Landau Days-09, Черноголовка

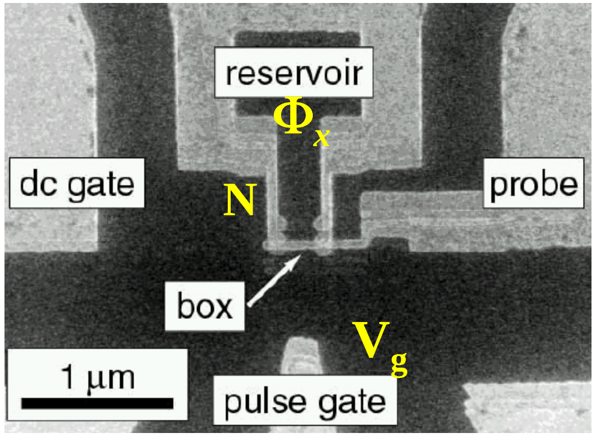
Period-doubling bifurcation readout for a Josephson qubit

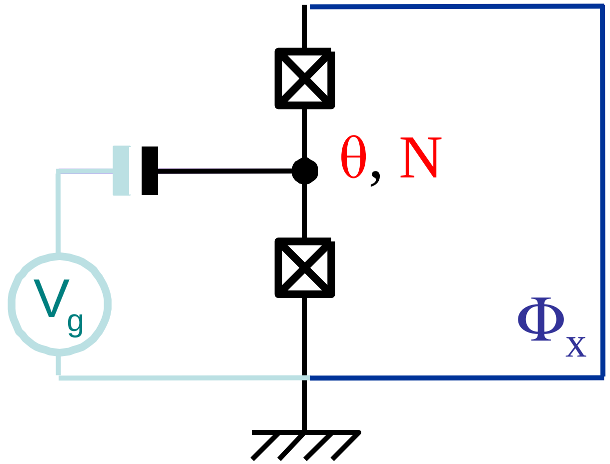
Alexander B. Zorin *(PTB, Braunschweig)*

Yuriy Makhlin *(Landau Institute)*

- quantum readout
- parametric driving and bifurcation
- stationary states, switching curve
- discussion

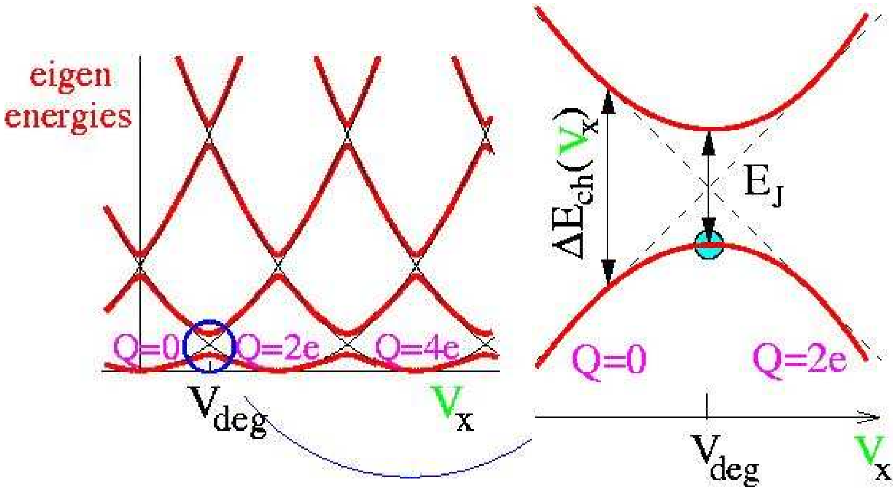
Josephson charge qubits





$$H = E_C\left(N - \frac{C_g V_g}{2e}\right)^2 - E_J \cos\left(\pi \frac{\Phi_x}{\Phi_0}\right) \cos\theta$$

tunable E_J



2 states only, e.g. for $E_C \gg E_J$

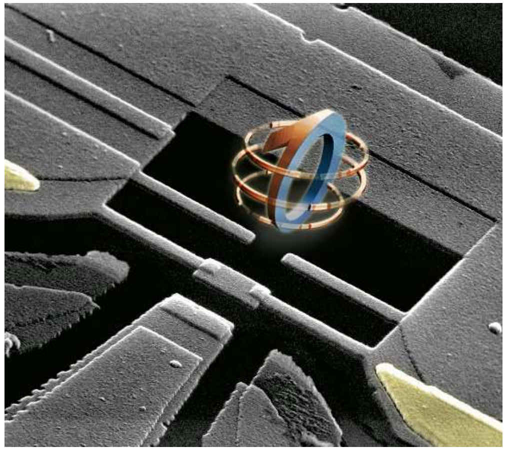
$$H = -\frac{1}{2} E_{ch}(V_g)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$

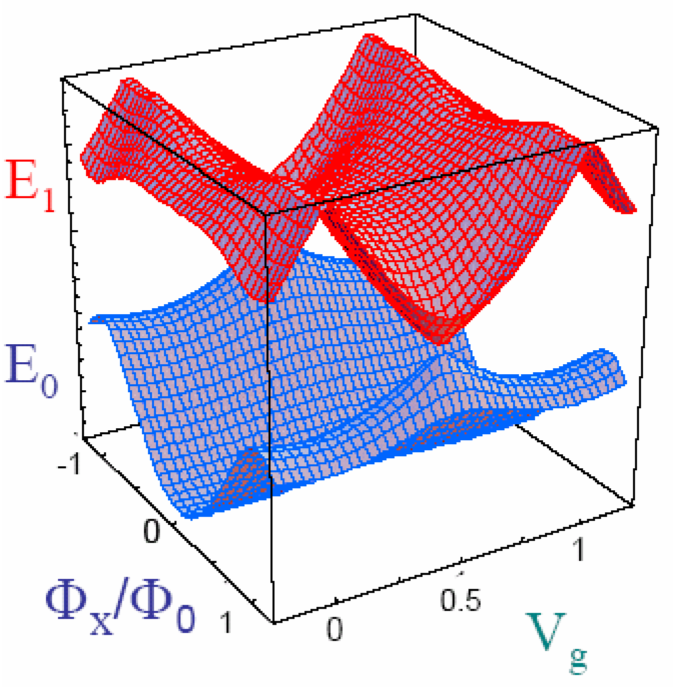
Charge-phase qubit

Vion et al. (Saclay)

