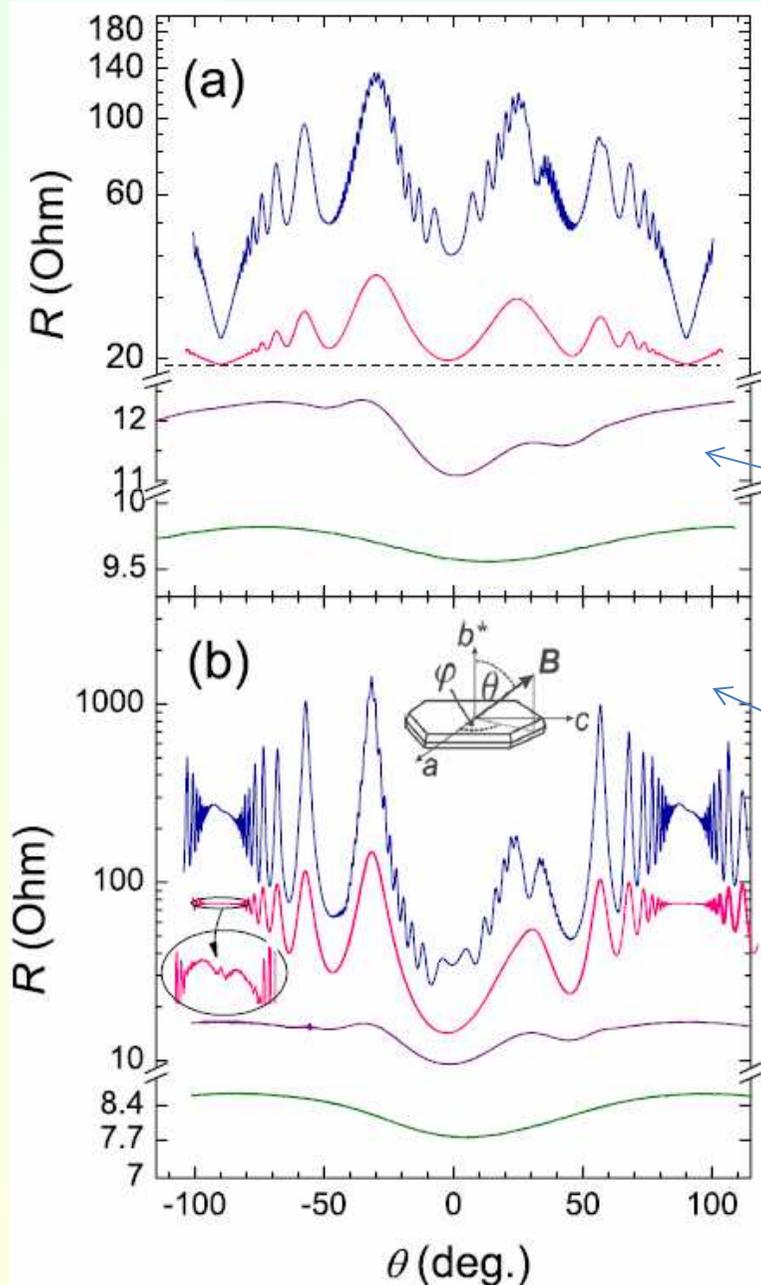


P. Moses & R. McKenzie,
PRB **60**, 7998 (1999)

Prediction: high MR at $B \parallel$ layers in both cases

- disagrees with the experimental results below!

Angular dependence of magnetoresistance



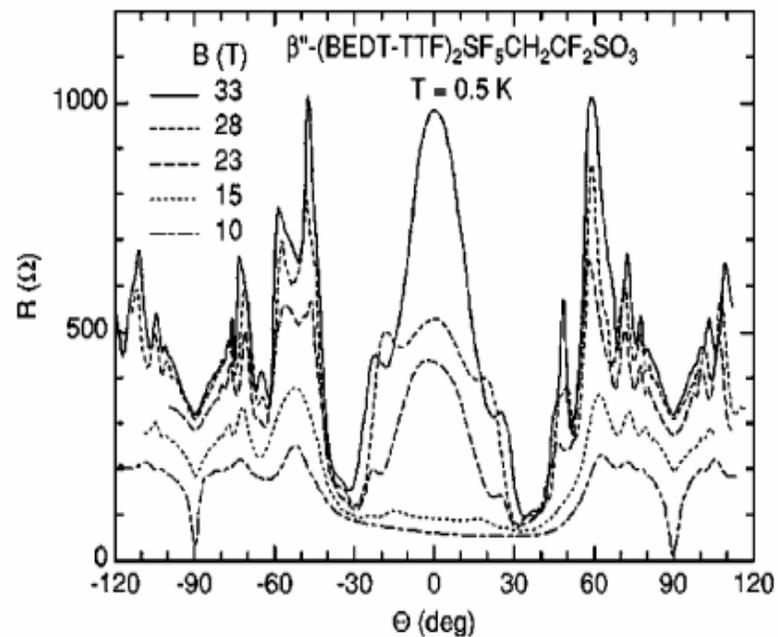
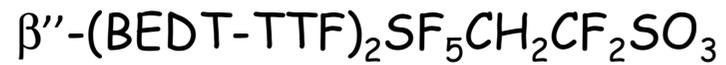
MR in clean and dirty samples have very different angle-dependence. In dirty sample there is a deep minimum of MR when $B \parallel$ conducting layers. In clean sample there is maximum of MR at $B \parallel$ conducting layers.

