PseudoGap Superconductivity near Localization Threshold

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Superconductivity v/s Localization

- Granular systems with Coulomb interaction

K.Efetov 1980 et al ***"Bosonic mechanism"***

- Coulomb-induced suppression of Tc in uniform films ***"Fermionic mechanism"***

A.Finkelstein 1987 et al

- Competition of Cooper pairing and localization (no Coulomb)

Imry-Strongin, **Ma-Lee**, Kotliar-Kapitulnik, Bulaevsky-Sadovsky(mid-80's)

Ghosal, Randeria, Trivedi 1998-2001

There will be no grains and no Coulomb in this talk !

Plan of the talk

1. Motivation from experiments
2. BCS-like theory for critical eigenstates

   - transition temperature
   - local order parameter
3. Superconductivity with pseudogap

   - transition temperature v/s pseudogap
   - tunnelling conductance
   - spectral weight
4. Conclusions and open problems

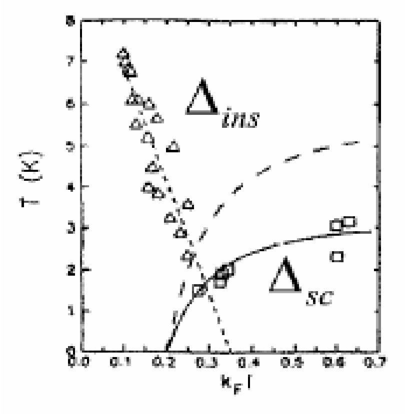
Major exp. data calling for a new theory

- Activated resistivity in insulating a-InO_x

D.Shahar-Z.Ovadyahu 1992,

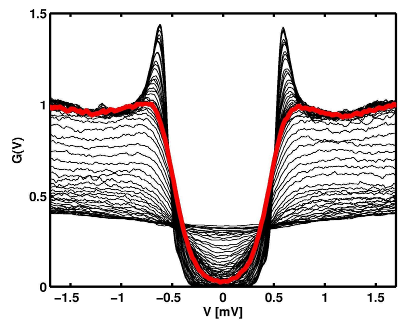
V.Gantmakher et al 1996

T_0 = 3 – 15 K



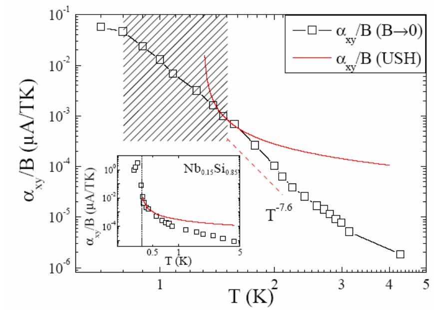
- Local tunnelling data

B.Sacepe et al 2007-8



- Nernst effect above T_c

P.Spathis, H.Aubin et al 2008



Class of relevant materials

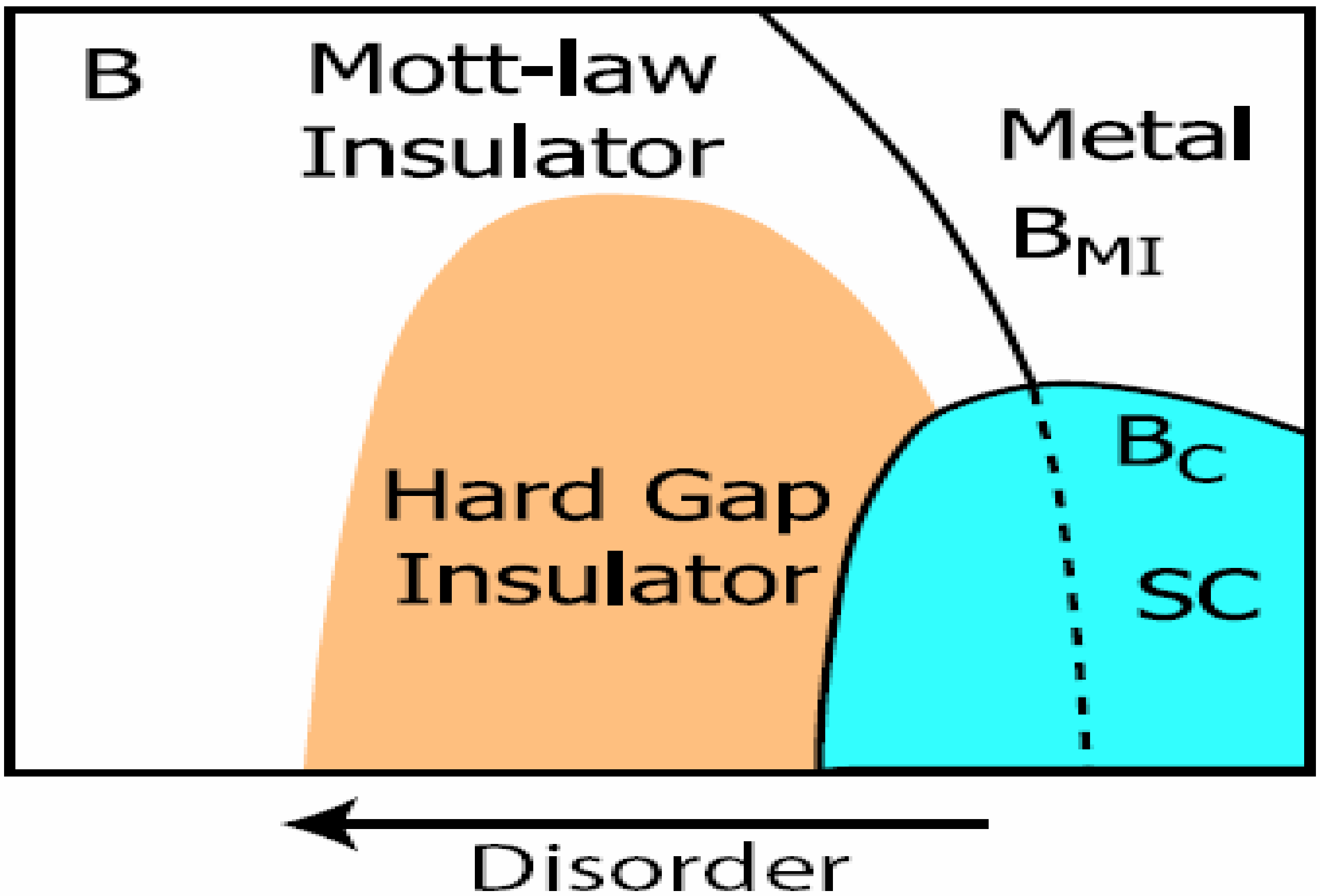
- Amorphously disordered
  (no structural grains)
- Low carrier density
  (around 10^{21} cm^{-3} at low temp.)

Examples:

InO_x NbN_x thick films or bulk (+ B-doped Diamond?)

TiN thin films Be, Bi (ultra thin films)

Phase Diagram



Theoretical model

Simplest BCS attraction model,

but for critical (or weakly) localized electrons

$$H = H_0 - g \int d^3r \, \Psi_{\uparrow}^{\dagger} \Psi_{\downarrow}^{\dagger} \Psi_{\downarrow} \Psi_{\uparrow}$$

$$\boldsymbol{\Psi} = \Sigma \, c_j \, \Psi_j \, (r)$$ Basis of localized eigenfunctions

M. Ma and P. Lee (1985) : S-I transition at $\delta_L \approx T_c$

Superconductivity at the Localization Threshold: $\delta_L \to 0$

Consider Fermi energy very close to the mobility edge:

single-electron states are extended but **fractal**

and populate small fraction of the whole volume

How BCS theory should be modified to account for eigenstate's fractality ?

Method: combination of analitic theory and numerical data for Anderson mobility edge model

Mean-Field Eq. for T_c

$$\Delta(r) = \int K_T(r,r')\Delta(r')d^d r' \quad (9)$$

where kernel $\hat{K}_T$ is equal to

$$K_T(r,r') = \frac{\lambda}{2\nu_0}\sum_{ij}\frac{\tanh\frac{\xi_i}{2T} + \tanh\frac{\xi_j}{2T}}{\xi_i + \xi_j}\psi_i(r)\psi_j(r)\psi_i(r')\psi_j(r') \quad (10)$$

Standard averaging over space $\Delta(r) \to \overline{\Delta}$ leads to "Anderson theorem" result: totally incorrect in the present situation.

