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Effect of spin-charge interaction induced due to a curvature of free fermion spectrum on magnetic properties of 1D correlated systems.

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- We develop a bosonization approach for finding magnetic susceptibility of 1D attractive two component Fermi gas at the onset of magnetization taking into account the curvature effects.
It is shown that the curvature of free dispersion at Fermi points couples the spin and charge modes and leads to a linear critical behavior and finite susceptibility for a wide range of models.

- Effect of curvature on correlations functions is investigated

•Bosonization approach

$$\Psi_\sigma(x) = \Psi_{\sigma+}(x)\exp(ik_F x) + \Psi_{\sigma-}(x)\exp(-ik_F x),$$

$$H = H_0 + H_{int}, \quad H_0 = v_F(\Psi^\dagger_{\sigma+}(-i\partial/\partial x)\Psi_{\sigma+} - \Psi^\dagger_{\sigma-}(-i\partial/\partial x)\Psi_{\sigma-}) + h.c.$$

$$\Psi_{\sigma,\pm} \propto \exp\left[\pm i(4\pi)^{1/2}\varphi_{\sigma,\pm}(x)\right],$$

$$\varphi_{\sigma\pm} = \frac{1}{2}(\varphi_\sigma \mp \int_{-\infty}^{x}\pi_\sigma(x')dx').$$

$$\Psi^\dagger_\pm(y)\Psi_\pm(x) \;=:\; \exp\left[\mp i2\sqrt{\pi}(\varphi_\pm(y) - \varphi_\pm(x))\right] : \frac{\mp i/2\pi}{x - y \mp ia_0}.$$

•Spin – Charge separation

$$H = H_c + H_s$$

$$H_c = \frac{u_c}{2}[K_c \Pi_c^2 + K_c^{-1}(\partial_x \Phi_c)^2]$$

$$H_s = \frac{u_s}{2}[\Pi_s^2 + (\partial_x \Phi_s)^2] - \frac{m}{\pi a_0} \cos \beta_s \Phi_s$$

$$\beta_s^2 = 8\pi K_s, \quad m = \frac{g_\perp}{4\pi a_0}$$

$$K_s = 1 - \frac{g_\parallel}{4\pi v_s} + 0(g_\parallel^2)$$

•The Hubbard model

$$\hat{H} = -t \sum_{j,\sigma} (c^{+}_{j+1,\sigma} c_{j,\sigma} + c^{+}_{j,\sigma} c_{j+1,\sigma}) + U \sum_{j} n_{j\uparrow} n_{j\downarrow}$$

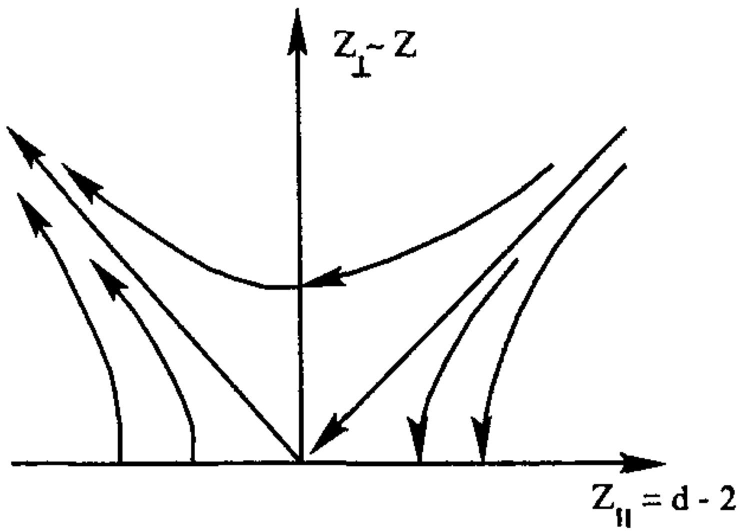
$$u_\rho K_\rho = u_\sigma K_\sigma = v_F$$

$$u_\rho / K_\rho = v_F (1 + \frac{U}{\pi v_F})$$

$$u_\sigma / K_\sigma = v_F (1 - \frac{U}{\pi v_F})$$

$$g_{1\perp} = U$$

$$
\begin{aligned}
S &= S_0 + S_1 \\
&= \frac{1}{2} \int \mathrm{d}^2\mathbf{x}\ (\nabla\Phi)^2 + z \int \frac{\mathrm{d}^2\mathbf{x}}{a^2} \cos\beta\Phi(\mathbf{x})
\end{aligned}
$$



$$d \equiv \beta^2/4\pi$$

•Luther-Emery point

β_s^2 is close to 4π.

$$H_{MT} = H_0 + H_{\mathrm{int}}$$
$$= \sum_k \psi^\dagger(k)(ku_s\hat{\tau}_3 - m\hat{\tau}_2 - \mu)\psi(k) + 2g\sum_k J(k)\bar{J}(-k) \qquad \mu = \mu_B \mathrm{H}$$

weakly interacting massive Thirring fermions.

