Quantum divisibility test and its application in mesoscopic physics

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We present a quantum algorithm to transform the cardinality of a set of charged particles flowing along a quantum wire into a binary number. The setup performing this task (for at most N particles) involves log2 N quantum bits serving as counters and a sequential read out. Applications include a divisibility check to experimentally test the size of a finite train of particles in a quantum wire with a one-shot measurement and a scheme allowing to entangle multi-particle wave functions and generating Bell states, Greenberger-Horne-Zeilinger states, or Dicke states in a Mach-Zehnder interferometer.

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a quantum algorithm to transform the cardinality of a set of charged particles flowing along a quantum wire into a binary number.

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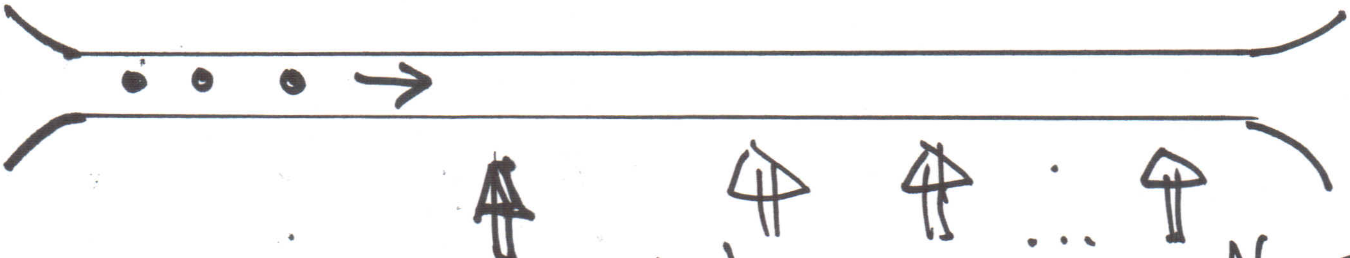
Applications include:

a divisibility check to experimentally test the size of a finite train of particles in a quantum wire with a one-shot measurement

and

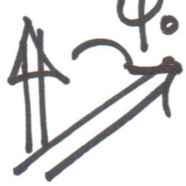
a scheme allowing to entangle multi-particle wave functions and generating Bell states, Greenberger-Horne-Zeilinger states, or Dicke states in a Mach-Zehnder interferometer.

Counting of $n < N$



spin $\frac{1}{2}$

N_m spins or N_m measurements iterations



φ_0 — after 1 electron passed $\quad \varphi_0 < \frac{\pi}{N}$

$$P_\uparrow = \cos^2(n\varphi_0/2)$$

measuring $P_\uparrow = \dfrac{\langle m_\uparrow \rangle}{N}$

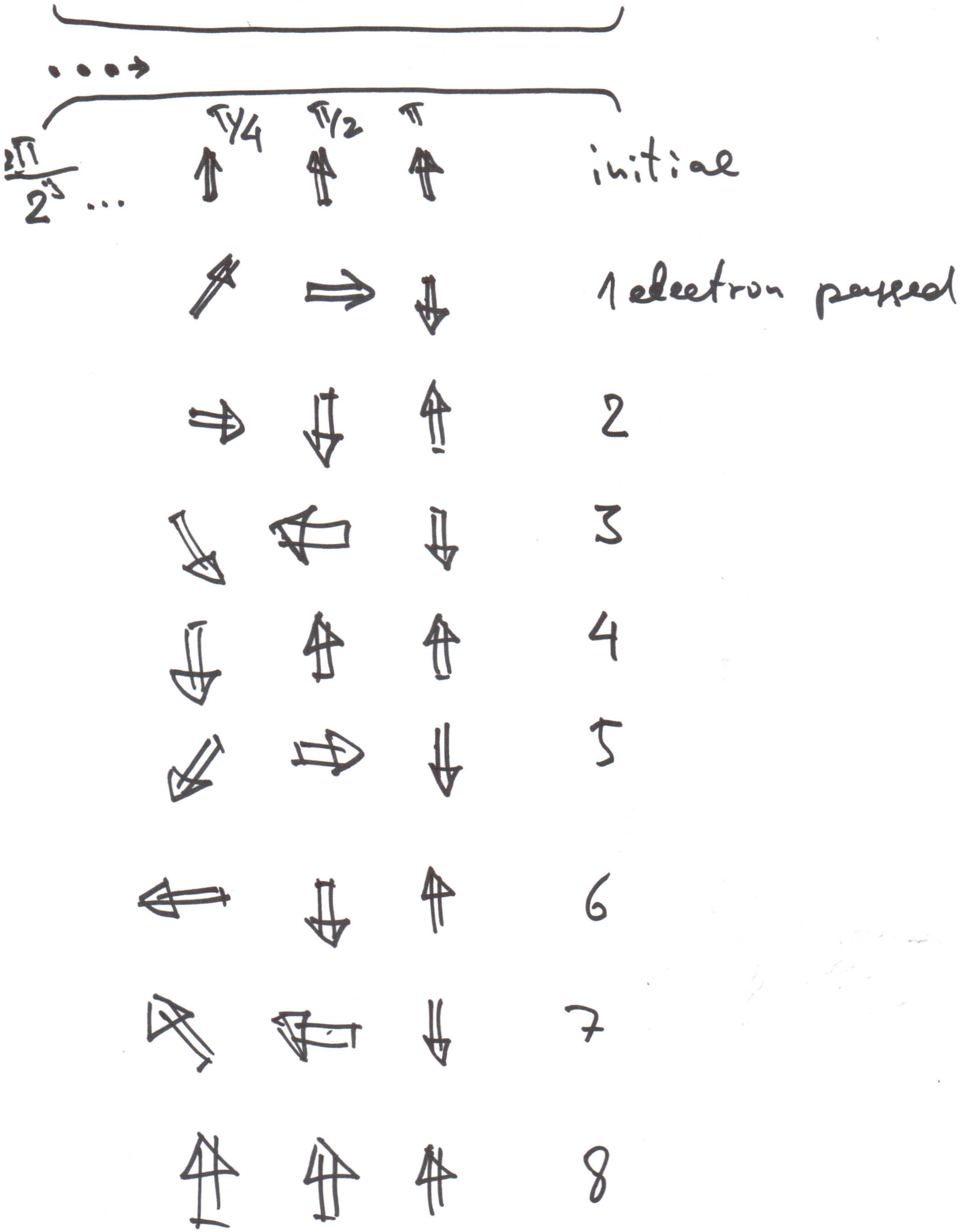
$$n = \frac{2}{\varphi_0} \arccos\sqrt{P_\uparrow}$$