The reason: critical eigenstates $\psi_j(r)$ are strongly correlated in real 3D space, they fill some small **submanifold** of the whole space only.

In fact one should define T_c as the divergence temperature of the Cooper ladder

$$\mathcal{C} = \left(1 - \hat{K}\right)^{-1}$$

Thus averaging procedure should be applied to $\mathcal{C}$ instead of K

We expand $\mathcal{C}$ in powers of K and average over disorder realizations. Keeping main sequence of resulting diagramms only, we come to the following equation for determination of T_c:

$$\Phi(\xi) = \frac{\lambda}{2} \int \frac{d\xi' \tanh(\xi'/2T)}{\xi'} M(\xi - \xi') \Phi(\xi') \qquad (11)$$

$$M(\omega) = \mathcal{V}\overline{M_{ij}} = \int \overline{\psi_i^2(r)\psi_j^2(r)} d^d r \quad \text{for} \quad |\xi_i - \xi_j| = \omega$$

For critical eigenstates

$$L_{\text{loc}} \to \infty$$

one finds

$$M(\omega) = \left(\frac{E_0}{\omega}\right)^{\gamma}$$

$$\langle M_i \rangle \approx 3\ell^{-(d-d_2)} L^{-d_2}.$$

where

$$\gamma = 1 - \frac{D_2}{d}$$

$$E_0 = 1/\nu_0 \ell^3$$

is a measure of fractality

$D_2 \approx 1.3$ in 3D

Usual "dirty superconductor":

$$M(\omega) = 1 \qquad \gamma = 0$$

3D Anderson model: $\gamma = 0.57$

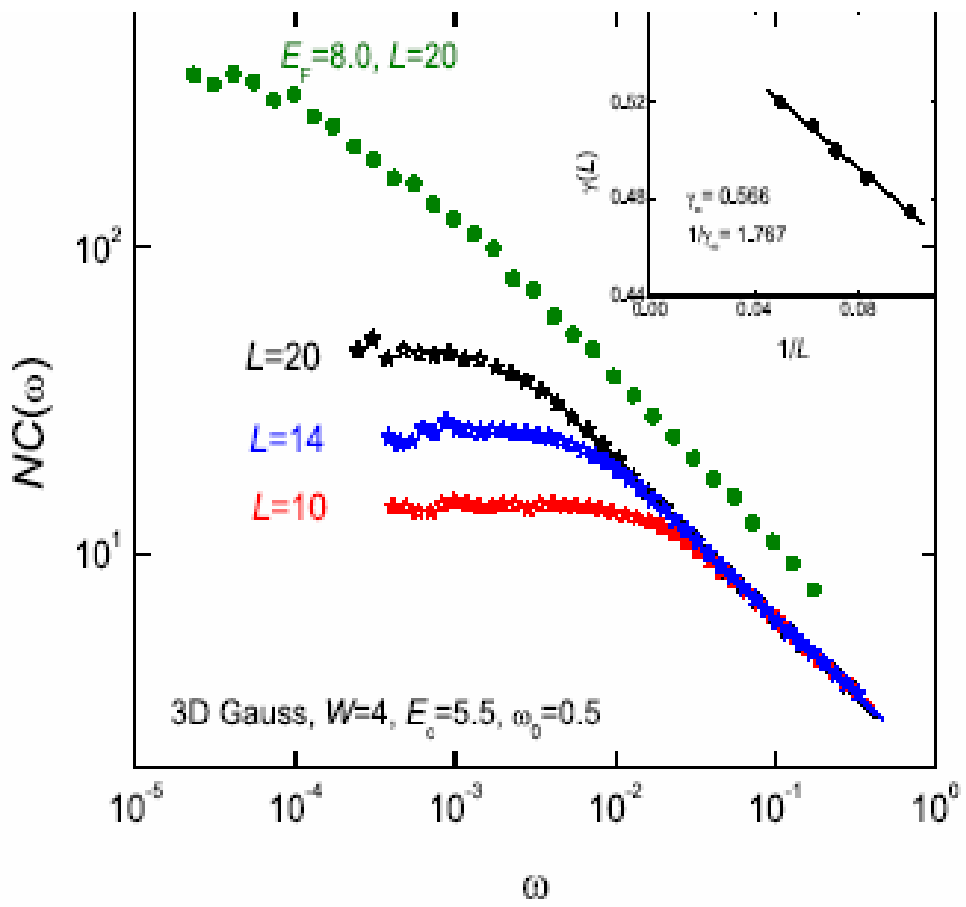


FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

Modified mean-field approximation for critical temperature T_c

$$\Delta(\xi) = \frac{\lambda}{2} \int d\zeta \eta(\zeta) M(\xi - \zeta) \Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1} \tanh(\xi_i / 2T).$$

$$T_c^0(\lambda, \gamma) = E_0 \lambda^{1/\gamma} C(\gamma)$$

For small λ this T_c is higher than BCS value !

arxiv:0810.2915 Y.Yanase & N.Yorozu: T_c for doped diamond, Si and SiC

Virial expansion method

(A.Larkin & D.Khmelnitsky 1970)

$$F = \sum_{n=1}^{\infty} \mathcal{F}^{(n)} = \sum_i F_i + \sum_{i>j} (F_{ij} - F_i - F_j)$$

$$+ \sum_{i>j>k} (F_{ijk} - F_{ij} - F_{jk} - F_{ik} + F_i + F_j + F_k) + \dots$$

$$V_\Delta = -\sum_j (\Delta S_j^+ + \Delta^* S_j^-)$$

$$\chi(T) = -\frac{\partial^2 F}{\partial \Delta \partial \Delta^*} = \sum_{M=1} \chi_M(T)$$

$$\chi_1 = \sum_i \chi_i^{(1)} \qquad (148)$$

$$\chi_2 = \sum_{n>m} (\chi_{nm}^{(2)} - \chi_n^{(1)} - \chi_m^{(1)})$$

$$\chi_3 = \sum_{n>m>l} (\chi_{nml}^{(3)} - \chi_{nl}^{(2)} - \chi_{ml}^{(2)} - \chi_{nm}^{(2)} + \chi_n^{(1)} + \chi_m^{(1)} + \chi_l^{(1)})$$

$$\lim_{n\to\infty} \frac{\chi^{(n+1)}(T_c)}{\chi^{(n)}(T_c)} = 1 \quad \Rightarrow \quad \frac{\chi^{(3)}(T_c)}{\chi^{(2)}(T_c)} = 1$$

T_c from 3 different calculations

Modified MFA equation leads to:

