Wave kinetic in fiber laser with random Rayleigh backscattering

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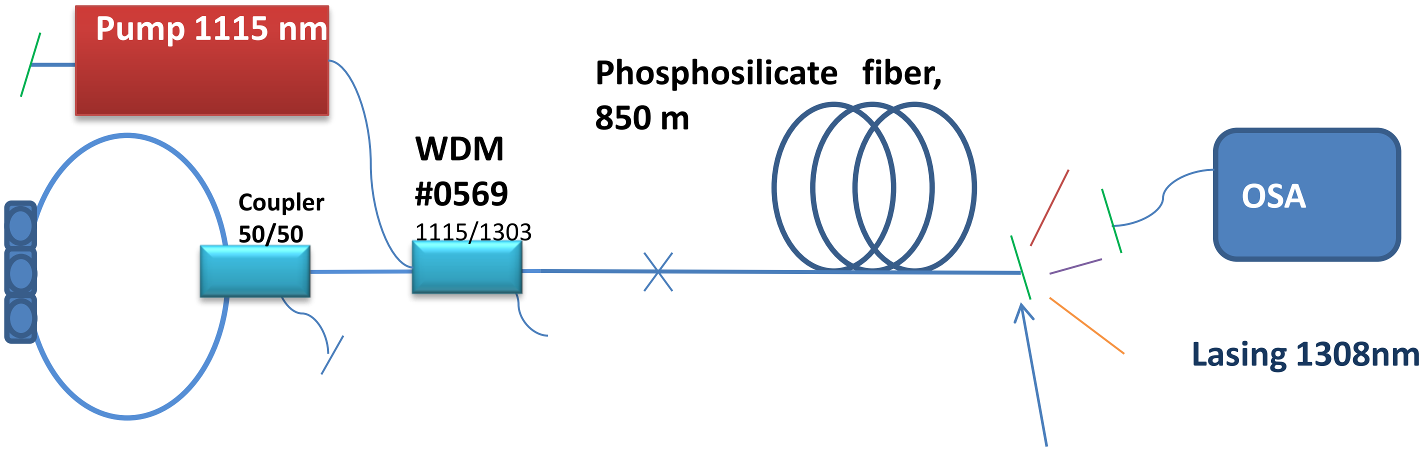
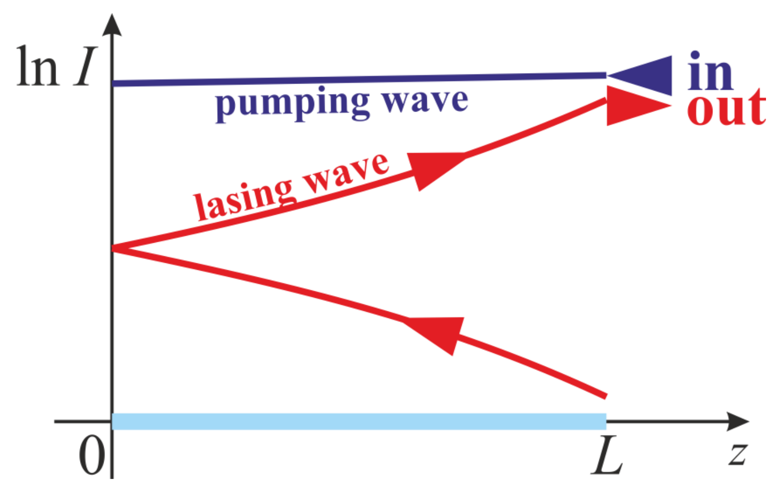
Outline

- 1. Experimental setup
- 2. General model of the experiment. Formulation of the reduced model
- 3. Qualitative picture
- 4. Kinetic description for the reduced model
- 5. Results

1. Experimental setup



Equivalent scheme:



2. General model of the experiment

$$\left[n\partial_t + \left(\partial_z - \frac{1}{2}P\hat{g} + \frac{1}{2}\alpha\right)\right]A_+ \;=\; \frac{i}{2}\beta_2\,\partial_t{}^2 A_+ + i\gamma\, A_+|A_+|^2 + rA_-$$

