Non-equilibrium tunnel junctions under high-voltage injection

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Outline

- Tunnel junction under injection are used for measurements of electron-phonon relaxation time and energy relaxation

  Review: *Giazotto et al*., Rev Mod Phys. (2006);
  Recent: *Karvonen & Maasilta*, PRL (2007);
  Timofeev et al., PRL (2009)

- Quasi-equilibrium is usually assumed
- Is it really so?
- What if it is not the case?

Al tunnel junctions under injection

Timofeev et al., PRL (2009)

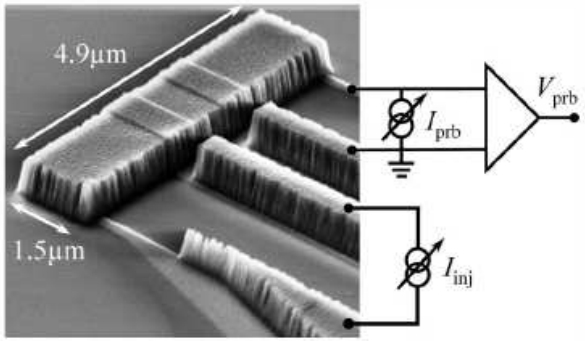


FIG. 1. A typical sample (sample *C*) for measuring energy relaxation in an Al superconducting bar. The circuits indicate injection of hot qp's and probing the island temperature.

TABLE I. Sample dimensions and junction resistances.

Sample	Volume (μm^3)	R_1, R_2, R_3, R_4 (kΩ)
A	$21 \times 1.5 \times 0.44$	840, 4, 4, 1160
B	$4.9 \times 1.5 \times 0.44$	760, 5.7, 5.7, 1290
C	$4.9 \times 1.5 \times 0.44$	485, 20, 20, 980

Probe IV curves

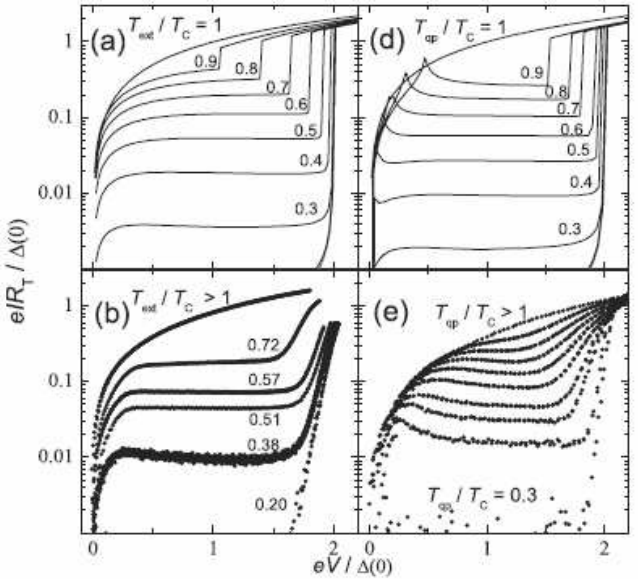


FIG. 2. Tunnel currents for a superconductor in equilibrium and quasiequilibrium. (a) Theoretical and (b) experimental *I-V* curves of a junction at several bath temperatures when $T_{qp} = T_{ext}$ (sample *A*). (c) Theoretical and experimental currents at $eV = \Delta$ (sample *B*). The two theory lines correspond to pair-breaking parameters $\gamma = 10^{-3}$ (upper curve) and $\gamma = 10^{-4}$ (lower curve). (d) Calculated *I-V* curves when the leads and the island have different temperatures $T_{qp} \neq T_{ext}$. (e) The measured *I-V* curves under a few injection conditions (sample *C*).