$$T_c = (6.5 \pm 0.8)\lambda^{1.77}$$

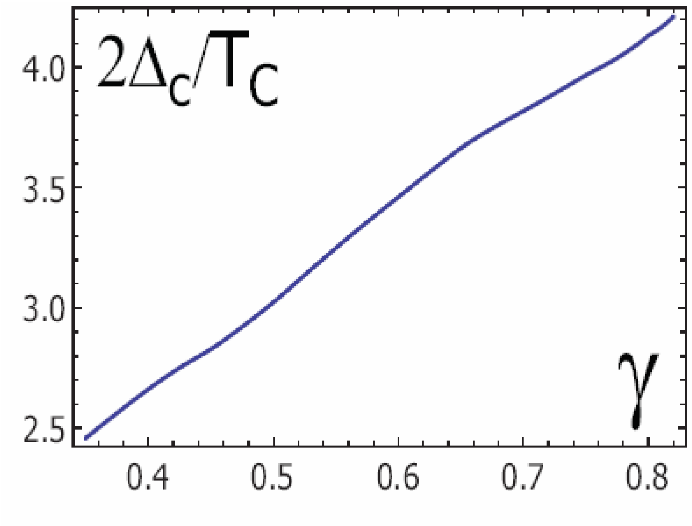
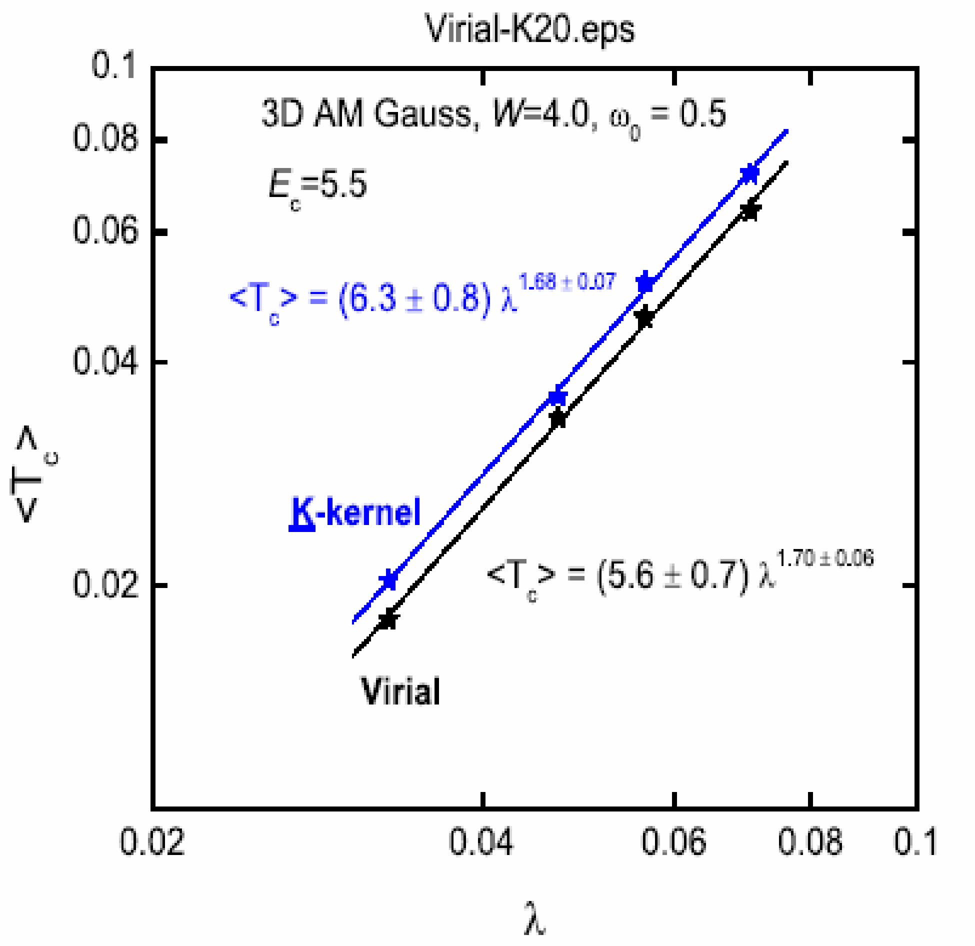


FIG. 16: Ratio $2\Delta(0)/T_c$ as function of γ.



Order parameter in real space

$$\tilde{\Delta}(\mathbf{r}) = \frac{g}{2} \sum_k \Delta_k \eta_k \psi_k^2(\mathbf{r})$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1} \tanh(\xi_i/2T).$$

$\Delta_k \Rightarrow \Delta(\xi)$ for $\xi = \xi_k$

$$\overline{(\tilde{\Delta}(\mathbf{r}))^2} \equiv \frac{1}{\mathcal{V}} \int d^d\mathbf{r} \tilde{\Delta}^2(\mathbf{r}) = \lambda \int_0^\infty d\xi \eta(\xi) \Delta_c^2(\xi)$$

$$\overline{\tilde{\Delta}(\mathbf{r})} \equiv \frac{1}{\mathcal{V}} \int d^d\mathbf{r} \tilde{\Delta}(\mathbf{r}) = \lambda \int_0^\infty d\xi \eta(\xi) \Delta_c(\xi)$$

Fluctuations of SC order parameter

With Prob = p << 1 $\Delta(r) = \Delta$, otherwise $\Delta(r) = 0$ ⇨

SC fraction = $$\frac{\left(\overline{\tilde{\Delta}(\mathbf{r})}\right)^2}{\overline{(\tilde{\Delta}(\mathbf{r}))^2}} = \lambda Q(\gamma) = \frac{Q(\gamma)}{C^{\gamma}(\gamma)}\left(\frac{T_c}{E_0}\right)^{\gamma} \ll 1$$

prefactor ≈ 1.7 for γ = 0.57

Higher moments: $$\frac{\left(\overline{\tilde{\Delta}(\mathbf{r})}\right)^n}{\overline{(\tilde{\Delta}(\mathbf{r}))^n}} \propto (T_c/E_0)^{(1-d_n/d)(n-1)}$$

$$\langle P_q \rangle \sim \ell^{-(d-d_q)(q-1)} L^{-d_q(q-1)} \propto L^{-d_q(q-1)}$$

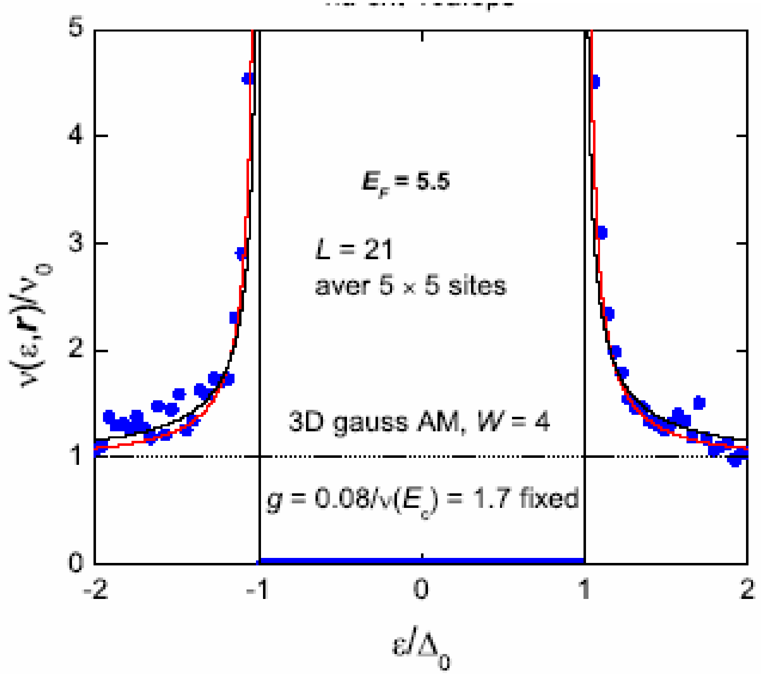
Tunnelling DoS

$$\nu(\varepsilon, \mathbf{r}) = \frac{1}{2} \sum_j \left(1 + \frac{\xi_j}{\varepsilon}\right) [\delta(\varepsilon - \varepsilon_j) + \delta(\varepsilon + \varepsilon_j)] \, \psi_j^2(\mathbf{r})$$

Average DoS:

$$\nu(\varepsilon) = \nu_0 \left| \frac{d\xi(\varepsilon)}{d\varepsilon} \right|$$

$$\varepsilon(\xi) = \sqrt{\xi^2 + \Delta^2(\xi)}$$



Asymmetry in loc DoS:

$$\nu_a(\varepsilon, \mathbf{r}) = \frac{1}{2}(\nu(\varepsilon, \mathbf{r}) - \nu(-\varepsilon, \mathbf{r})).$$

$$\overline{\nu_a^2(\varepsilon, \mathbf{r})} = \frac{1}{2} \left(\nu(\varepsilon) \frac{\xi(\varepsilon)}{\varepsilon} \right)^2 [M(0) - M(2\xi(\varepsilon))]$$

Neglected : off-diagonal terms

$$M_{ijkl} = \int d\mathbf{r}\psi_i^*(\mathbf{r})\psi_j^*(\mathbf{r})\psi_k(\mathbf{r})\psi_l(\mathbf{r}) \,,$$

Non-pair-wise terms with 3 or 4 different eigenstates were omitted

To estimate the accuracy we derived effective Ginzburg-Landau functional taking these terms into account