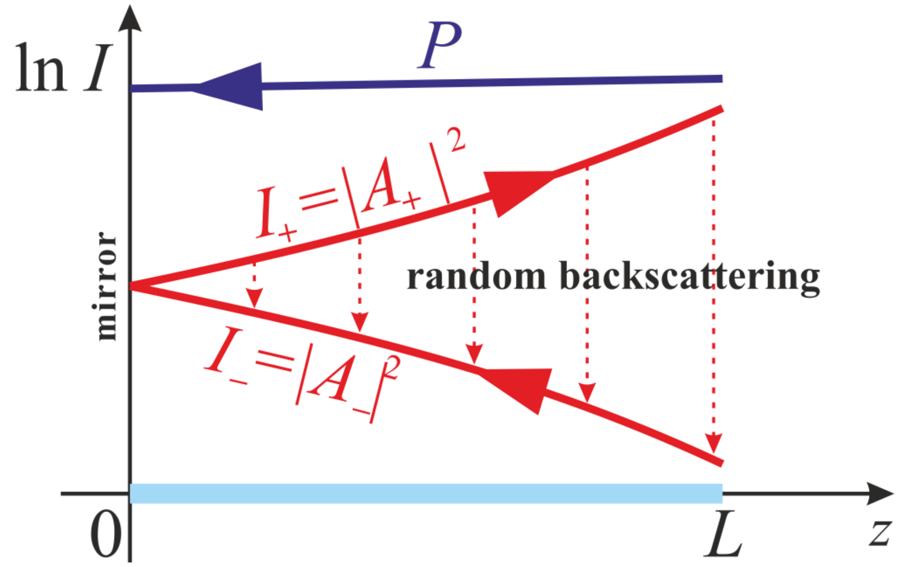
$$\left[n\partial_t - \left(\partial_z - \frac{1}{2}P\hat{g} + \frac{1}{2}\alpha\right)\right]A_- \;=\; \frac{i}{2}\beta_2\,\partial_t{}^2 A_- + i\gamma\, A_-|A_-|^2 + r^* A_+$$

n – refractive index
β_2 – chromatic dispersion
α – linear losses
γ – Kerr nonlinearity

r – random backscatters

P(z) – pumping intensity profile

$\hat{g}$ – gain coefficient



2. Parameters of the model

Gain coefficient:

in frequency domain

$$g(\omega) \;=\; g_R\left(1-\frac{(\omega-\omega_S)^2}{{\Delta_g}^2}\right)$$

$g_R = 1.35\ 1/W/km$

$\Delta_g \approx 5.5\ 1/ps$

The width of the lasing spectrum

$\Delta \approx 0.7 \div 1.3\ 1/ps$

Random backscatters:

statistics in a sense of spatial average

$$\langle r(z)\, r^*(z')\rangle \;=\; D\,\delta(z-z')$$

nature of the backscattering:

$D \approx \alpha/600$

linear losses

$\alpha \approx 0.18\ 1/km$

$$\left[n\partial_t + \left(\partial_z - \frac{1}{2}P\hat{g} + \frac{1}{2}\alpha\right)\right]A_+ \;=\; \;=\; \frac{i}{2}\beta_2\,{\partial_t}^2 A_+ + i\gamma\, A_+|A_+|^2 + rA_-$$

Length of the fiber

$L = 850\ m$

Pumping wave intensity:

$P_{th} = 5.5\ W, \quad P < 10W$

Lasing intensity:

$I_{out} \le 2.2\ W$

Kerr nonlinearity:

$\gamma \approx 3.45\ 1/W/km$

2.1 Balance equations

Equations on full intensity

$$\pm\partial_z I_\pm = g_0 P I_\pm - \alpha I_\pm + D I_\mp$$

$$-\partial_z P = -\frac{\omega_p}{\omega_s} g_R I P - \alpha_p P, \qquad I = I_+ + I_-,$$

final width was neglected, $\Delta \ll \Delta_g$ is assumed

Threshold condition

$$1 = D\int_0^L dz_1 \exp\left[2\,\frac{\exp(-\alpha_p L)\,[\exp(\alpha_p z)-1]}{\alpha_p} g_R P_{th} - 2\alpha_p \alpha z\right]$$

