Tunneling conductance due to a discrete spectrum of Andreev states

P. A. Ioselevich, M. V. Feigel'man

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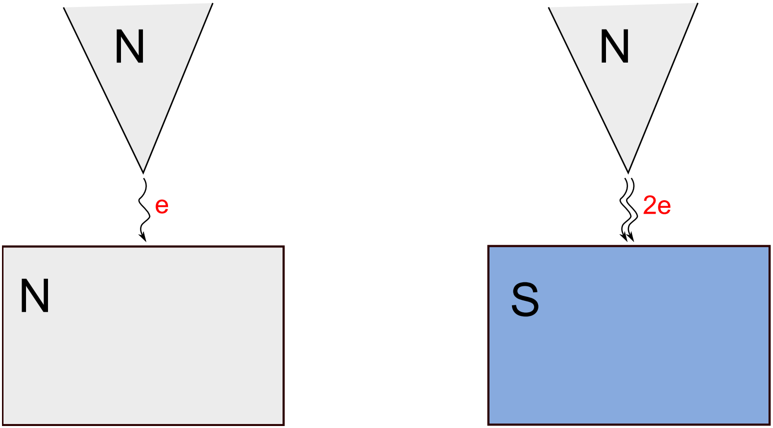
Chernogolovka

Tunneling conductance and Andreev reflection



- For simple tunneling DoS directly translates into conductance

$$G(V) \equiv \frac{dI}{dV} \propto |H_T|^2 \cdot \mathrm{DoS}(eV)$$

- Andreev reflection is a second-order process $G \sim |H_T|^4$, that becomes important at subgap voltages.

Blonder-Tinkham-Klapwijk formula

Conductance is expressed through the S-matrix describing the tunneling junction

$$I = \frac{2e}{h} \int_{-\infty}^{\infty} T_A(E) \left[f_0(E - eV) - f_0(E) \right] dE$$

where

$$T_A(E) = \mathrm{tr} \left(S_{eh}(E)^\dagger S_{eh}(E) \right)$$

describes the probability of Andreev reflection at the junction. Andreev bound states E_j in the probed superconductor should produce resonant peaks in T_A, which must be treated non-perturbatively.

Kubo-type formula

The probabilities $T_A(E)$ can be written as

$$T_A = \mathrm{tr}\left(\begin{pmatrix} \hat{v} & 0 \\ 0 & 0 \end{pmatrix} \check{G}_E^A \begin{pmatrix} 0 & 0 \\ 0 & \hat{v}^* \end{pmatrix} \check{G}_E^R \right)$$

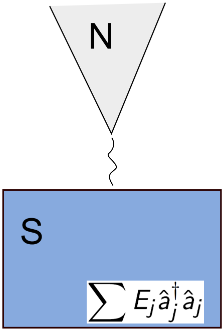
where $\hat{v}$ is the velocity operator in the lead and $\hat{v}^*$ is its time-reversal image. The explicit matrices are in Nambu space. The Green's functions are exact:

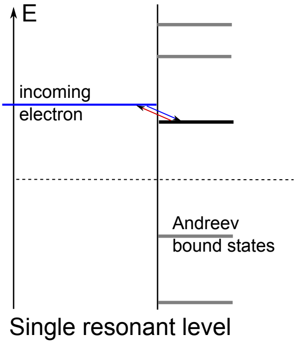
$$\check{G} = \check{G}_0 + \check{T}\check{G}_0$$

$$\check{T} = \frac{\check{G}_0 \check{H}_T \check{G}_S \check{H}_T}{1 - \check{G}_0 \check{H}_T \check{G}_S \check{H}_T}$$

with G_0 and G_S denoting unperturbed Green's functions of the lead and the SC system, respectively. Green's functions without superscript are retarded.