$F[\Psi(\mathbf{r})]$ defined in terms of an envelope function

$$\Psi(\mathbf{r}) = \Delta(\mathbf{r})/\tilde{\Delta}(\mathbf{r})$$

$$\tilde{\Delta}(\mathbf{r}) = \frac{g}{2}\sum_k \Delta_k \eta_k \psi_k^2(\mathbf{r})$$

$$F_{GL}[\Psi(\mathbf{r})] = \nu_0 T_c^2 \int d\mathbf{r} \left(a(\mathbf{r})\Psi^2(\mathbf{r}) + \frac{b}{2}\Psi^4(\mathbf{r}) + C|\nabla\Psi(\mathbf{r})|^2 \right)$$

$$\mathrm{Gi} \sim \frac{b^2}{C^3(\nu_0 T_c)^2} \sim 1 \qquad \mathrm{Gi}_d \sim \frac{W^2}{C^3} \sim 1$$

Superconductivity at the Mobility Edge: major features

- Critical temperature T_c is well-defined through the whole system in spite of strong $\Delta(r)$ fluctuations
- Local DoS strongly fluctuates in real space; it results in asymmetric tunnel conductance

  $G(V,r) \neq G(-V,r)$
- Both thermal (Gi) and mesoscopic (Gi_d) fluctuational parameters of the GL functional are of order unity

Superconductivity with Pseudogap

Now we move Fermi-level into the range of localized eigenstates

Local pairing in addition to collective pairing

Local pairing energy

1. Parity gap in ultrasmall grains

   K. Matveev and A. Larkin 1997

$\Delta \ll \delta:$

No many-body correlations

$\Delta_P = \frac{1}{2}\lambda\delta$

$\lambda_R = \lambda/(1 - \lambda \log(\epsilon_0/\delta)).$

$$\Delta_P = \frac{\delta}{2 \ln \frac{\delta}{\Delta}}$$

2. Parity gap for Anderson-localized eigenstates

The increase of thermodynamic potential Ω due to addition of *odd* electron to the ground-state is

$$\delta\Omega_{\rm oe} = \xi_{m+1} = \xi_{m+1} - \tilde{\xi}_{m+1} + \tilde{\xi}_{m+1} = \frac{g}{2} M_{m+1} + O(\mathcal{V}^{-1})$$

$$\tilde{\xi}_j = \xi_j - \frac{g}{2} M_j.$$

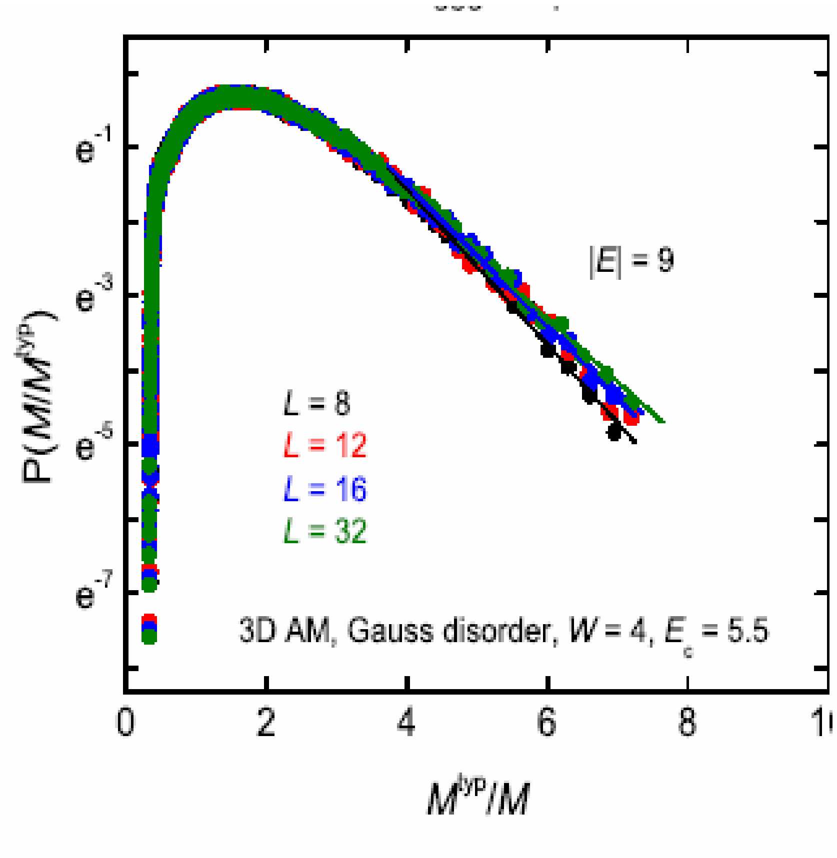
Energy of two single-particle excitations after depairing:

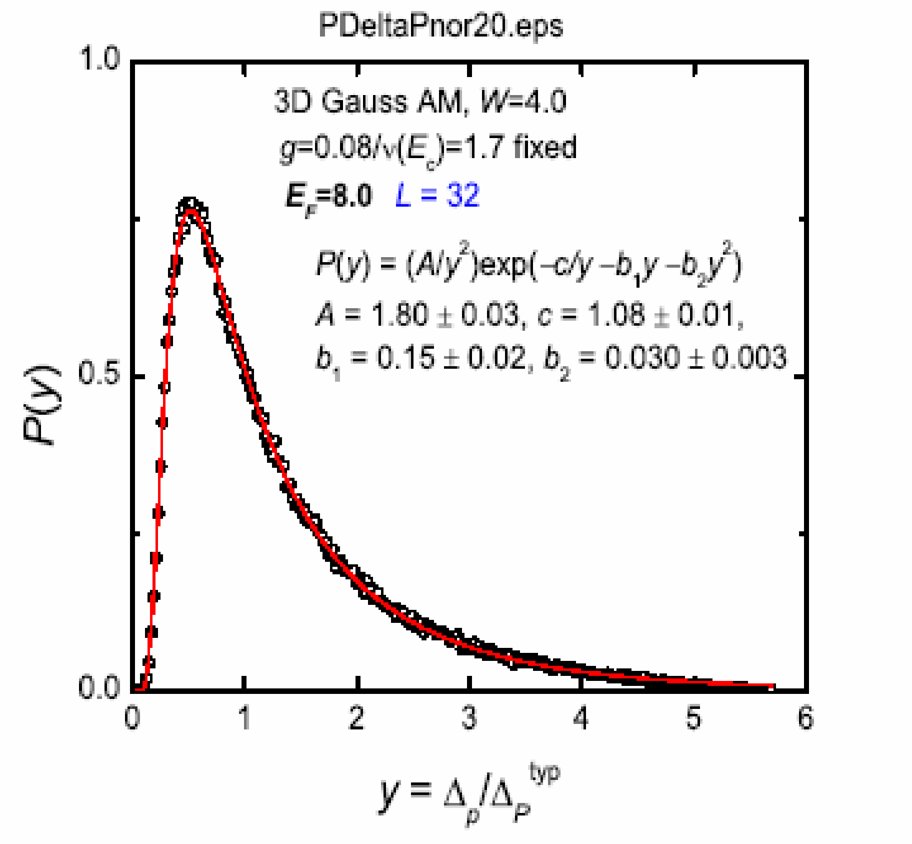
$$2\Delta_P = \xi_{m+1} - \xi_m + g M_m = \frac{g}{2}(M_m + M_{m+1}) + O(\mathcal{V}^{-1})$$

