Charge Relaxation Resistivity
In The Coulomb Blockade Problem

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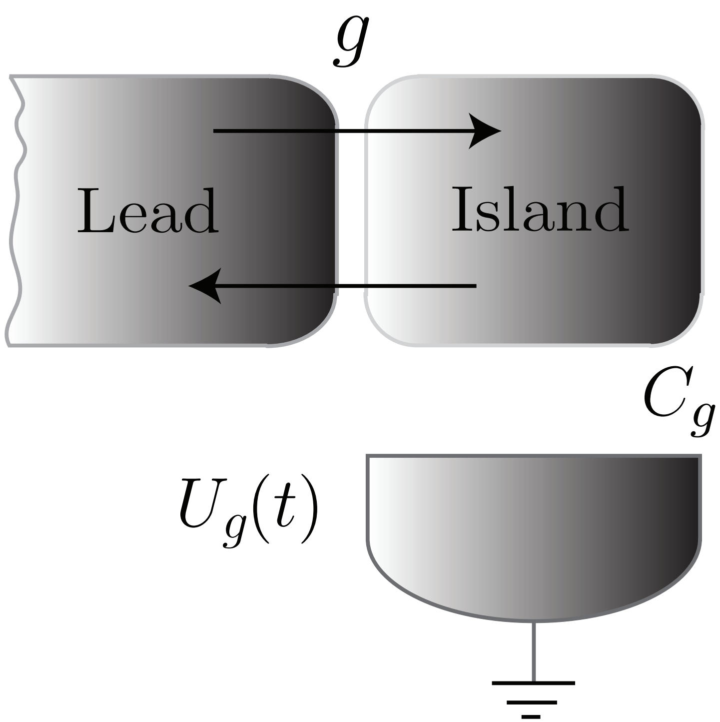
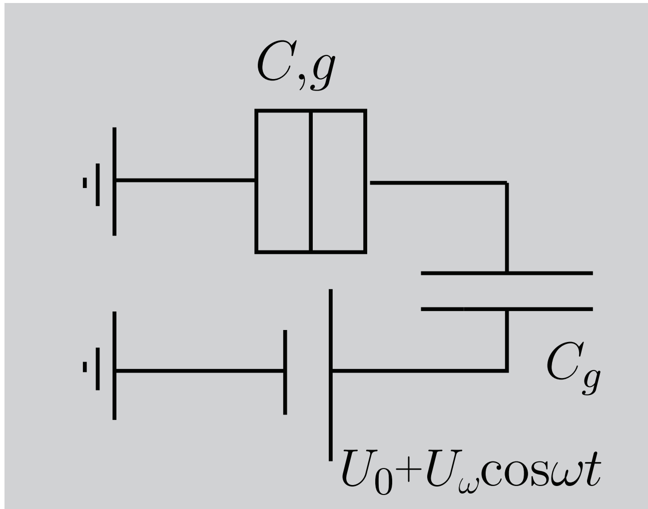
Landau ITP

The Set-Up





AC-gate voltage

$$U_g(t) = U_0 + U_\omega \cos \omega t$$

results in AC tunneling current $I(t)$.

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SEB admittance

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SEB admittance

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Energy dissipation

$$\mathcal{W}_\omega = \frac{C_g}{2C} \text{Re}\,\mathcal{G}(\omega)|U_\omega|^2, \qquad U_\omega \to 0$$

Parameters and scales

- Effective number of transport channels N_{ch}
- Effective channel conductance $g_{ch} \ll 1$
- Tunneling conductance $g = g_{ch} N_{ch}$
- Charging energy $E_c = e^2/2C$
- Level spacing on the island δ
- External charge $q = C_g U_0$
- Temperature $\max\{\delta, g\delta\} \ll T \ll E_c$

Throughout all our calculations: $T = 0 \Leftrightarrow T \leq \delta$

Motivation

- Experiments on admittance in a single electron box
  - Low temperatures $T \leqslant \delta$, quantum dot in 2DEG

    Gabelli, Fève, Berroir, Plaçais, Cavanna, Etienne, Jin, Glattli,

    Science 313, 499 (2006)
  - Intermediate temperatures $\delta \leqslant T < E_c$, metallic island Persson, Wilson, Sandberg,

    Johansson, Delsing, arXiv:0902.4316

- Theory on admittance in single electron box
  - Low temperatures $T \leqslant \delta$

    Büttiker, Thomas, Pretre, Phys. Lett. A180, 364 (1993)
  - High temperatures $T \geq \delta$

    Büttiker, Nigg, Phys. Rev. B77, 085312 (2008)

The problem

High temperatures $T \gg \max\{gE_c, E_c\}$ SEB admittance (classical)

$$\mathcal{G}(\omega) = -i\omega C_g + C_g C R \omega^2 + \mathcal{O}(\omega^3), \qquad R = \frac{h}{e^2}\frac{1}{g}$$

Energy dissipation rate (classical)

$$\mathcal{W}_\omega = \omega^2 C_g^2 R |U_\omega|^2$$

Low temperatures $T \leq \delta$

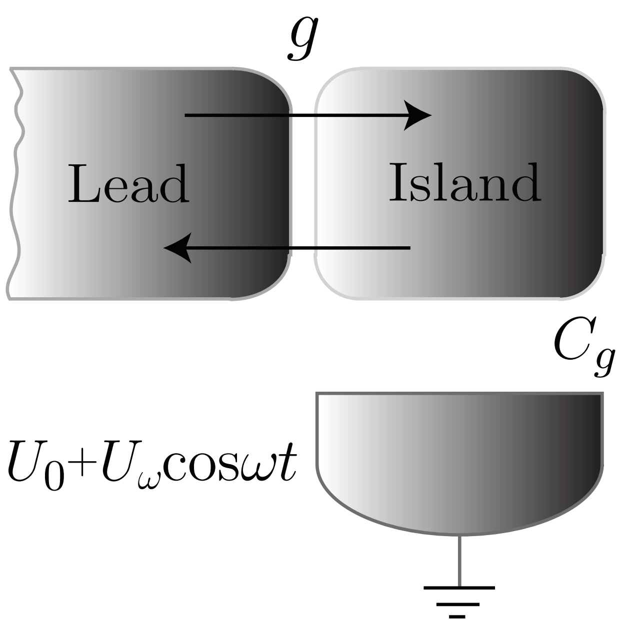
$$\mathcal{G}(\omega) = -i\omega C_\mu + C_\mu C R_q \omega^2 + \mathcal{O}(\omega^3),$$

$$\mathcal{W}_\omega = \omega^2 C_\mu^2 R_q |U_\omega|^2$$

C_μ - *mesoscopic capacitance*, R_q *charge relaxation resistance*. M. Buttiker, et. al. 1993

What is the SEB admittance, R_q, and C_μ
at intermediate temperatures $\max\{\delta, g\delta\} \ll T \ll E_c$?