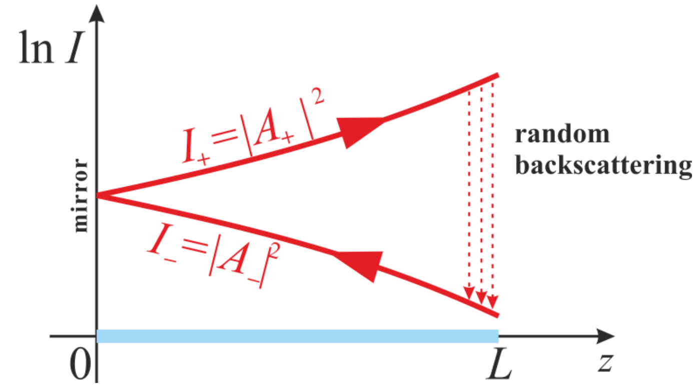
Solution above the threshold

$$I = \frac{C\ exp\left(\alpha_p(z-L)\right)}{\frac{P_{in}}{P_{th}}\ exp\left(C\, g_R P_{th} z_{eff}\right) - \frac{\omega_p}{\omega_s}\frac{I_{out}}{P_{th}}} I_{out}, \qquad P = \frac{C\ \exp(\alpha(z-L))}{\frac{P_{in}}{P_{th}} - \frac{\omega_p}{\omega_s}\frac{I_{out}}{P_{th}}\ \exp\left(-C\, g_R P_{th} z_{eff}\right)} P_{in}\,,$$

$$C = \frac{P_{in}}{P_{th}} - \frac{\omega_p}{\omega_s}\frac{I_{out}}{P_{th}}, \qquad z_{eff} = \frac{1-\exp\left(-\alpha_p(L-z)\right)}{\alpha_p}\ .$$

2.2 Role of random Rayleigh backscattering

$$\left[n\partial_t - \left(\partial_z - \frac{1}{2}P\hat{g} + \frac{1}{2}\alpha\right)\right]A_- = \frac{i}{2}\beta_2\,\partial_t{}^2 A_- + i\gamma A_-|A_-|^2 + r^* A_+$$



Weak mirror with random phase

$$A_\omega \equiv A_{+\omega}$$

$$A_\omega(0) = R(\omega)A_\omega(L),$$

$$\langle R(\omega)R^*(\omega')\rangle = \frac{D\exp[\Pi(L)g(\omega) - \alpha L]}{2\left(g_R \mathrm{P_{in}} - in(\omega - \omega')\right)}, \qquad \Pi(z) = \int_0^z dz_1 P(z_1).$$

Mean strength of the mirror

$$\langle|R(\omega)^2|\rangle = \left(D/2g_R P_{in}\right) exp[\Pi(\mathrm{L})\mathrm{g}(\omega) - \alpha\mathrm{L}]$$

$$\langle|R(\omega)^2|\rangle \leq 10^{-2}$$

Correlation frequency

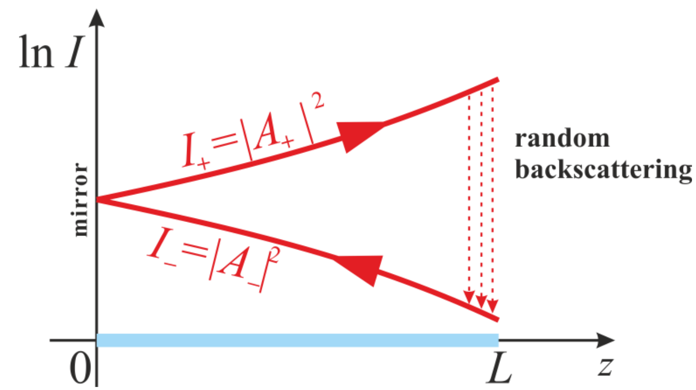
$$\delta\omega_r \sim g_R P_{in}/n.$$

2.3 Spontaneous emission

Balance equations on spectrum
(in absence of nonlinearity)

$$\partial_z I_\omega = g(\omega)PI_\omega - \alpha I_\omega + DI_{\omega-} + 2g(\omega)P\hbar\omega$$

$$-\partial_z I_{\omega-} = g(\omega)PI_{\omega-} - \alpha I_{\omega-} + DI_\omega + 2g(\omega)P\hbar\omega$$