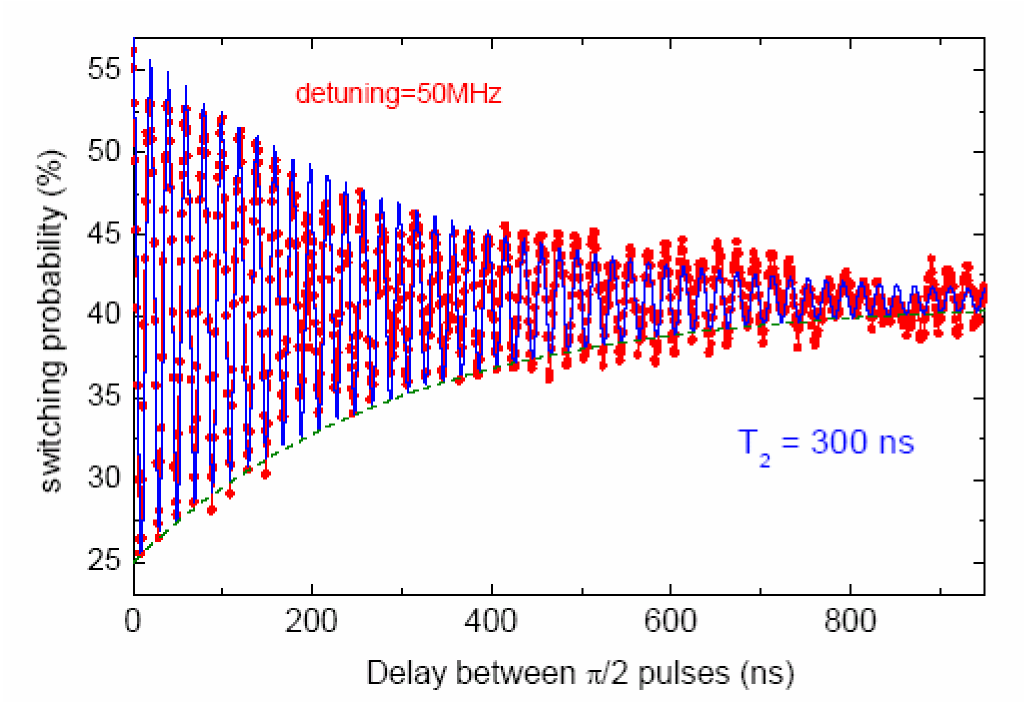
$$H = -\frac{1}{2} E_{ch}(V_g)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$

Quantronium

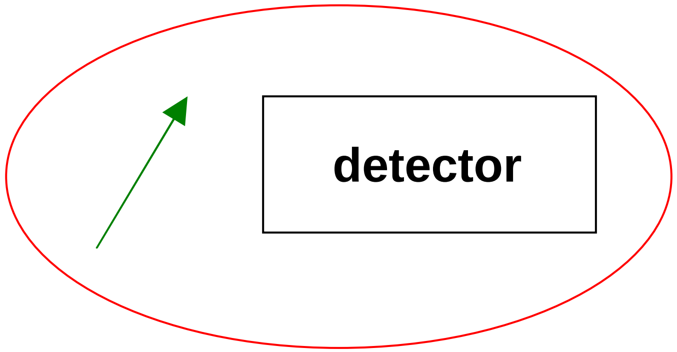




operation at a saddle point



Quantum measurement



$$a|0\rangle + b|1\rangle$$

quality of detection:

- reliable ?   single-shot?
- QND? back-action

- fast?

