Many-particle correlations in non-equilibrium Luttinger liquids and singular Fredholm determinants

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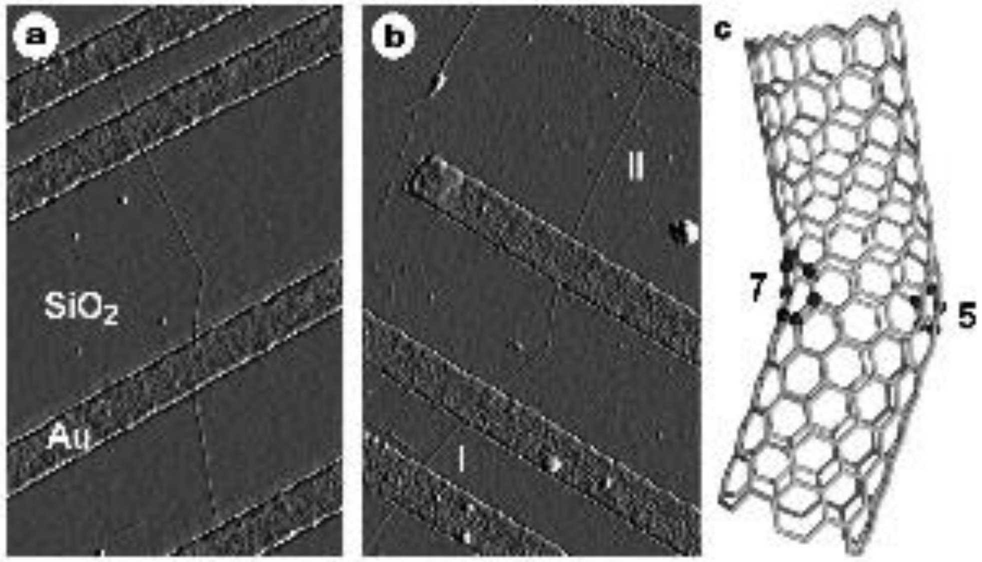
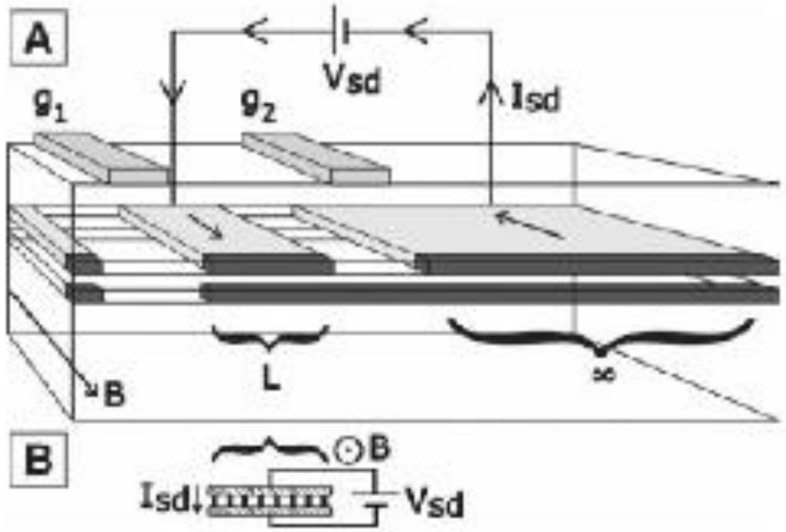
Outline

1) Introduction: **1D** world, Luttinger liquids, bosonization.
2) Summary of non-equilibrium LL: functional bosonization and operator approach.
3) Single-particle correlation functions and Toeplitz determinants.
4) Single-particle correlation functions: Szegő limit theorem and generalized Fisher-Hartwig conjecture.
5) Many-particle correlation functions and more complicated determinants.
6) Generalized Szegő and yet another generalization of Fisher-Hartwig.
7) Two particle correlations at the exit from LL wire.
8) Conclusions.

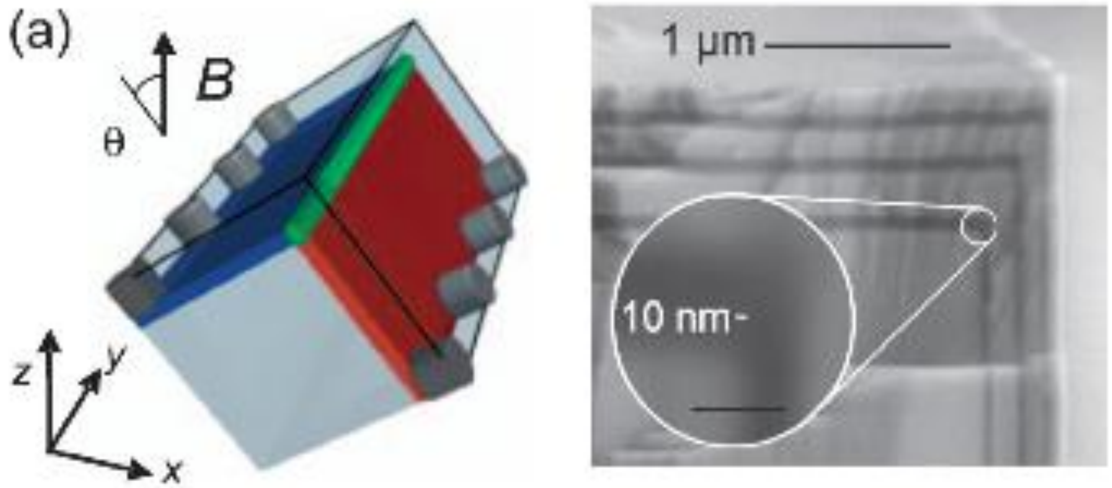
Luttinger Liquids

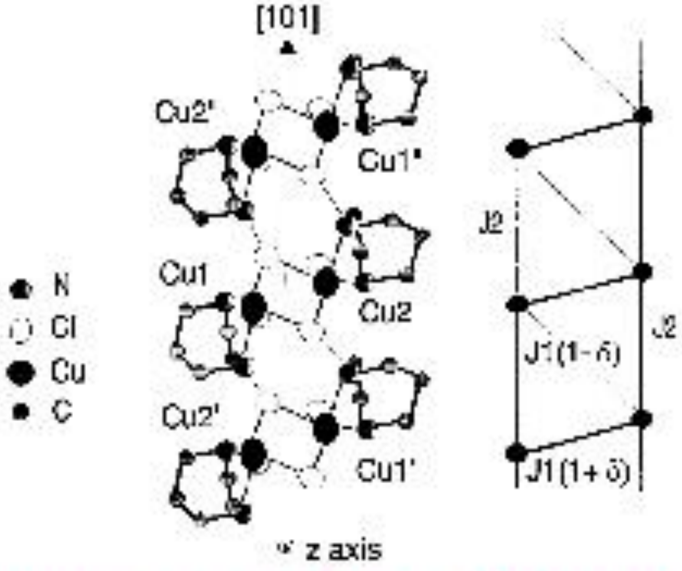




Z. Yao, H. W. Ch. Postma, et, al., Nature 402, 273 (1999).

O. M. Auslaender, A. Yacoby, et. al. Science 295, 825 (2002).

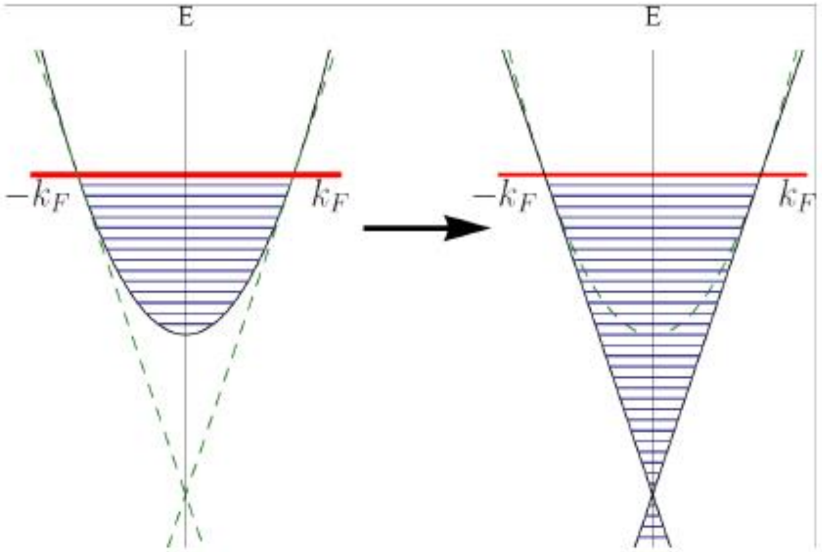




M. Grayson, L. Steinke, et. al., Phys. Rev. B 76, 201304 (2007)

G. Chaboussant, P. A. Crowell, et. al., Phys. Rev. B 55, 3046 (1997)

Bosonization



S. Coleman (1975)
S. Mandelstam (1975)
D.C. Mattis and E.H. Lieb (1965)
D.C. Mattis (1974)
F.D.M. Haldane (1981)

Hamiltonian

$$H = \int dx \psi^+(x) \frac{\hat{p}^2 - k_F^2}{2m} \psi(x) + \frac{g}{2} \int dx (\rho_L(x) + \rho_R(x))^2 \to \frac{V_F}{K} \sum_{\eta, q, \eta q > 0} \eta q \tilde{b}^+_{\eta,q} \tilde{b}_{\eta,q} .$$