Dirty sample - almost no MR at $B \parallel$ layers;
- MR scales to a function of B

Clean sample (strong MR at $B \parallel$ layers)

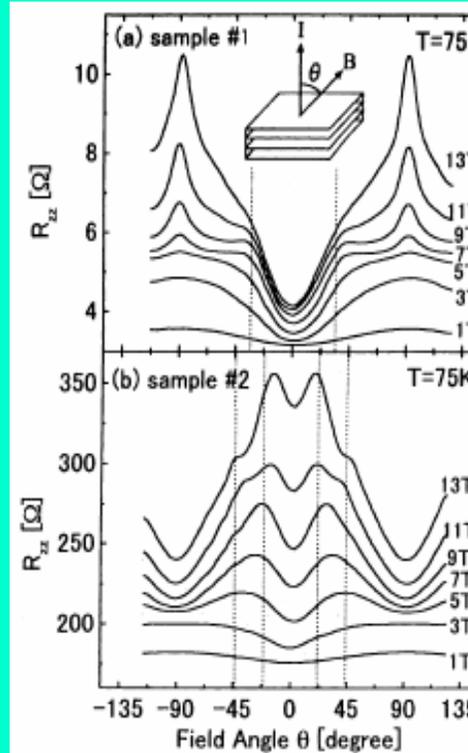
FIG. 1. (Color online) (a) Angle-dependent interlayer magnetoresistance of a relatively dirty sample, 1, of α -(BEDT-TTF) $_2$ KHg(SCN) $_4$ in the high-pressure metallic state recorded at $T=1.4$ K at magnetic fields (bottom to top): 0.12, 0.5, 3, and 15 T; $\varphi \approx 20^\circ$. (b) Same for a very clean sample, 2. The upper inset illustrates the definition of angles θ and φ ; the lower inset: enlarged fragment of the 3 T curve showing a small "coherence peak." [PRB 79, 165120 (2009).]

Similar features: other q2D systems



J. Wosnitzer et al., **65**, 180506(R) (2002)

GaAs/AlGaAs superlattice

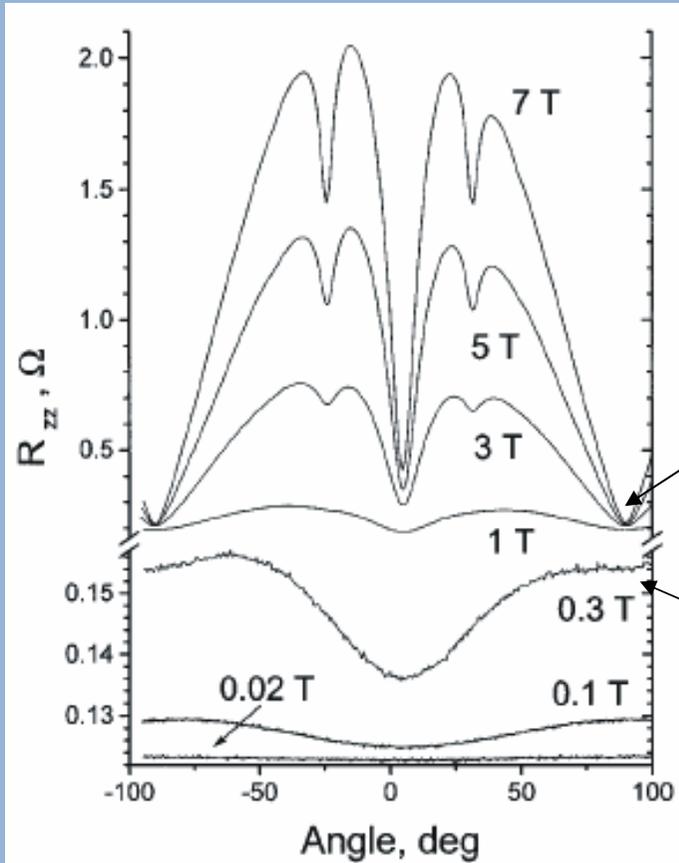


coherent

incoherent

M. Kuraguchi et al.,
Synth. Met. **133-134**, 113 (2003)

Similar behaviour: (TMTSF)₂PF₆ in the metallic state



anomalous MR

conventional MR

E. Chashechkina & P. Chaikin,
PRL **80**, 2181 (1998)

What is the nature of this
magnetic field - induced
coherent - incoherent
crossover?

First ideas. Origin of the term *coherent-incoherent crossover*

Dimensional crossover in B|| layers in quasi-1D layered metals.

Electron dispersion in quasi-1D metals

$$\mathcal{E}(k) = v_F (|k| - k_F) - 2t_y \cos(k_y b) - 2t_z \cos(k_z d),$$

$$v_F k_F \gg t_y \gg t_z. \quad \text{Vector potential } A_z = H_y x.$$

$$\text{Hamiltonian } \hat{H} = v_F (\hat{k}_x - k_F) + t_y \cos(\hat{k}_y b) + t_z \cos\left[d(\hat{k}_z - eH_{\parallel} x / c)\right]$$

When $eH_{\parallel} l_{\tau} / c > 2\pi / d$ the electron energy does not depend on k_z ,
i.e. the effective interchain transfer integral $t_z \rightarrow 0$.

Coherent-incoherent crossover in B|| layers in quasi-1D metals.

[S.P. Strong, D.G. Clarke, and P.W. Anderson, PRL **73**, 1007; PRL **72**, 3218 (1994)]

Electron transport between two chains, where the ground state is the Luttinger liquid, becomes incoherent when the interchain transfer integral become less than critical value: $t_z \ll \exp(-\text{const} / U^5)$. Magnetic field reduce t_z and leads to the crossover from coherent to the incoherent interchain transport.