Hamiltonian



$$H = H_0 + H_c + H_t$$

Coulomb interaction

$$H_0 = \sum_k \varepsilon_k^{(a)} a_k^\dagger a_k + \sum_\alpha \varepsilon_\alpha^{(d)} d_\alpha^\dagger d_\alpha,$$

$$H_t = \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha + \text{h.c.}$$

$$H_c = E_c \left(\hat{n}_d - q\right)^2, \qquad \hat{n}_d = \sum_\alpha d_\alpha^\dagger d_\alpha, \qquad E_c = \frac{e^2}{2C}, \qquad q = \frac{C_g U_g}{e}$$

Hierarchy of scales

$$\max\{g\delta, \delta\} \ll T \ll E_c \ll E_{th},$$

Dissipation and admittance

$$H = \sum_{\boldsymbol{k}} \varepsilon_k^{(a)} a_k^\dagger a_k + \sum_{\alpha} \varepsilon_\alpha^{(d)} d_\alpha^\dagger d_\alpha + \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_{\boldsymbol{\alpha}} + \mathbf{h}.\mathrm{c}. + E_{\boldsymbol{c}}\big(\hat{n}_{\boldsymbol{d}} - q(t)\big)^2,$$

$$\hat{n}_{\boldsymbol{d}} = \sum d_{\boldsymbol{\alpha}}^\dagger d_{\boldsymbol{\alpha}}, \qquad q(t) = \frac{C_g U_g(t)}{e}, \qquad U_g(t) = U_0 + U_\omega \cos\omega t.$$

$$W(\omega) = -\frac{C_g^2}{C^2}\omega |U_\omega|^2 \mathrm{Im}\Pi_R(\omega),$$ **Dissipation**

$$\Pi_R(t) = i\Theta(t)\langle[\hat{n}_d(t), \hat{n}_d(0)]\rangle,$$ **Polarization operator**

$$\mathcal{G}(\omega) = -i\omega C_g\left(1 + \frac{\Pi^R(\omega)}{C}\right)$$ **Admittance**

SEB energy dissipation rate at $\max\{\delta, g\delta\} \ll T \ll E_c$

$$\mathcal{W}_\omega = \omega^2 \mathcal{A}(T) |U_\omega|^2, \qquad \omega \to 0$$

Weak coupling regime	Strong coupling regime
$g \gg 1$	$g \ll 1$, $\lvert q - k - 1/2 \rvert \ll 1$
$\mathcal{A} = \frac{h}{e^2} \frac{C_g^2}{g(T)} \left(1 - D g^2(T) e^{-\frac{g(T)}{2}} \cos 2\pi q\right)$	$\mathcal{A} = \frac{h}{e^2} \frac{C_g^2 E_c^2}{2g\Delta T} \frac{\sinh \frac{\widehat{\Delta}}{T}}{\cosh^4 \frac{\widehat{\Delta}}{2T}}$
where $g(T) = g - 2\ln \frac{gE_c}{6DT}$	where $\Delta = E_c(2k + 1 - 2q)$, $\widehat{\Delta} = \frac{\Delta}{1 + g\lambda}$ $\lambda = \frac{1}{2\pi^2} \ln \frac{E_c}{\max\{T, \lvert\widehat{\Delta}\rvert\}}$
weak Coulomb oscillations	*developed Coulomb blockade*

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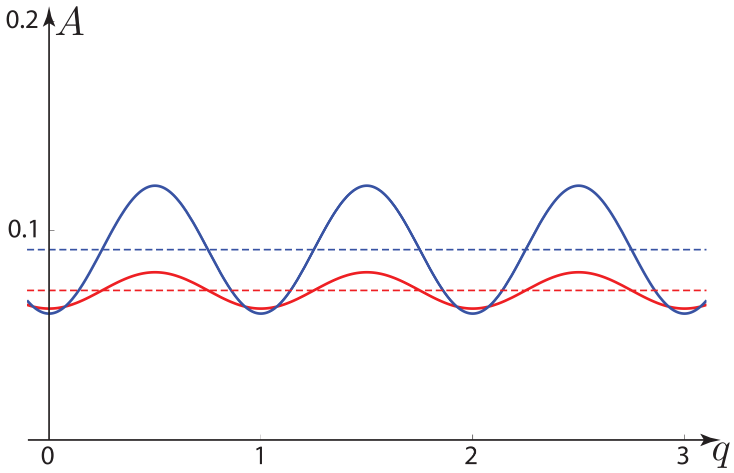	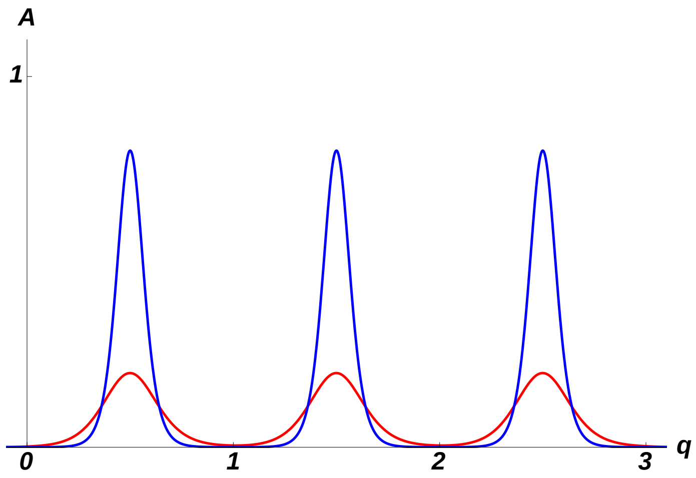
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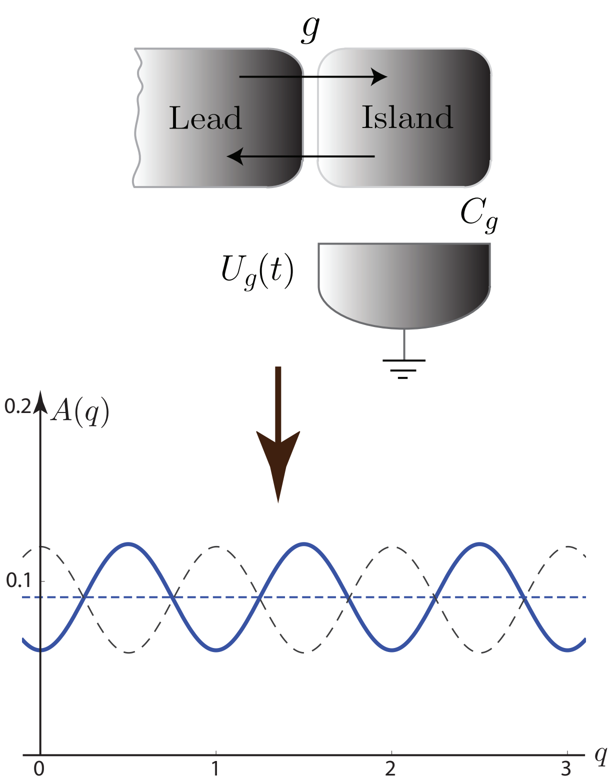
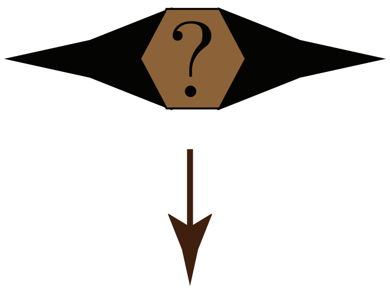
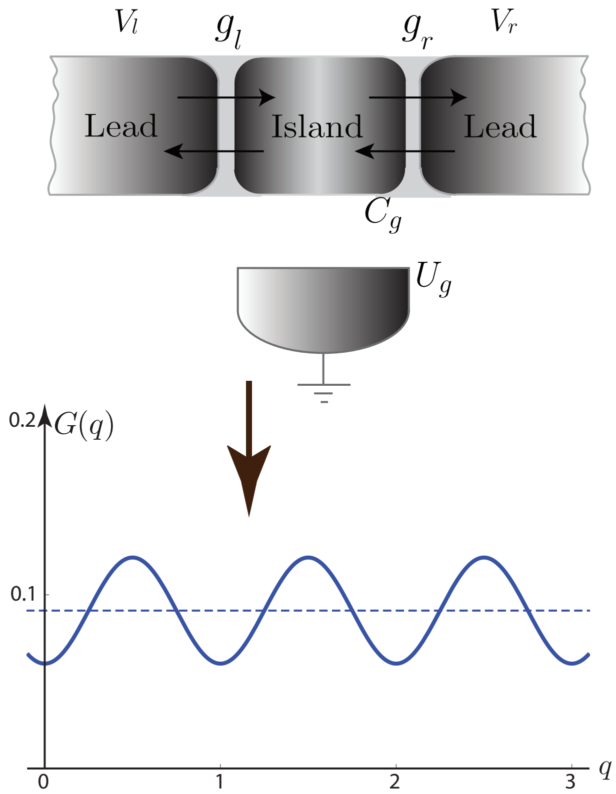
At $g \gg 1$ one expects classical-type resistance.