The data suggests electronic quasi-equilibrium

Goal: E-ph energy transfer as a function of electronic temperature

Measurements: Electronic temperature *vs* injected power

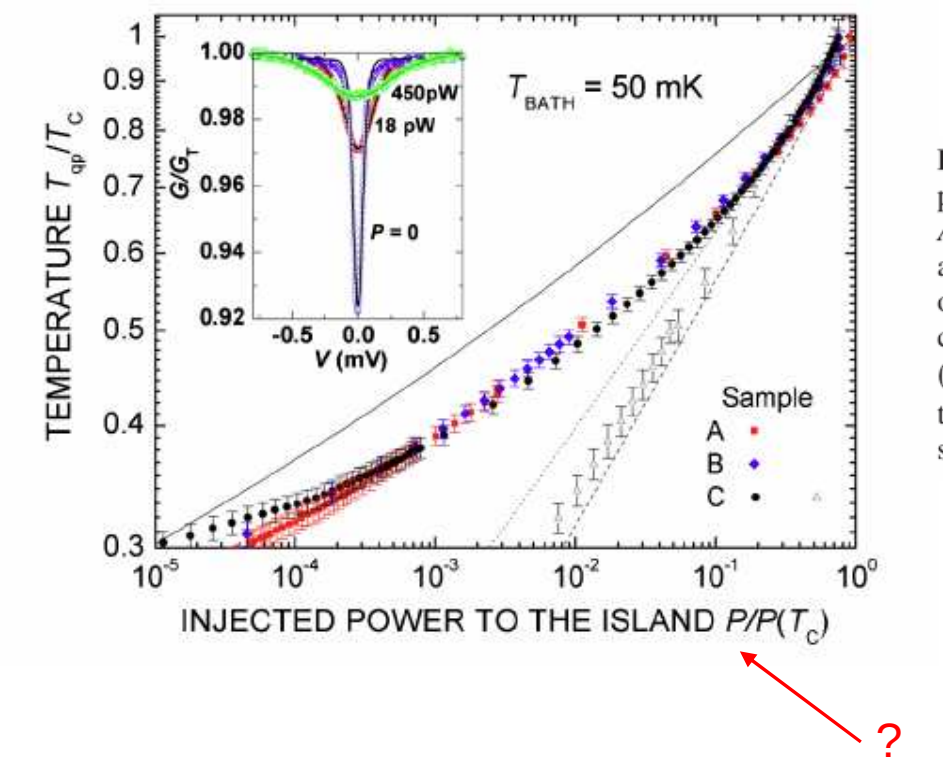
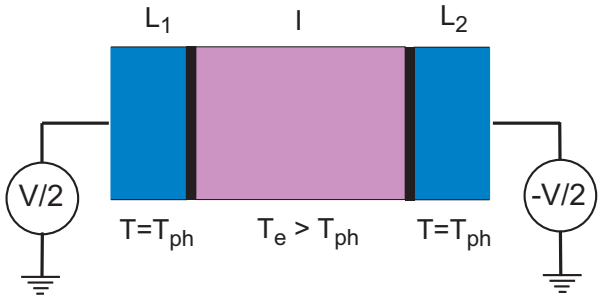


FIG. 3 (color online). Energy relaxation from theory and experiment. The data in the superconducting state are from samples *A* (squares), *B* (diamonds), and *C* (circles). The open triangles are from sample *C* in the normal state. The solid line is the result of Eq. (2) in the superconducting state. The dotted line indicates $P/P(T_C) = (T_{qp}/T_C)^5$, and the dashed line $P/P(T_C) = (T_{qp}/T_C)^4$. The inset shows three Coulomb peaks measured in the normal state under different levels of power injection; the solid lines are theoretical fits to them.