Tilded bosons are linear functions of

$$b^+_{\eta,q} = \sqrt{\frac{2\pi}{L|q|}} \rho_{\eta,q} , \qquad b_{\eta,q} = \sqrt{\frac{2\pi}{L|q|}} \rho_{\eta,-q} , \qquad \left[b_{\eta,q}, b^+_{\eta',q'} \right] = \delta_{\eta\eta'} \delta_{qq'} .$$

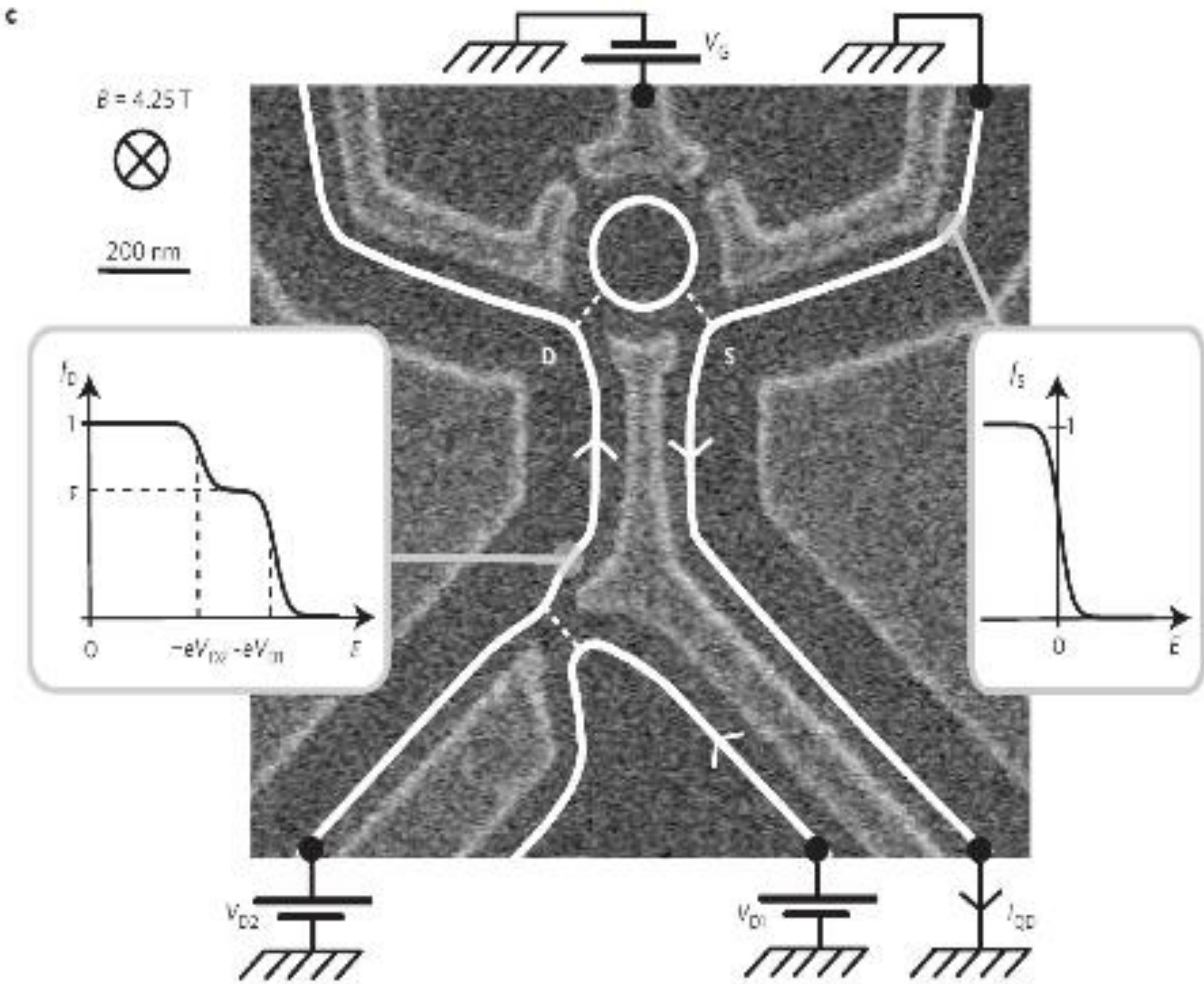
Fermions and bosons

$$\psi(x) \sim \psi_R(x) e^{ik_F x} + \psi_L(x) e^{-ik_F x} ,$$

$$\rho_\eta(x) = \frac{\eta}{2\pi} \partial_x \varphi_\eta(x) .$$

$$\psi^+_\eta(x) = \sqrt{\frac{\Lambda}{4\pi}} e^{-i\varphi_\eta(x)} F^+_\eta .$$