$$\langle M_i \rangle = 3\ell^{-(d-d_2)} L_{\rm loc}^{-d_2},$$

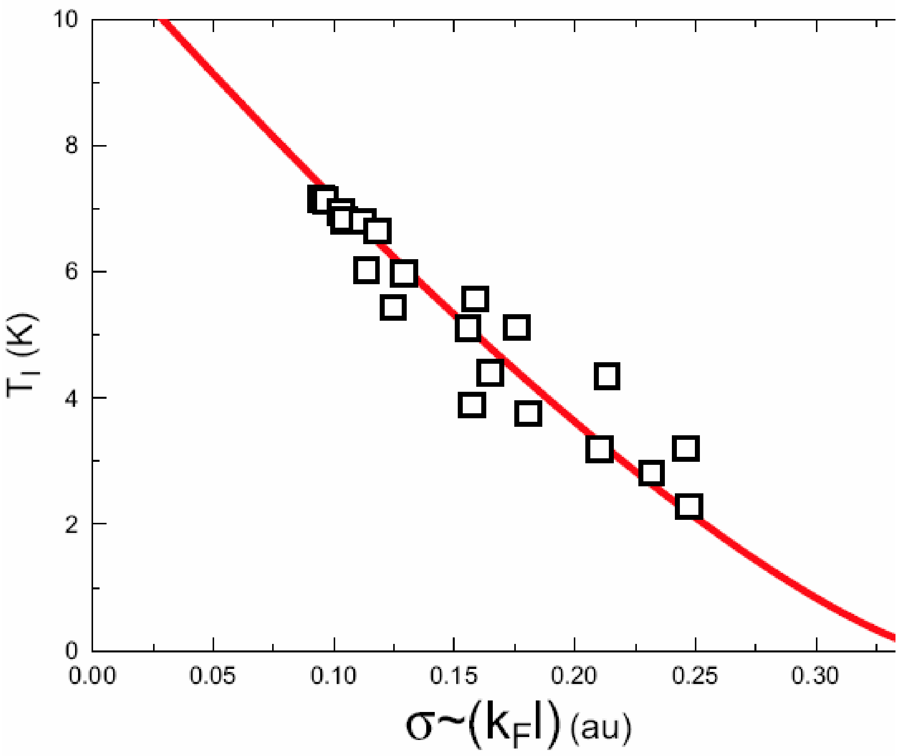
$$\Delta_P = \frac{3}{2} g \ell^{-3} (L_{loc}/\ell)^{-d_2} = \frac{3\lambda}{2} E_0 \left(\frac{E_c - E_F}{E_0}\right)^{\nu d_2}$$

P(M) distribution





Activation energy T_I from Shahar-Ovadyahu exp. and fit to theory



The fit was obtained with single fitting parameter

$$A \approx 0.5\lambda E_0$$

Example of consistent choice:

λ **= 0.05** E_0 **= 400 K**

Critical temperature in the pseudogap regime

MFA:

$$\Delta(\xi) = \frac{\lambda}{2} \int d\zeta \eta(\zeta) M(\xi - \zeta) \Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1} \tanh(\xi_i / 2T).$$

Here we use M(ω) specific for localized states

MFA is OK as long as $Z \sim \nu_0 T_c L_{loc}^d$ **is large**

Correlation function M(ω)

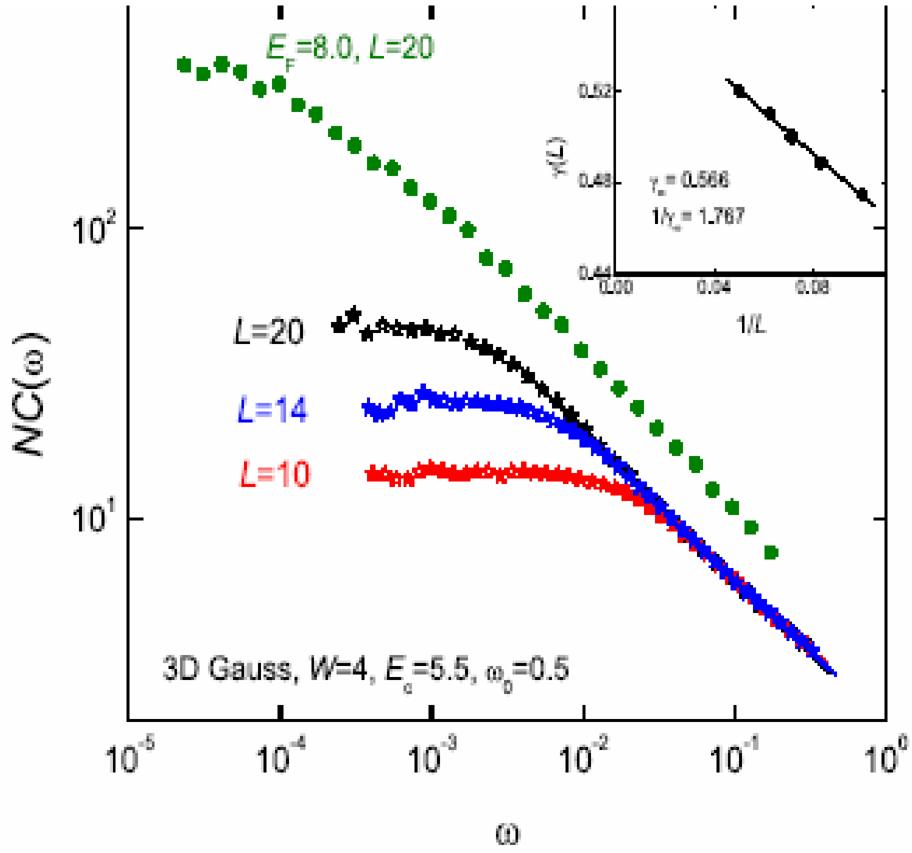


FIG. 2: (Color online) Correlation function $M(\omega)$ for 3DAM with Guassian disorder and lattice sizes $L = 10, 14, 20$ at the mobility edge $E = 5.5$ (red, blue and black points) and at the energy $E = 8$ inside localized band (green points). Inset shows γ values for $L = 10.12.14.16.20$.

No saturation at **$\omega < \delta_L$** :

$M(\omega) \sim \ln^2 (\delta_L / \omega)$

(Cuevas & Kravtsov PRB,2007)

Superconductivity with **$T_c < \delta_L$** is possible

This region was not found previously

Here "local gap" exceeds **SC gap :**

$$\Delta_P = \frac{1}{2D^\gamma(\gamma)}\delta_L \left(\frac{\Delta(0)}{\delta_L}\right)^\gamma$$

Critical temperature in the pseudogap regime

MFA:

$$\Delta(\xi) = \frac{\lambda}{2}\int d\zeta \eta(\zeta) M(\xi - \zeta)\Delta(\zeta)$$

$$\eta_i \equiv \eta_{ii} = \xi_i^{-1}\tanh(\xi_i/2T).$$

We need to estimate $Z \sim \nu_0 T_c L_{loc}^d$

$$R_\omega \approx 2L_{\rm loc}\ln\frac{\delta_L}{\omega} \gg L_{\rm loc}$$

$$R_0^2 = \frac{\sum_{ij} r_{ij}^2 M_{ij}}{\sum_{ij} M_{ij}}$$

$$Z_{\rm eff} \equiv \nu_0 T_c R_0^3 = 8\frac{T_c}{\delta_L}\ln^3\frac{\delta_L}{T_c}$$

It is nearly constant in a very broad range of $\frac{\delta_L}{T_c}$

Virial expansion results:

T_c versus Pseudogap

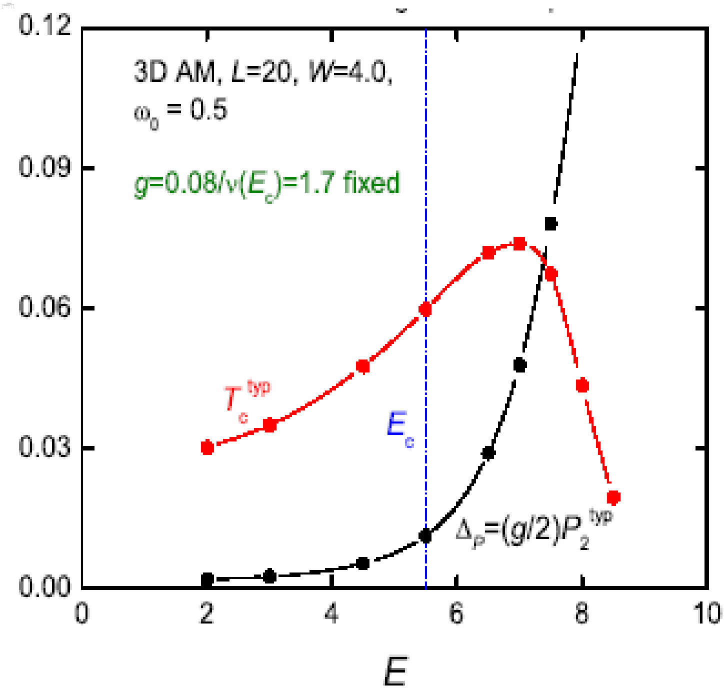


FIG. 25: (Color online) Virial expansion results for T_c (red points) and typical pseudogap Δ_P (black) as functions of E_F. The model with fixed value of the attraction coupling constant $g = 1.7$ was used; pairing susceptibilities were calculated using equations derived in Appendix B.