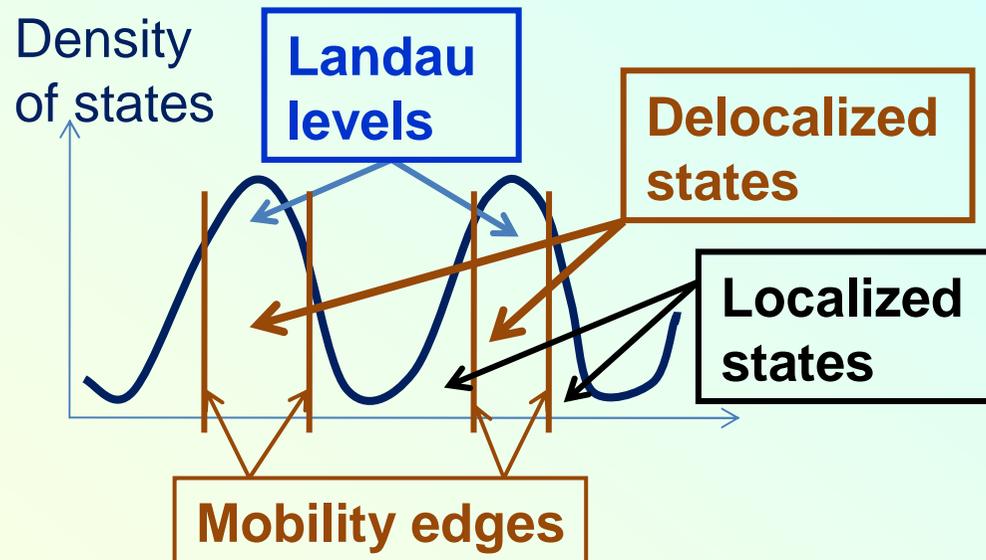
! This scenario predicts the anomalous magnetoresistance for clean samples, while in experiment it occurs first in dirty samples.

Disorder-driven coherent-incoherent crossover

What happens with the interlayer magnetoresistance when the interlayer tunneling time \hbar/t_z is longer than the mean scattering time τ due to impurity scattering? The in-plane momentum is not conserved during the tunneling time. Does the angular dependence of magnetoresistance change? Does the metal-insulator transition happen and how it goes?

1. No Anderson transition happens in one particular direction. In the case of Anderson localization the system becomes insulating in all directions, Quantum corrections (weak localization and Altshuler-Aronov effect) also predict the metal-insulator crossover in all direction [**A.A. Abrikosov, PRB 50, 1415 (1994)**]. On contrary, in all experiments in layered metals the anomalous behavior is shown by the interlayer conductivity σ_{zz} only.
2. The angular magnetoresistance oscillations survive at $t_z \ll \hbar/\tau$ (so-called *weakly incoherent regime*), being the same as in coherent case [**R. H. McKenzie and P. Moses, Phys. Rev. Lett. 81, 4492 (1998)**]
3. The Boltzmann transport equation is valid even at $t_z \ll \hbar/\tau$ [**D. B. Gutman and D. L. Maslov, Phys. Rev. Lett. 99, 196602 (2007)**].

Landau level quantization and localized states between LLs as possible reason of incoherent magnetotransport



When the Fermi level μ is in the region of localized states, the system is insulating, and conductivity has the activation temperature dependence $\sim \exp(-h\omega_c / T)$ or the variable hopping range behavior $\sim \exp[-(T_0 / T)^{1/2}]$.