result	0	1
state	|0>	|1>
probability	$\|a\|^2$	$\|b\|^2$

Measurement as entanglement

unitary evolution of qubit + detector

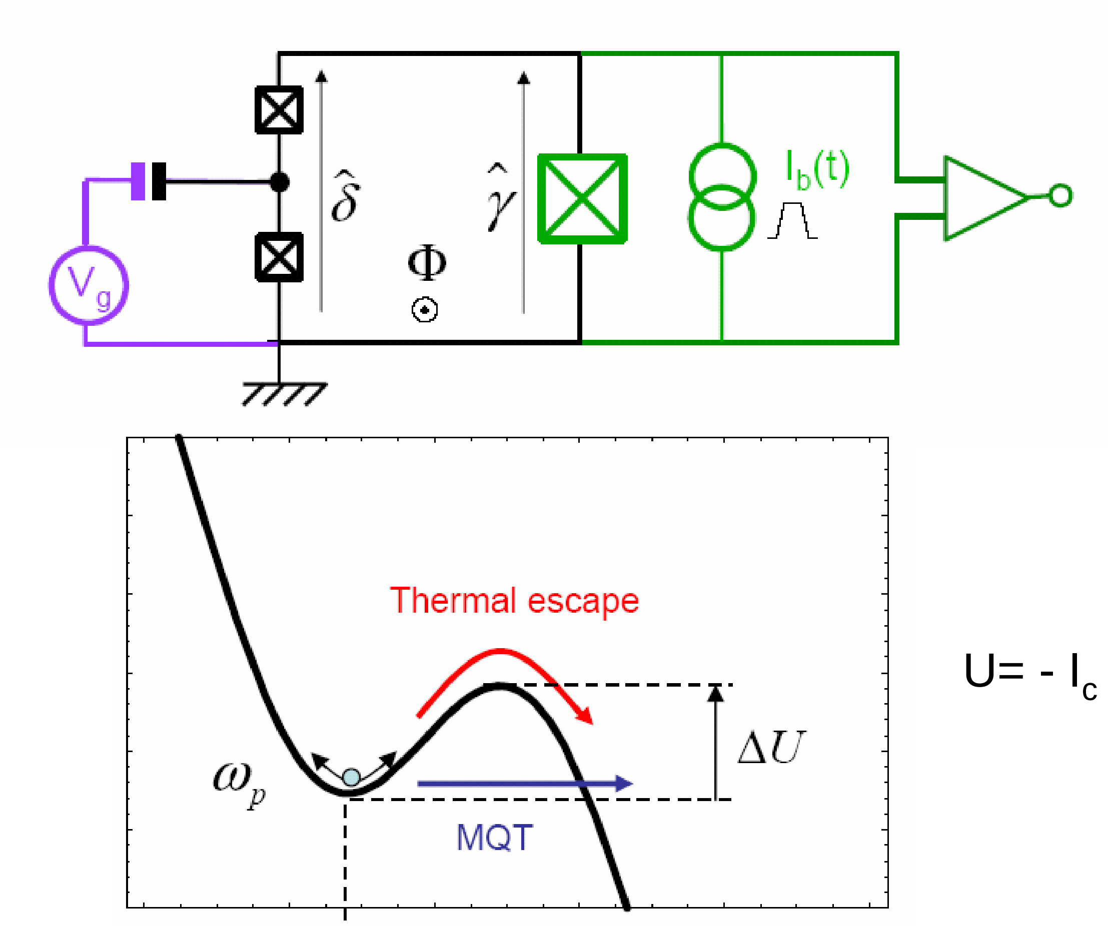
$$(a|0\rangle + b|1\rangle) \otimes |M\rangle$$

$$\Downarrow$$

$$a|0\rangle \otimes |M_0\rangle + b|1\rangle \otimes |M_1\rangle$$

$|M_i$i --- macroscopically distinct states

Quantum readout for Josephson qubits



$U = -I_c \cos\varphi - I\,\varphi$

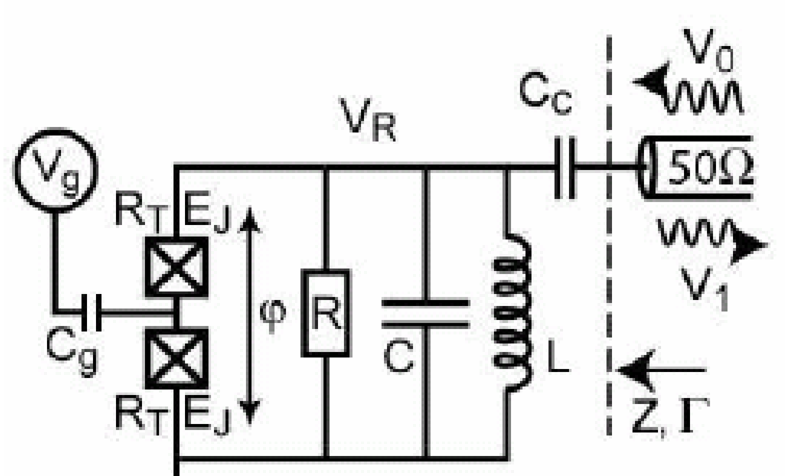
<u>switching readout</u> – monitor the critical current

- no signal at optimal point
- strong back-action by voltage pulse (no QND)
- threshold readout