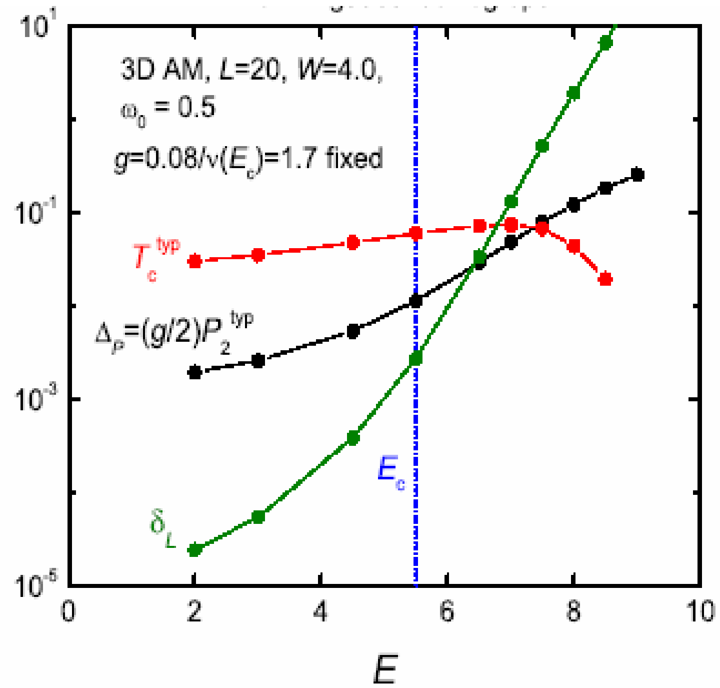


FIG. 26: (Color online) Virial results for T_c (red points), typical pseudogap Δ_P (black) and the corresponding level spacing δ_L (green), as functions of E_F on semi-logarithmic scale.

Transition exists even at $\delta_L \gg T_{c0}$

Single-electron states suppressed by pseudogap

„Pseudo spin" representation:

$$S^+_\mu = a^+_{\mu\uparrow} a^+_{\mu\downarrow} \qquad S^-_\mu = a_{\mu\uparrow} a_{\mu\downarrow}$$

$$2S^z_\mu = a^+_{\mu\uparrow} a_{\mu\uparrow} + a^+_{\mu\downarrow} a_{\mu\downarrow}$$

H_{BCS} acts on Even sector:
all states which are
2-filled or empty

$$\hat{H} = \sum_\mu 2\xi_\mu S^z_\mu - g \sum_{\mu,\nu} M_{\mu\nu} S^+_\mu S^-_\nu + \sum_{B_\mu} \left(\xi_\mu + \frac{G_\mu}{2}\right)$$

B: „blocked" states

$$\bar{M}_{\mu\nu} = \frac{1}{\nu V} M(\xi_\mu - \xi_\nu)$$

D.S ↑ ↖ total volume

"Pseudospin" approximation

$$Z \sim \nu_0 T_c L^d_{loc}$$

Effective number of interacting neighbours

Third Scenario

- Bosonic mechanism: preformed Cooper pairs + competition Josephson v/s Coulomb – S I T in arrays
- Fermionic mechanism: suppressed Cooper attraction, no paring – S M T

- Pseudospin mechanism: individually localized pairs

- S I T in amorphous media

SIT occurs at small Z and lead to paired insulator

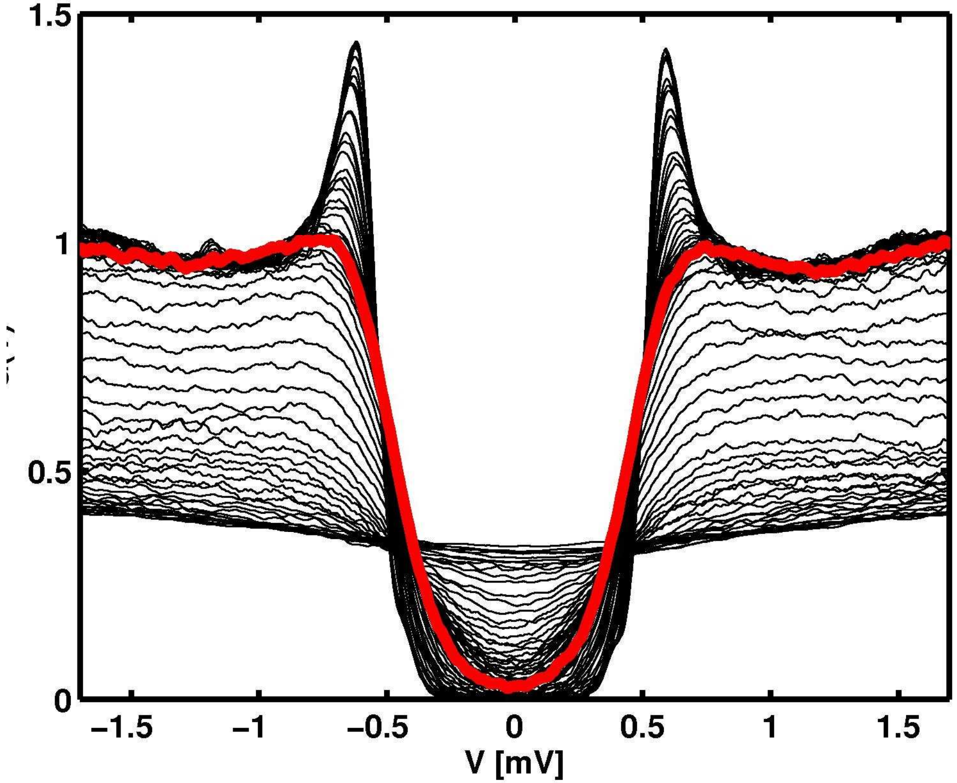
$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y)$$

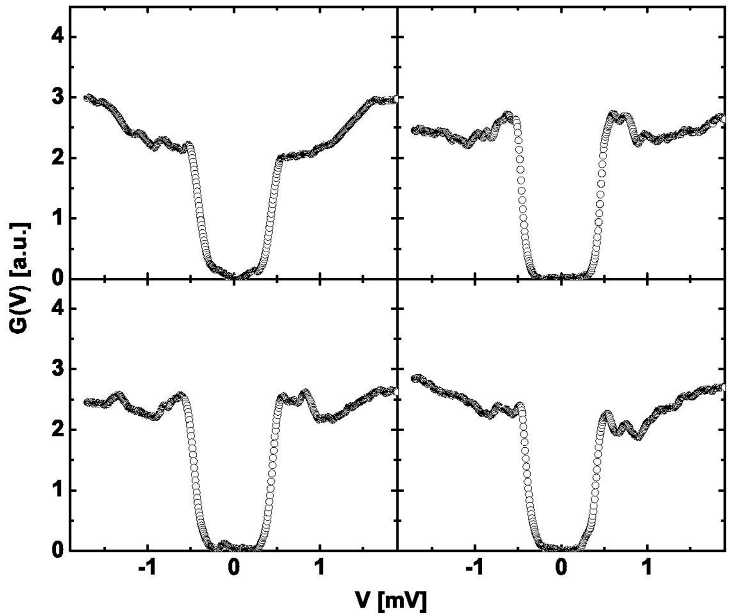
How to describe this quantum phase transition ?