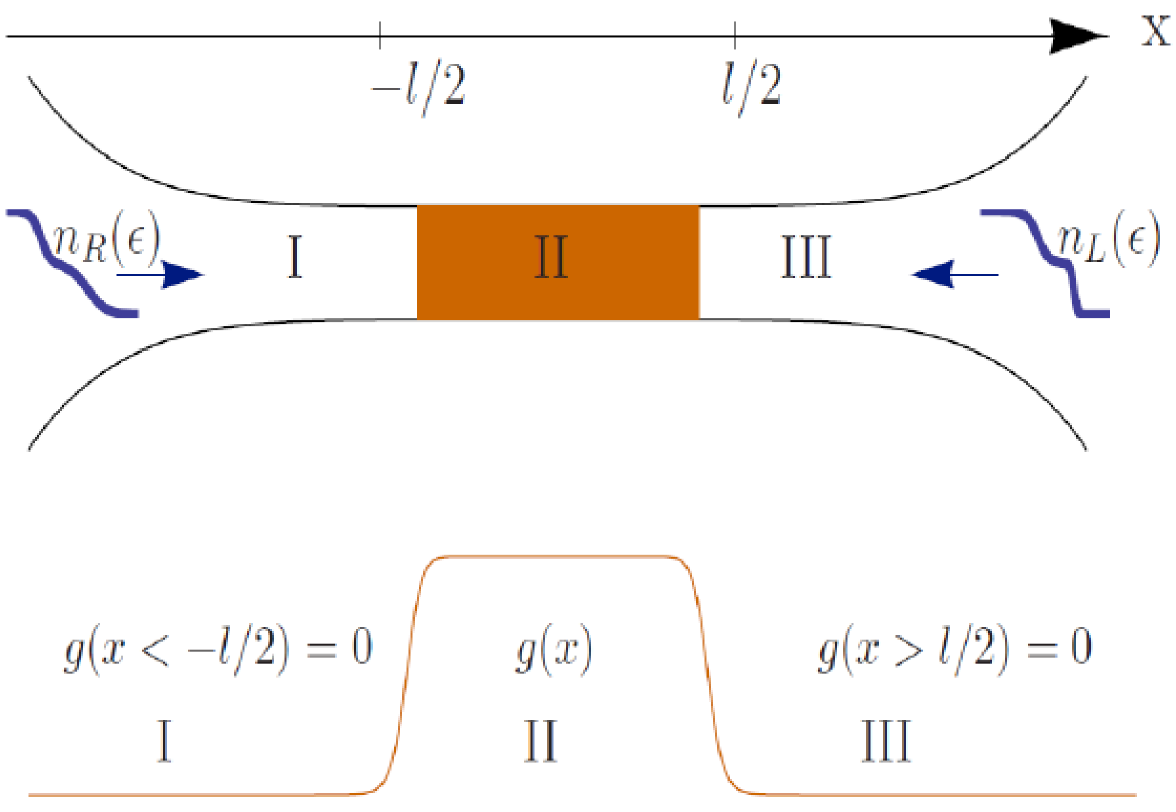
Non-equilibrium Luttinger Liquids

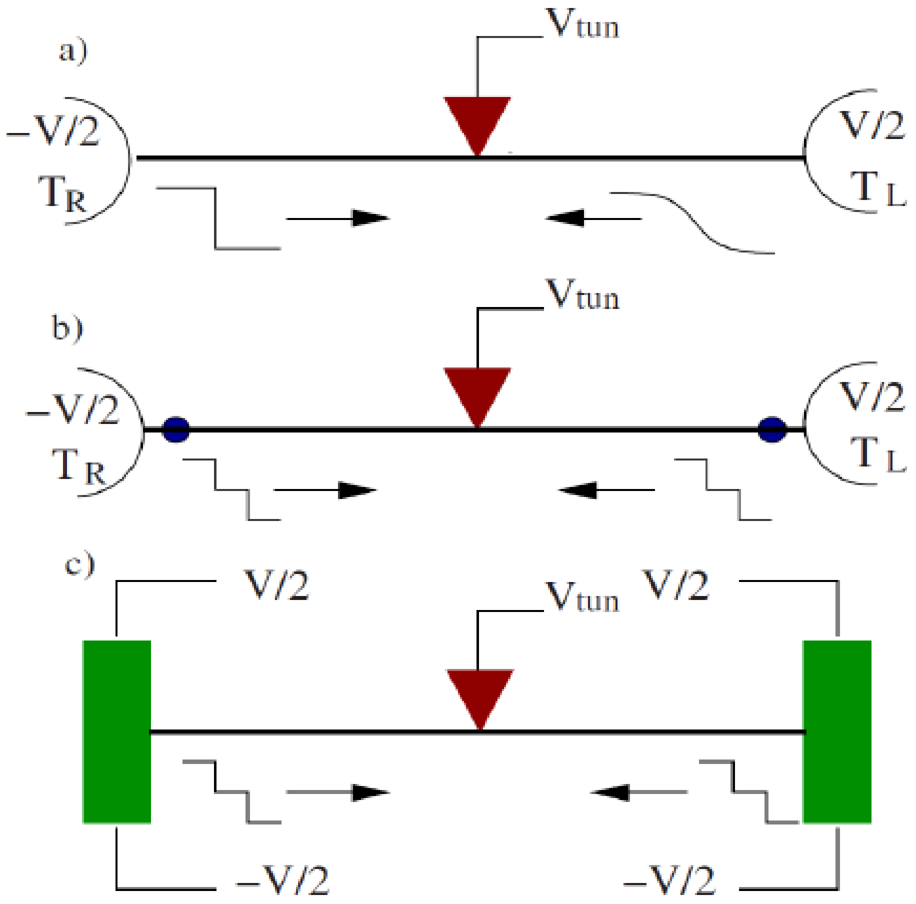


Non-equilibrium edge-channel spectroscopy

C. Altimiras, H. le Sueur, et. al. , Nature Physics 6, 34 (2010); Phys. Rev. Lett. 105, 226804 (2010).

The model





$$H = V_F \sum_{k,\eta} \eta k \left(a^+_{\eta,k} a_{\eta,k} - n^0_\eta(k) \right) + \frac{1}{2} \int dx g(x) (\rho_L(x) + \rho_R(x))^2$$

The interaction constant is smooth on the scale of the Fermi wave-length but sharp on scale of the relevant bosonic wave-length.

$$g(x) = g\Theta\left(l/2 - |x|\right)$$

Non-equilibrium LL: summary

Functional bosonization [Gutman, Gefen, and Mirlin, (2010)]
Operator approach [I.P., Gutman, and Mirlin, (2011)]

Any correlation function is a determinant!

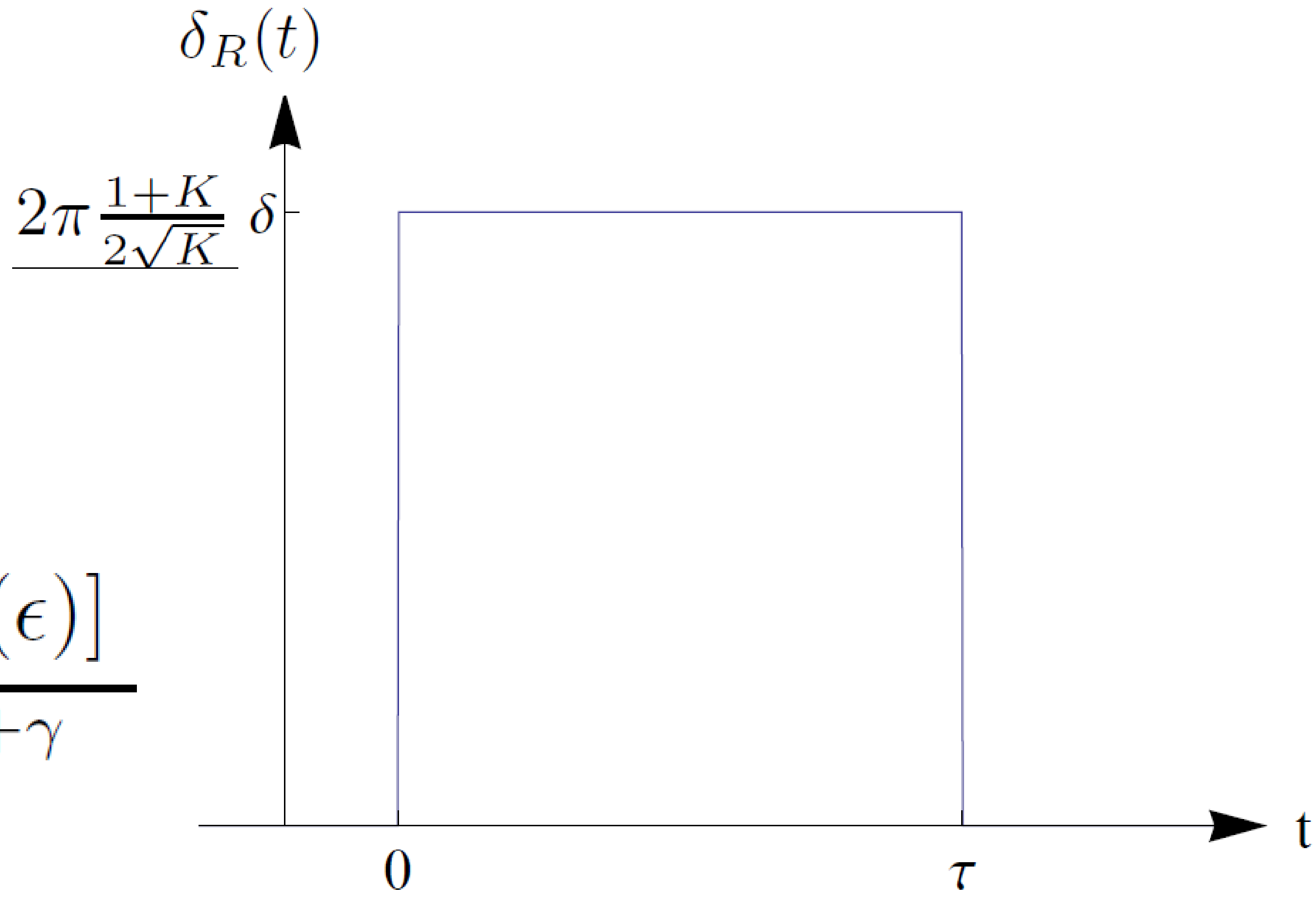
$$M = M(T=0)\overline{\Delta}[\delta_R(t), n_R(\epsilon)]\overline{\Delta}[\delta_L(t), n_L(\epsilon)]$$

$$\overline{\Delta}[\delta(t), n(\epsilon)] = \frac{\det\left[1 + \left(e^{i\delta(t)} - 1\right) n(\epsilon)\right]}{\det\left[1 + \left(e^{i\delta(t)} - 1\right) n_{T=0}(\epsilon)\right]}$$