$$A = C_g^2 R, \quad R = 1/g$$

On the other hand
the conductance of a two terminal set-up :







Renormalization of C_g

Main conclusion

Dissipation and admittance are universally factorized in both $g \ll 1, g \gg 1$ limits

$$\mathcal{G}(\omega) = -i\omega C_{\rm eff}(T) + \frac{C}{C_g} C_g^2(T) R_q(T)\omega^2 + \mathcal{O}(\omega^3)$$

Energy dissipation rate

$$\mathcal{W}_\omega = \omega^2 C_g^2(T) R_q(T) |U_\omega|^2$$

Correlators $C_g(T), C_{\rm eff}(T), R_q(T)$ are defined for $0 < g < \infty$

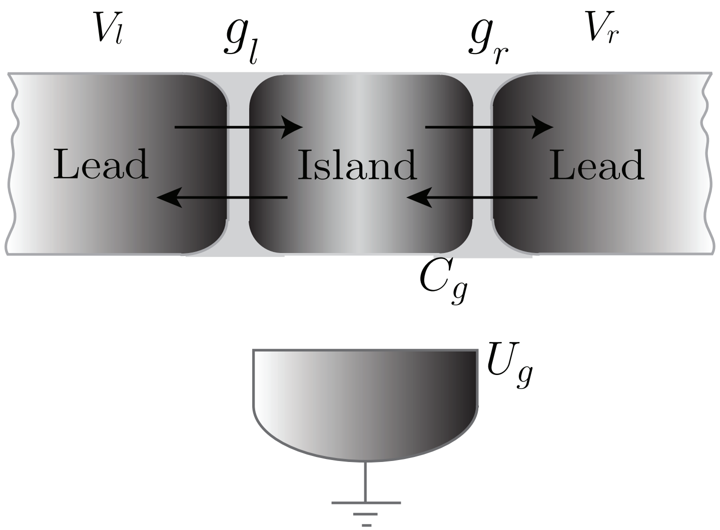
The SEB parameters become temperature dependent.

If $C_{\rm eff}(T) \to C_g$, $C_g(T) \to C_g$ and $R_q(T) \to R$ then it coincides with classical result

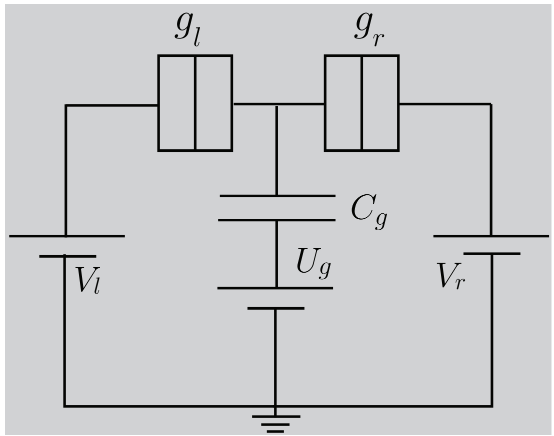
What is the physical meaning of $C_{\rm eff}(T)$, $C_g(T)$ and $R_q(T)$?