Single-level resonance





At energies close to a single level $|j_0\rangle$, conductance is dominated by its contribution. At resonance the Green's function has to be calculated non-perturbatively. We keep only the $|j_0\rangle$-term in G_S:

$$\check{G}_0 \check{H}_T \check{G}_S \check{H}_T = \frac{\check{G}_0 |\tau\rangle\langle\tau|}{E - E_{j_0} + i0} \quad \text{and} \quad \check{T} = \frac{\check{G}_0 |\tau\rangle\langle\tau|}{E - E_{j_0} + i0 - \langle\tau|\check{G}_0|\tau\rangle}$$

where $|\tau\rangle \equiv \check{H}_T |j_0\rangle$.

Single-level resonance result

$$G = \frac{2e^2}{h} T_A = \frac{2e^2}{h} \frac{\langle \tau_e | g_E^A \hat{v} g_E^R | \tau_e \rangle \langle \tau_h | g_{-E}^{R*} \hat{v}^* g_{-E}^{A*} | \tau_h \rangle}{(E - E_{j_0})^2 + W^2}$$

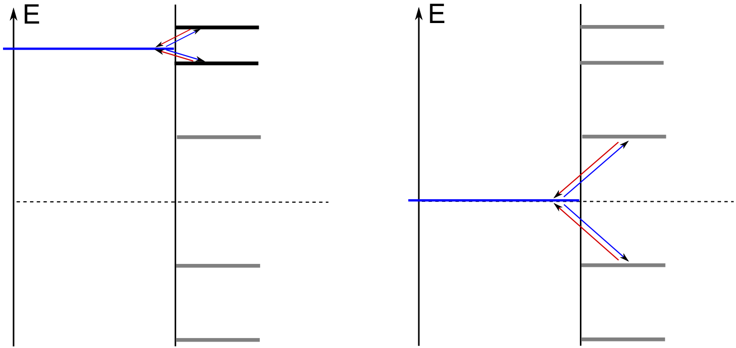
$$W = \pi (n_e + n_h) \quad \text{with} \quad \begin{cases} n_e = \langle \tau_e | \frac{g_E^R - g_E^A}{2\pi i} | \tau_e \rangle \\ n_h = \langle \tau_h^* | \frac{g_{-E}^R - g_{-E}^A}{2\pi i} | \tau_h^* \rangle \end{cases}$$

The zero-temperature conductance assumes the Lorentz form

$$G(eV) = \frac{2e^2}{h} \frac{1}{1 + \frac{(eV - E_{j_0})^2}{W^2}} T_A^* \quad \text{with} \quad T_A^* = \frac{4 n_e n_h}{(n_e + n_h)^2}$$

For a point-contact n_e, n_h are proportional to electron and hole densities; $W = \pi |t_0^2| \nu_M |\psi_0|^2$. The peak T_A^* is largest for an electron-hole symmetric level. In particular, $T_A^* \equiv 1$ for a Majorana level which obeys $|\tau_e\rangle = |\tau_h^*\rangle$.

Interference effects



Multiple interfering processes

Destructive interference of opposite levels, G=0

If several levels are close to E then amplitudes of Andreev reflection through all of them must be added together. This is most obvious at $E \simeq 0$ and most pronounced for a single-channel contact.

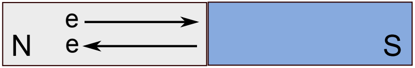
Single-channel topological arguments

BdG-symmetry at zero energy: $\det S = \pm 1$

single-channel zero-bias conductance: $G = (1 - \det S) G_Q$

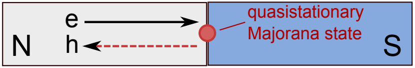
zero-bias conductance is quantized! Reflection of electrons is either completely normal or completely Andreev-type

trivial N-S junction



$$S_{triv} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix} \qquad \det S_{triv} = +1, \quad G = 0$$

topological N-S junction



$$S_{topo} = \begin{pmatrix} 0 & e^{i\lambda} \\ e^{-i\lambda} & 0 \end{pmatrix} \qquad \det S_{topo} = -1, \quad G = 2G_Q$$