•Bogolubov transformation:

$$\psi(k) = \exp(\mathrm{i}\gamma(k)\hat{\tau}^1/2)\ \Psi(k), \quad \Psi(k) = \begin{pmatrix} a_+(k) \\ a_-(k) \end{pmatrix}$$

$$\tan[\gamma(k)/2] = m/ku_s.$$

$$H_0 = \sum[\varepsilon_+(k)a_+^\dagger(k)a_+(k) + \varepsilon_-(k)a_-^\dagger(k)a_-(k)]$$

$$\varepsilon_\pm(k) = \pm\sqrt{k^2u_s^2 + m^2} - \mu$$

•Magnetic field. C-IC transition

$$H_s = \frac{u_s}{2}[\Pi_s^2 + (\partial_x \Phi_s)^2] - \frac{\beta_s \mu_B \mathrm{H}}{2\pi}\partial_x \Phi_s - \frac{m}{\pi a_0}\cos \beta_s \Phi_s$$

$$\mathrm{M(H)} \sim (\mathrm{H} - \mathrm{H_C})^{1/2}, \quad \chi_s(\mathrm{H}) \sim (\mathrm{H} - \mathrm{H_C})^{-1/2}, \quad (\mathrm{H} \to \mathrm{H_C} + 0)$$

(Japaridze, Nersesian; Pokrovskii, Talapov)

This is a consequence of the fact that solitons appearing in the spin sector above the critical field h_c behave themselves as free fermions. They have a quadratic dispersion, and the soliton density is proportional to the magnetization m. The kinetic energy of solitons is thus proportional to m^3 and minimization of their total energy $E \sim [-(h-h_c)m + const * m^3]$ in the field $h > h_c$ gives the square root dependence $m \sim \sqrt{h-h_c}$

•Nonlinearity of fermion spectrum

$$H \to H + \delta H, \qquad \delta H = -\Gamma \Psi^\dagger \frac{\partial^2}{\partial x^2} \Psi,$$

$$\Gamma = \frac{1}{2} \frac{\partial^2 \epsilon}{\partial k^2}|_{k_F} \ \approx \ \cos \frac{\pi \rho}{2}.$$

$$\delta H = \left(\frac{\pi}{2}\right)^{1/2} \Gamma : \left\{ \left(\frac{\partial \varphi}{\partial x}\right)^3 + 3 \frac{\partial \varphi}{\partial x} \left[\left(\frac{\partial \sigma}{\partial x}\right)^2 + \pi_\varphi^2 + \pi_\sigma^2 \right] + 6 \pi_\varphi \pi_\sigma \left(\frac{\partial \sigma}{\partial x}\right) \right\} : .$$

•Electric current

$$j = -\delta H / \delta A = \Psi^\dagger \sigma_z \Psi = (\pi/2)^{1/2} \pi_\varphi \propto \partial \varphi / \partial t,$$

$$j \Rightarrow j + \delta j, \qquad \delta j = \Gamma \Psi^\dagger (-i \frac{\partial}{\partial x}) \Psi \sim -\Gamma : (\frac{\partial \varphi}{\partial x} \pi_\varphi + \frac{\partial \sigma}{\partial x} \pi_\sigma) :,$$

•Repulsive case : U > 0

$$H = H(\varphi) + H(\sigma)$$

$$H(\varphi) \approx \frac{a}{2}(\partial_x \varphi)^2 + \frac{b}{2}\pi_\varphi^2; \qquad H(\sigma) \approx \frac{c}{2}(\partial_x \sigma)^2 + \frac{d}{2}\pi_\sigma^2.$$

•Spin excitations current

$$H_\sigma \propto \pi_\sigma^2 + (\frac{\partial \sigma}{\partial x})^2 = (cd)^{1/2} \sum_k |\, k \,|\, a_k^\dagger a_k,$$

$$|\Omega| = a_k^{+}|0> \Longrightarrow <\Omega|j|\Omega> = \Gamma k$$

Brazovskii, S. M., Nozieres (95)