Generally $\quad \Delta[A(t), B(\epsilon)] = \det\left[1 + \widehat{A}\widehat{B}\right]$

Single-particle correlation functions and Toeplitz determinants

Green function of right fermions in the wire
(left-movers at zero temperature, smooth boundaries)



$$G_R^{\gtrless}(\tau) = \mp \frac{i\Lambda}{2\pi u} \, \frac{\overline{\Delta}[\delta_R(t), n_R(\epsilon)]}{(1 \pm i\Lambda\tau)^{1+\gamma}}$$

$$\gamma = (1-K)^2/2K$$

After time discretization

$$\Delta_N[\delta] = \det[f(t_j - t_k)]\,, \;\; 0 \le j,k \le N-1 \qquad N = \tau\Lambda/\pi$$

$$f(\epsilon) = [1 + n(\epsilon)(e^{-i\delta} - 1)]e^{-i\frac{\delta}{2}\frac{\epsilon}{\Lambda}}$$

Szegö limit theorem
(Szegö, 1952; Widom 1976)

$$f(z) = e^{V_0 + V_+(z) + V_-(z)}, \qquad V_+(z) = \sum_{k>0} V_k z^k, \qquad V_-(z) = \sum_{k<0} V_k z^k$$

$$z = e^{i\epsilon/2\Lambda}$$

$$\Delta_N[f] \to e^{NV_0} \exp\left[\sum_{k=1}^{-\infty} k V_k V_{-k}\right] = e^{NV_0} \det\left[T[f]T[f^{-1}]\right]$$

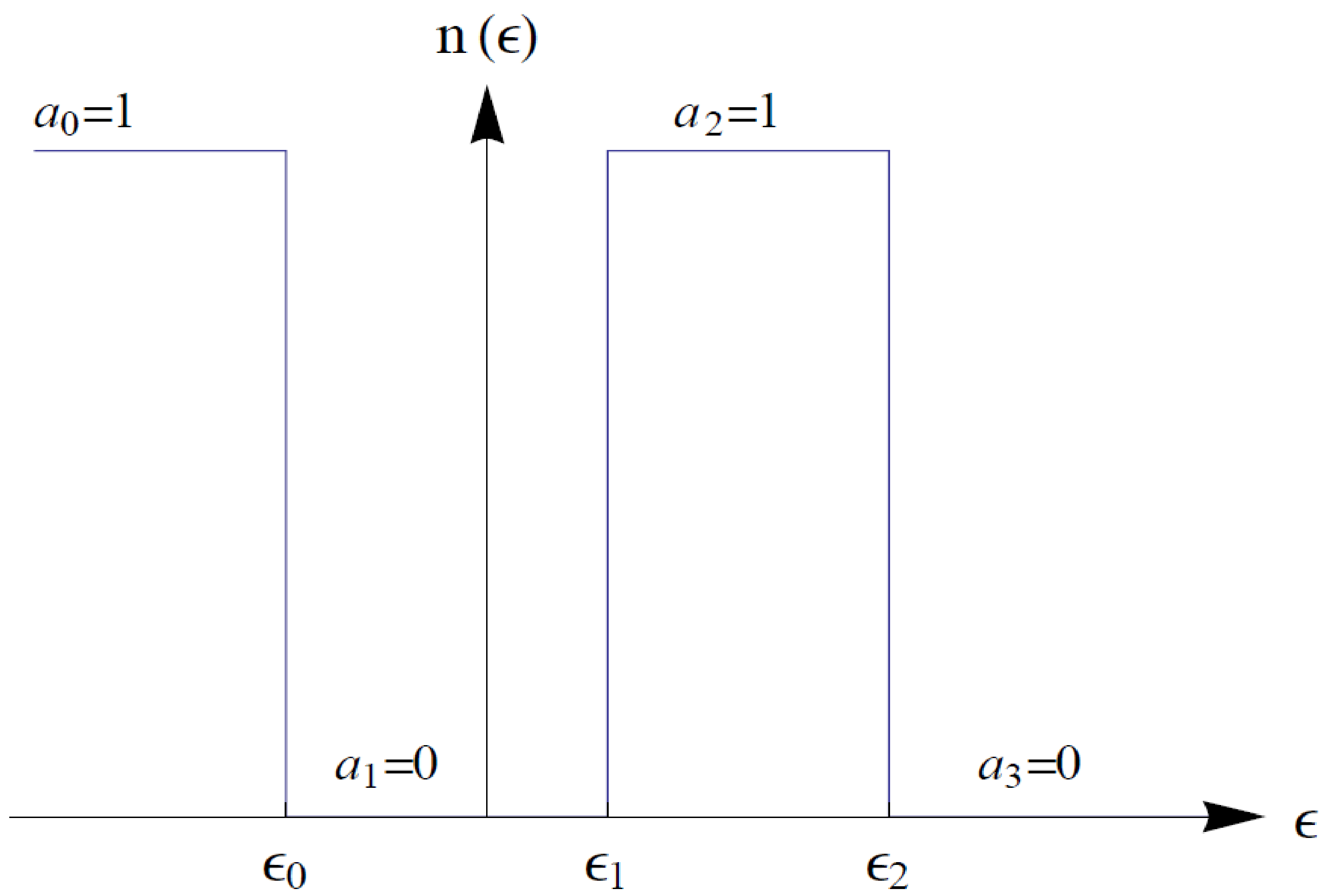
$$\det\left[T[f]T[f^{-1}]\right] = \det\left[e^{T[V_-]} e^{T[V_+]} e^{-T[V_-]} e^{-T[V_+]}\right] = e^{\mathrm{Tr}[T[V_-],T[V_+]]}$$

In physical terms this translates into long time asymptotics of correlation functions.
They are boring for smooth energy distribution

Fermi edges and Fisher-Hartwig conjecture

Interesting things happens if the energy distribution has Fermi-edges

$$n(\epsilon) = \begin{cases} 1 \equiv a_0 \,, & \epsilon < \epsilon_0 \\ a_1 \,, & \epsilon_0 < \epsilon < \epsilon_1 \\ \dots & \\ a_m \,, & \epsilon_{m-1} < \epsilon < \epsilon_m \\ 0 \equiv a_{m+1} \,, & \epsilon_m < \epsilon \,. \end{cases}$$