Quantronics
Ithier, 2005

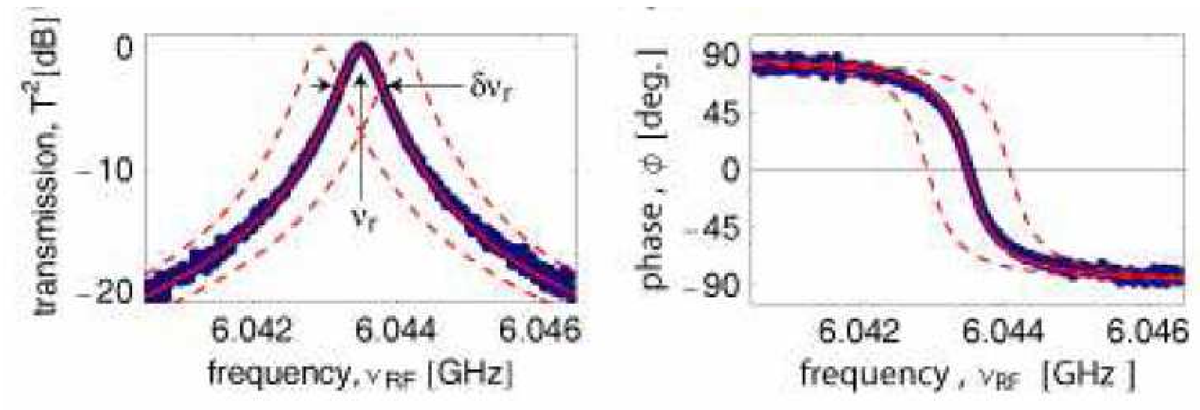
Quantum readout for Josephson qubits

<u>dispersive readout</u> – monitor the eigen-frequency of LC-oscillator



Sillanpää et al. 2005

monitor reflection / transmission amplitude / phase



Wallraff et al. 2004

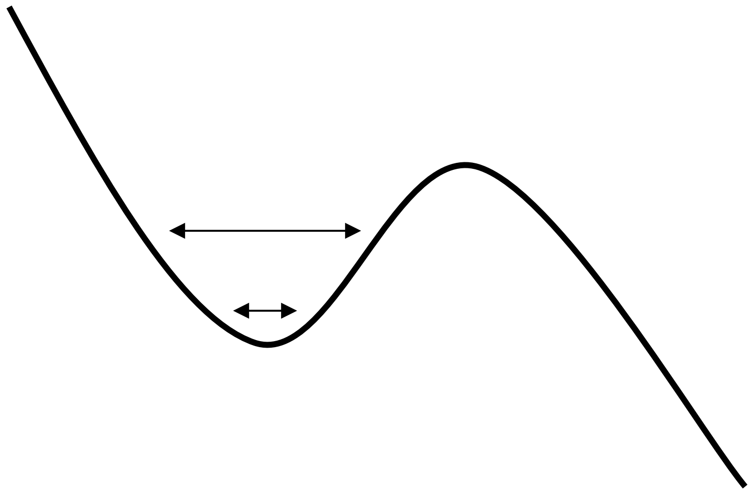
Quantum readout for Josephson qubits

Josephson bifurcation amplifier (Siddiqi et al.) – dynamical switching

exploits nonlinearity of JJ

switching between two oscillating states
(different amplitudes and phases)

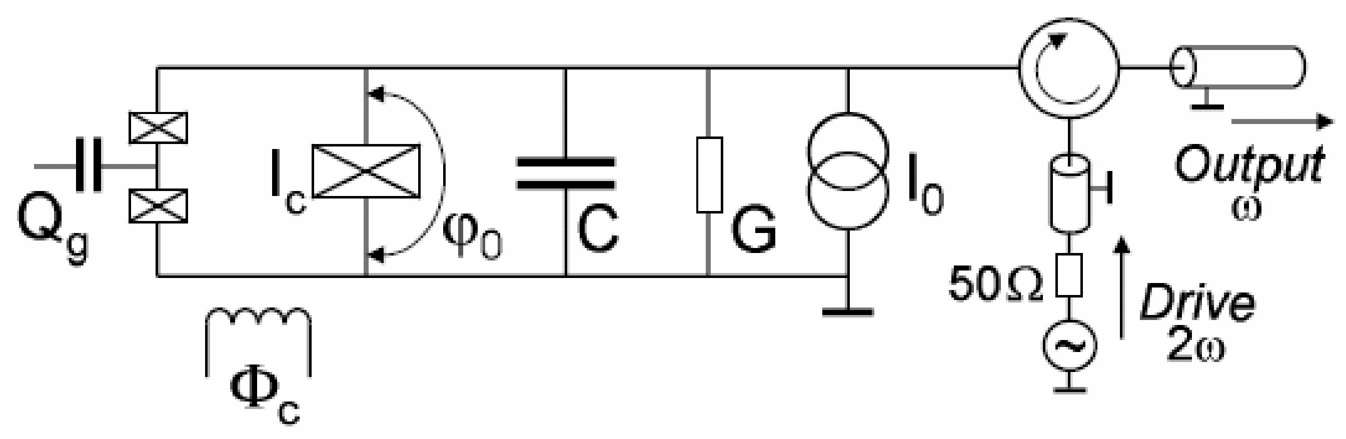
advantages:
- no dc voltage generated, close to QND
- higher repetition rate
- qubit always close to optimal point



Siddiqi et al. 2003

cf. Ithier, thesis 2005

Parametric bifuraction for qubit readout



$$\frac{\hbar C}{2e}\frac{d^2\varphi}{dt^2} + \frac{\hbar G}{2e}\frac{d\varphi}{dt} + I_c \sin\varphi = I_0 + I_{ac}$$

$I_0 = I_c \sin \varphi_0$

$\varphi = \varphi_0 + x$

$\tau = \omega t$

$$\ddot{x} + x = \xi x - 2\theta\dot{x} + \beta x^2 + \gamma x^3 - \mu x^4 + 3P \cos 2\tau.$$

$$\xi = 1 - \kappa \ll 1, \qquad \theta = G/2\omega C \equiv 1/2Q \ll 1,$$

$$\kappa = (\omega_p/\omega)^2$$

$$\beta = 12\mu = (\kappa \tan\varphi_0)/2, \quad \gamma = \kappa/6, \quad 3P = \kappa I_A/I_c$$

Landau, Lifshitz, Mechanics
diff. driving: Dykman et al. ‘98

Method of slowly-varying amplitudes

$$x \equiv y - P\cos 2\tau$$

$$y = u\cos\tau + v\sin\tau, \qquad \dot{y} = -u\sin\tau + v\cos\tau.$$

$$\dot{u} = -\theta u - \frac{1}{2}(\beta P + \xi + \frac{3}{4}\gamma A^2 + \frac{3}{2}\gamma P^2)v - \mu(Pv^3 + \frac{3}{4}P^3 v).$$

$$\dot{v} = -\theta v - \frac{1}{2}(\beta P - \xi - \frac{3}{4}\gamma A^2 - \frac{3}{2}\gamma P^2)u - \mu(Pu^3 + \frac{3}{4}P^3 u).$$

$A^2 = u^2 + v^2$

Migulin et al., 1978

Equations of motion

$$\dot{u} = -\frac{\partial H}{\partial v} - \theta u + \xi_u(t)\,,$$
$$\dot{v} = \frac{\partial H}{\partial u} - \theta v + \xi_v(t)\,.$$

Hamiltonian

$$H = \frac{\tilde{\xi}}{4}A^2 - \frac{\tilde{\beta}P}{4}A^2\cos 2\varphi + \frac{3}{32}\gamma A^4 - \frac{\mu P}{4}A^4\cos 2\varphi\,,$$

where $\tilde{\beta} = \beta + \frac{3}{2}\mu P^2$, $\tilde{\xi} = \xi + \frac{3}{2}\gamma P^2$, $A^2 = u^2 + v^2$, $u = A\cos\varphi$, $v = A\sin\varphi$