$$eV \sim 10^2 \Delta$$

Conditions for quasi-equilibrium

Normal junction



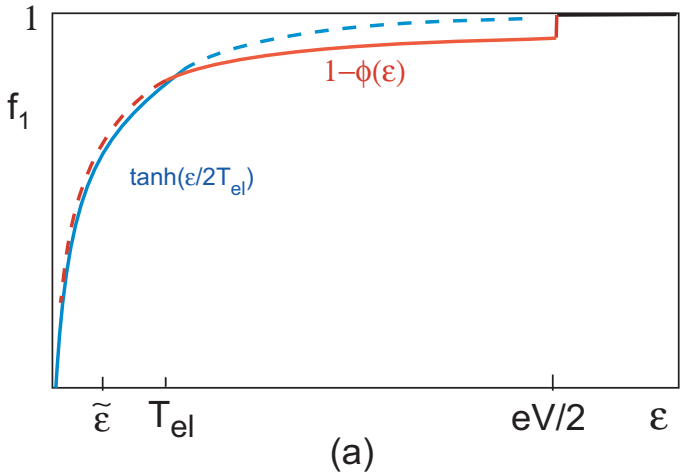
- Quasiequilibrium with $T_e \neq T_{ph}$ exists only if $\gamma_{e-e}(T_e) \gg \gamma_{e-ph}(T_e)$.
  γ_{e-e} and γ_{e-ph} are the e-e and e-ph relaxation rates

Nevertheless:

- Injection creates deviation from equilibrium $\phi(\epsilon)$ in the energy range $0 < \epsilon < eV/2$.
- Though $\phi(\epsilon)$ is small for small contact resistance R,
- $\phi(\epsilon)$ increases with lowering ϵ. Assume that $\phi \sim 1$ at certain $\tilde{\epsilon}$

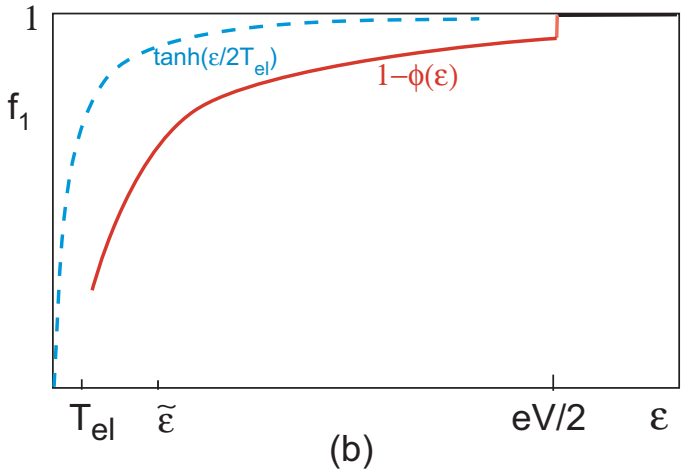
A. $\tilde{\epsilon} \ll T_e$

Thermal behavior at low energies matches with the non-equilibrium correction



B. $\tilde{\epsilon} \gg T_e$

Thermal behavior does not exist



Electron-phonon relaxation data

TABLE I. Characteristic quasiparticle times and associated parameters. All data are taken from Ref. 9 unless otherwise referenced.

Metal	T_c (K)	ω_D [a] (K)	μ [a] (K)	ω_D^2/μ (K)	$\hbar^2\omega_D^2/\mu k T_c$	$Z_1(0)$	$\alpha^2(2\Delta(0))F(2\Delta(0))$	$10^3 b$ (meV^{-2})	$10^9\tau_0$ (sec)
Pb	7.19	105	10.9	0.101	0.0141	2.55	0.154	5.72	0.196
In	3.40	108	10.0	0.117	0.0344	1.81	0.0110	9.43	0.799
Sn	3.75	200	11.6	0.344	0.0916	1.72	0.003 41	2.32	2.30
Hg	4.19	71.9	8.29	0.0624	0.0149	2.63	0.564	78.4	0.0747
Tl	2.33	78.5	9.46	0.0651	0.028	1.80	0.007 09	13.2	1.76
Ta	4.48	240	18.0	0.321	0.0715	1.69	0.003 60	1.73	1.78
Nb [b]	9.2	275	6.18	1.22	0.133	2.84	0.037	4.0	0.149
Al [c]	1.19	428	13.5	1.36	1.14	1.43	0.000 039 3	0.317	438.
Zn [d]	0.875	327	10.9	0.981	1.12	1.34	0.000 024 2	0.420	780.
$Pb_{60}Tl_{40}$	6.0					2.38	0.132	27.9	0.0647
$Pb_{40}Tl_{60}$	4.7					2.15	0.0744	28.7	0.118
$Pb_{60}Bi_{20}Tl_{20}$	7.26					2.81	0.661	21.2	0.0567
$Pb_{90}Bi_{10}$	7.55–8.05					2.66	0.564	21.4	0.043