SET instead of SEB

Single electron transistor (SET)



Equivalent circuit for SET



If $V_l = V_r = 0$ SET is equivalent to SEB with $g = g_r + g_r$

Charge relaxation resistance

SEB/SET	SET
Charge relaxation resistance	Conductance
$R_q(T) = \frac{h}{e^2}\frac{1}{g'(T)}$	$G(T) = \frac{e^2}{h}\frac{g_l g_r}{(g_l+g_r)^2} g'(T)$

Weak coupling regime	Strong coupling regime
$g \gg 1$	$g \ll 1, \lvert q-k-1/2\rvert \ll 1$
$g'(T) = g(T) - Dg^2(T)e^{-g(T)/2}\cos 2\pi q$ where $g(T) = g - 2\ln\frac{gE_c}{6DT}$ and $D = (\pi^2/3)\exp(-\gamma-1)$	$g'(T) = \frac{\hat{g}}{2}\frac{\hat{\Delta}}{T\sinh\frac{\hat{\Delta}}{T}}$ where $\Delta = E_c(2k+1-2q)$, $\hat{\Delta} = \frac{\Delta}{1+g\lambda}$ $\lambda = \frac{1}{2\pi^2}\ln\frac{E_c}{\max\{T,\lvert\hat{\Delta}\rvert\}}$, and $\hat{g} = \frac{g}{1+g\lambda}$
Altland, Glazman, Kamenev, Meyer, Ann. of Phys. (N.Y) 321, 2566 Guinea, Schön, EPL 1, 585 (1986); Bulgadaev, JETP Lett. 45, 622 (1987)	Schoeller, Schön, Phys. Rev. B50, 18436 (1994)

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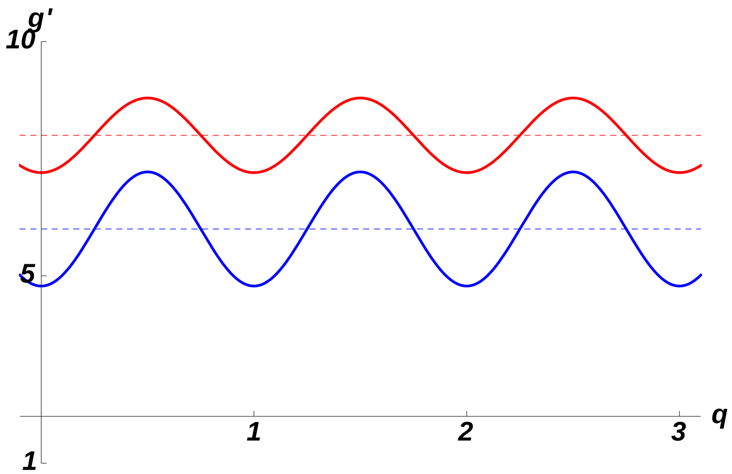	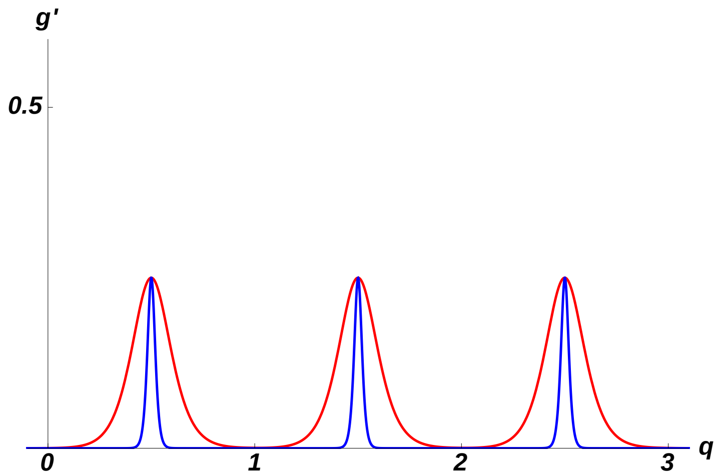
weak Coulomb oscillations	*developed Coulomb blockade*

Renormalized gate capacitance

$$C_g(T) = \frac{\partial q'(T)}{\partial U_0} \equiv C_g \frac{\partial q'(T)}{\partial q}$$

For SET

$$q' = Q + \frac{(g_l + g_r)^2}{2\pi g_l g_r} p.v. \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \frac{\partial S_I(\omega, V_{dc})}{\partial V_{dc}} \Bigg|_{V_{dc}=0}$$

Burmistrov, Pruisken, Phys. Rev. Lett. 101, 056801 (2008)

where non-symmetrized current noise

$$S_I(\omega, V_{dc}) = \int_0^{\infty} dt\, e^{-i\omega t} \langle \hat{I}(t) \hat{I}(0) \rangle, \qquad \hat{I}(t) = \frac{d\hat{n}_d(t)}{dt}$$

Renormalized gate capacitance

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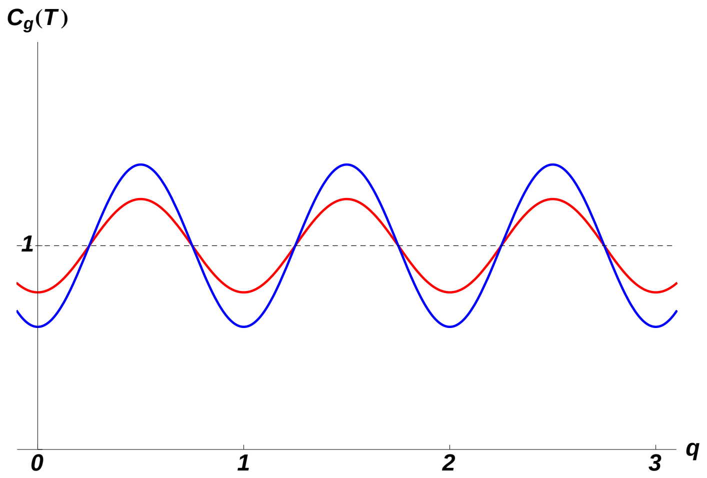
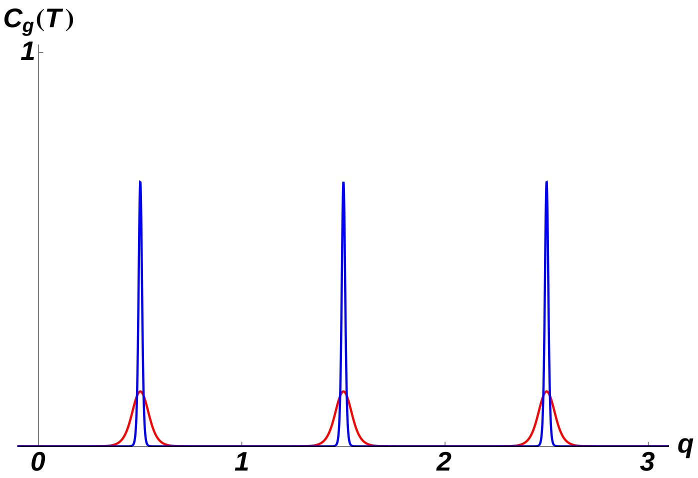
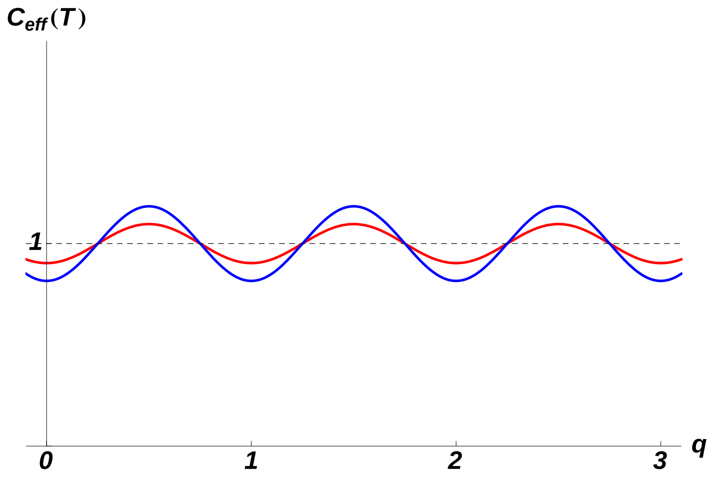
Weak coupling regime	Strong coupling regime
$g \gg 1$	$g \ll 1, \|q-k-1/2\| \ll 1$
$q'(T) = q - \frac{D}{4\pi} g^2(T) e^{-g(T)/2} \sin 2\pi q$	$q'(T) = k + \frac{1}{2} - \frac{1}{2}\tanh\frac{\bar{\Delta}}{2T}$
Burmistrov, Pruisken,	Phys. Rev. Lett. 101, 056801 (2008)
	