Equations of motion in polar coordinates:

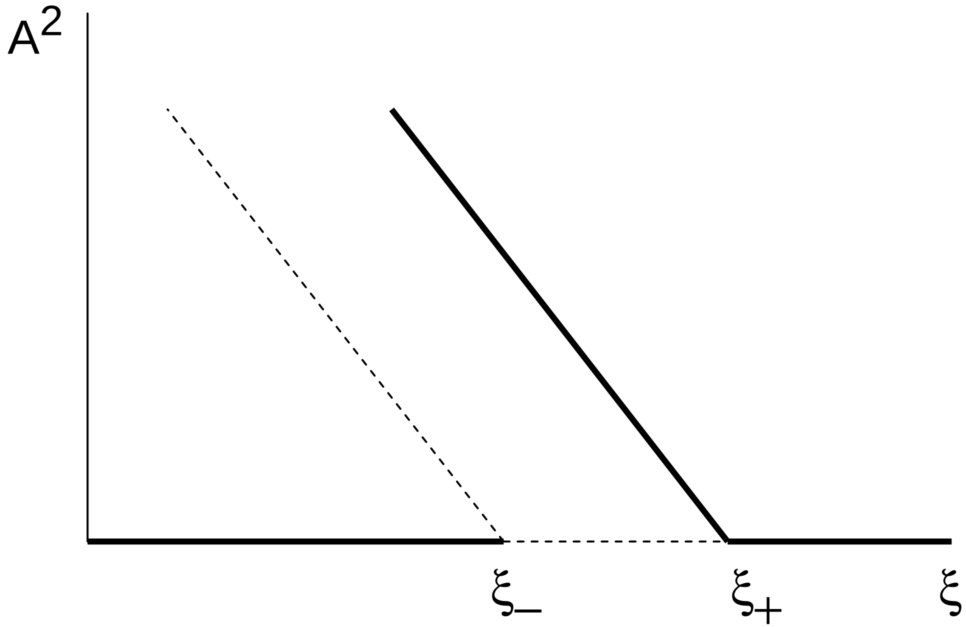
$$\frac{d}{dt}(A^2) = -PA^2\sin 2\varphi(\tilde{\beta} + \mu A^2) - 2\theta A^2\,,$$
$$\dot{\varphi} = -\frac{\tilde{\beta}P}{2}\cos 2\varphi - \mu P A^2\cos 2\varphi + \frac{\tilde{\xi}}{2} + \frac{3}{8}\gamma A^2\,.$$

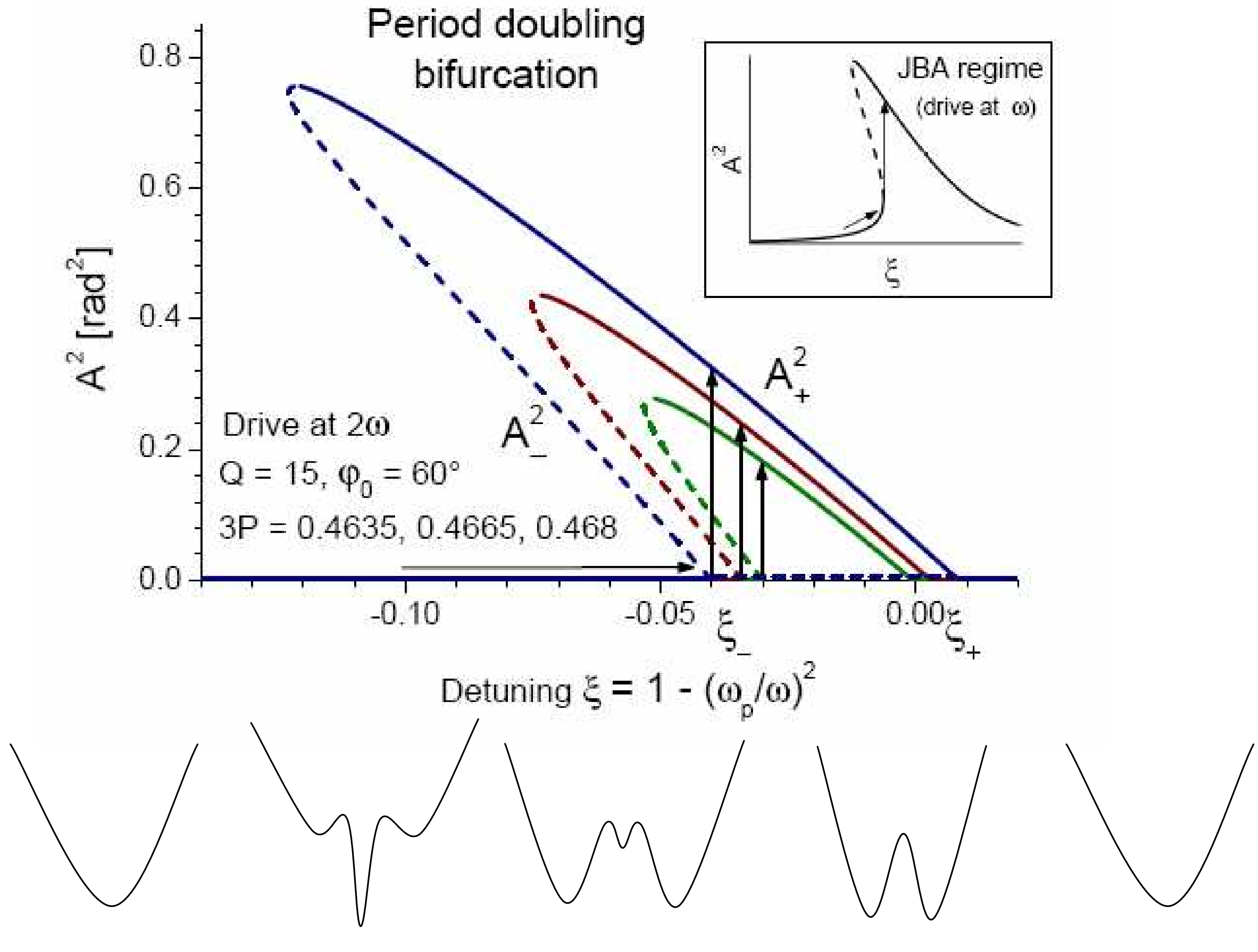
without quartic term

μ=0

stationary solutions:

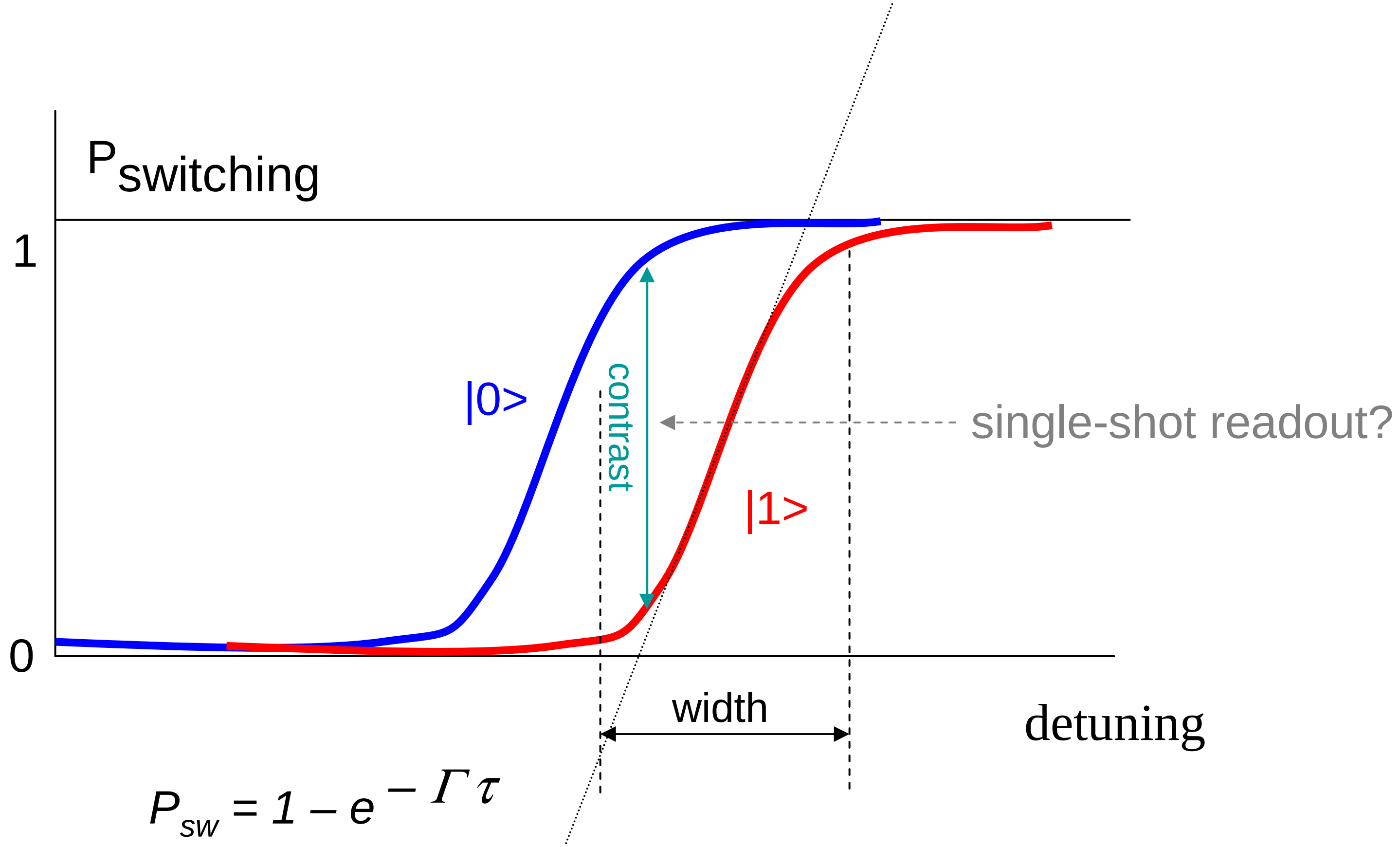
A=0 or $$A_{\pm}^2 = \frac{4}{3\gamma}\left(-\xi - \frac{3}{2}\gamma P^2 \pm \sqrt{\beta^2 P^2 - 4\theta^2}\right)$$