Kaplan et al., PRB (1976)

In Al at ~1 K: $\gamma_{e-e} \sim 10^9$ s^{-1} *Shinozaki, Rinderer (1988)*

$\gamma_{e-e}/\gamma_{e-ph} > 10^2$

γ_{e-ph} overtakes γ_{e-e} at energies $\epsilon^*/k_B \gg 1$ K

In other materials ϵ^* can be considerably smaller.

Distribution function

Assumption: No spatial dependence in the island, bulk sample

Odd and even components

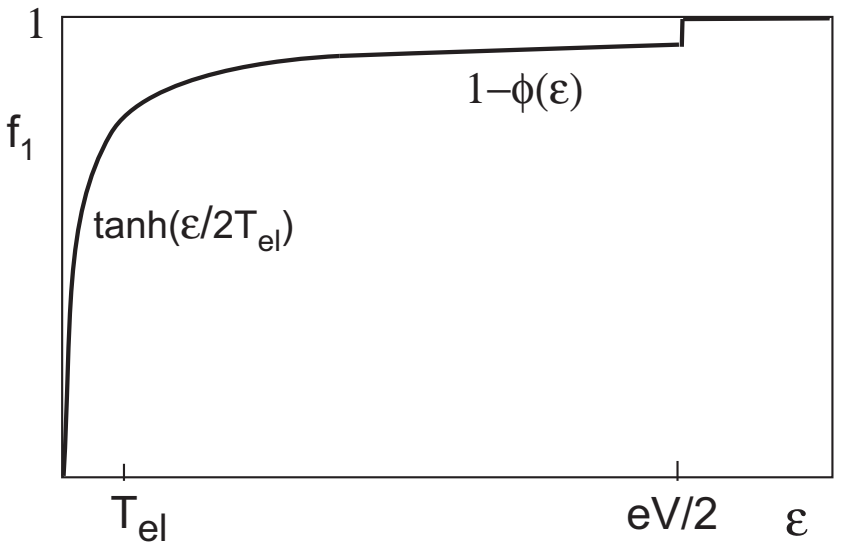
$$f_1(\epsilon) = n(-\epsilon) - n(\epsilon) \ , \ f_2 = 1 - n(\epsilon) - n(-\epsilon)$$

Even component is zero due to symmetry

Assume that $\tilde{\epsilon} \ll T_e$

$$f_1 = \tanh \frac{\epsilon}{2T_e} + \delta f \ , \ \delta f \ll 1$$

$$\delta f = -\phi(\epsilon)\Theta[(eV/2)^2 - \epsilon^2]$$



Kinetic equation for the odd component of the distribution function

$$J_1^{\mathsf{T}} + J_1^{\mathsf{e}} + J_1^{\mathsf{ph}} = 0$$

1. Tunnel “collision” integral

$$J_1^{\mathsf{T}} = -4\eta \left[f_1 - f_1^{(L)} \right] \approx -4\eta \left[1 - f_1^{(L)} \right]$$

Tunneling rate $\eta = \dfrac{1}{4\nu_i(E_F) e^2 \Omega_i R}$

In the leads

$$f_{1,2}^{(L)} = \frac{1}{2} \left[\tanh \frac{\epsilon - eV/2}{2T_L} \pm \tanh \frac{\epsilon + eV/2}{2T_L} \right]$$

Zero temperature phonon bath $T_L = T_{ph} = 0$

$$f_1^{(L)} = \Theta[\epsilon^2 - (eV/2)^2]\mathrm{sign}(\epsilon)$$

$$J_1^{\mathsf{T}} \approx -4\eta\Theta[(eV/2)^2 - \epsilon^2]\mathrm{sign}(\epsilon) \qquad \delta f = -\phi(\epsilon)\Theta[(eV/2)^2 - \epsilon^2]$$

2. Electron-phonon collision integral

Sound velocity $s \sim 5000$ m/s and $\ell \sim 20$ nm.