Cayley tree model is solved (L.Ioffe & M.Mezard)

Strong local pseudogap above T_c :

experiment B.Sacepe et al





At T=Tc - almost fully developed gap but no coherence peak

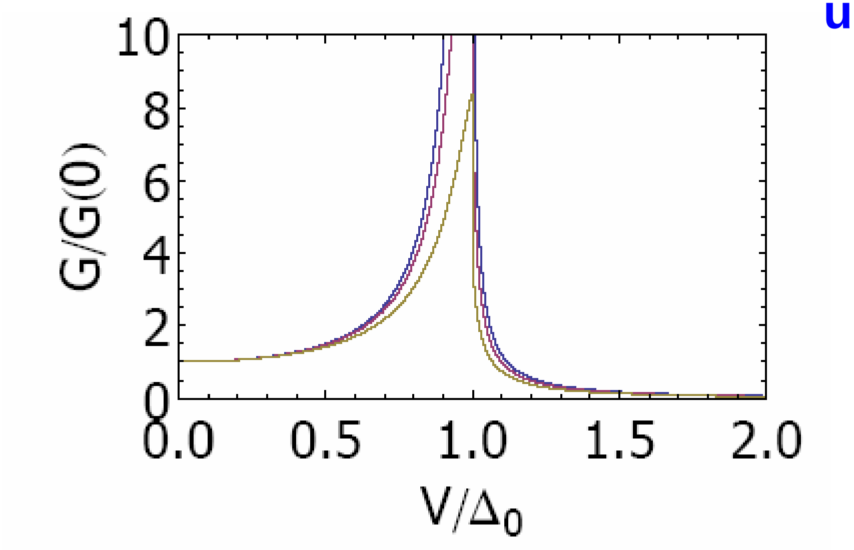
Point-contact spectroscopy

Generalization of the Blonder-Tinkham-Klapwijk formula for pseudogaped SC

1e transport: $eV_c = \Delta_P + \Delta$ Scales as G_t

2e transport: $2eV_c = 2\Delta$ Scales as $(G_t)^2$

unvisible in tunnelling regime $G_t << 1$



Double-peak structure at moderate G_t

Full Spectral Weight K(T)

$$K^{tot}(T) = \frac{2}{\pi}\int_0^{\Omega_{max}} \Re\sigma(\omega,T)d\omega + \rho_s(T) \equiv K(T) + \rho_s(T)$$

is usually (BCS) const across Tc : contributions from superconductive response and from DoS suppression cancel each other.

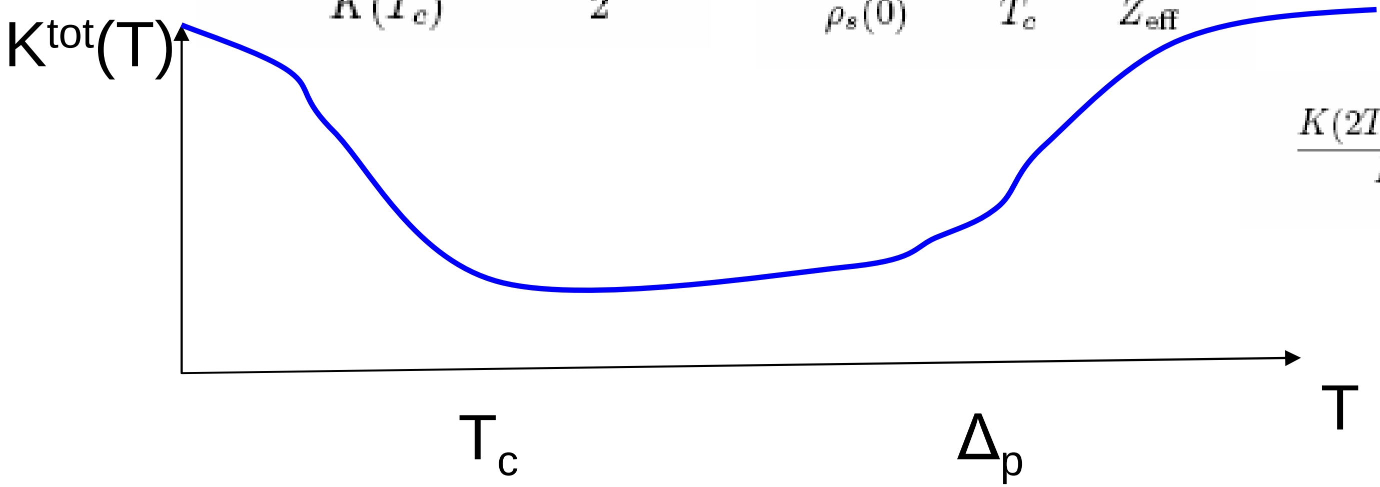
It is NOT the case for underdoped HTSC :

Experiment: D.Basov et al 1994 Theory: L.Ioffe & A.Millis 1999

The same effect is even more pronounced in Pseudogaped SC:

$$\frac{K(T_c) - K(0)}{K(T_c)} \sim \frac{1}{2}.$$

$$\frac{K(T_c)}{\rho_s(0)} \sim \frac{gM_{ij}}{T_c} \sim \frac{1}{Z_{\rm eff}}$$

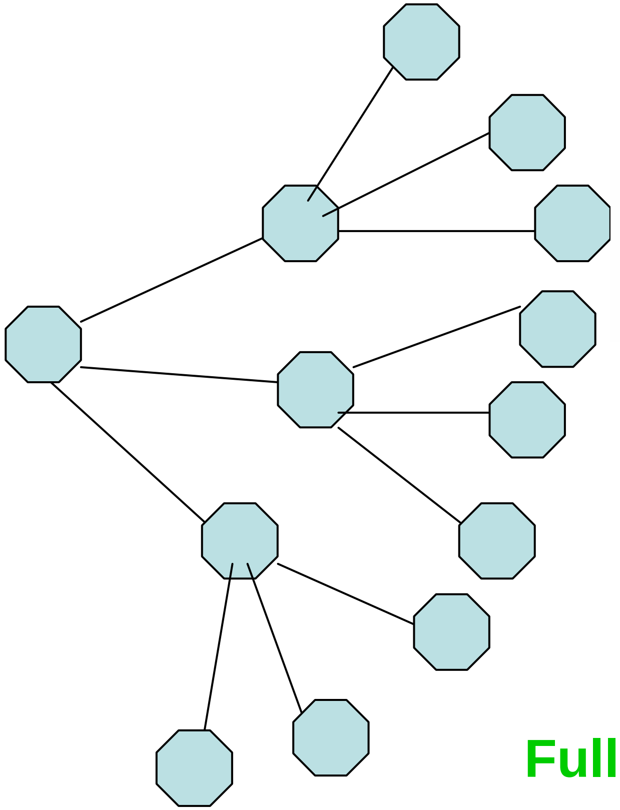


$$\frac{K(2T_c) - K(T_c)}{K(T_c)} \sim \frac{T_c}{\Delta_P} \ll 1$$

Qualitative features of “Pseudogaped Superconductivity”:

- STM DoS evolution with T
- Double-peak structure in point-contact conuctance
- Nonconservation of full spectral weight across T_c

S-I transition on Cayley tree



example with branching number $q = 3$

$$H = 2\sum_i \xi_i s_i^z - \sum_{ij} M_{ij}(s_i^x s_j^x + s_i^y s_j^y) \quad (1)$$