weak Coulomb oscillations	*developed Coulomb blockade*

Effective capacitance: $g \gg 1$

$$C_{\text{eff}}(T) = \frac{\partial Q(T)}{\partial U_0}$$

$$Q(T) = q - \frac{g^2}{\pi} e^{-g/2} \ln \frac{E_c}{T} \sin 2\pi q$$

Wang, Grabert, Phys. Rev. B53, 12621 (1996)



Effective capacitance: $g \ll 1$, $|q-k-1/2| \ll 1$

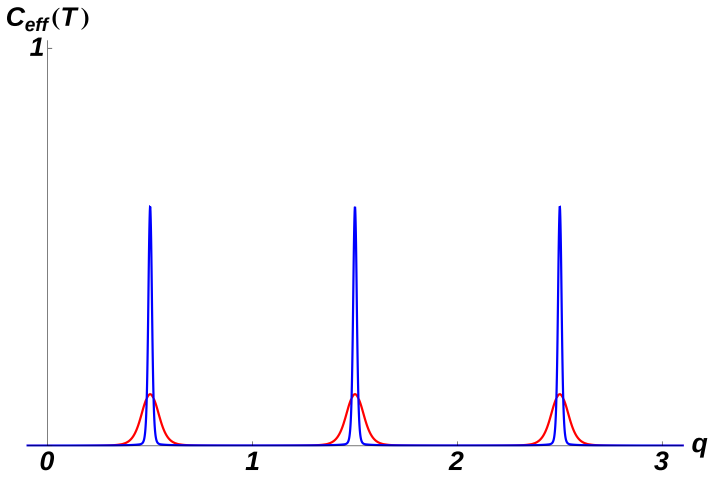
$$C_{\text{eff}}(T) = \frac{\partial Q(T)}{\partial U_0}$$

$$Q(T) = k + \frac{1}{2} - \frac{1}{2}\frac{\tanh \widehat{\Delta}/(2T)}{1+g\lambda}$$

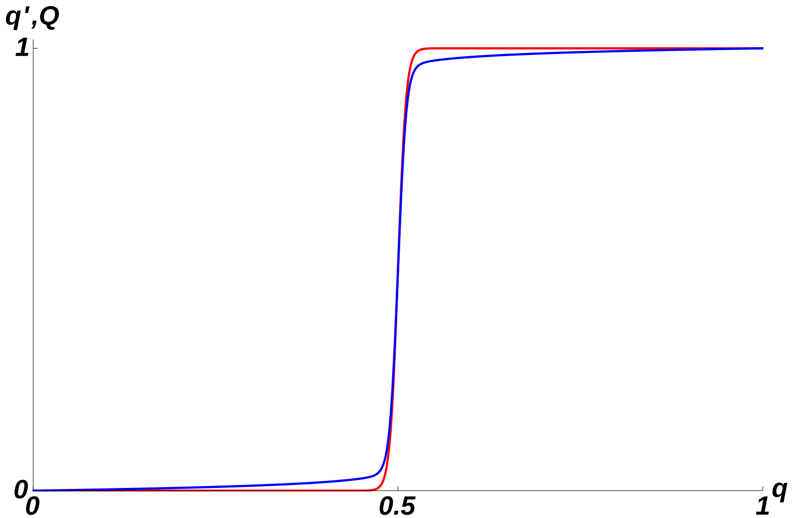
where $\Delta = E_c(2k+1-2q)$, $\widehat{\Delta} = \frac{\Delta}{1+g\lambda}$, $\widehat{g} = \frac{g}{1+g\lambda}$, and

$$\lambda = \frac{1}{2\pi^2}\ln\frac{E_c}{\max\{T, |\widehat{\Delta}|\}}$$

Matveev, Sov. Phys. JETP72, 892 (1991); Schoeller, Schön, Phys. Rev. B50, 18436 (1994).



$C_g(T)$ versus $C_{\rm eff}(T)$: $g \ll 1$, $|q-k-1/2| \ll 1$



At $T=0$:

$$q' = k + \frac{1}{2} + \frac{1}{2}\mathrm{sgn}\left(q-k-\frac{1}{2}\right), \qquad Q = k + \frac{1}{2} + \frac{1}{2}\frac{\mathrm{sgn}\left(q-k-\frac{1}{2}\right)}{1+\frac{g}{2\pi^2}\ln\frac{1}{|2k+1-2q|}}$$

$$C_g = \delta\left(q-k-\frac{1}{2}\right)$$

q' is integer quantized at $T=0$ but Q is not !