Spontaneous emission is relevant only in regions with small intensity

$$A_\omega \equiv A_{+\omega}$$

Boundary conditions

$$I_\omega(0) = \langle |R(\omega)^2| \rangle I_\omega(L) + \delta I_{\omega,se},$$

$$\delta I_{\omega,se} = 2\hbar\omega g(\omega) \int_0^L exp[\Pi(z)g(\omega) - \alpha z]P(z)dz$$

2.4 Reduced model

Comoving reference frame

$\psi(\mathrm{z,t}) = A\left(\mathrm{z}, \mathrm{t} + \frac{z}{v_g}\right)\exp(ik_s z - i\omega_s t),$

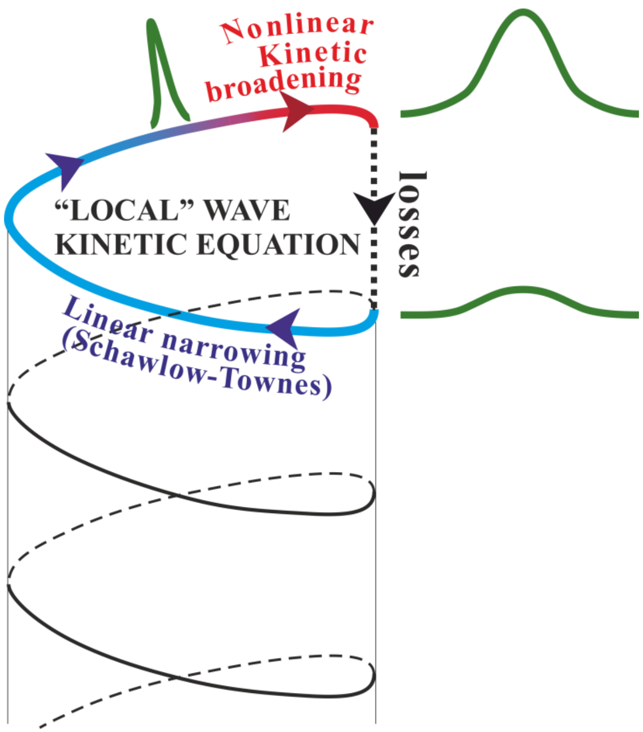
group velocity v_g at carrying frequency ω_s,

$$\mathrm{i}\left(\partial_\mathrm{z} - \frac{1}{2}\mathrm{P}\hat{\mathrm{g}} + \frac{1}{2}\alpha\right)\psi = \frac{1}{2}\beta\,{\partial_t}^2\,\psi + \gamma\psi|\psi|^2, \qquad 0 < z < L$$

Boundary conditions on correlation function

$$I_\omega(0) = \langle|R(\omega)^2|\rangle I_\omega(L) + \delta I_{\omega,se}$$

3. Qualitative picture



Small lasing intensity:

Schawlow-Townes process.

- **broadening due to spontaneous emission**
  acts in region with weakest intensity
- **narrowing due to selective gain**
  acts along the whole fiber

Large lasing intensity

Nonlinear process

- **broadening due to nonlinearity**
  acts in region with highest intensity
- **narrowing due to selective gain**
  acts along the whole fiber

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4. Kinetic description for the reduced model

$$i\left(\partial_z - \frac{1}{2}P\hat{g} + \frac{1}{2}\alpha\right)\psi = \frac{1}{2}\beta\,\partial_t{}^2\,\psi + \gamma\psi|\psi|^2$$