•Strong coupling limit

$$\begin{aligned}\Omega \; &= \; \frac{v_\uparrow}{2}\left[(\partial_x\phi_\uparrow)^2+(\partial_x\theta_\uparrow)^2\right]+\frac{v_p}{2}\left[(\partial_x\phi_p)^2+(\partial_x\theta_p)^2\right]\\ &- \; \frac{h}{2}\frac{\partial_x\phi_\uparrow}{\sqrt{\pi}}+W\cos\sqrt{4}\pi\phi_\uparrow-\mu\frac{(\partial_x\phi_\uparrow+2\partial_x\phi_p)}{\sqrt{\pi}}.\end{aligned}$$

$$\Pi_{c(s)}=\partial_x\theta_{c(s)},\; \left[\phi_{c(s)}(x),\Pi_{c(s)}(y)\right]=i\delta(x-y).$$

$$\mathcal{N}=(\partial_x\phi_\uparrow+2\partial_x\phi_p)/\sqrt{\pi}$$

At the critical field $h_{cr}=2\Delta$, where 2Δ is equal to the binding energy of the pairs, the low-momentum dispersion relation for spin-$\uparrow$ fermions is $E_\uparrow(k)=\sqrt{v_\uparrow^2k^2+\Delta^2}-\Delta\simeq v_\uparrow^2k^2/2\Delta$, with $v_\uparrow$ being their velocity. The bound pairs disperse linearly with velocity $v_p\neq 0$. Magnetization density is expressed as follows:

$$m=\frac{1}{2}\delta n_\uparrow=-\delta n_p,$$

- At a fixed μ the fields $\partial_x \phi_p, \partial_x \theta_p$ and $\partial_x \phi_\uparrow, \partial_x \theta_\uparrow$ are decoupled and we have sine-Gordon like square root dependence of magnetization on the field $m \sim \sqrt{h - h_{cr}}$, for $h \to h_{cr} + 0$.

- At a constant number of particles we have a constraint $\langle \mathcal{N} \rangle = 0$,,$\to$ $\chi = \partial m / \partial h|_{h_{cr}} = 1/\pi v_p$

For strong coupling Hubbard model $(-U \gg t)$ the Bethe Ansatz inverse susceptibility is given by $\chi^{-1}(|U| \to \infty) = 2\pi^2 \nu (1-\nu)^2/|U|$ (Woynarovich91), which at a low filling factor ν tends to our strong coupling result , with $v_p = 2\pi\nu/|U|$.

•Spin gap case (U < 0 Hubbard model)

$$S_E = \int dx d\tau \left\{ \sum_{\alpha=c,s} \frac{u_\alpha}{2K_\alpha}\left[(\partial_x\phi_\alpha)^2 + \frac{1}{u_\alpha^2}(\partial_\tau\phi_\alpha)^2\right] \right.$$

$$+\frac{g_s}{2\pi}\cos\left(\sqrt{8\pi}\phi_s\right) - \frac{h}{v_F}\frac{\partial_x\phi_s}{\sqrt{2\pi}} + \frac{\sqrt{2\pi}\kappa}{K_s K_c v_F}\partial_x\phi_s\partial_\tau\phi_s\partial_\tau\phi_c$$

$$\left. + \frac{\sqrt{\pi}\kappa}{\sqrt{2}v_F}\partial_x\phi_c\left[(\partial_x\phi_s)^2 + (\partial_\tau\phi_s)^2/K_s^2\right] + \cdots \right\}$$

$$H = H_\sigma^0 + \frac{2g_{1\perp}}{(2\pi\alpha)^2}\int dx\ \cos(2\sqrt{2}\phi_\sigma)$$

$$u_\rho K_\rho = u_\sigma K_\sigma = v_F$$

$$u_\rho/K_\rho = v_F(1+\frac{U}{\pi v_F})$$

$$H = \frac{1}{2\pi}\int dx[uK(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

$$u_\sigma/K_\sigma = v_F(1-\frac{U}{\pi v_F})$$

$g_{1\perp} = U$ •(for the Hubbard model)

For finding the susceptibility at a given number of particles we have to impose a constraint: $\langle \partial_x \phi_c \rangle = 0$, which allows us to integrate out the charge modes. We calculate the ground state energy at the onset of magnetization, confining ourselves to the terms proportional to m^2. For extracting these terms we write: $\partial_x \phi_s =: \partial_x \phi_s : + \sqrt{2\pi} m$. This ammounts to separation of $\partial_x \phi_s$ into its mean part and fluctuations at $h > h_{cr}$. Then, after integrating out charge degrees of freedom, the Euclidean action is $S_{eff} = S_s^0 + S_\kappa$, where:

$$S_s^0 = \frac{1}{2K_s} \int \Big[(: \partial_\tau \phi_s :)^2 + (: \partial_x \phi_s :)^2 + 2\pi m^2 + \frac{g_s K_s}{\pi} \cos\left(\sqrt{8\pi} : \phi_s : + 4\pi m x\right) \Big] \, \mathrm{d}\tau \mathrm{d}x \ ,$$

and it does not give rise to an m^2 contribution in the ground state energy (Japaridze,Nersesyan;Pokrovski,Talapov)