Effective action

$$H = \sum_{k} \varepsilon_k^{(a)} a_k^\dagger a_k + \sum_{\alpha} \varepsilon_\alpha^{(d)} d_\alpha^\dagger d_\alpha + \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha + \text{h.c.} + E_c\big(\hat{n}_d - q(t)\big)^2,$$

$$\hat{n}_d = \sum d_\alpha^\dagger d_\alpha, \qquad q(t) = \frac{C_g U_g(t)}{e}, \qquad U_g(t) = U_0 + U_\omega \cos\omega t.$$

AES-model

Ambegaokar Eckern Schön, PRL 1983

$$S_{AES} = S_c + S_{gt} + S_g + S_d$$

$$S_c = \frac{1}{4E_c} \int_0^\beta \dot{\varphi}^2 d\tau$$

$$S_{gt} = -2\pi i q \int_0^\beta \dot{\varphi} d\tau, \quad S_g = i C_g \int_0^\beta \dot{\varphi}(\tau) U(\tau) d\tau, \quad q = C_g U_0$$

$$S_d = -\frac{g}{4} \int_0^\beta \alpha(\tau_{12}) e^{i(\varphi(\tau_1) - \varphi(\tau_2))} d\tau_1 d\tau_2, \quad \alpha(\tau_{12}) = \frac{1}{\sin^2 \pi T(\tau_1 - \tau_2)}$$

S_c -charging	S_g	Coupling with gate
S_d Non-linear dissipative term	S_{gt}	topological term

Effective action

$$H = \sum_{\boldsymbol{k}} \varepsilon_k^{(a)} a_k^\dagger a_k + \sum_\alpha \varepsilon_\alpha^{(d)} d_\alpha^\dagger d_\alpha + \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha + \mathrm{h.c.} + E_c \big(\hat{n}_d - q(t)\big)^2,$$

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S_c -charging

S_d Non-linear dissipative term

S_g Coupling with gate

S_{gt} topological term

Tunneling conductance g

$$\hat{g}_{kk'} = (2\pi)^2 \left[\delta(\varepsilon_k^{(a)})\delta(\varepsilon_{k'}^{(a)})\right]^{1/2} \sum_\alpha t_{k\alpha}\delta(\varepsilon_\alpha^{(d)})t^\dagger_{\alpha k'},$$

$$\hat{\hat{g}}_{\alpha\alpha'} = (2\pi)^2 \left[\delta(\varepsilon_\alpha^{(d)})\delta(\varepsilon_{\alpha'}^{(d)})\right]^{1/2} \sum_k t^\dagger_{\alpha k}\delta(\varepsilon_k^{(a)})t_{k\alpha'},$$

$$g_{\mathrm{ch}} = \frac{\mathrm{tr}(\hat{\mathrm{g}}^2)}{\mathrm{tr}\hat{\mathrm{g}}}, \qquad N_{\mathrm{ch}} = \frac{(\mathrm{tr}\hat{\mathrm{g}})^2}{\mathrm{tr}(\hat{\mathrm{g}}^2)}.$$

effective channel conductance

effective number of channels

$$g_{\mathrm{ch}} \ll 1$$

$$g = \mathrm{tr}\hat{\mathrm{g}} = \mathrm{tr}\hat{\hat{\mathrm{g}}} = \mathrm{g}_{\mathrm{ch}}\mathrm{N}_{\mathrm{ch}}.$$

Formal definition of observables

$$S_{AES} = S_c + S_{gt} + S_g - \frac{g}{4}\int_0^\beta \alpha(\tau_{12}) e^{i[\varphi(\tau_1)-\varphi(\tau_2)]} d\tau_1 d\tau_2, \quad \alpha(\tau_{12}) = \frac{1}{\sin^2 \pi T(\tau_1 - \tau_2)}$$

Field-correlator

$$K(\tau_{12}) = -\frac{g}{4}\alpha(\tau_{12})\langle e^{i[\varphi(\tau_1)-\varphi(\tau_2)]}\rangle,$$

Observable $g'(T)$

$$g'(T) = 4\pi \mathrm{Im} \frac{\partial K^R(\omega)}{\partial \omega}\Big|_{\omega=0}$$

G. Schön, E. Mottola E. Ben-Jacob PRL, 1983

Observable q'

$$q'(T) = Q + \mathrm{Re} \frac{\partial K^R(\omega)}{\partial \omega}\Big|_{\omega=0}$$

Q - average charge in the island

I. Burmistrov, A. Pruisken, PRL, 2008

Perturbation theory in $1/g$

Existence of instanton solutions

$$\varphi_W(\beta) = \varphi(0) + 2\pi W$$

$$e^{i\varphi(\{z\},\tau)} = \prod_{a=1}^{|W|} \left[\frac{e^{2\pi i\tau T} - z_a}{1 - \bar{z}_a e^{2\pi i\tau T}} \right]^{\operatorname{sgn} W}$$

Korshunov, JETP Lett.1987; Bulgadaev, Phys.Lett. 1987

Action on instanton configuration

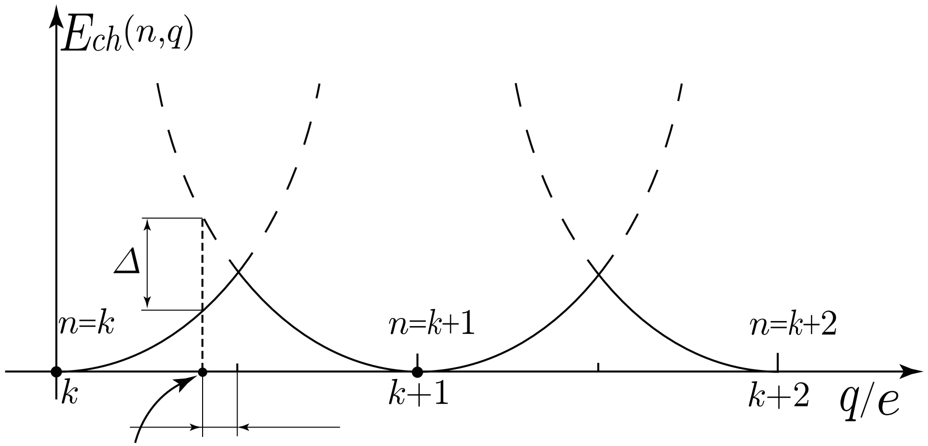
$$S_g[\varphi_W] + S_d[\varphi_W] = -2\pi i W q + \frac{g}{2}|W|$$

Ward identity for electron polarization operator

$$\Pi(\tau) = -C^2 \langle \mathcal{T}_\tau \dot{\varphi}(\tau)\dot{\varphi}(0)\rangle$$

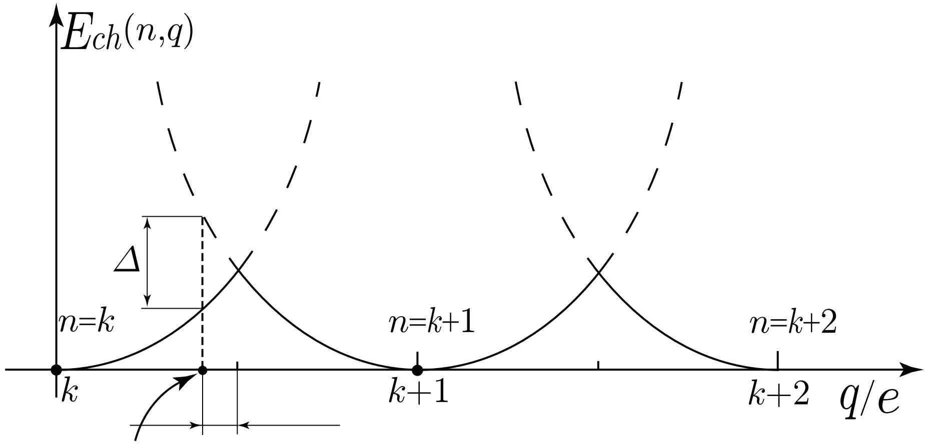
Strong coupling regime: $g \ll 1, |q-k-1/2| \ll 1$