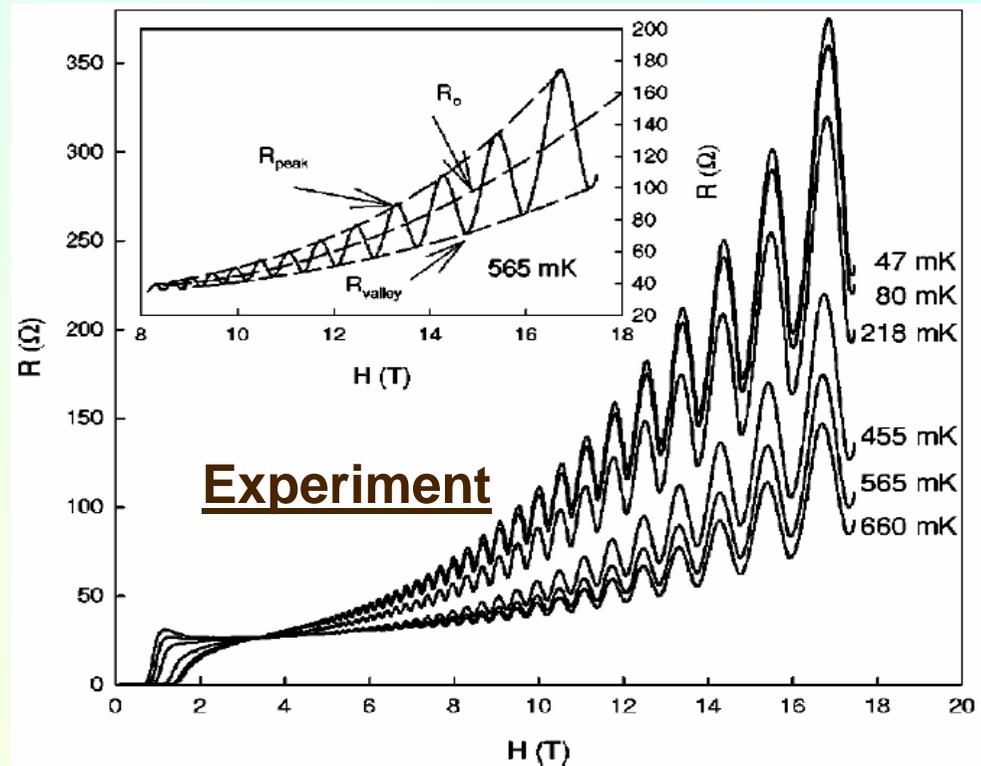
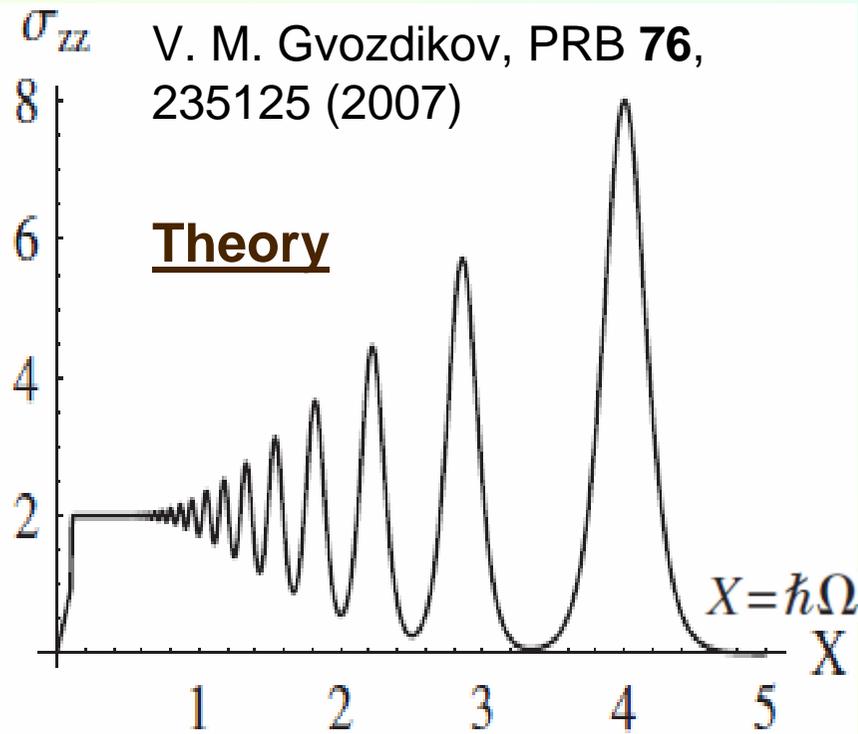
Problems with this explanation:

1. The experimental in-plane conductivity has metallic temperature dependence, which contradicts the localization of electrons.
2. This theory predicts the decrease of conductivity **only** when the Fermi level μ is between the LLs (the minima of conductivity decrease). When μ is on the LL, the maxima of conductivity increases with increase of magnetic field \mathbf{B} due to the increase of the DoS. This prediction contradicts the experiments.

Landau level quantization and localized states between LLs as possible reason of incoherent magnetotransport (2)

Theory predicts the decrease of conductivity only when the Fermi level μ is between the LLs (the minima of conductivity). When μ is on the LL, the maxima of conductivity increases with increase of magnetic field B due to the increase of the DoS. This prediction contradicts the experiments.

F. Zuo et al., PRB **60**, 6296 (1999)

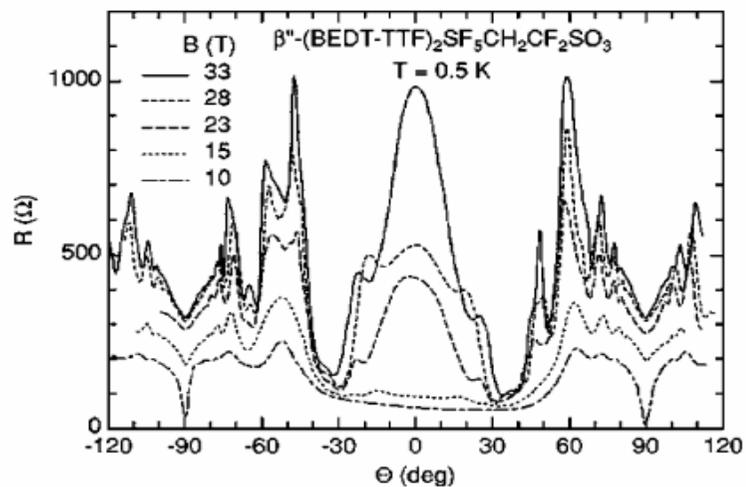
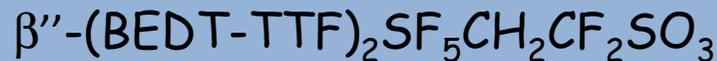


Other models of incoherent conductivity channel.

1. Interlayer tunneling via resonance impurity [A.A. Abrikosov, Physica C **317-318**, 154 (1999); applied to describe high-T_c cuprates].
2. Boson-assisted interlayer tunneling [A. F. Ho and A. J. Schofield, PRB **71**, 045101 (2005); D.B. Gutman, D.L. Maslov, PRB **77**, 035115 (2008)].