To have $\delta P^\uparrow = P^\uparrow(n+1) - P^\uparrow(n) \gg$

$$\gg \left(\frac{\langle(\delta m^\uparrow)^2\rangle}{N_m}\right)^{1/2} = [P^\uparrow(1-P^\uparrow)/N_m]^{1/2}$$

$$N_m \gg \frac{N^2}{\pi^2} \; (\gg 1)$$

Effective counting



$$\varphi_j = \frac{2\pi}{2^j}\cdot n$$

$$e^{i\varphi_j/2}|\uparrow\rangle + e^{-i\varphi_j}|\downarrow\rangle$$

like in quantum Fourier transformation

$$|n\rangle \to \frac{1}{\sqrt{N}}\sum_{q=0}^{N-1} e^{2\pi i q n/N}|q\rangle$$

$$|n_k n_{k-1}\dots n_1\rangle \to$$

$$\to \frac{1}{2^{k/2}}\left(|0\rangle + e^{2\pi i \frac{n_1}{2}}|1\rangle\right)\left(|0\rangle + e^{2\pi i\left[\frac{n_2}{2}+\frac{n_1}{4}\right]}|1\rangle\right)\dots\left(|0\rangle + e^{2\pi i[0,n_k\dots n_1]}|1\rangle\right) =$$

$$= \frac{1}{2^{k/2}}\bigotimes_{q=1}^{k}\left[|0\rangle + e^{2\pi i n/2^q}|1\rangle\right]$$

We measure state of spins, starting 1-st, and rotate the others to $\Delta\varphi_j = -\frac{2\pi}{2^j}(n_{j-1}\dots n_1)$

$$\varphi_j + \Delta\varphi_j = \frac{2\pi}{2^j} n_j \cdot 2^{j-1} = \pi \cdot n_j$$

So, unrotated spin means $n_j = 0$
rotated to π means $n_j = 1$

We got binary representation of

$$n = (n_k n_{k-1} \dots n_1)$$

Simple single shot check of divisibility to 2^K

if at least one spin is detected „down" the number n is NOT divisible to $2K$

if all „up"