Exact solution for single channel setup

$$\check{H}_T = |\theta\rangle t \langle\sigma| - |\theta^*\rangle t^* \langle\sigma^*| + h.c.$$

$$\check{G}_0 \check{H}_T \check{G}_S \check{H}_T = \check{G}_0 \begin{pmatrix} |\theta\rangle & |\theta^*\rangle \end{pmatrix} N_0^{-1} \check{\Sigma} \begin{pmatrix} \langle\theta| \\ \langle\theta^*| \end{pmatrix}$$

with a 2×2 matrix $\check{\Sigma}$:

$$\check{\Sigma} = \begin{pmatrix} \Sigma_e & \Sigma_A \\ \Sigma_A^* & \Sigma_h \end{pmatrix}$$

$$\Sigma_e = t^2 N_0 \sum_{j>0} \frac{|\langle\sigma|j\rangle|^2}{E - E_j} + \frac{|\langle\sigma|j^*\rangle|^2}{E + E_j}$$

$$\Sigma_h = t^2 N_0 \sum_{j>0} \frac{|\langle\sigma|j^*\rangle|^2}{E - E_j} + \frac{|\langle\sigma|j\rangle|^2}{E + E_j}$$

$$\Sigma_A = -t^2 N_0 \sum_{j>0} \frac{\langle\sigma|j\rangle\langle j|\sigma^*\rangle}{E - E_j} + \frac{\langle\sigma|j^*\rangle\langle j^*|\sigma^*\rangle}{E + E_j}$$

with a normalization factor $N_0 = \mathrm{Im}\langle\theta|g_E^R|\theta\rangle$. The Σ_A amplitude describes the amplitude of Andreev reflection in the lowest order in H_T.

Exact solution for single channel setup - 2

$$\Sigma_e = t^2 N_0 \sum_{j>0} \frac{|\langle\sigma|j\rangle|^2}{E - E_j} + \frac{|\langle\sigma|j^*\rangle|^2}{E + E_j} \qquad \Sigma_h = t^2 N_0 \sum_{j>0} \frac{|\langle\sigma|j^*\rangle|^2}{E - E_j} + \frac{|\langle\sigma|j\rangle|^2}{E + E_j}$$

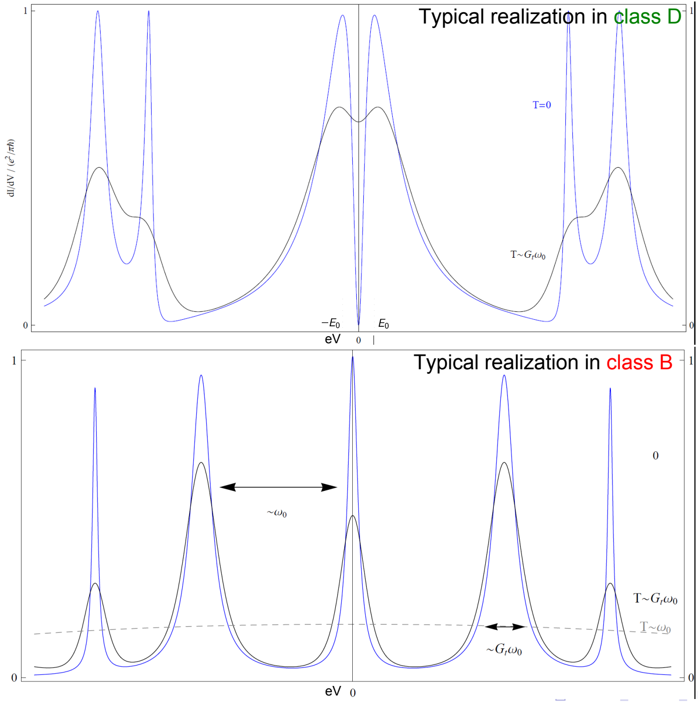
$$\Sigma_A = -t^2 N_0 \sum_{j>0} \frac{\langle\sigma|j\rangle\langle j|\sigma^*\rangle}{E - E_j} + \frac{\langle\sigma|j^*\rangle\langle j^*|\sigma^*\rangle}{E + E_j}$$