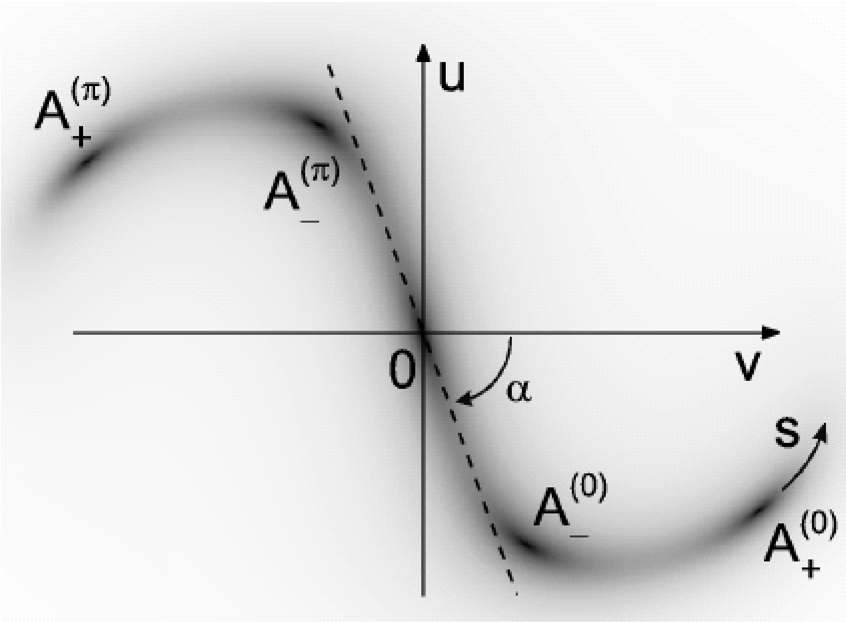
Period doubling bifurcation
JBA regime
(drive at ω)
A²
ξ
0.8
0.6
0.4
0.2
0.0
A² [rad²]
A₊²
A₋²
Drive at 2ω
Q = 15, φ₀ = 60°
3P = 0.4635, 0.4665, 0.468
-0.10
-0.05
0.00
ξ₋
ξ₊
Detuning ξ = 1 - (ωp/ω)²

Switching curves



$$P_{sw} = 1 - e^{-\Gamma \tau}$$

Velocity profile and stationary oscillating states



0, A_+ – stable states

A_- – unstable state

near origin: φ relaxes fast, (at rate θ)

A – slow degree of freedom

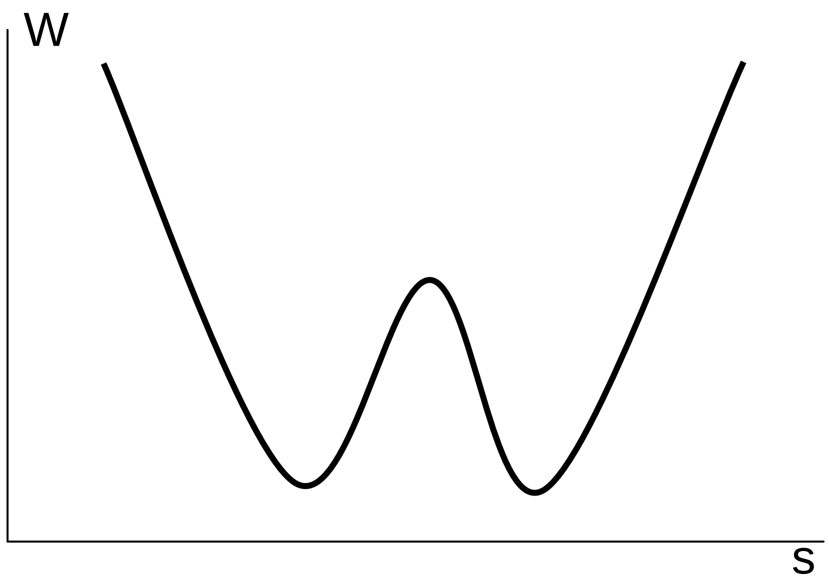
2D Focker-Planck eq. => 1D FPE

Near bifurcation $(\xi \approx \xi_{-})$

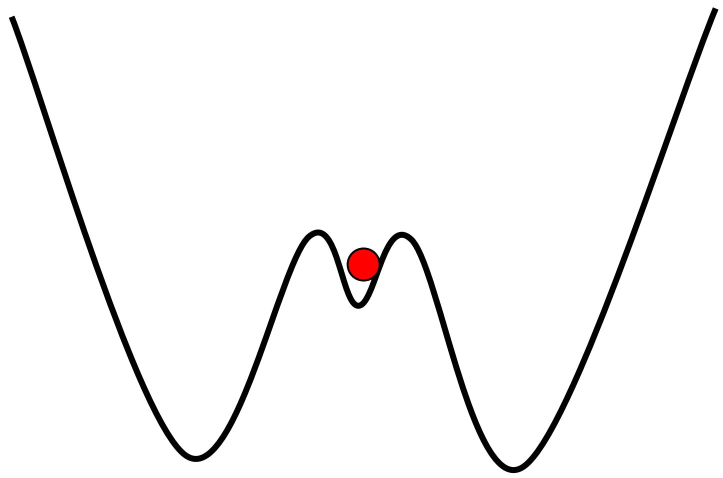
$$\cos 2\alpha = (\xi + {}^{3}/_{2}\gamma P^{2})/P(\beta + {}^{3}/_{2}\mu P^{2})$$

$$ds/dt = -\,dW(s)/ds + \xi(s)$$

W(s) ≠ U(s) !

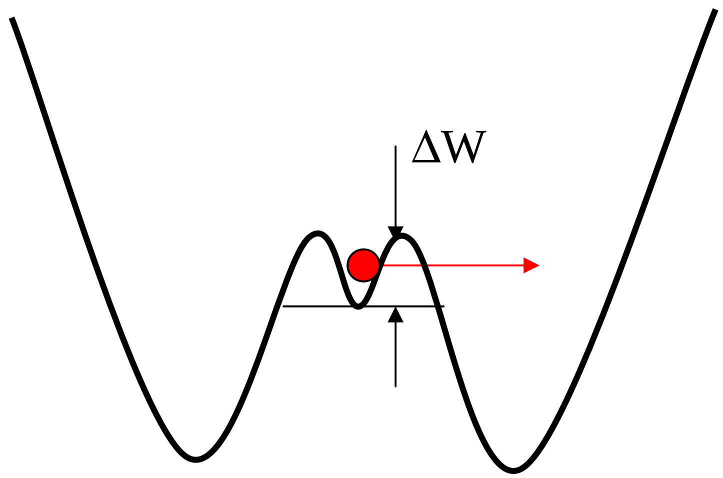


=>



Tunneling, switching curves

Focker-Planck equation for P(s)