$g \ll 1 \;\Leftrightarrow$ **Pronounced Coulomb blockade**



Rigorous treatment is available for:

$$\Delta \ll E_c \sim \frac{e^2}{C}$$



Only 2 closest states contribute to transport

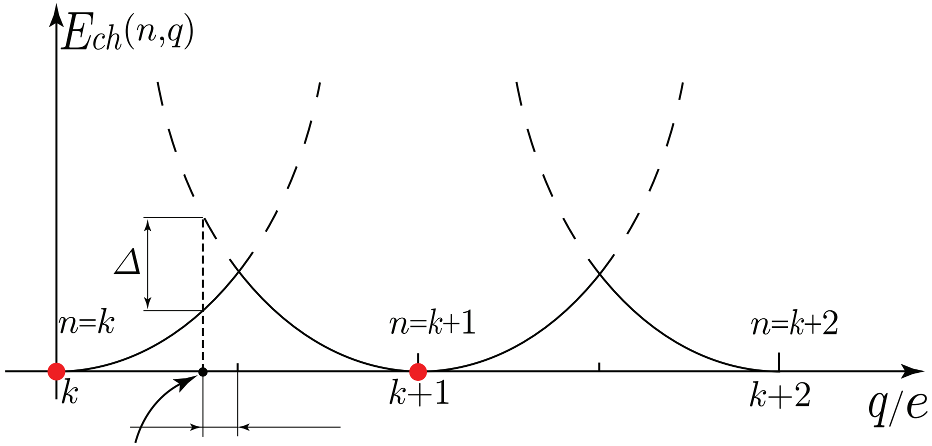
$$0: \; n = k$$

$$1: \; n = k + 1$$

Kondo-type-Hamiltonian in the truncated Hilbert space

$$H = H_0 + \sum_{k,\alpha} t_{k\alpha} a_k^\dagger d_\alpha s^+ + \text{h.c.} + \Delta s_z + \frac{\Delta^2}{4E_c} + \frac{E_c}{4}$$

Matveev, JETP, 1991



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Matveev, JETP, 1991

Strong coupling regime: $g \ll 1, |q-k-1/2| \ll 1$

Effective action

$$S = \beta E_c (k-q)^2 + \frac{\beta\Delta}{2} + \Delta \int_0^\beta s^z(\tau) d\tau + \frac{g}{4} \int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12}) s^-(\tau_1) s^+(\tau_2)$$

Correspondence with AES-phase field:

$$\dot{\varphi} \to s_z, \quad e^{i\varphi} \to s^+, \quad e^{-i\varphi} \to s^-$$

A. Larkin V. Melnikov JETP 1971

Key correlators in terms of spin-variables:

- *Correlator* $K(\tau)$

$$K(\tau) = -\frac{g}{4}\alpha(\tau)\langle s^+(\tau) s^-(0)\rangle$$

- *Polarization operator* $\Pi(\tau)$

$$\Pi(\tau) = \langle s^z(\tau) s^z(0)\rangle$$

s- operators are described in terms of Abrikosov pseudo-fermions

$$s^i = \psi^\dagger_\alpha S^i_{\alpha\beta}\psi_\beta$$

Effective action

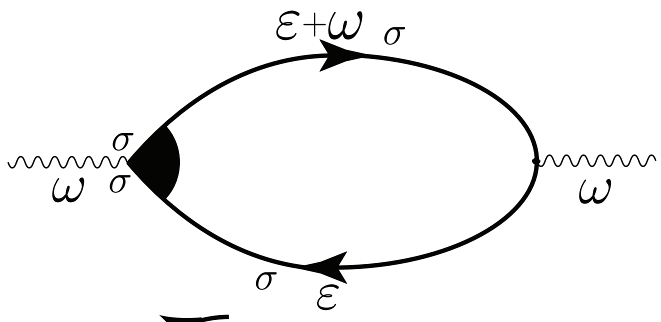
$$\mathcal{S} = \int_0^\beta d\tau \bar{\psi}_{pf}\left(\partial_\tau + \frac{\sigma_z\Delta}{2} - \eta\right)\psi_{pf} + \frac{g}{4}\int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12})[\bar{\psi}_{pf}(\tau_1)\sigma_-\psi_{pf}(\tau_1)][\bar{\psi}_{pf}(\tau_2)\sigma_+\psi_{pf}(\tau_2)]$$

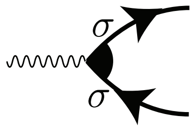
$$Z = \frac{\partial}{\partial e^\eta} Z_{pf}\Big|_{\eta\to-\infty}$$

$$\langle\mathcal{O}\rangle = \lim_{\eta\to-\infty}\left\{\langle\mathcal{O}\rangle_{pf} + \frac{Z_{pf}}{Z}\frac{\partial}{\partial e^\eta}\langle\mathcal{O}\rangle_{pf}\right\}$$

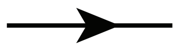
Strong coupling regime, field theory

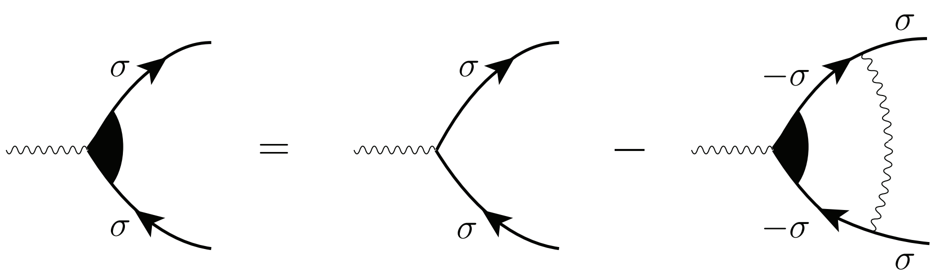
Partial summation of some infinite classes of diagrams yields non-perturbative result