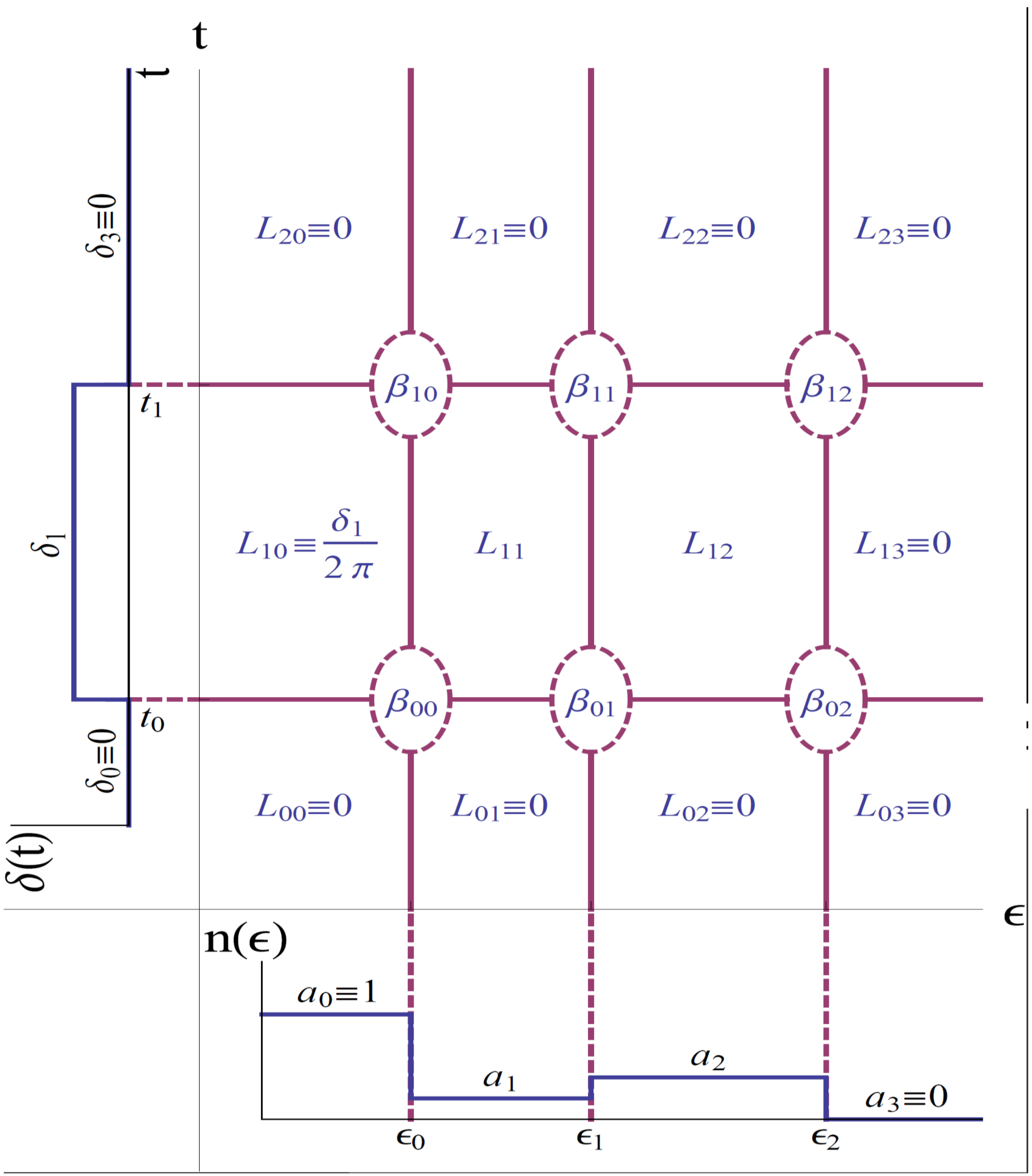


Fermi edges and Fisher-Hartwig conjecture

$$\Delta_N[f] \to e^{NV_0} \exp\left[\sum_{k=1}^{-\infty} k V_k V_{-k}\right] \qquad f(\epsilon) = [1 + n(\epsilon)(e^{-i\delta} - 1)]e^{-i\frac{\delta}{2}\frac{\epsilon}{\Lambda}}$$

Jumps in distribution function lead to logarithmic divergence of the sum in the exponent and power-law behavior of the determinant.

Fermi edges and Fisher-Hartwig conjecture



$$L_{lp} = \frac{i}{2\pi} \log \left[1 + (e^{-i\delta_l} - 1) a_p\right]$$

$$0 \le l \le 2\,,\ 0 \le p \le m$$

$$\beta_{lp} = L_{l+1,j} + L_{l,p+1} - L_{l,p} - L_{l+1,p+1}$$

Fermi edges and Fisher-Hartwig conjecture

$$\overline{\Delta}[\delta(t), n(\epsilon)] = C \exp\left[i \sum_{l\ ,p} t_l \beta_{lp} \epsilon_p\right] \prod_{\substack{l,p=0 \\ p<l}}^{1} \prod_{\substack{r,q=0 \\ q<r}}^{m} [\tau(\epsilon_r - \epsilon_q)]^{\gamma_{lp;qr}}$$

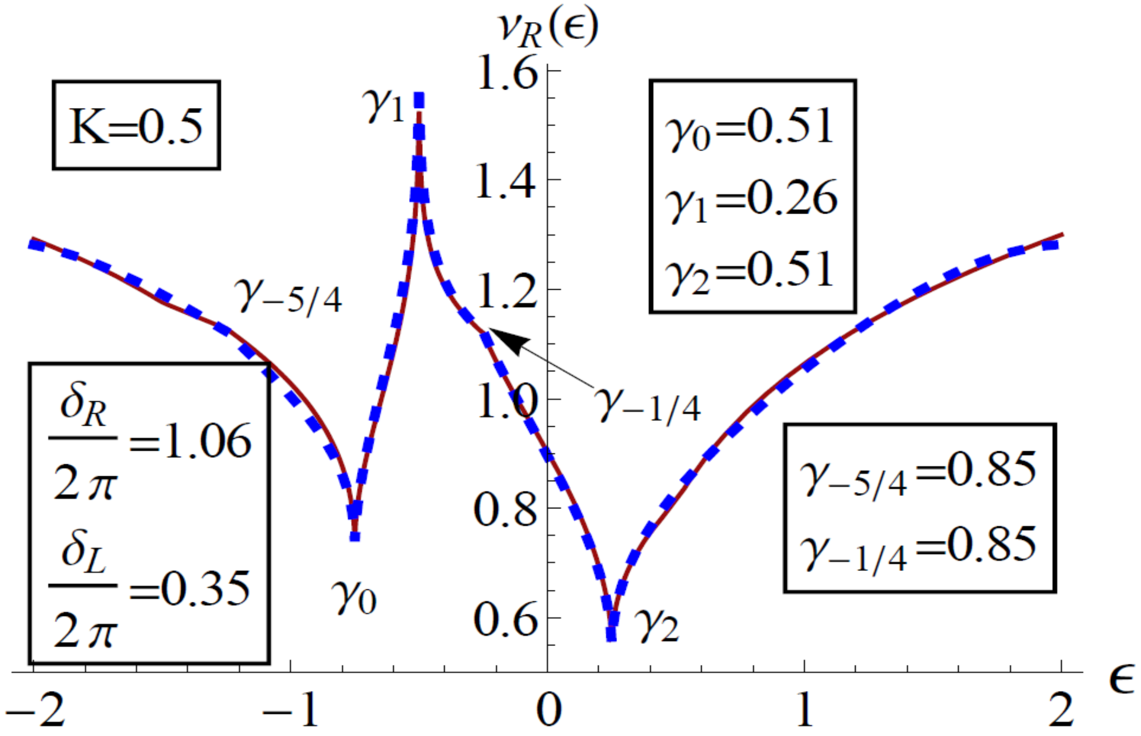
$$\gamma_{lp;rq} = -\beta_{lr}\beta_{pq} - \beta_{pr}\beta_{lq}$$

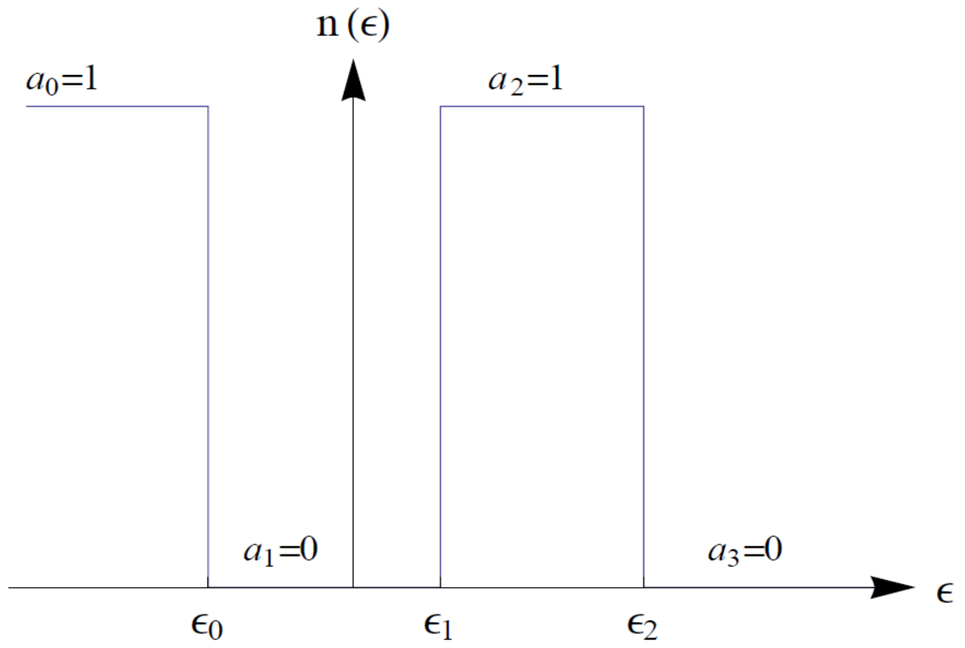
Fermi edges and Fisher-Hartwig conjecture