Eq.(1) contains random energies ξ_i

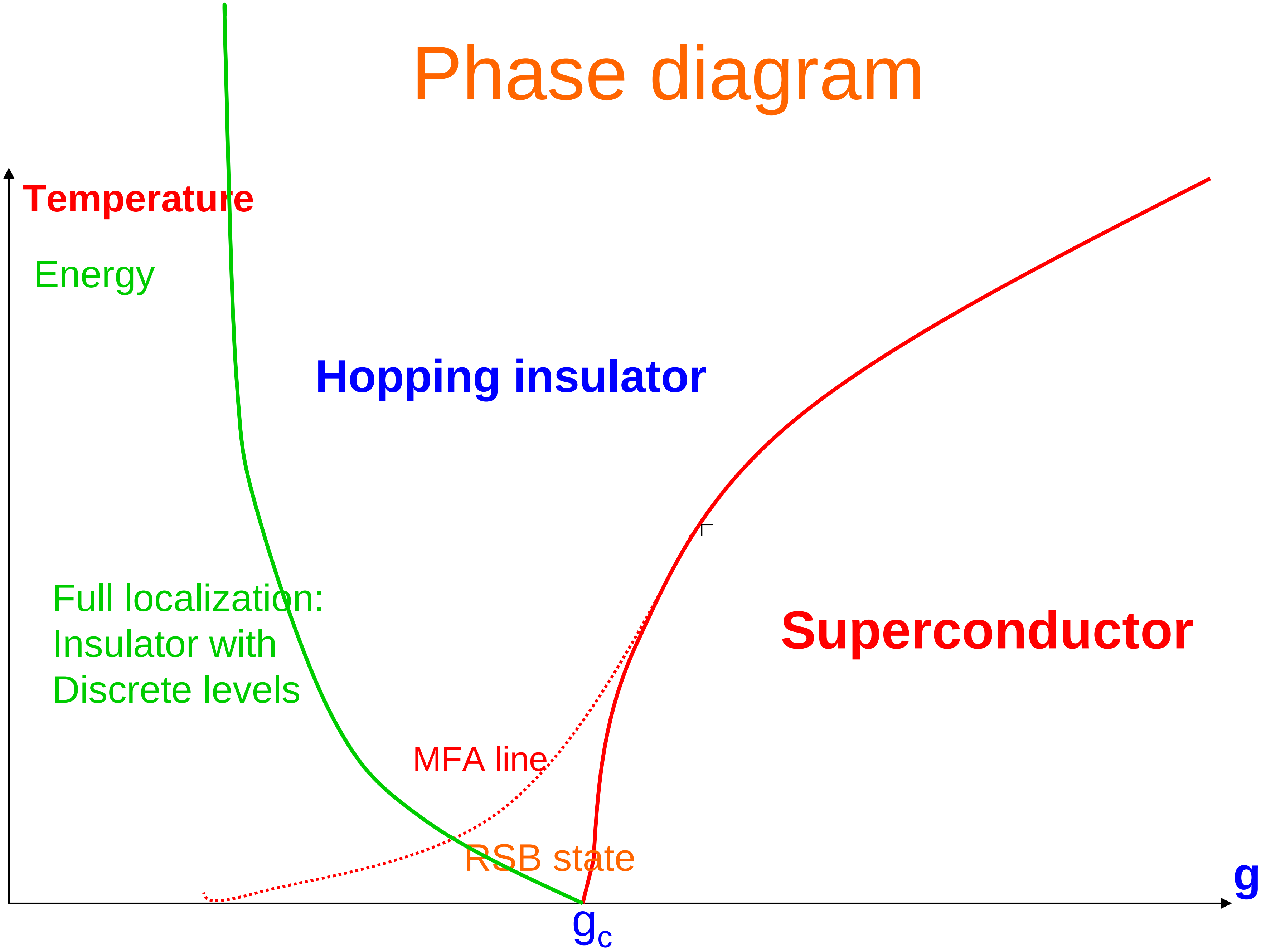
Large bandwidth W

$M_{ij} = M$ for nearest neighbours

Full self-consistent equation can be written for distribution functions of local fields Δ_i

Control parameter: $g = qM/W$

Phase diagram



Conclusions

Pairing on nearly-critical states produces fractal superconductivity with relatively high T_c but very small superconductive density

Pairing of electrons on localized states leads to hard gap and Arrhenius resistivity for 1e transport

Pseudogap behaviour is generic near S-I transition, with "insulating gap" above T_c

New type of S-I phase transition is described (on Cayley tree, at least)

Major unsolved problems (theor)

1. How to include magnetic field into the "fractal" scheme ?

- 2. Transition between pseudogap SC and insulator.   Why Cooper pair transport is activated ?
- 3. Rectangular gap in local tunnelling ?
- 4. Size-dependence of SIT (Kowal-Ovadyahu 2007)

Coulomb enchancement near mobility edge ??

Normally, Coulomb interaction is overscreened,
with universal effective coupling constant ~ 1

Condition of universal screening: $2\sigma/(T_c\kappa) \sim (\xi_0/a_{\rm scr})^2/\kappa \gg 1$

$a_{\rm scr}$ is the Thomas-Fermi screening length, $\sigma = (e^2 k_F/6\pi^2)\,(k_F l)$

Example of a-InO_x bad dirty metal with $k_F l \sim 0.3$

$e^2 k_F \sim 5000K$ the ratio $\sigma/T_c \sim 10$

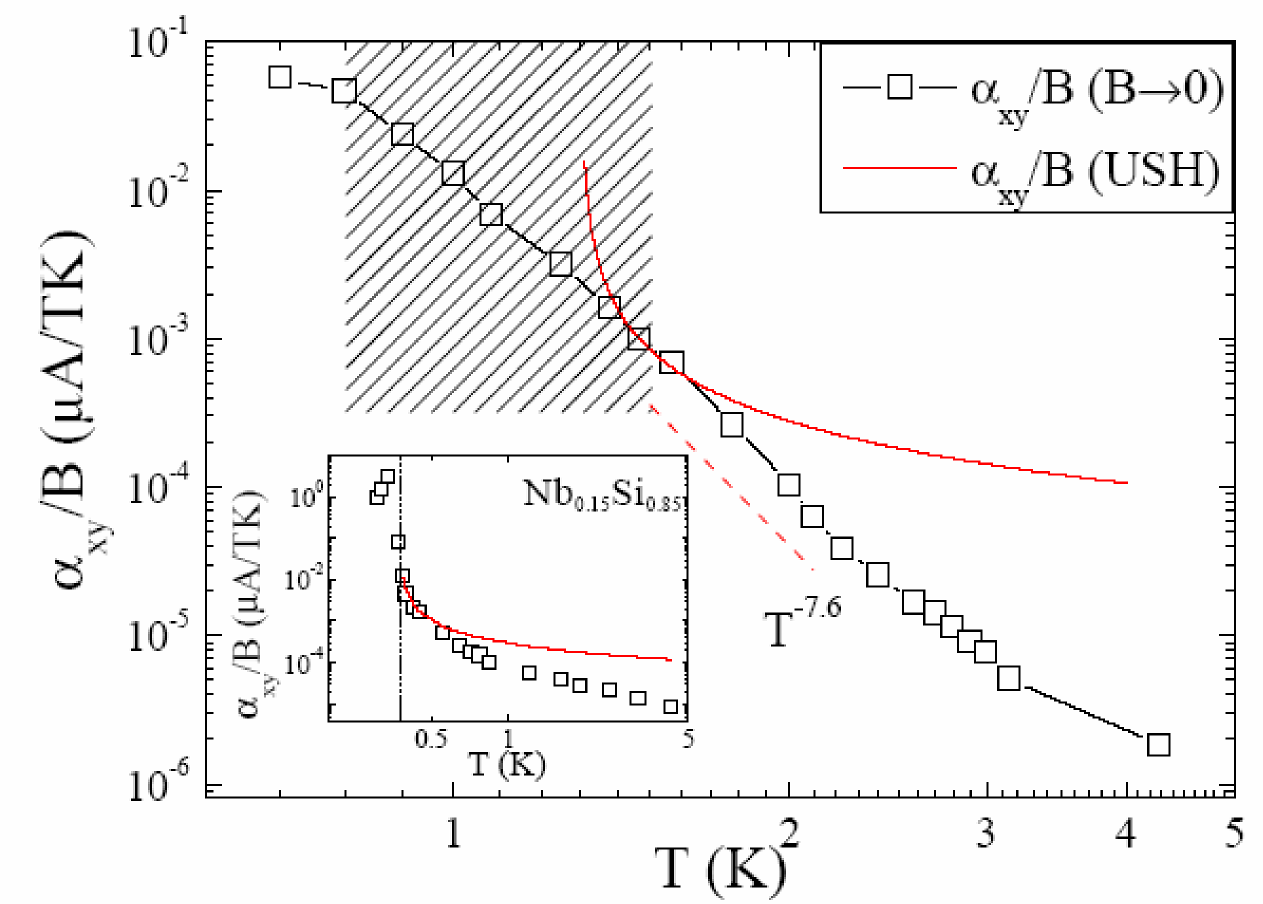
dielectric constant $\kappa \geq 30$

Effective Couloomb potential is weak:

$$\mu \sim 2\sigma/(T_c\kappa) < 1$$

Nernst coeff. in a-InOx

P.Spathis, H. Aubin *et al* 2007



Similarity to underdoped HTSC

Exponent 7.6 ??

No way to describe InO_x data by Gaussian fluctuations contrary to NbSi case: ***M.Serbyn et al, Phys.Rev.Lett. 102, 067001 (2009)***
K.Michaeli and A.Finkelstein arxiv:0902.2732

"Phase fluctuations" ? Where the amplitude comes from ?