Surface plasmon resonance between two close metallic grains

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Outline

- Problem formulation
- Qualitative estimations
- Analytic solutions
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- Conclusions
- Further research

Problem formulation

δ – gap width;

a – granule size;

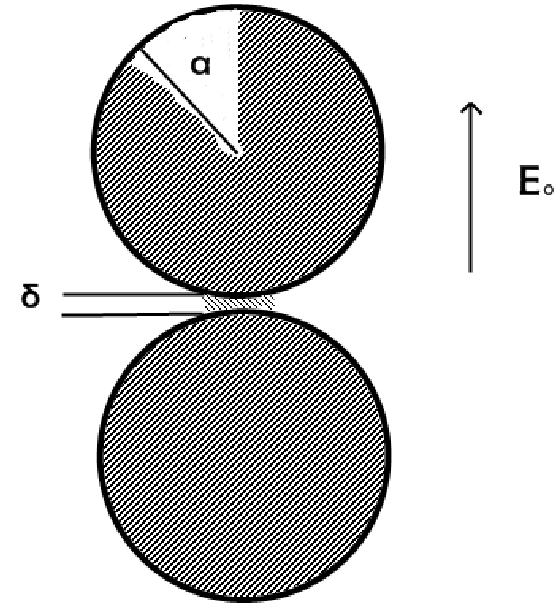
$\lambda \gg a \gg \delta$

$\varepsilon(\omega) = \varepsilon' + i\varepsilon''$;

$\varepsilon' \gg \varepsilon''$;

$\varepsilon' < 0$;

$|\varepsilon'| \gg 1$



Equations

$$\lambda \gg a; \quad a - \text{granule size}$$

$$\mathbf{E} = -\nabla \phi;$$

$$\nabla^2 \phi = 0$$

Boundary conditions:

$$\phi_{in} = \phi_{out}; \; \varepsilon \frac{\partial \phi_{in}}{\partial n} = \frac{\partial \phi_{out}}{\partial n}$$

Resonance on single granule

$$\mathbf{E} = \mathbf{E}_0 + a^3 \frac{\varepsilon - 1}{\varepsilon + 2}\left(\frac{3(\mathbf{E}_0, \mathbf{n})\mathbf{n} - \mathbf{E}_0}{r^3}\right)$$

Resonance conditions:

$\varepsilon = -2$ – single sphere; $\left(\varepsilon = -\frac{l+1}{l}\right)$

$\varepsilon = -1$ – single cylinder

Field enhancement - $\varepsilon' / \varepsilon''$

Permittivity

Drude-Lorentz formula:

$$\varepsilon \sim -\left(\frac{\omega_p}{\omega}\right)^2 \left[1 + i/(\omega\tau)\right]; \qquad \omega\tau \gg 1$$

ω_p – plasma frequency;

τ – relaxation time.

$$\varepsilon \sim -2 \quad \rightarrow \quad \omega \sim \omega_p$$

Qualitative estimations. Resonance condition.

For close spheres (or cylinders) gap width $\sim \delta$ at distances $\sim \sqrt{\delta a}$.

Propagating wave in narrow gap: $\beta \sim \dfrac{1}{\delta |\varepsilon|}$.

Resonance condition: $\varepsilon_{res} \sim -\dfrac{1}{n}\sqrt{\dfrac{a}{\delta}}$. $\left(n \ll \sqrt{\dfrac{a}{\delta}} \right)$

Qualitative estimations. Field enhancement.

E_c – field in the gap; $E(\rho)$ – field on the line between grains;

E_0 – external field

$\Delta\phi$ – potential difference between grains at $\sqrt{a\delta} < \rho < a$

$\Delta\phi_{in}$ – potential change inside grain at $\sqrt{a\delta} < \rho < a$

$$E(\rho) \sim \frac{\Delta\phi a}{\rho^2};$$

$$\Delta\phi_{in} \sim \frac{E}{\varepsilon}\rho \sim E\rho\sqrt{\frac{\delta}{a}} \ll \Delta\phi$$

$$d \sim E_c a^2 \delta$$

Qualitative estimations. Field enhancement.