Classical Fisher-Hartwig conjecture:
choose the branches of logarithm providing the slowest decay
(Fisher & Hartwig, 1969; Deift, Its, Krasovsky, 2011)

Generalized Fisher-Hartwig conjecture:
sum over branches (Gutman, Gefen, Mirlin, 2011;
I.P. , Gutman, Mirlin, 2012)

$$\overline{\Delta}[\delta(t), n(\epsilon)] = \sum C[\beta] \exp\left[i \sum_{l,p} t_l \beta_{lp} \epsilon_p\right] \prod_{\substack{l,p=0 \\ p<l}}^{1} \prod_{\substack{r,q=0 \\ q<r}}^{m} [\tau(\epsilon_r - \epsilon_q)]^{\gamma_{lp;qr}}$$





Higher correlation functions and Fredholm determinants

$$\overline{\Delta}[\delta(t), n(\epsilon)] = \frac{\det\left[1 + \left(e^{i\delta(t)} - 1\right) n(\epsilon)\right]}{\det\left[1 + \left(e^{i\delta(t)} - 1\right) n_{T=0}(\epsilon)\right]}$$

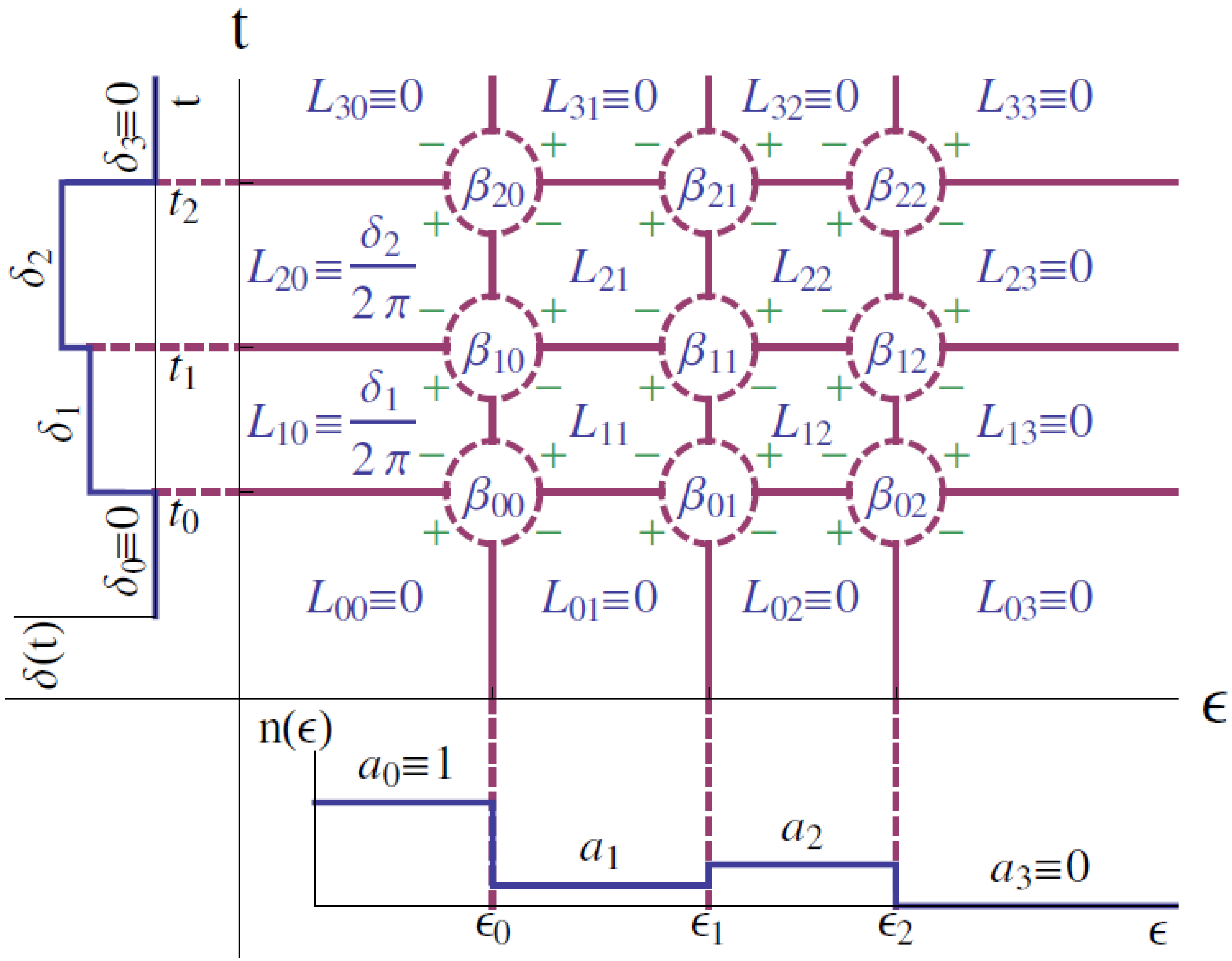
$$\delta(t) = \begin{cases} 0 \equiv \delta_0 \,, & t < t_0 \\ \delta_1 \,, & t_0 < t < t_1 \\ \dots & \\ \delta_k \,, & t_{k-1} < t < t_k \\ 0 \equiv \delta_{k+1} \,, & t_k < t \,. \end{cases}$$

Generalized Szegö

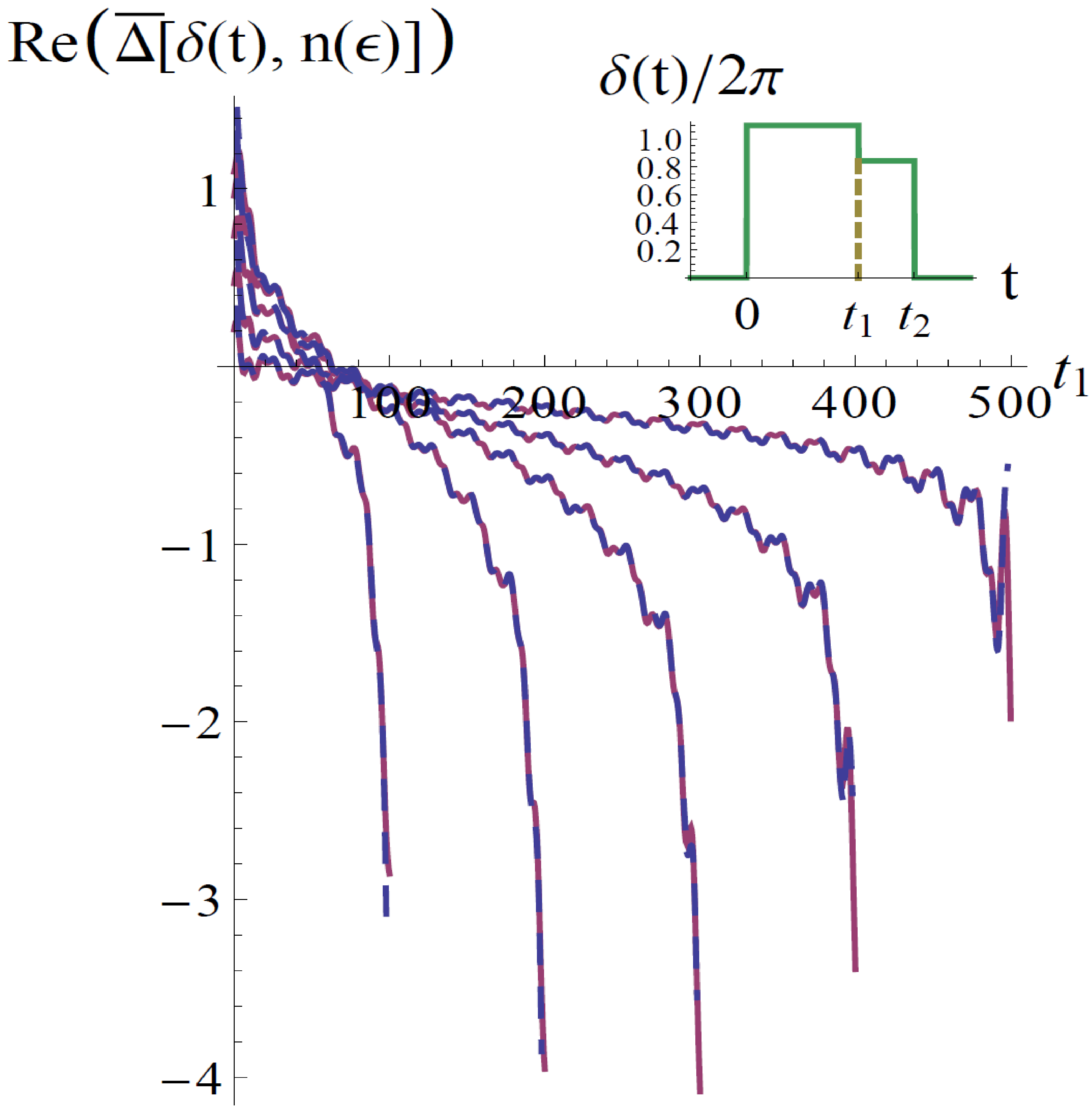
$$\det \begin{pmatrix} f_0 & f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 \\ f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\ f_{-2} & f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 & f_5 \\ f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 \\ f_{-4} & f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 & f_2 & f_3 \\ f_{-5} & f_{-4} & f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 & f_2 \\ f_{-6} & f_{-5} & f_{-4} & f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 \\ f_{-7} & f_{-6} & f_{-5} & f_{-4} & f_{-3} & f_{-2} & f_{-1} & f_0 \end{pmatrix} \to e^{NV_{f0}} \det \left[T[f]T[f^{-1}]\right]$$