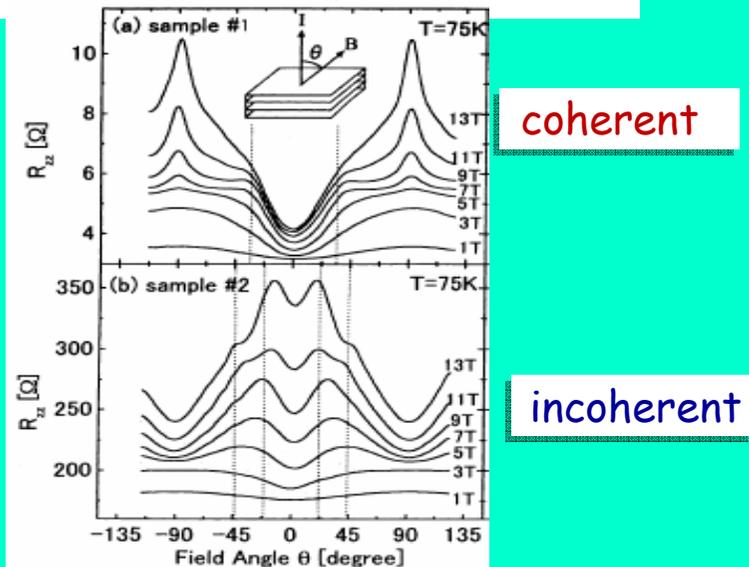
All these models cannot explain

- 1). Strong magnetoresistance in field, perpendicular to conducting layers.
- 2). Metallic temperature dependence of the incoherent conductivity.



J. Wosnitza et al., **65**, 180506(R) (2002)

GaAs/AlGaAs superlattice



M. Kuraguchi et al.,
Synth. Met. **133-134**, 113 (2003)

Model:
Two conduction channels

$$\sigma(\omega_c, \tau) = \sigma_{\text{coh}}(\omega_c \tau) + \sigma_i(\tau)$$

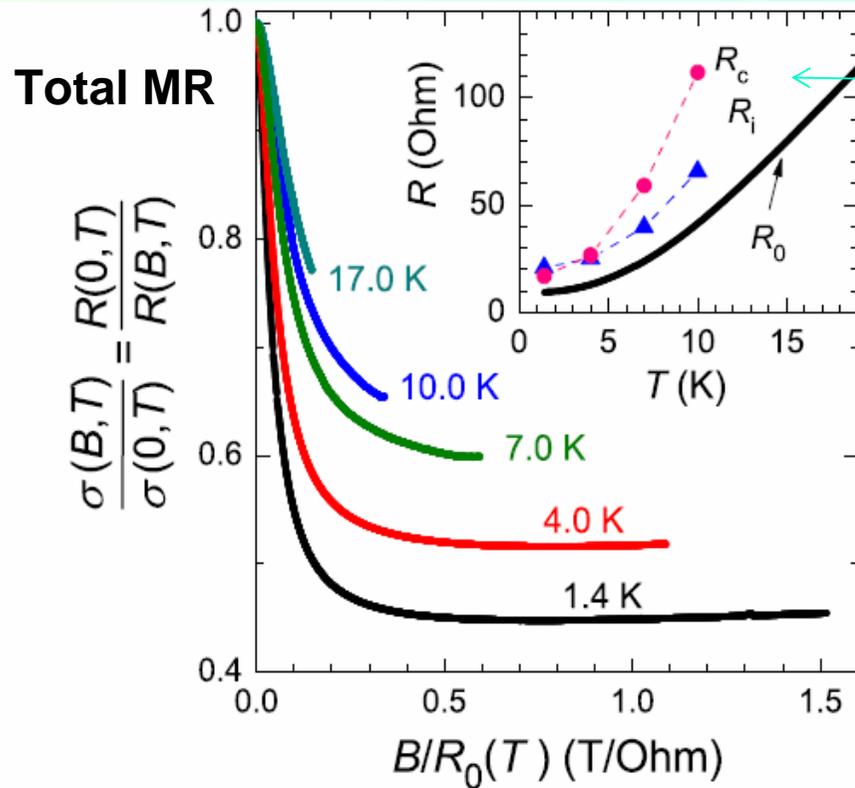


FIG. 2. (Color online). Kohler plot of the normalized interlayer conductivity of sample 1 for the field aligned parallel to conducting layers obtained from field sweeps at different temperatures. Inset: temperature dependence of the zero-field resistance R_0 (thick line) and the resistances of the coherent, $R_c \propto 1/\sigma_c$, (circles) and incoherent, $R_i \propto 1/\sigma_i$, (triangles) channels, see text.

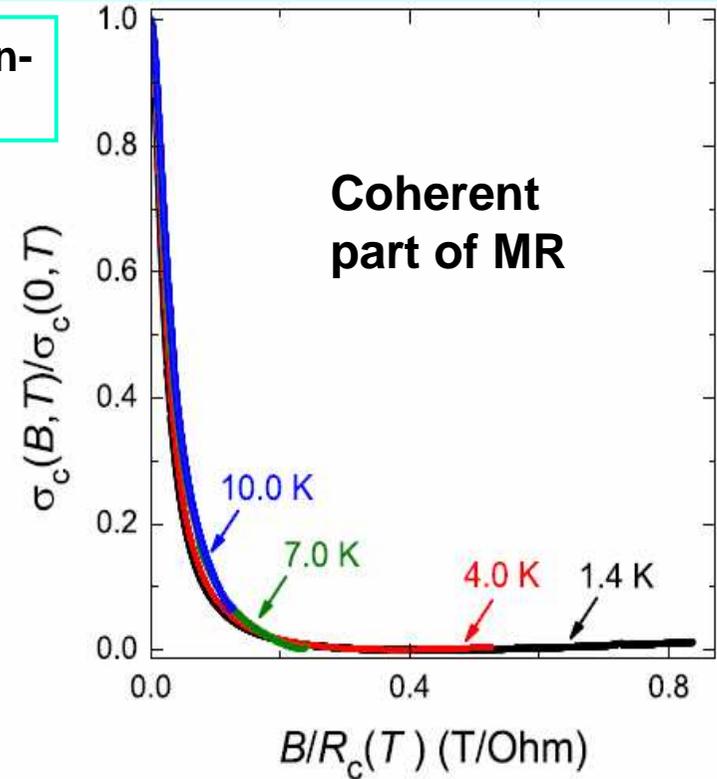
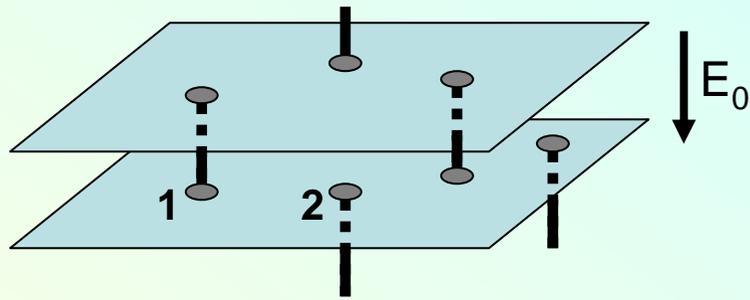