Clean limit

$$q\ell = \ell\epsilon/\hbar s \sim (0.5\ \mathrm{K}^{-1})\ \epsilon/k_B > 1$$

Therefore, at $\epsilon/k_B \sim eV/k_B \gg 1$ K the clean limit is appropriate.

If $T_{ph} = 0$, $J_1^{(ph)}$ in the island vanishes for

$$f_1(\epsilon) = \mathrm{sign}(\epsilon)$$

Linear part

For $|\epsilon|, |\epsilon + \omega| \gg T_e$ and $\epsilon > 0$, neglecting exponentially small terms,

$$J_1^{(ph)} = -\frac{\gamma_{ph}(T_e)}{T_e^3}\left[\frac{\epsilon^3 \delta f(\epsilon)}{3} - \int_0^\infty d\omega\, \omega^2 \delta f(\epsilon + \omega)\right]$$

Here

$$\gamma_{ph}(T_e) = \pi \lambda_{ph} T_e^3 / 2\hbar (s p_F)^2$$

is the electron-phonon relaxation rate at the electronic temperature T_e;
λ_{ph} is the interaction constant.

3. Electron-electron collision integral

Energy conservation

The electron-electron collision integral satisfies the energy conservation

$$\int_{-\infty}^{\infty} \epsilon J_1^{(e)}(\epsilon)\, d\epsilon = 0$$

Clean limit

Clean limit is appropriate if

$$\epsilon\tau/\hbar > (\hbar/p_F\ell)^2$$

Linear part

For large energies $|\epsilon|, |\epsilon_1|, |\epsilon_2|, |\epsilon_3| \gg T_e$ and $\epsilon > 0$ the collision integral is

$$J_1^{(e)} = -\frac{\gamma_e(T_e)}{T_e^2}\left[\frac{\epsilon^2 \delta f(\epsilon)}{2} - 3\int_0^{\infty} d\omega\, \omega\, \delta f(\epsilon+\omega)\right].$$

Here $\gamma_e(T_e) = \pi\lambda_e T_e^2/8\hbar E_F$ is the electron-electron relaxation rate at T_e,
λ_e is the interaction constant.

Full kinetic equation

$$\frac{1}{\epsilon_{ph}^3}\left[\frac{\epsilon^3}{3}\phi(\epsilon)-\int_0^{eV/2-\epsilon}d\omega\,\omega^2\phi(\epsilon+\omega)\right]$$
$$+\frac{1}{\epsilon_e^2}\left[\frac{\epsilon^2}{2}\phi(\epsilon)-3\int_0^{eV/2-\epsilon}d\omega\,\omega\,\phi(\epsilon+\omega)\right]=1$$

Here

$$\epsilon_{ph}^3=\frac{4\eta T_e^3}{\gamma_{ph}(T_e)}\ ,\ \epsilon_e^2=\frac{4\eta T_e^2}{\gamma_e(T_e)}$$

The characteristic energy scale

$$\epsilon^*=\epsilon_{ph}^3/\epsilon_e^2\sim T_e[\gamma_e(T_e)/\gamma_{ph}(T_e)]$$

such that $\gamma_{ph}(\epsilon^*)=\gamma_e(\epsilon^*)$.

$\epsilon>\epsilon^*$: electron-phonon interaction dominates

$\epsilon<\epsilon^*$: electron-electron interaction dominates.