$$\det \begin{pmatrix} f_0 & f_1 & f_2 & f_3 & f_4 & f_5 & f_6 & f_7 \\ f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 & f_5 & f_6 \\ f_{-2} & f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 & f_5 \\ f_{-3} & f_{-2} & f_{-1} & f_0 & f_1 & f_2 & f_3 & f_4 \\ \hline g_{-4} & g_{-3} & g_{-2} & g_{-1} & g_0 & g_1 & g_2 & g_3 \\ g_{-5} & g_{-4} & g_{-3} & g_{-2} & g_{-1} & g_0 & g_1 & g_2 \\ g_{-6} & g_{-5} & g_{-4} & g_{-3} & g_{-2} & g_{-1} & g_0 & g_1 \\ g_{-7} & g_{-6} & g_{-5} & g_{-4} & g_{-3} & g_{-2} & g_{-1} & g_0 \end{pmatrix} \to e^{NV_{f0}+MV_{g0}} \det \left[T[f]T[gf^{-1}]T[g^{-1}]\right]$$

Multiple Fermi-edges and power-laws

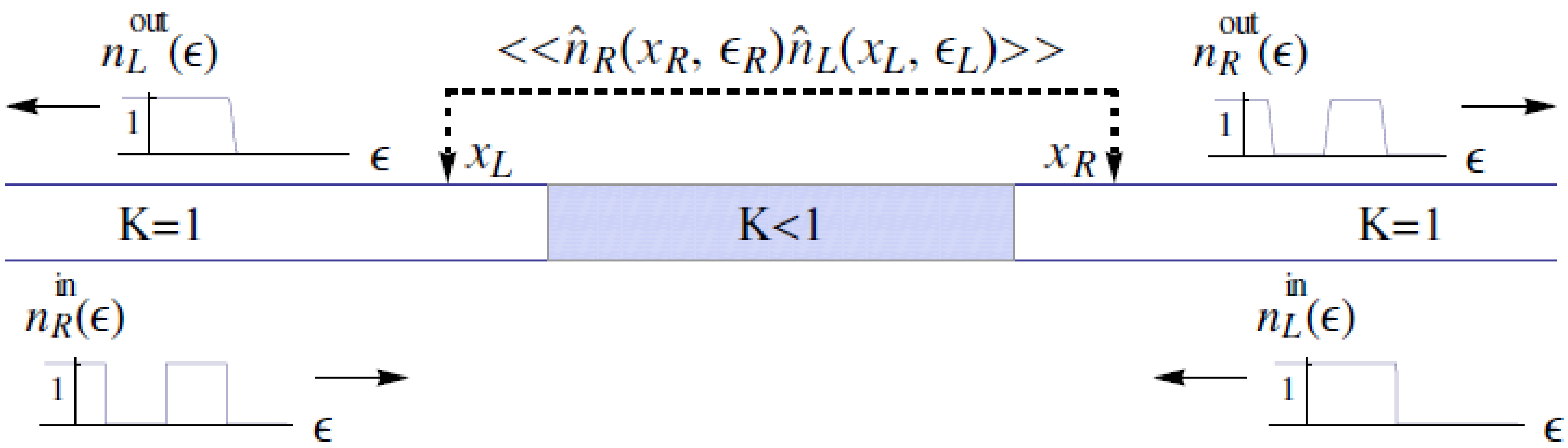


$$\overline{\Delta}[\delta(t), n(\epsilon)] = \sum_{\beta} C[\widehat{\beta}] \exp\left[i \sum_{l,p} t_l \beta_{lp} \epsilon_p \right] \prod_{\substack{l,p=0 \\ p<l}}^{k} \prod_{\substack{r,q=0 \\ q<r}}^{m} [(t_l - t_p)(\epsilon_r - \epsilon_q)]^{\gamma_{lp;qr}}$$