FIG. 3. (Color online). Kohler plot of the coherent part of interlayer conductivity of sample 1 in the field parallel to the layers at temperatures 1.4 to 10 K. The coherent conductivity has been determined from the data in Fig. 2: $\frac{\sigma_c(B, T)}{\sigma_c(0, T)} = \frac{\sigma(B, T)/\sigma(0, T) - \sigma_i(T)/\sigma(0, T)}{1 - \sigma_i(T)/\sigma(0, T)}$. The resistance $R_c(T)$ corresponding to the coherent channel at zero magnetic field is taken from the inset in Fig. 2.

Kohler plots of magnetoresistance ($B//\text{layers}$) [PRB 79, 165120 (2009).]

The model of the incoherent conductivity channel



The resistance through each short-cut (hopping center) contains two in-series elements:

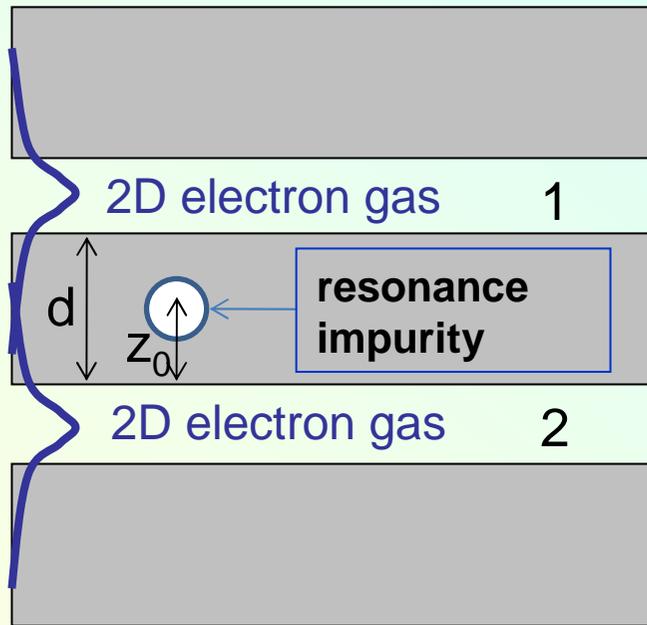
$$R_{\perp} = R_{hc} + R_{\parallel}.$$

The hopping-center resistance R_{hc} is almost independent of magnetic field and has nonmetallic temperature dependence. The dependence $R_{hc}(T, B)$ is determined by the nature of the hopping center.

The in-plane resistance R_{\parallel} depends on the magnetic field \perp to the conducting layers according to the standard theory: $R_{\parallel} \sim B_{\perp}^2$.

R_{\parallel} has the metallic temperature dependence. It can be calculated in the limit when the concentration of short-cuts $n_i = 1/l_i^3$ is much less than the concentration of normal impurities $n_{\tau} = 1/l_{\tau}^3$. Then the resistance R_{\parallel} is determined by the normal in-plane conductivity σ_{\parallel} .

Resonance impurities and interlayer e⁻ transport



Without impurities the transparency coefficient between layers 1 and 2 is exponentially small.

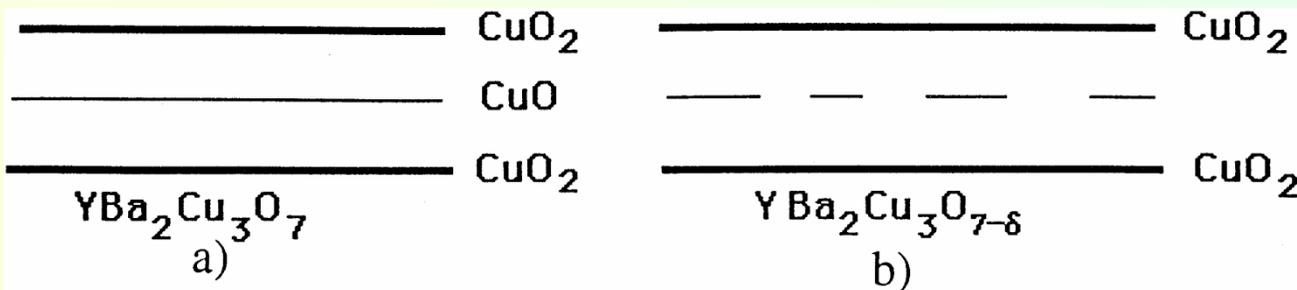
$$p \propto \exp(-\alpha d), \quad \alpha \equiv \sqrt{2m(U - \varepsilon)}.$$

This transparency coefficient p becomes ~ 1 if
 a). The impurity energy level is very close to the Fermi level in 2D electron gas.

b). The impurity is located almost in the middle of the potential barrier: $z_0 = d/2$.