Integral kinetic equation can be transformed into differential eq. by triple differentiation over $d\varepsilon$

Only e-e interaction, $\epsilon_{ph} \to \infty$

$$\frac{d^2}{d\epsilon^2}\left[\epsilon^2\phi_e(\epsilon)\right] = 6\phi_e(\epsilon)$$

Boundary conditions at $\epsilon = eV/2$:

$$\epsilon^2\phi_e(\epsilon) = 2\epsilon_e^2 \ , \ (d/d\epsilon)[\epsilon^2\phi_e(\epsilon)] = 0$$

Solution

$$\phi_e = \frac{4\epsilon_e^2}{5}\left[\frac{3}{2}\frac{(eV/2)^2}{\epsilon^4} + \frac{\epsilon}{(eV/2)^3}\right] .$$

Only e-ph interaction, $\epsilon_e \to \infty$

$$\frac{d^3}{d\epsilon^3}\left[\epsilon^3\phi_{ph}(\epsilon)\right] + 6\phi_{ph}(\epsilon) = 0$$

Boundary conditions

$$\epsilon^3\phi_{ph}(\epsilon) = 3\epsilon_{ph}^3 \ , \ (d/d\epsilon)[\epsilon^3\phi_{ph}(\epsilon)] = (d^2/d\epsilon^2)[\epsilon^3\phi_{ph}(\epsilon)] = 0$$

Solution

$$\phi_{ph}(\epsilon) = \frac{18}{11}\left(\epsilon_{ph}/\epsilon\right)^3 \mathcal{F}\left(eV/2\epsilon\right) ,$$

$$\mathcal{F}(x) \equiv x + \frac{1}{x^2}\left[\frac{5}{6}\cos(\sqrt{2}\ln x) + \frac{\sqrt{2}}{3}\sin(\sqrt{2}\ln x)\right] .$$

Volkov & Zaitsev (1975)

❑ Crossover from e-e to e-ph dominated relaxation at

$$eV \sim \epsilon^*$$

❑ The main term in both cases is $\phi(\epsilon) \sim (\tilde{\epsilon}/\epsilon)^4$

$$\tilde{\epsilon} = \begin{cases} (\epsilon_e eV)^{1/2}\,, & eV \ll \epsilon^* \\ (\epsilon_{ph}^3 eV)^{1/4}\,, & eV \gg \epsilon^* \end{cases}$$

Deviation from equilibrium $\phi \sim 1$ for $\epsilon < \tilde{\epsilon}$.

To match this correction to the thermal part at lower energies one needs $\tilde{\epsilon} \ll T_e$.

❑ The latter is the condition of quasi-equilibrium:

$$eV \ll \begin{cases} T_e^2/\epsilon_e\,, & eV \ll \epsilon^* \\ T_e^4/\epsilon_{ph}^3\,, & eV \gg \epsilon^* \end{cases}$$

Energy transfer to the phonon bath

Rate of the energy transfer

$$2\nu \int_0^{eV/2} \epsilon J_1^{(p)}\, d\epsilon + 2\nu \int_0^{eV/2} \epsilon J_1^{(T)}\, d\epsilon = 0$$

The electron-electron collision integral vanishes due to the energy conservation.

$$\overbrace{P_{eq}(T_e) + P_{neq}} = P_V$$

Determines T_e as a function of the injected power.

For $eV \gg \Delta$ and $\phi \ll 1$,

$$P_V = V^2/4R\Omega_i$$

Contribution from thermal electrons

$P_{eq}(T_e)$, is determined by the electron-phonon interaction in the low-energy domain $\epsilon \sim T_e$ with the thermal part of the distribution function, $\tanh(\epsilon/2T_e)$.

For a clean normal metal,

$$P_{eq}(T_e) = \Sigma T_e^5$$

where

$$\Sigma = \pi\nu\lambda\Gamma(5)\zeta(5)/(sp_F)^2\hbar$$

Wellstood, Urbina & Clarke, PRB (1994)

Contribution from non-equilibrium electrons

$$P_{neq} = \frac{8\nu\eta}{\epsilon_{ph}^3}\int_0^{eV/2}\epsilon\, d\epsilon\left[\frac{\epsilon^3\phi(\epsilon)}{3} - \int_0^{eV/2-\epsilon} d\omega\,\omega^2\phi(\epsilon+\omega)\right].$$

The main contribution comes from $\epsilon \sim eV$.