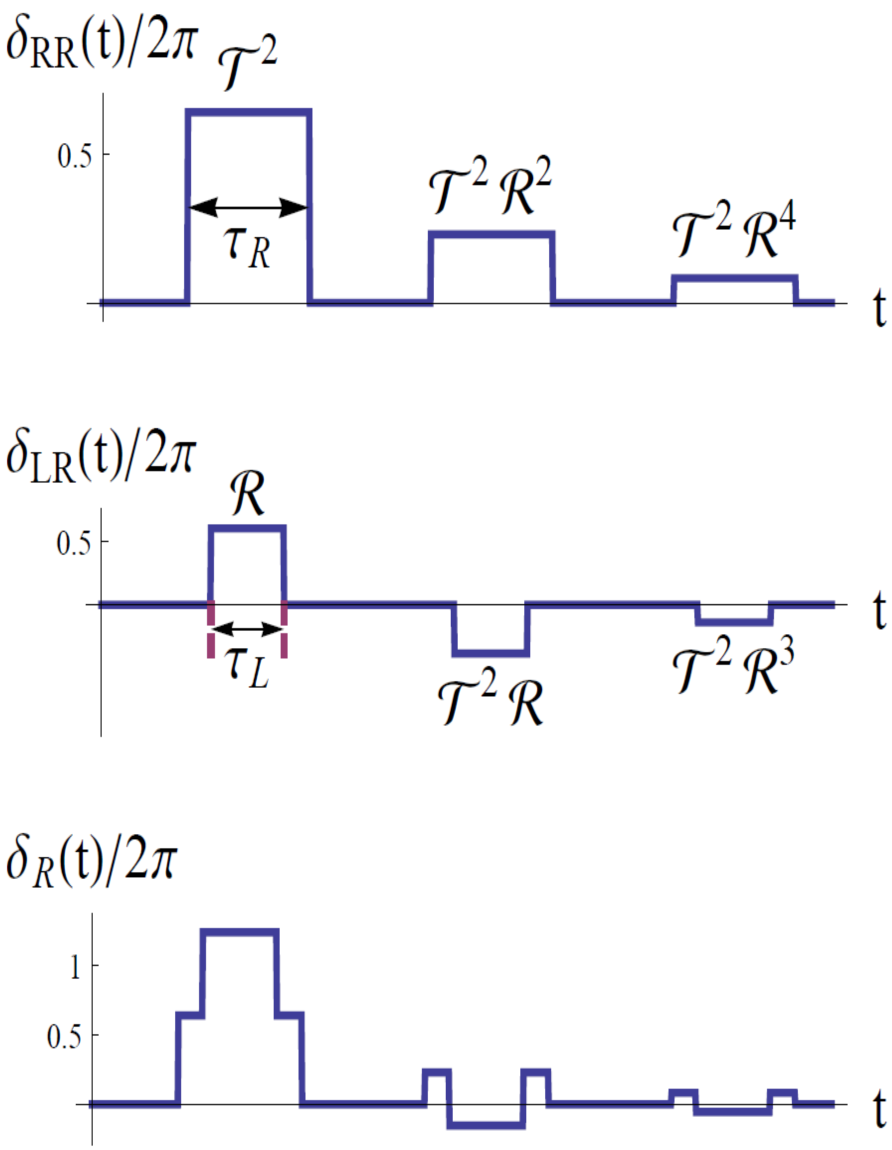


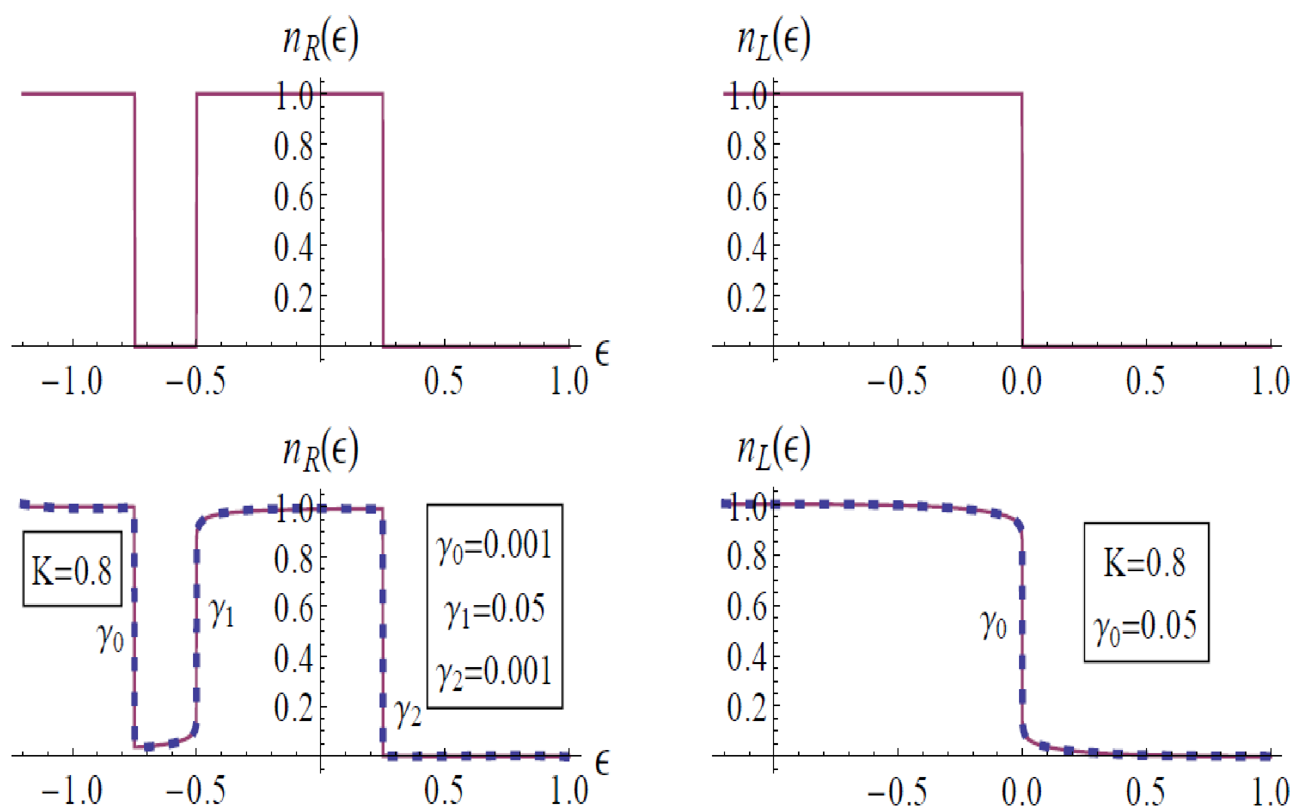
Re($\overline{\Delta}[\delta(t), n(\epsilon)]$)
1
−1
−2
−3
−4
100
200
300
400
500
t_1
$\delta(t)/2\pi$
1.0
0.8
0.6
0.4
0.2
t
0
t_1
t_2

Correlations at the output of the wire



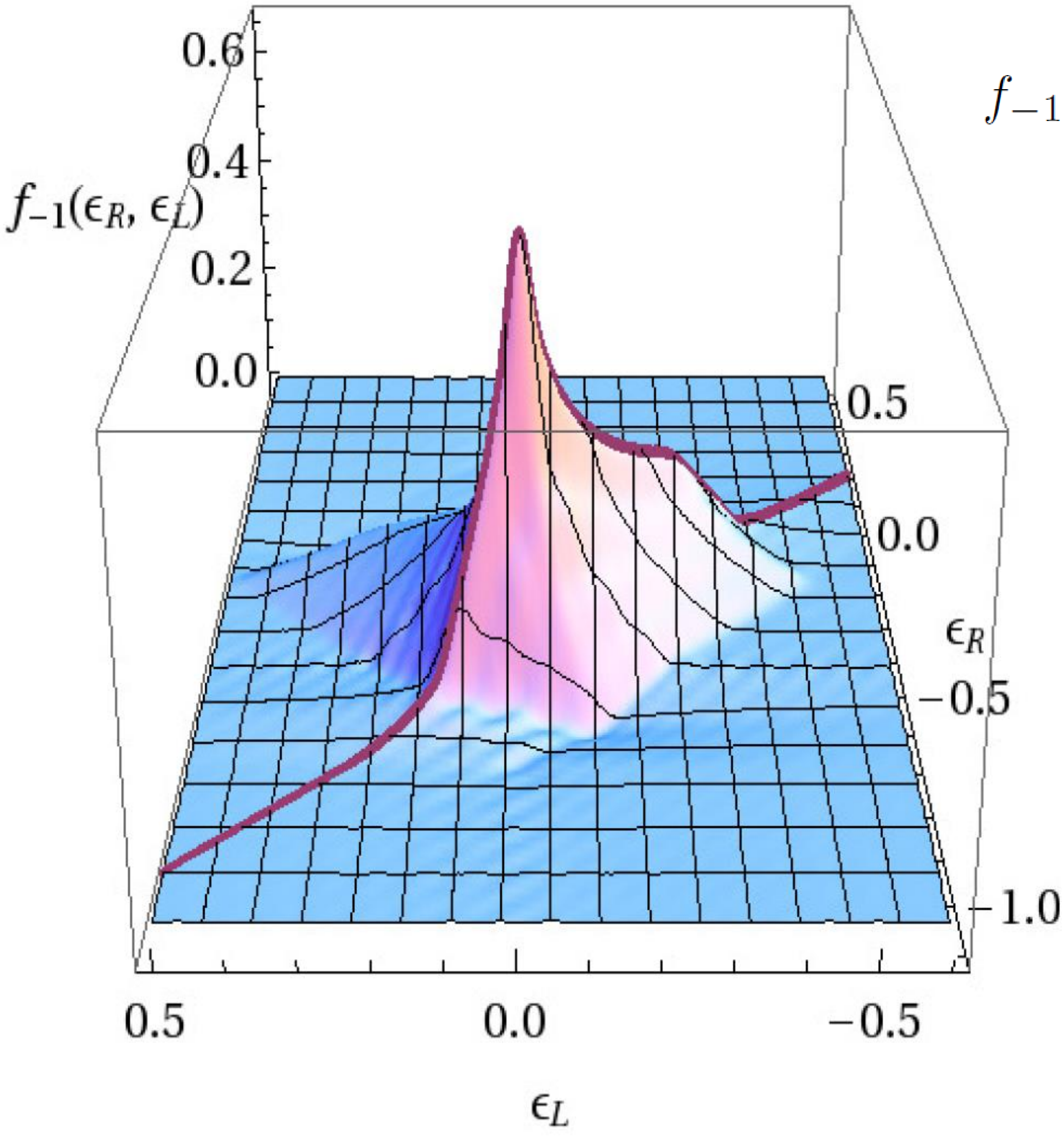
Fermionic distributions at the output of the wire





$$f(\epsilon_R, \epsilon_L, x_R + x_L) = \langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle = ?$$

Correlations at the output of the wire



$$f_{-1}(\epsilon_R, \epsilon_L) \sim |\epsilon_R + \epsilon_L - \epsilon_1|^{1-\alpha_1-\alpha_2}$$

Outlook

- Mathematical proof of generalized and “even further generalized” Fisher-Hartwig conjectures.

- Application of mathematical results to other physical systems

- Electronic backscattering.

- Effects of curvature of fermionic spectrum.