Collision integral for the NLSE is pure zero

V.E.Zakharov, Turbulence in integrable systems, 1999

4.1 Field description and perturbation theory

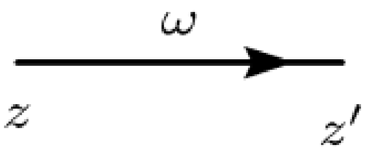
Action

$$S_{\psi p} = \int_0^L dz \int dt \left[\bar{p}\left(\left(\partial_z - \frac{1}{2}P(z)\hat{g}\right)\psi + \frac{i}{2}\beta_2\partial_t^2\psi + i\gamma\,\psi|\psi|^2\right) + \right.$$

$$\left. + p\left(\left(\partial_z - \frac{1}{2}P(z)\hat{g}\right)\bar{\psi} - \frac{i}{2}\beta_2\partial_t^2\psi - i\gamma\,\bar{\psi}|\psi|^2\right)\right]$$

Green function

$G(z,t,z',t') = \langle\psi(z,t)\bar{p}(z',t')\rangle$,

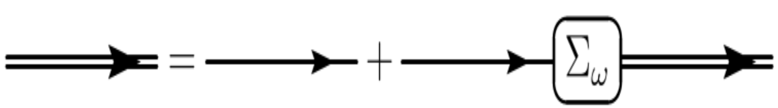


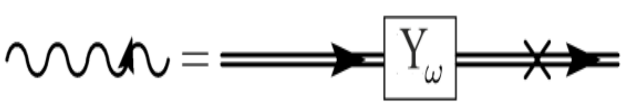
Pair correlation function

$F(z,z';t-t') = \langle\psi(z,t)\,\psi^*(z',t')\rangle$

Spectrum

$$F_\omega(z) \equiv I_\omega(z) = \int dt\, e^{i\omega t}\; F(z,t)$$



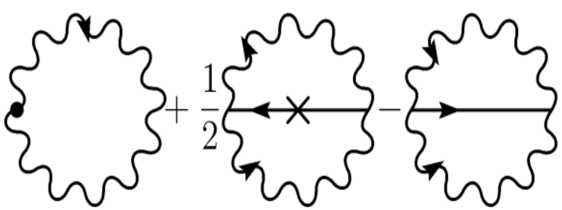


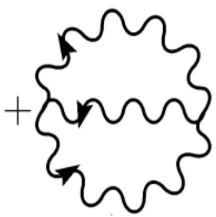
4.2 Kinetic equation

General form of kinetic equation

$$(\partial_z - P(z)g(\omega))F_\omega(z) = \int_0^z dy\,(\Sigma_\omega(z,y)F_\omega(y,z) + G(z,y,\omega)Y_\omega(y,z) + c.c.)$$

Expansion in small nonlinearity

$\Sigma_\omega = -i$  $+ \ldots$

$\mathrm{Y}_\omega = \chi +$  $+ \ldots$

$$(\partial_z - g(\omega)\,P + \alpha)F_\omega(z) = 2\gamma^2 \int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^2}\,\delta(\omega + \omega_1 - \omega_2 - \omega_3)\frac{g_R P(z)}{g_R^2 P^2(z) + \Phi^2} \times$$

$$\times \left[F_\omega F_{\omega_2} F_{\omega_3} + F_{\omega_1} F_{\omega_2} F_{\omega_3} - F_\omega F_{\omega_1} F_{\omega_2} - F_\omega F_{\omega_1} F_{\omega_3}\right],$$

$$\Phi = \beta(\omega^2 + \omega_1^2 - \omega_2^2 - \omega_3^2)$$

Assumptions:

i) slowly varying pumping, $\ln' P \ll g_R P$

ii) small width of the spectrum, $\Delta \ll \Delta_g$

4.2 Parameters in kinetic equation

$$(\partial_z - g(\omega)\,P + \alpha)F_\omega(z) = 2\gamma^2 \int \frac{d\omega_1 d\omega_2 d\omega_3}{(2\pi)^2}\,\delta(\omega + \omega_1 - \omega_2 - \omega_3)\,\frac{g_R P(z)}{g_R^2 P^2(z) + \Phi^2} \times$$

$$\times \left[F_\omega F_{\omega_2} F_{\omega_3} + F_{\omega_1} F_{\omega_2} F_{\omega_3} - F_\omega F_{\omega_1} F_{\omega_2} - F_\omega F_{\omega_1} F_{\omega_3}\right],$$