$\Pi(\omega) = \sum_{\sigma}$ 

$\Gamma_\sigma(\varepsilon, \varepsilon+\omega, \omega) =$ 

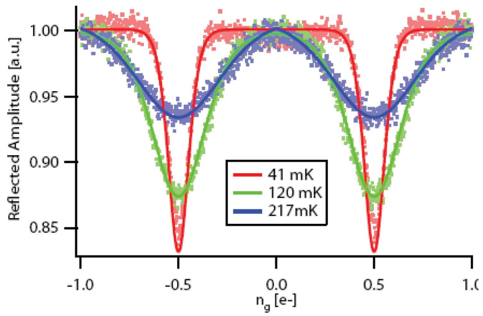
-exact vertex

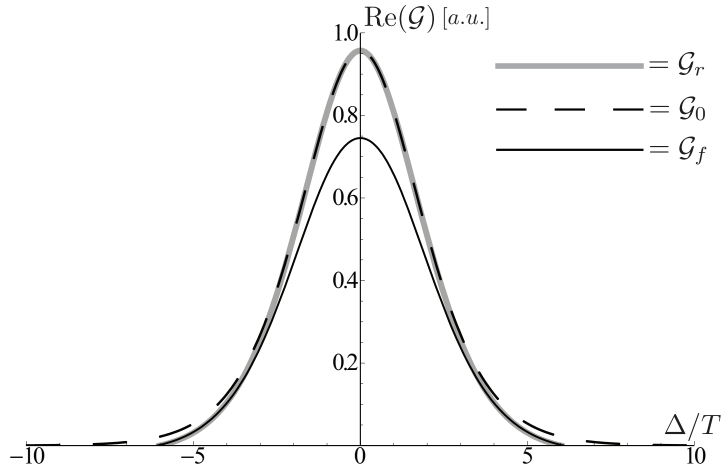
$G_\sigma(\omega) =$  -exact Green's function



Comparison with experiment of Persson et al. arxiv:0902.4316

$E_c = 700$ mK (15 GHz)
$\omega = 30$ mK (0.6 GHz)





$$\mathcal{G}_f(\omega) = \frac{C_g}{C}\frac{1}{(1+g\lambda)^2}\frac{\widehat{g}}{4\pi\coth\frac{\widehat{\Delta}}{2T}}\frac{-i\omega\coth\frac{\widehat{\Delta}}{2T}+\frac{1}{\pi}\widehat{F}^R(\omega)}{-i\omega+\frac{\widehat{g}\widehat{\Delta}}{2\pi}\coth\frac{\widehat{\Delta}}{2T}}$$

$$\mathcal{G}_0(\omega) = \frac{C_g}{C}\frac{g}{4\pi}\frac{\beta\Delta}{\sinh\beta\Delta}\frac{-i\omega}{-i\omega+\frac{g\Delta}{2\pi}\coth\frac{\Delta}{2T}}, \quad \omega \ll \max\{\Delta, T\}$$

where $\Delta = E_c(2k+1-2q)$, $\widehat{\Delta} = \frac{\Delta}{1+g\lambda}$, $\widehat{g} = \frac{g}{1+g\lambda}$, $\lambda = \frac{1}{2\pi^2}\ln\frac{E_c}{\max\{T, |\widehat{\Delta}|, |\omega|\}}$, and

$$\widehat{F}^R(\omega) = \sum_{\sigma=\pm 1}\left[(\widehat{\Delta}+\sigma\omega)\psi\left(\frac{\omega+\sigma\widehat{\Delta}}{2\pi T i}\right) - \widehat{\Delta}\psi\left(\frac{i\sigma\widehat{\Delta}}{2\pi T}\right)\right]$$

$g\lambda \approx 0.03$

Conclusions

- Energy dissipation in SEB at $\max\{\delta, g\delta\} \ll T \ll E_c$ in weak ($g \gg 1$) and strong ($g \ll 1$, $|q-k-1/2| \ll 1$) coupling regimes was studied.

- In both cases it can be expressed in terms of two physical observables $g'(T)$ and $q'(T)$:

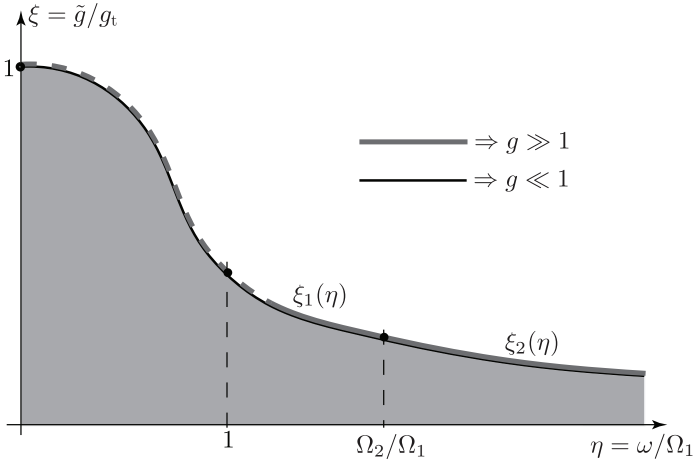
$$\mathcal{W}_\omega = \omega^2 C_g^2(T) R_q(T) |U_\omega|^2, \;\; R_q(T) = \frac{h}{e^2}\frac{1}{g'(T)}, \; C_g(T) = \frac{\partial q'(T)}{\partial U_0}$$

- We expect that this result holds in general for $\max\{\delta, g\delta\} \ll T \ll E_c$.

Comparsion with the internal energy losses

Thouless conductance $g_t = E_{Th}/\delta$

$$W_\omega^c \sim \frac{\hbar}{g_\mathrm{t} e^2} a^2 \omega^2 \max\left\{1, \frac{\omega^{3/2}}{\omega_0^{3/2}}\right\} |U_\omega|^2, \quad \omega_0 = \frac{E_c \hbar c^2}{g_\mathrm{t} e^4}$$



	$\omega_0 \leqslant \Omega$	$\omega_0 > \Omega$
Ω_1	ω_0	Ω
Ω_2	Ω	ω_0
ξ_1	$1/\eta^{3/2}$	$1/\eta^2$
ξ_2	$\frac{1}{\eta^{7/2}}\left(\frac{\Omega_2}{\Omega_1}\right)^2$	$\frac{1}{\eta^{7/2}}\left(\frac{\Omega_2}{\Omega_1}\right)^{3/2}$

	Ω	$\tilde{g}$
$g \gg 1$	$gE_c/\hbar$	g
$g \ll 1, \Delta \ll T$	$gT/\hbar$	$g\left(\frac{T}{E_c}\right)^2$
$g \ll 1, \Delta \gg T$	$g\Delta/\hbar$	$g\frac{\Delta T}{E_c^2} e^{\Delta/T}$