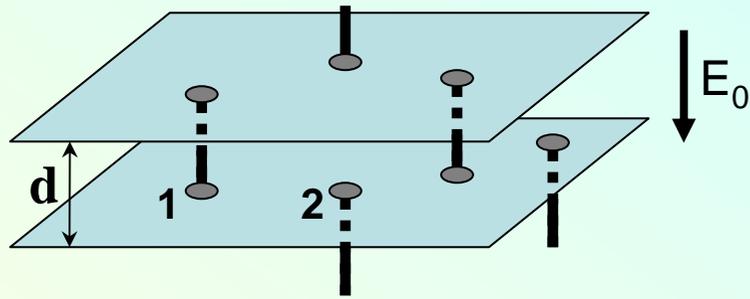
$$p \propto \left[(1 - \varepsilon / E_F) \exp(\alpha d) + A \exp[\alpha(d - 2z_0)] \right]^{-1}.$$

Originally, the resonance-impurity interlayer electron transport was proposed in high-T_c superconductor [A.A. Abrikosov, Physica C **317-318**, 154 (1999)].



CuO layers in underdoped SC serve as resonance impurities.

The incoherent conductivity channel (basic formulas)



The in-plane electron transport between short-cuts is given by macroscopic in-plane conductivity:

$$n_i = 1/l_i^3 \ll n_d = 1/l_\tau^3, \rightarrow j = \sigma_{\parallel} d E$$

The stationary current density $j(\mathbf{r}-\mathbf{r}_i)$ and electric field $E(\mathbf{r}-\mathbf{r}_i)$ around each short-cut at point $\mathbf{r} = \mathbf{r}_i$ is axially symmetric and satisfies $\text{div } j(\mathbf{r}) = 0$:

$$E(\mathbf{r} - \mathbf{r}_i) = \frac{j(\mathbf{r} - \mathbf{r}_i)}{\sigma_{\parallel} d} = \frac{I_0}{2\pi\sigma_{\parallel} d} \frac{(\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^2},$$

The voltage between two closest short-cuts to adjacent layers is

$$I_0 R_{\parallel}(T) \approx 2 \int_{l_\tau}^{l_i} E(r) dr = \frac{I_0 \ln(l_i/l_\tau)}{\pi\sigma_{\parallel} d}.$$

The total resistance due to each short-cut

$$R_{\perp} = R_{\text{hc}} + R_{\parallel}, \text{ where } R_{\parallel} = \ln(l_i/l_\tau)/\pi\sigma_{\parallel} d.$$

The total incoherent part of conductivity:

σ_{\parallel} depends on magnetic field \perp layers and has metallic T -dependence.

$$\sigma_i = \frac{\pi\sigma_{\parallel} n_i d^3}{\pi d \sigma_{\parallel} R_{\text{hc}} + \ln(l_i/l_\tau)}.$$

Analysis of the result and comparison with experiment

The total interlayer conductivity is a sum of coherent and incoherent parts:

$$\sigma(\omega_c, \tau) = \sigma_{\text{coh}}(\omega_c \tau) + \sigma_i(\tau).$$

The **coherent** part $\sigma_{\text{coh}}(\omega_c \tau)$ is given by standard formulas. It shows AMRO and is suppressed by the magnetic field \parallel layers:

$$\sigma_{\text{coh}\perp}(\omega, B_{\parallel}) = \sigma_{\text{coh}\perp}(0) \frac{1 - i(\omega_{H\parallel} - \omega)\tau}{1 + (\omega_{H\parallel} - \omega)^2 \tau^2}, \quad \omega_{H\parallel} = \frac{eB_{\parallel}}{m^* c}, \quad \sigma_{\text{coh}\perp}(0) \approx e^2 \langle v_z^2 \rangle \tau g_0(E_F).$$

For the **incoherent** part of interlayer conductivity σ_i we obtain:

$$\sigma_i = \frac{\pi \sigma_{\parallel}(B_{\perp}, T) n_i [R_{hc}] d^3}{\pi \sigma_{\parallel}(B_{\perp}, T) R_{hc}(T) d + \ln(l_i / l_{\tau})}.$$