$$\Phi = \beta(\omega^2 + \omega_1^2 - \omega_2^2 - \omega_3^2)$$

1) **Spectrum squeezing due to gain dispersion** $2g_R \frac{\Delta^2}{\Delta_g^2} \int_0^L P(z)\mathrm{dz} \approx 0.2 \div 0.7$
2) **Dispersion vs. pumping:** $\frac{\beta\Delta^2}{g_R P} \lesssim 0.3$
3) **Integral nonlinearity:** $\gamma \int_0^z I(z)\mathrm{dz} \approx 0.5$
4) **Nonlinearity vs. dispersion** $\frac{\gamma I_{out}}{\beta_2\,\Delta^2} \leq 0.7$
5) **Length where backscattering occurs:** $\delta_r \approx 50m$

4.3 Small nonlinearity, small dispersion

$$\frac{\beta\Delta^2}{g_R P} \ll 1, \qquad \gamma \int_0^z I(z)\mathrm{d}z \ll 1$$

Boundary condition: $F(z=0,t) = \kappa\, F(L,t); \quad \kappa\ \exp[\Pi(z) g_R] = (1+\eta)$

$$F(L) = \exp[\Pi(z) g_R]\, F(0) + \delta F(L);$$

$$\eta F(L) + \delta F(L) = 0$$

Local equation on $F(L,t)$: $\eta\, F + 2\alpha L \partial_t^2 F + \frac{\gamma^2}{8 g_R^2}(F|F|^2 - F I^2)$

$$F(L,t) = \frac{4\, g_R\sqrt{2\alpha L}\Delta}{\gamma\, ch(\Delta\, t)}, \qquad \gamma I = 4\sqrt{2} g_R \sqrt{2\alpha L}\Delta, \qquad \eta = 2\alpha L \Delta^2$$