Energy dissipation rate:

$$Q \sim \omega \varepsilon'' E_c{}^2 \left(1/\varepsilon'\right)^2 \left(a\delta\right)^{3/2} \sim \omega \varepsilon'' E_c{}^2 a^{1/2} \delta^{5/2}$$

$$Q \sim \omega d\, E_0$$

$$\frac{E_c}{E_0} \sim \frac{1}{\varepsilon''}\left(\frac{a}{\delta}\right)^{3/2} \quad - \quad \text{spheres}$$

$$\frac{E_c}{E_0} \sim \frac{1}{\varepsilon''}\frac{a}{\delta} \quad - \quad \text{cylinders}$$

Analytic solutions

Bispherical coordinates

$$x=\frac{C\sin\eta\cos\varphi}{\cosh\xi-\cos\eta};\quad y=\frac{C\sin\eta\sin\varphi}{\cosh\xi-\cos\eta};\quad z=\frac{C\sinh\xi}{\cosh\xi-\cos\eta}.$$

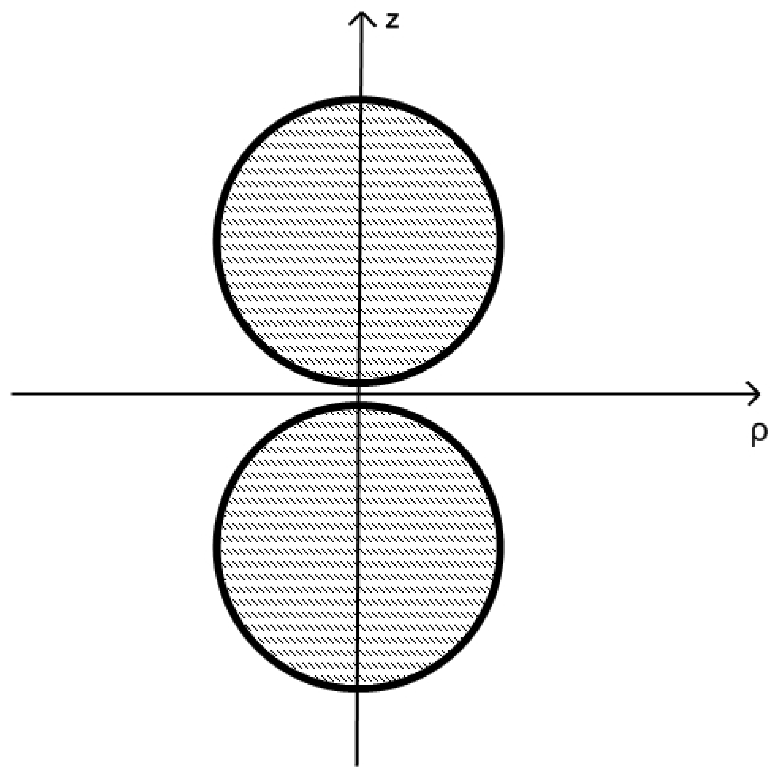
$$-\infty<\xi<\infty;\ \ 0<\eta<\pi;\quad 0<\varphi<2\pi$$

$$\varepsilon+i\eta=\ln\frac{(z+i\rho)+C}{(z+i\rho)-C}$$

Coordinate surface $\xi=const$:

$$x^2+y^2+\left(z-C\coth\xi\right)^2=\left(C/\sinh\xi\right)^2$$

Two granules



Analytic solution.
Two close spheres.

Resonance condition :

$$\left(\varepsilon_{res}\sqrt{\delta / a}\right)^{-1} = -(n+1/2) + \frac{1}{\ln\left[\sqrt{\delta / a}(n+1)^2\right]}$$

Field enhancement :

$$\frac{E_c}{E_0} = \frac{G}{\varepsilon - \varepsilon_{res}} + G_{bg};$$

$$G = \frac{8\pi^2}{3\ln(a/\delta)}\left(\frac{a}{\delta}\right)^{3/2}; \quad G_{bg} = -2\left(\frac{a}{\delta}\right)^{1/2}$$

Bicylindrical coordinates

$$x = \frac{C \sin\eta}{\cosh\xi - \cos\eta}; \quad y = \frac{C \sinh\xi}{\cosh\xi - \cos\eta}$$