$$\check{T} = \check{G}_0 \left(|\theta\rangle \quad |\theta^*\rangle\right) \frac{N_0^{-1}\check{\Sigma}}{1 - i\check{\Sigma}} \begin{pmatrix} \langle\theta| \\ \langle\theta^*| \end{pmatrix}$$

which we substitute into the Kubo formula and find

$$G(eV) = \frac{2e^2}{h} \frac{4|\Sigma_A^2|}{\left(1 - \Sigma_e\Sigma_h + |\Sigma_A^2|\right)^2 + (\Sigma_e + \Sigma_h)^2}.$$

Away from resonances $\Sigma_{e,h,A} \ll 1$ and $G(eV) = \frac{2e^2}{h} \times 4|\Sigma_A^2|$ which is the perturbative answer. If there is no zero level, we also find $G(0) \propto |\Sigma_A(0)|^2 = 0$ due to the electron-hole symmetry of the spectrum.



Typical realization in class D
dI/dV / (e²/πħ)
T=0
T~G_t ω_0
−E_0
E_0
eV
0
1
Typical realization in class B
~ω_0
0
T~G_t ω_0
T~ω_0
~G_t ω_0
eV

Low-energy behavior

At low energies we may keep only the lowest level at E_0 and its partner at $-E_0$. Dropping the small contributions of other levels we get

$$G(E) = \frac{2e^2}{h} \frac{1}{\left[\frac{E^2 - \tilde{E}_0^2}{2EW}\right]^2 + 1} T_A^*$$

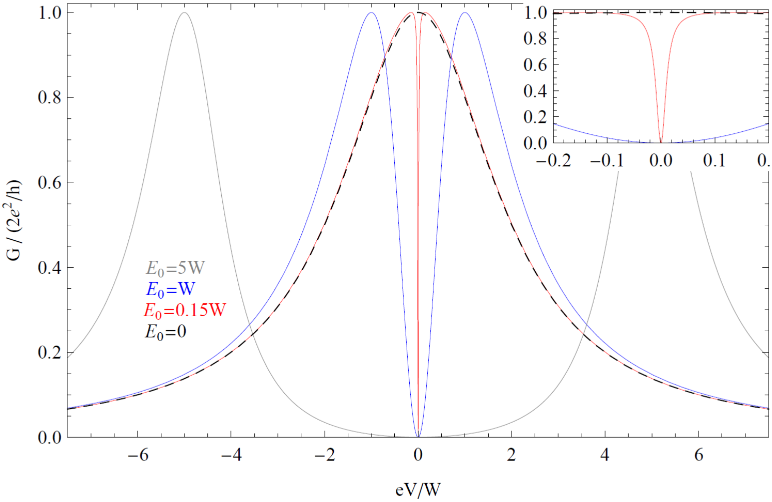
where

$$\tilde{E}_0^2 = E_0^2 + (|t_1|^2 - |t_2|^2)^2 \qquad W = |t_1^2| + |t_2^2|$$

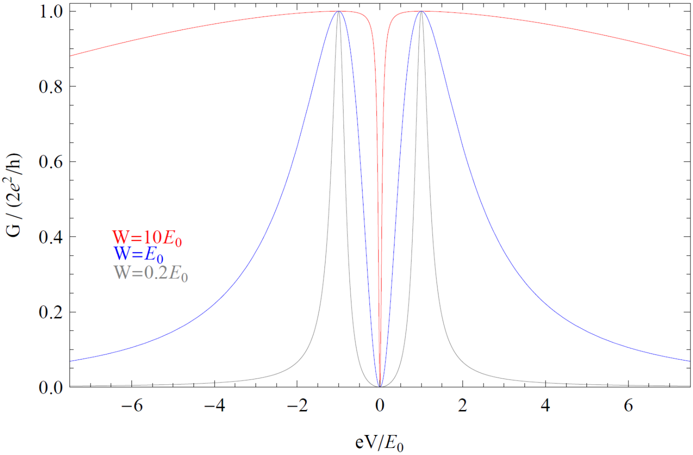
$$T_A^* = \frac{4|t_1 t_2|^2}{(|t_1|^2 + |t_2|^2)^2}$$

$$t_1 = t\sqrt{N_0}\langle\sigma|j\rangle, \qquad t_2 = t\sqrt{N_0}\langle\sigma|j^*\rangle$$