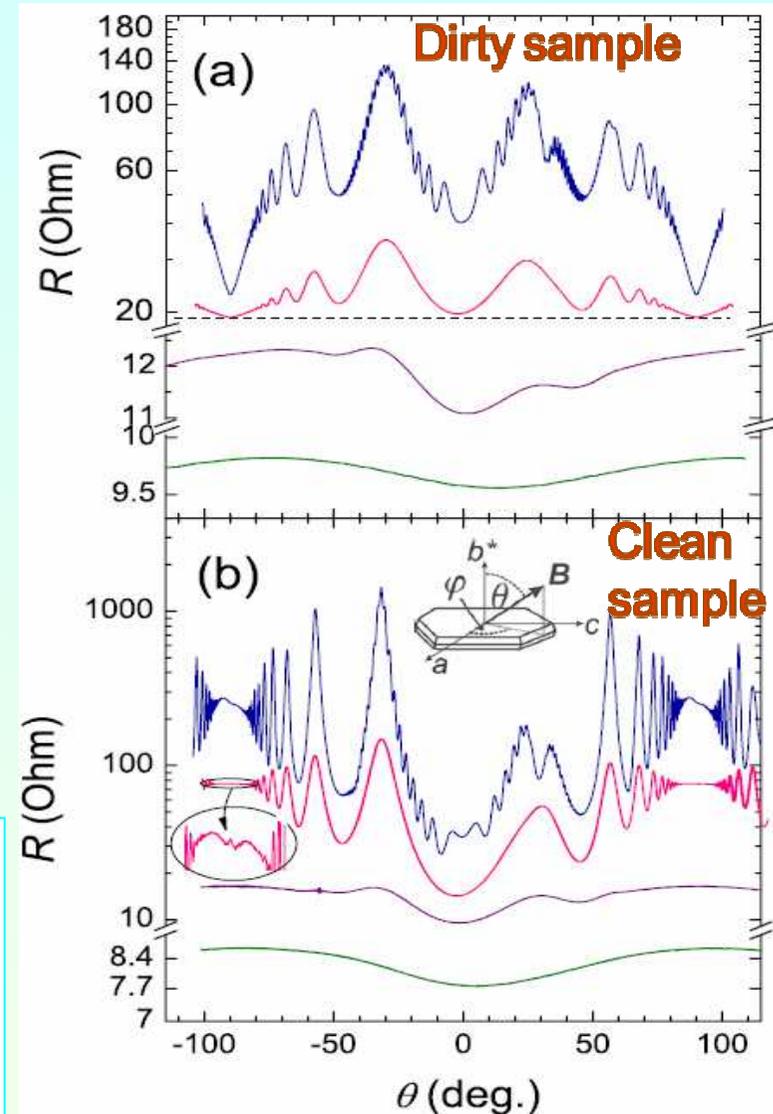
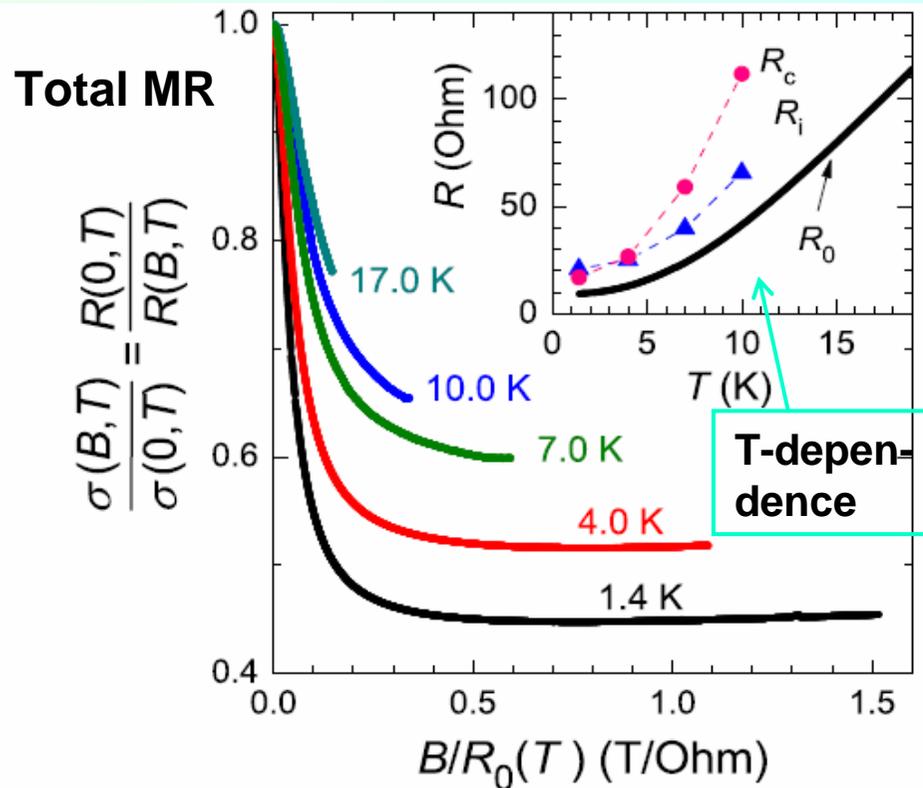
The (T, B) dependence of short-cut resistance R_{hc} and their distribution $n_i [R_{hc}]$ are determined by the nature of short-cuts .

The in-plane conductivity σ_{\parallel} does not depend on B_{\parallel} but only on B_{\perp} :

$$\sigma_{\parallel}(\omega, B_{\perp}, T) = \sigma_{\parallel}(T) \frac{1 - i(\omega_H - \omega)\tau}{1 + (\omega_H - \omega)^2 \tau^2}, \quad \omega_H = \frac{eB_{\perp}}{m^* c}, \quad \sigma_{\parallel}(T) \sim \frac{n_e e^2 E_F h}{mT^2} + \sigma_{\parallel}^{\text{quant}}.$$

The incoherent part of interlayer conductivity σ_i does not show AMRO.

Experimental tests of the proposed model [PRB 79, 165120 (2009)]



Main tests of the model:

The incoherent part of conductivity is not sensitive to $B_{||}$ but depends strongly on B_{\perp} and has metallic temperature

Other tests in favor of the model: With the increase of disorder the role of the incoherent part of conductivity becomes stronger. Anisotropy grows with temperature decrease. Anomalous angular dependence of MR survives at $T \gg LL$ separation.

Conclusion

The model of incoherent electron interlayer transport is proposed, and the analytical formula for interlayer magnetoresistance is obtained. The predictions agree well with experimental observations in layered organic metals and heterostructures and explain the long-standing problems of the anomalous angle-dependence of “incoherent “ magnetoresistance in layered metals.

Outlook

The proposed model allows further theoretical and experimental study of the interlayer electron transport in various layered compounds and artificial structures:

1. Investigation of the nature and properties of short-cuts.
2. Calculation of quantum corrections.
3. Investigation of the effects of spin, strong electric and magnetic field, light radiation on the interlayer electronic transport.
4. Control of interlayer transport by chemical composition.