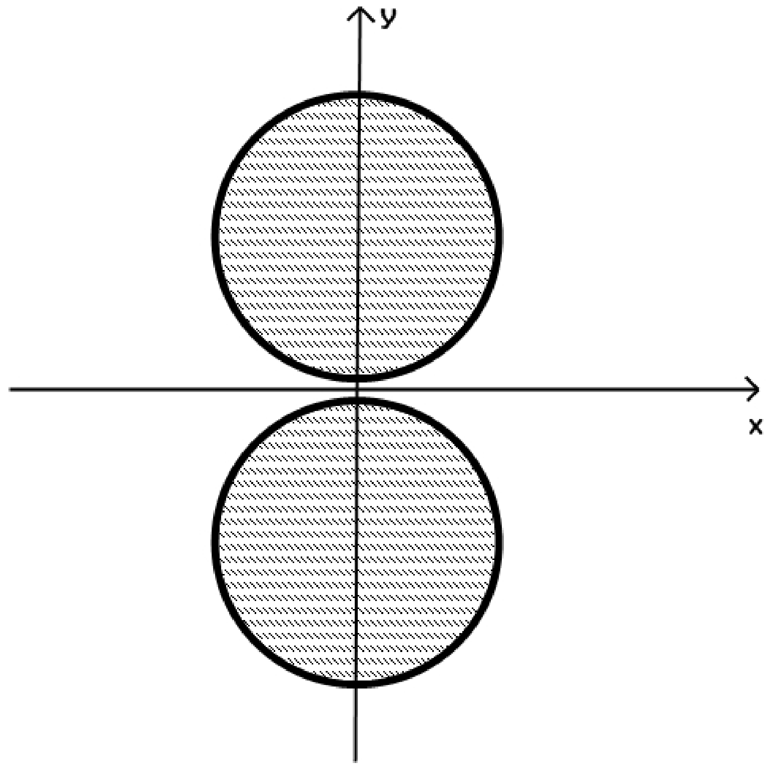
$$-\infty < \xi < \infty; \ \ 0 < \eta < 2\pi;$$

$$\varepsilon + i\eta = \ln \frac{(x+iy)+iC}{(x+iy)-iC}$$

Coordinate surface $\xi = const$:

$$x^2 + \left(y - C\coth\xi\right)^2 = \left(C / \sinh\xi\right)^2$$

Two granules



Analytic solution.
Two close cylinders.

Resonance condition :

$$\varepsilon_{res} = -\frac{1}{n}\sqrt{\frac{a}{\delta}}; \quad \left(\varepsilon_{res} = -\coth n\xi_0; \quad \sinh\frac{\xi_0}{2} = \frac{1}{2}\sqrt{\frac{\delta}{a}}. \right)$$

Field enhancement :

$$\frac{E_c}{E_0} = \frac{G}{\varepsilon - \varepsilon_{res}} + G_{bg};$$

$$G = 4\frac{a}{\delta}; \quad G_{bg} = 0.77\left(\frac{a}{\delta}\right)^{1/2}$$

Geometric enhancement

Metal granules in static field $(\varepsilon = \infty)$:

$$\frac{E_c}{E_0} \sim \frac{a}{\delta} \frac{1}{\ln(a/\delta)} \quad - \quad \text{spheres}$$

$$\frac{E_c}{E_0} \sim \sqrt{\frac{a}{\delta}} \quad - \quad \text{cylinders}$$

Radiation losses

Dipole radiation :

$$I \sim \frac{\omega^4}{c^3} d^2;$$

$$I \ll Q;$$

$$\varepsilon'' \gg \left(\frac{a}{\lambda}\right)^3 \sqrt{\frac{a}{\delta}} - \text{spheres;}$$

$$\varepsilon'' \gg \left(\frac{a}{\lambda}\right)^2 - \text{cylinders.}$$

Conclusions

- Resonance in narrow gap occurs at large negative values of permittivity, thus shifting the resonance frequency to visible light range.
- Resonance permittivity (frequency) is determined by the gap geometry (width and length). Field enhancement is determined by the system geometry (spheres, cylinders, etc.).
- Field enhancement in resonance is larger than possible geometric enhancement for the system.

Further research

- More complicated geometries, e.g. rod-like graines.
- Chains of granules, periodic structures.