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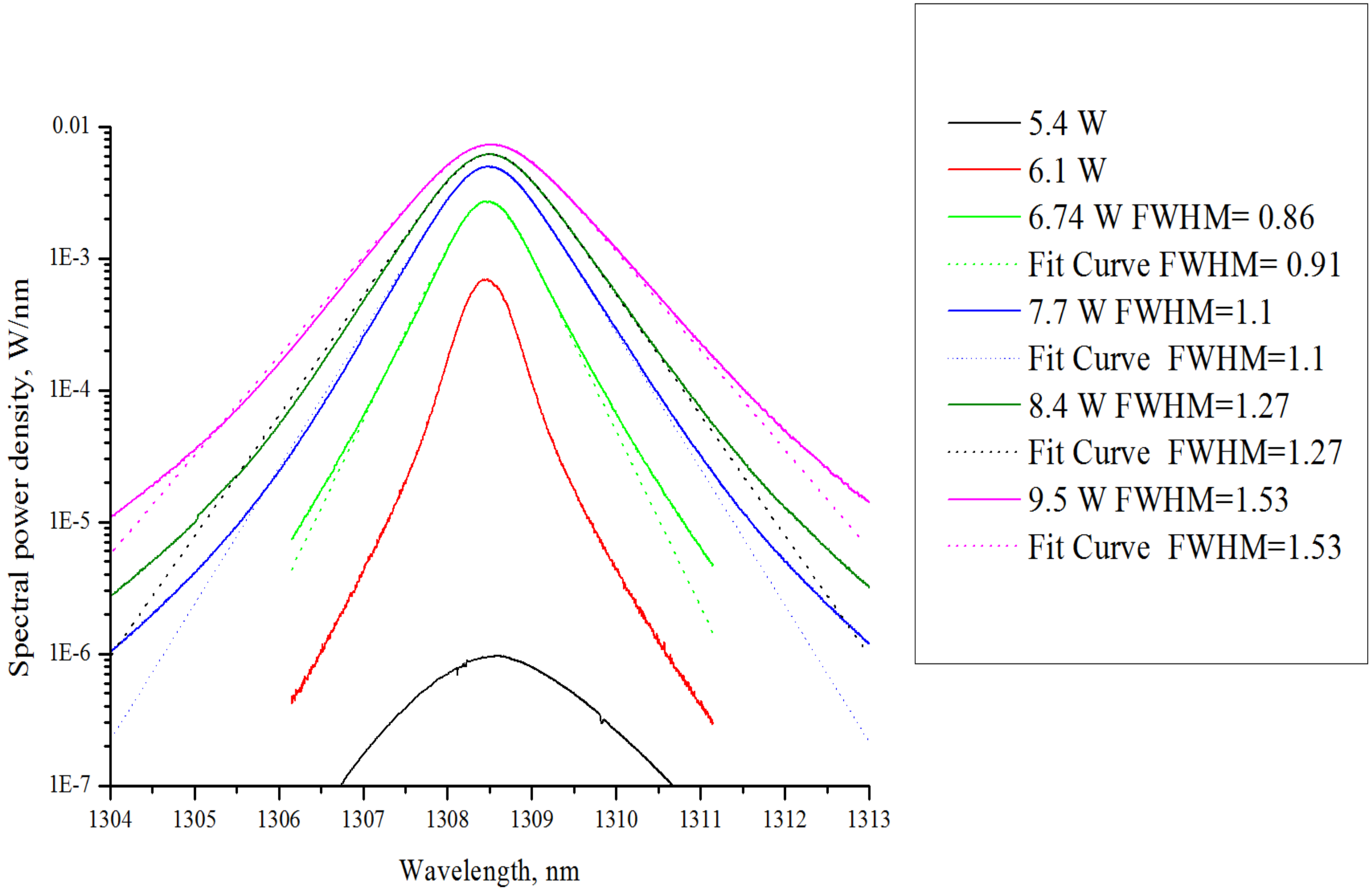
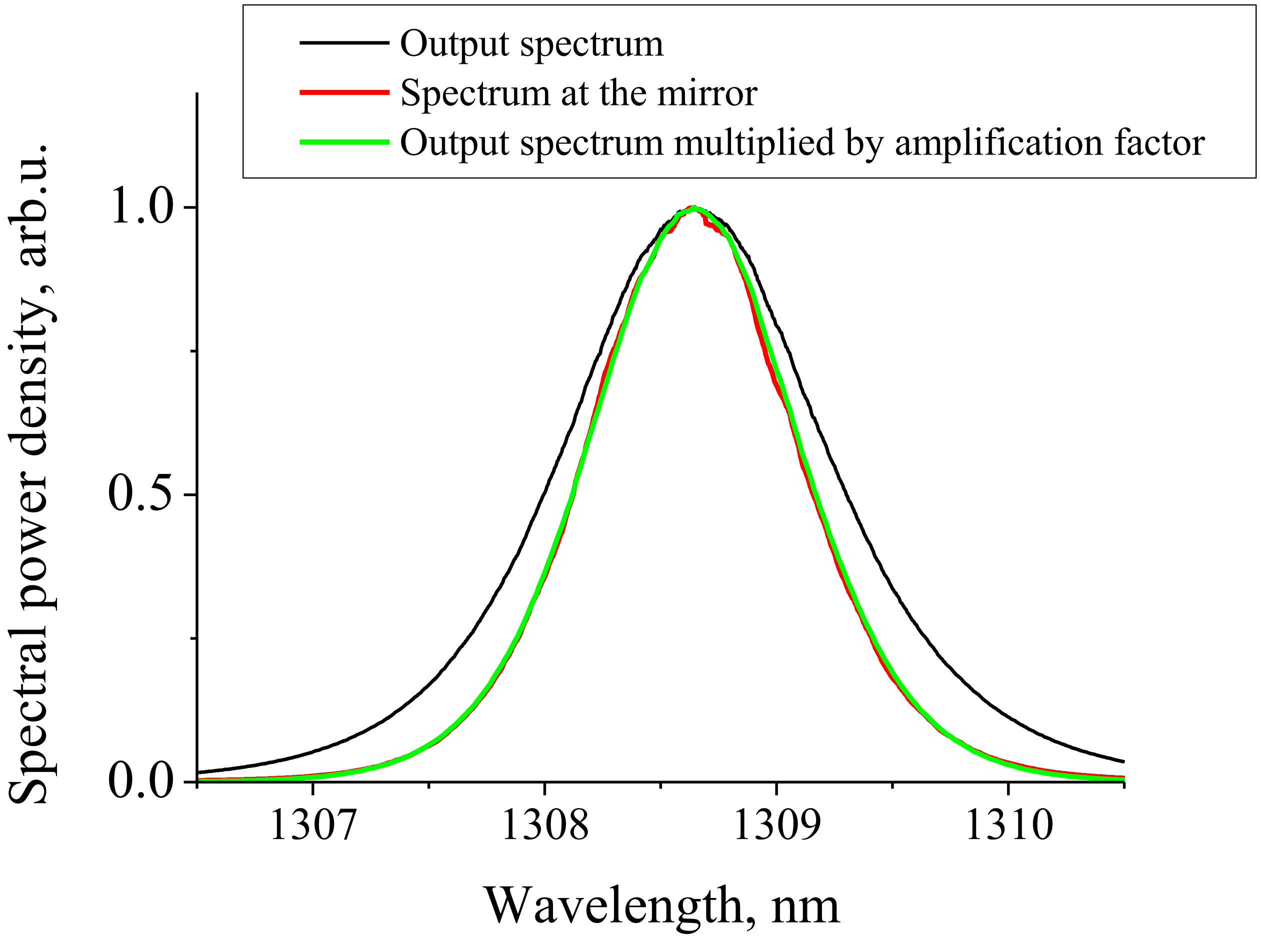
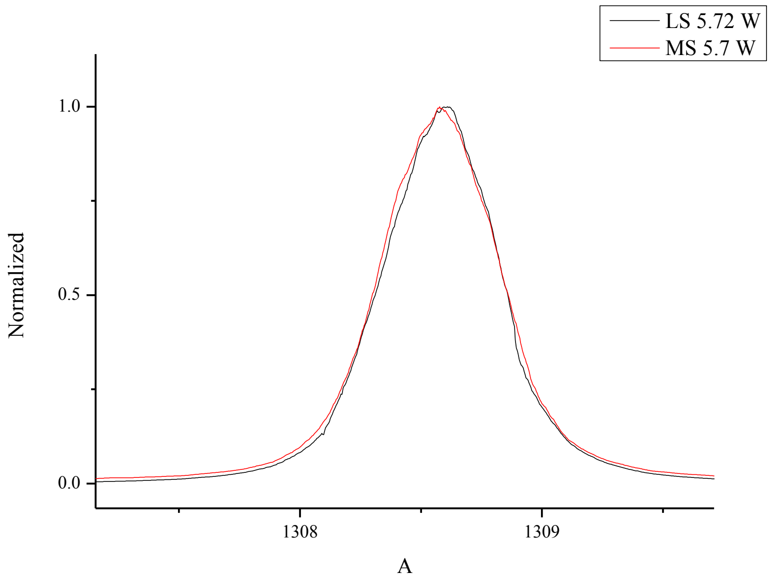
5. Results. 5.1. Experiment