then $n = 2^K \cdot C$ C- integer

$$P_\uparrow = \cos^2 \frac{\pi n}{2^j}$$

if $0 < n < 2^K$

$n = 2^m \cdot D$ D is odd

$0 \leq m < K$

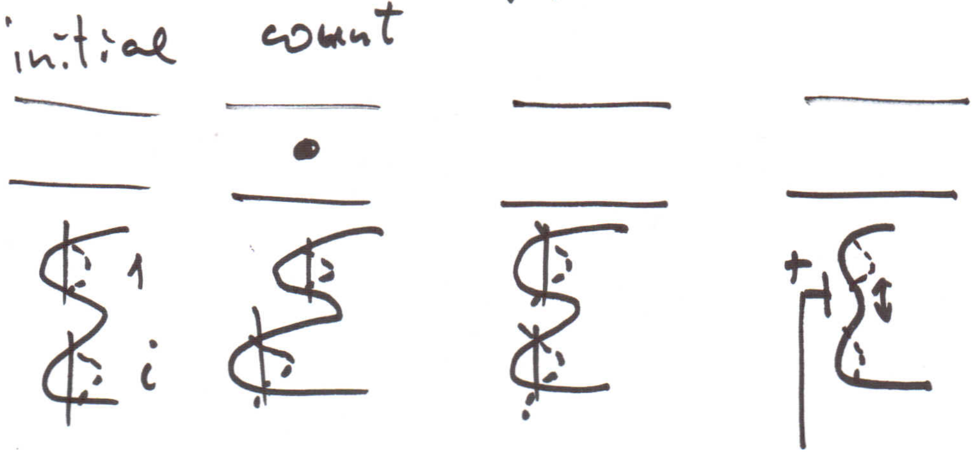
for $j^* = m+1$ ONLY

$$P_\uparrow = \cos^2\left(\frac{\pi \cdot 2^m}{2^{m+1}} \cdot D\right) = 0$$

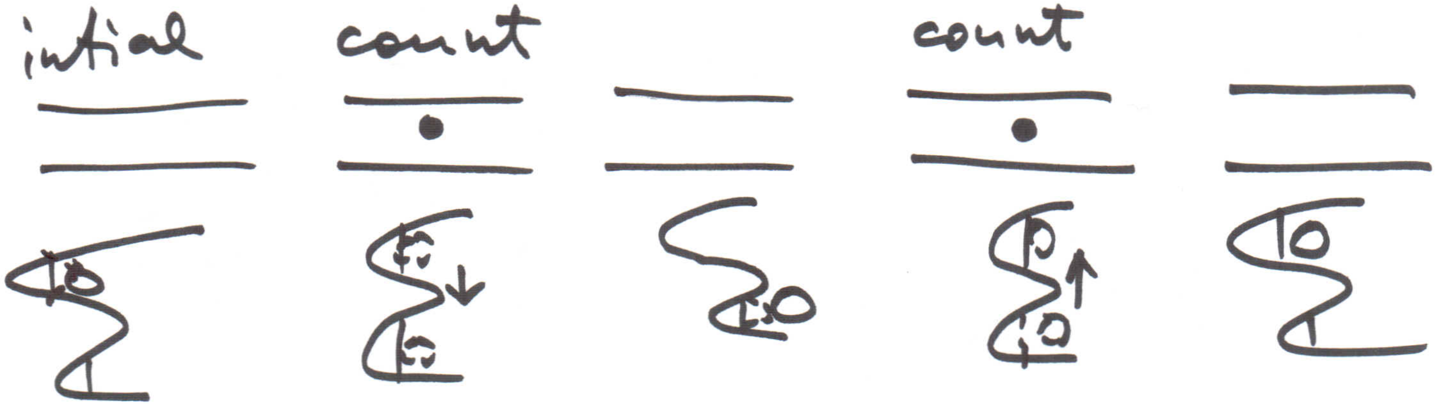
for all the other j Φ_j is a multiple of π $(j < j^*)$

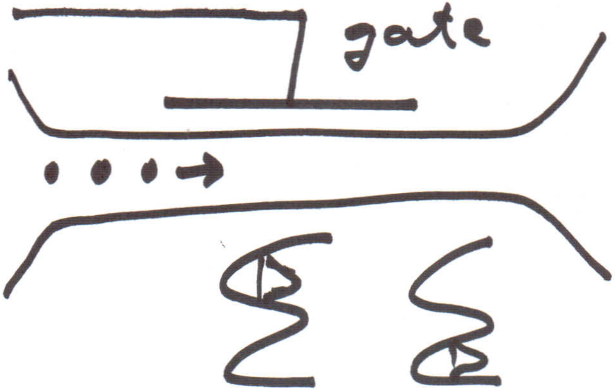
or a fraction of $\pi/2$ $(j > j^*)$

PHASE MANIPULATION



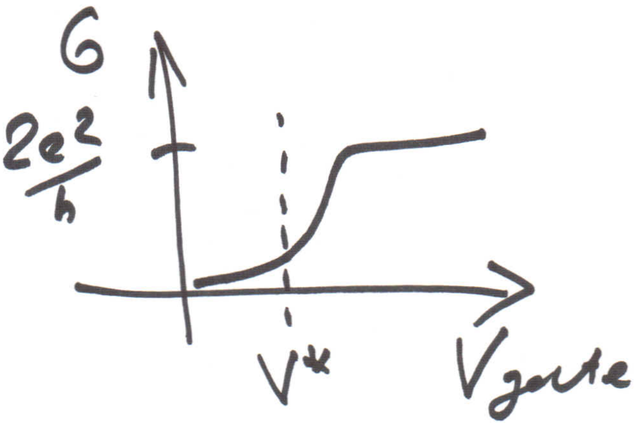
Probability tunneling





effective potential due to quantization +
+ gate potential

DD qubit potential



$$\psi_{4,A} = \{[(-1)|\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\downarrow\rangle]\otimes|0\rangle +$$

$$+(\mp i)[|\uparrow,\downarrow\downarrow\downarrow\rangle + \dots]\otimes|1\rangle +$$

$$+\dots$$