Retaining only contributions proportional to m^2, the term S_κ originating from the spin-charge interaction is given by:

$$S_\kappa = \frac{2m^2\kappa^2\pi^2}{v_F^2}\int \sum_{i,j=0,1}^{i\neq j} \Big[\partial^2_{x_i y_i} G_c(\mathbf{x},\mathbf{y}) :\ \partial_{x_i}\phi_s(\mathbf{x}) :: \partial_{y_i}\phi_s(\mathbf{y}) :$$
$$- \partial^2_{x_i y_j} G_c(\mathbf{x},\mathbf{y}) : \partial_{x_i}\phi_s(\mathbf{x}) :: \partial_{y_j}\phi_s(\mathbf{y}) :\Big]\ d\mathbf{x}d\mathbf{y}.$$

Here $\mathbf{x} = \{x,\tau\} \equiv \{x_0, x_1\}$, and $\mathbf{y} = \{y,\tau'\} \equiv \{y_0, y_1\}$, and the propagator for the charge sector is $G_c(\mathbf{x},\mathbf{y}) = -K_c/4\pi \ln((x-y)^2/a^2 + (\tau - \tau')^2/a^2 + 1)$, where a is a short distance cut-off.

•Using Eucidean invariance and explicit forms of Green functions:

$$\left\langle \partial_{x_i}\phi_s \partial_{y_j}\phi_s \right\rangle = \partial_{x_i}\partial_{y_j} G_s(r), \text{ where } r = \sqrt{(x-y)^2 + (\tau - \tau')^2}$$

$$\left\langle :\partial_{x_i}\phi_s :: \partial_{y_j}\phi_s : \right\rangle_{h_{cr}+0} = \left\langle \partial_{x_i}\phi_s \partial_{y_j}\phi_s \right\rangle_{h_{cr}-0} + O(m).$$

$$G_c(\mathbf{x}, \mathbf{y}) = -\frac{K_c}{4\pi} \ln((x-y)^2/a^2 + (\tau - \tau')^2/a^2 + 1)$$

$\Rightarrow$ m^2 contribution to the energy can be obtained by using a simplified effective action:

$$S_{eff} = S_s^0 - \frac{m^2 \kappa^2 \pi^2 K_c}{v_F^2} \int dx d\tau \left[(\partial_x \phi_s)^2 + (\partial_\tau \phi_s)^2 \right].$$

⟺ •Renormalization of the Luttinger parameter of spin sector:

$$K_s \to K_s \left(1 + 2m^2\kappa^2\pi^2/v_F^2\right).$$

⇓

$$\Delta\mathcal{E}_0(m^2) = \frac{\partial\mathcal{E}_0}{\partial K_s}\Delta K_s = \frac{2K_s m^2\kappa^2\pi^2}{v_F^2}\frac{\partial\mathcal{E}_0}{\partial K_s}.$$

⇓

$$v_F^2\chi^{-1} = 4K_s\kappa^2\pi^2\partial\mathcal{E}_0/\partial K_s$$

$\mathcal{E}_0$ is the ground state energy of the sine- Gordon model.

For $K_s \to 1$ we follow the RG results (Amit, Kosterlitz) $\to$ in the one-loop approximation $\mathcal{E}_0 = -\lambda\Delta^2/v_F$, where Δ is the soliton mass (gap in the excitation spectrum), and $\lambda > 0$ which wel fix later for the $SU(2)$ symmetric sine- Gordon case.

$$\Delta \simeq E_F \begin{cases} \exp\{-\frac{\arctan\sqrt{g_s^2/(2-2K_s)^2-1}}{\sqrt{g_s^2-(2-2K_s)^2}}\}; & \frac{|g_s|}{(2-2K_s)} \geq 1 \\ \exp\{-\frac{\mathrm{arctanh}\sqrt{1-g_s^2/(2-2K_s)^2}}{\sqrt{(2-2K_s)^2-g_s^2}}\}; & \frac{|g_s|}{(2-2K_s)} \leq 1 \end{cases}$$

$$g_s \ll 1, \quad K_s \sim 1$$

In the vicinity of the $SU(2)$ separatrix of the sine- Gordon RG flow

$$\chi^{-1} = \frac{4\lambda K_s \kappa^2 \pi^2 \Delta^2}{3(1-K_s)^2 v_F^3} = \frac{16\lambda K_s \kappa^2 \pi^2 \Delta^2}{3 v_F^3} \ln^2 \frac{\Delta}{E_F}$$