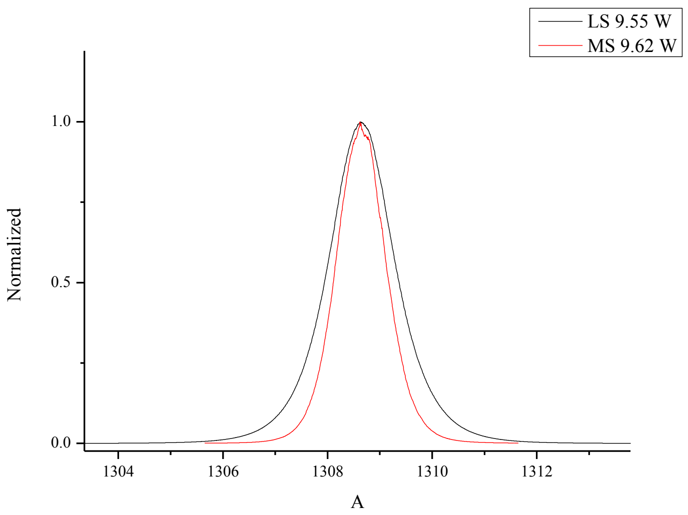


5.3. Results: Experiment vs Theory



5.3. Results: Experiment





5.2. Results: Experiment vs Theory