$\Gamma \sim \exp(-\Delta W/T_{\mathrm{eff}})$

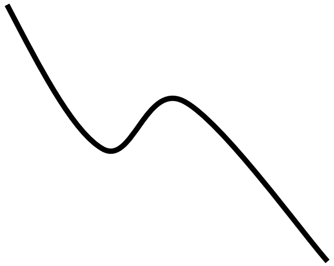
$W = a\,s^2 - b\,s^4$

$$\Delta W = \frac{1}{12\gamma\theta}\sqrt{(\tilde{\beta}P)^2 - 4\theta^2}\;(\xi - \xi_-)^2$$

$$T_{\mathrm{eff}} = GT\omega\left(\frac{\omega_p^2}{\omega^2 I_c \cos\varphi_0}\right)^2$$

width of switching curve $\xi - \xi_- \sim \sqrt{T}$

For a generic bifurcation (incl. JBA)



$$U(x) = \varepsilon x - a x^3$$

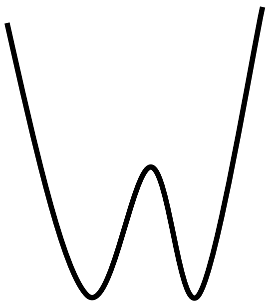
$\Delta U \sim \varepsilon^{3/2}$ controls $\Gamma \propto \exp(-\Delta U / T)$

width of switching curve $\delta\varepsilon \propto T^{2/3}$

For a parametric bifurcation - additional symmetry

at origin

u,v -> -u,-v shift by period of the drive

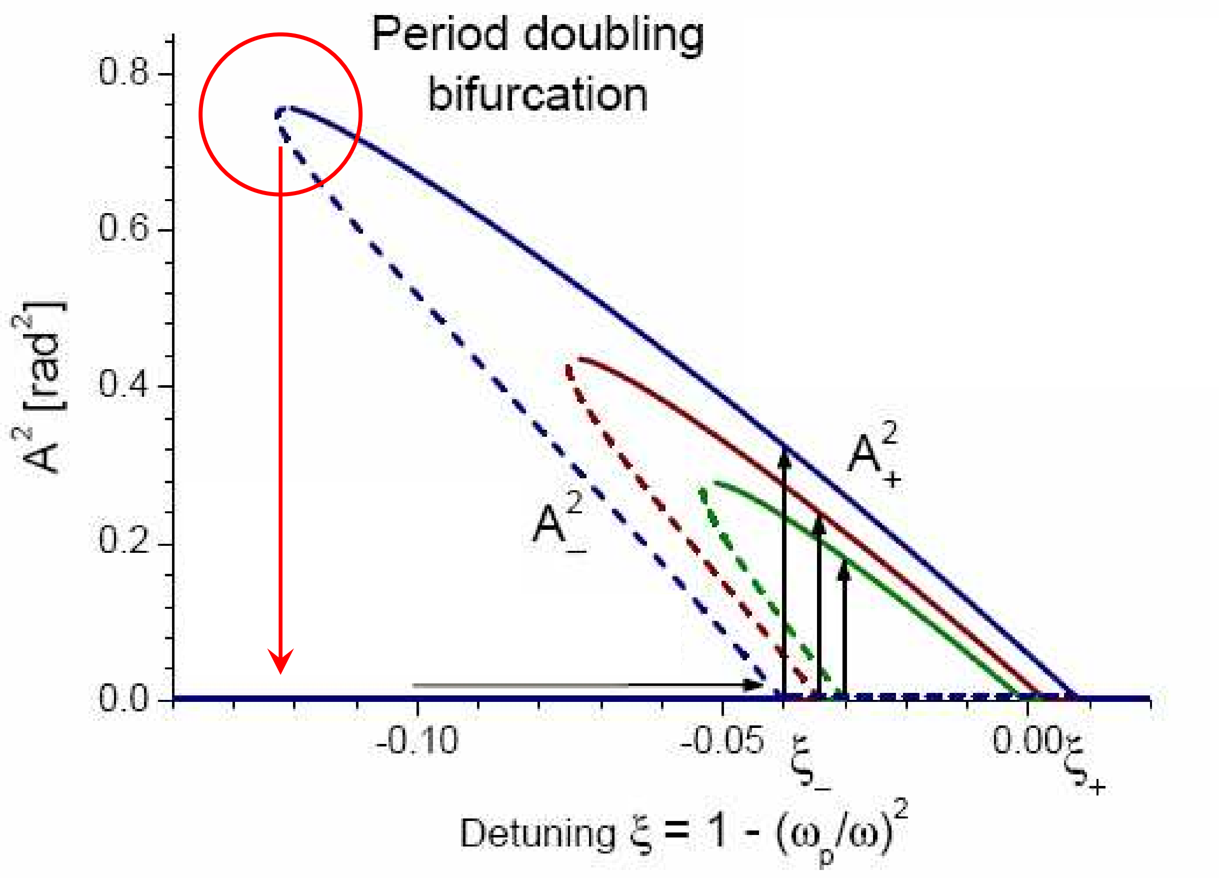


$$U = \varepsilon x^2 - a x^4$$

should be even !

$\Delta U \propto \varepsilon^2$ => $\delta\varepsilon \propto \sqrt{T}$.

Another operation mode



no mirror symmetry => generic case, stronger effect of cooling

Conclusions

Period-doubling bifurcation readout:

- towards quantum-limited detection?
- low back-action
- rich stability diagram
- various regimes of operation
- tuning amplitude or frequency

results

- bifurcations, tunnel rates,
- switching curves,
- stationary states for various parameters,
- temperature and driving dependence of the response with double period