The same result is valid in the vicinity of the Luther-Emery point $K_s \to 1/2$. By mapping the spin sector onto free massive fermions $\to$ $\chi^{-1} \propto \kappa^2 \partial \mathcal{E}_0 / \partial K_s \propto \kappa^2 \left\langle (\partial_t \phi_s)^2 / v_F^2 - (\partial_x \phi_s)^2 \right\rangle \propto$ $(\kappa \Delta \ln \Delta / E_F)^2$.

In the case of the Hubbard model with attractive interaction $U < 0$, one has $1 - K_s \simeq |U| / 2\pi v_F$ and this result is similar to the Bethe Ansatz calculation in the weak coupling limit (Woynarovich): $\chi^{-1}(|U \to 0) = 8\kappa^2 \pi^3 \Delta^2 / v_F U^2$. This implies that the factor λ is equal to $3/2\pi$ on the $SU(2)$ line.

•Correlation functions

$$\mathcal{H}_{eff} = \sum_{\beta=\pm} \frac{v_\beta}{2}\left[(\partial_x\phi_\beta)^2/K_\beta + K_\beta(\partial_x\theta_\beta)^2\right].$$

$$\phi_+ = \phi_c - \xi\phi_s,\ \theta_+ = \theta_c,\quad \phi_- = \phi_s,\ \theta_- = \theta_s + \xi\theta_c;$$

Penc,Solyom

and $v_\pm$, $K_\pm$ are the Bethe Ansatz velocities and Luttinger parameters for the $\pm$ sectors. For $m \to 0$ we have $v_- \propto m \to 0$, $K_- \to 1/2$ at any U and ν [24]. In the case of half filling ($\nu = 1$) one has $K_+ = 1$, $\xi = 0$ for all $|U|$, and there is an exact spin-charge separation so that the fields $\phi_\pm, \theta_\pm$ coincide with $\phi_{c,s}, \theta_{c,s}$. For $\nu < 1$ one has

$$K_+ = 1 + \frac{|U|}{2\pi v_F};\ \xi = \sqrt{\frac{8v_F}{|U|}}\cos\left(\frac{\pi\nu}{2}\right)\exp\left(-\frac{\pi v_F}{|U|}\right)$$

The limit of $m \to 0$ allows us to derive analytical expressions for the critical exponents of the correlation functions and make a number of physical conclusions. For the pair correlation function from Eq. (14) we obtain:

$$\langle \psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\psi_\downarrow(0)\psi_\uparrow(0)\rangle \propto \frac{\cos 2\pi m x}{x^{1/2+1/K_+}}; \ \ x \to \infty, \quad (17)$$

whereas for $h < h_{cr}$ it is $\sim x^{-1/K_+}$. There is a universal jump of 0.5 in the critical exponent, the result that is expected from the theory based on spin-charge separation. However, for the single fermion Green function we find:

$$\langle \psi_{\uparrow(\downarrow)}^\dagger(x)\psi_{\uparrow(\downarrow)}(0)\rangle \propto \frac{\cos k_{F\uparrow(\downarrow)}x}{x^{\nu_{\uparrow(\downarrow)}}}; \ \ x \to \infty, \quad (18)$$

The critical exponents:

$$\nu_\uparrow = 1/2 + K_+/4 + (1+\xi)^2/8 + (1-\xi)^2/4K_+$$

$$\nu_\downarrow = \nu_\uparrow + (1/K_+ - 1/2)\xi > \nu_\uparrow$$

. $\nu_\uparrow < \nu_\downarrow$ even in the limit of $m \to 0$, which is a clear signature of spin-charge coupling.

Conclusions

- We develop a bosonization approach for finding magnetic susceptibility of 1D attractive two component Fermi gas at the onset of magnetization taking into account the curvature effects. It is shown that the curvature of free dispersion at Fermi points couples the spin and charge modes and leads to a linear critical behavior and finite susceptibility for a wide range of models.

The curvature couples spin and charge modes for $m\to 0$ and changes critical properties of 1D spin gapped fermions at the onset of magnetization.