- Low voltages, $eV \ll \epsilon^*$

  ϕ_e can be used. The term ϵ^{-4} is cancelled out:

  $$P_{neq} = (eV/3\epsilon^*)P_V$$

  Therefore,

  $$P_{eq}(T_e) = P_V\left[1 - eV/3\epsilon^*\right] \quad \Rightarrow P_{eq} \approx P_V$$

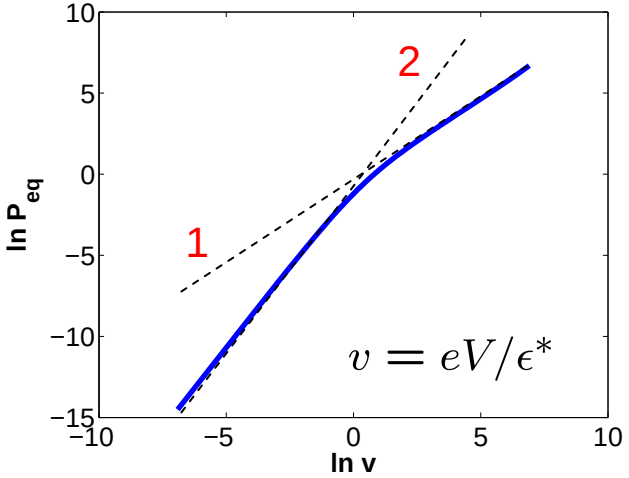
- High voltages, $eV \gg \epsilon^*$

  $$P_{eq}(T_e) = \frac{8\nu\eta}{\epsilon_{ph}^3}\int_0^{eV/2}\epsilon\, d\epsilon\left(\frac{\epsilon^3}{3}[\phi_{ph}(\epsilon) - \phi(\epsilon)] - \int_0^{eV/2-\epsilon}\omega^2 d\omega\,[\phi_{ph}(\epsilon+\omega) - \phi(\epsilon+\omega)]\right).$$

  For $\epsilon \sim eV \gg \epsilon^*$, $(\phi_{ph} - \phi)/\phi_{ph} \sim \epsilon^*/eV$.

  Therefore

  $$P_{eq}(T_e) \sim P_V(\epsilon^*/eV) = \nu\eta\epsilon^* eV \quad \Rightarrow P_{eq} \ll P_V$$



Conclusions

- Crossover at $eV \sim \epsilon^*$

$eV \ll \epsilon^*$	$eV \gg \epsilon^*$
E-e interaction dominates: Power is absorbed by thermal electrons	E-ph interaction dominates: Power is absorbed by high-energy electrons

- For Al samples

$$\gamma_e(T_c) \sim 10^8 \text{ s}^{-1} \ , \ \gamma_{ph}(T_c) \sim 10^6 \text{ s}^{-1}$$

In *Timofeev et al., PRL (2009)*, $\eta = 10 \text{ s}^{-1}$.

Therefore, $\epsilon^*/T_c \sim 10^2$. For $eV \sim 10^2\, T_c$,

$$eV < \epsilon^*$$

Quasi-equilibrium conditions are satisfied

❑ For other materials

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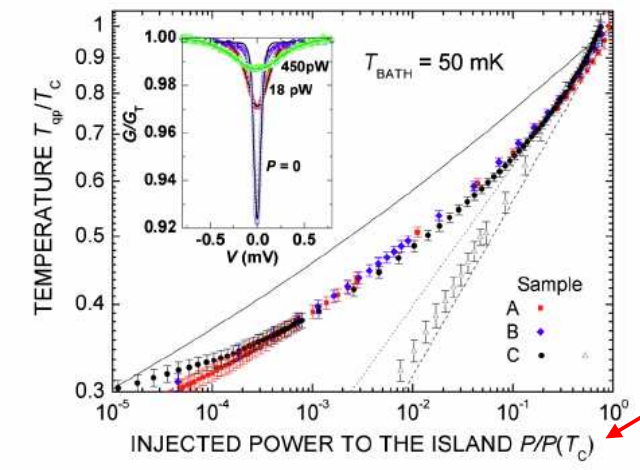
The crossover can thus be observed.



$$P_{eq}(T_e) \neq P_V$$

Injected power is NOT the power absorbed by thermal electrons !