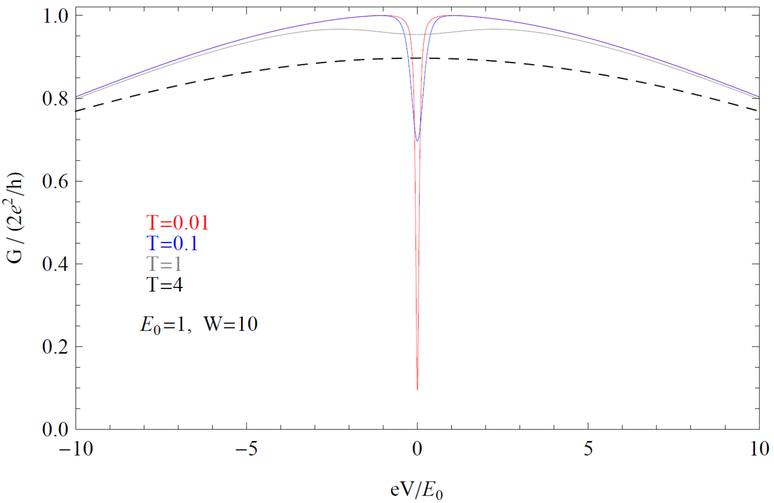
The result for G allows one to study a smooth transition between topologically trivial and non-trivial NS-junctions.



Here the contact strength W is fixed, and the level energy E_0 is varied. When $E_0 = 0$ a perfect Lorentz zero-bias peak is formed (Law, Lee, Ng 2009) . $I_\infty = \int G dV = const$ across the curves.



Here E_0 is fixed, while W varies. The dip width is parametrically small, $\delta \sim \frac{E_0^2}{W}$. At $W = E_0$ the poles of $G(E)$ meet at the imaginary energy axis, indicating a topological transition of the S-matrix (Pikulin, Nazarov 2011) – at lower E_0 one of two Majorana fermions „buries" in the system – its lifetime becomes parametrically large, $\tau \sim W^{-1}$.



At finite temperature the dip is smeared out, so that the topological (with a single Majorana) and trivial (without Majoranas or with a pair of them) systems become indistinguishable.

Possible applications

- Single-level resonance peaks could be observable in a vortex core insuperconductor if the Caroli-Matricon-de Gennes states are not spread into a continous band along the vortex core. This should be the case for strongly anisotropic materials, e.g. $\mathrm{NbSe_2}$ where the effective mass along the core is much larger than the other two, so that even small disorder localizes the CMdG states.
- The low-energy conductance for single-channel contacts should work for many systems hosting Majorana fermions. This includes the vortex core on top of a superconducting topological insulator surface, as well as topological superconductor wires based on nanowires etc. The single-level resonances should be observable for those systems as well.

Conclusions [P I, M. V. Feigel'man, New J. Phys. 15 (2013) 055011]

- A general formula for resonant Andreev reflection from discrete levels has been derived. Peaks have Lorentz form with width scaling as normal conductance and heights only depending on the electron-hole profile of the levels and usually being close to $2e^2/h$.
- An exact expression has been derived for single-channel tunneling contacts. At $T = 0$ the zero-bias conductance is quantized to 0 or $2e^2/h$ in agreement with topological arguments.
- A situation of two low-lying Andreev levels (or Majorana fermions) has been studied – if their splitting is weak, $G(V)$ has a Lorentz shape with a very narrow dip to zero on top of the peak at $V = 0$. Finite temperature smears the dip out, so that the topological and trivial cases become indistinguishable.