Light Propagation in Active Metamaterials with Mixed Positive and Negative Refractive Index

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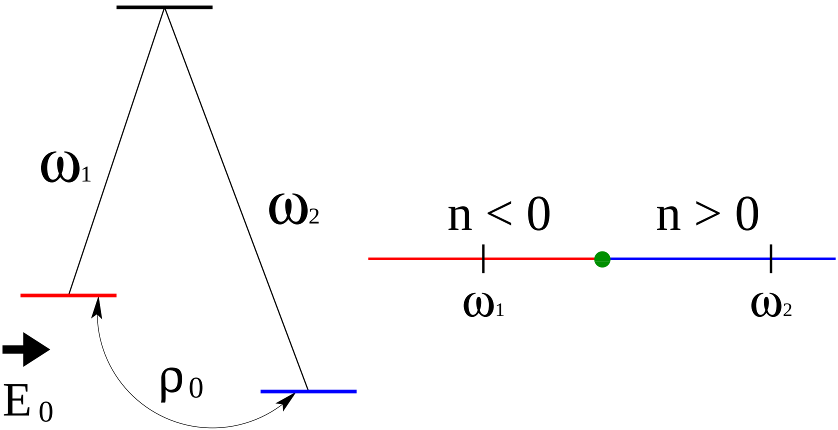
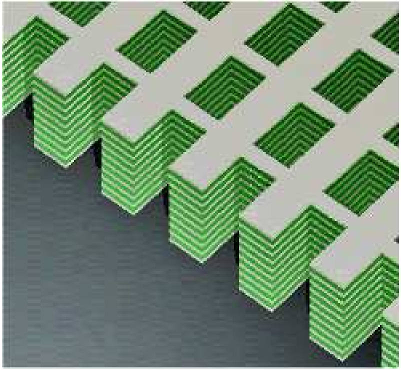
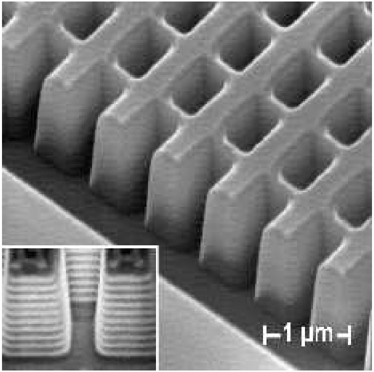
Basics of a Maxwell-Bloch Equations in Λ-configuration.

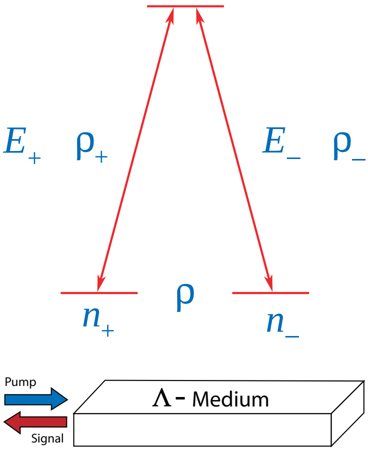


Fishnet structure. Sensitive to polarization.





Test for negative refraction.



Maxwell-Bloch Equations.

$$\frac{\partial E_\pm}{\partial x} \pm \frac{1}{v_g}\frac{\partial E_\pm}{\partial t} = \int_{-\infty}^{\infty} \rho_\pm \, g(\lambda)\, d\lambda$$

$$\frac{\partial \rho_+}{\partial t} - 2i\lambda\rho_+ = \frac{1}{2}\left[E_+(\mathcal{N} - n_+) - E_-\mu^*\right]$$

$$\frac{\partial \rho_-}{\partial t} - 2i\lambda\rho_- = \frac{1}{2}\left[E_-(\mathcal{N} - n_-) - E_+\mu\right]$$

$$\frac{\partial \mu}{\partial t} = \frac{1}{2}\left[{E_+}^*\rho_- + E_-{\rho_+}^*\right] \qquad \frac{\partial n_\pm}{\partial t} = \frac{1}{2}\left[E_\pm{\rho_\pm}^* + {E_\pm}^*\rho_\pm\right]$$

$$\frac{\partial \mathcal{N}}{\partial t} = -\frac{1}{2}\left[E_+{\rho_+}^* + {E_+}^*\rho_+ + E_-{\rho_-}^* + {E_-}^*\rho_-\right]$$

$$g(\lambda) = \frac{\varepsilon}{\pi(\varepsilon^2 + \lambda^2)} \qquad n_+ + n_- + \mathcal{N} = 1$$

Conserved quantities (traces of $\hat{\rho}$, $\hat{\rho}^2$, and $\hat{\rho}^3$).

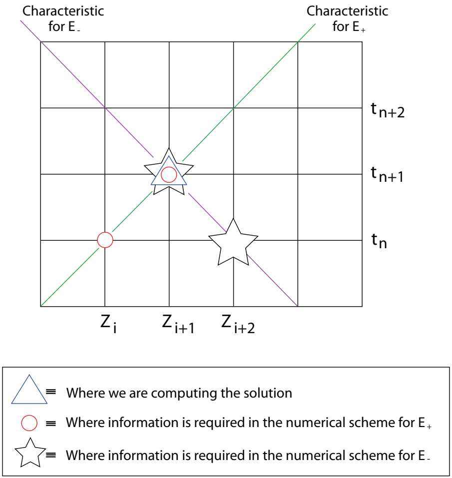
$$I_1 = n_+ + n_- + \mathcal{N} \quad \text{(conservation of particles)}$$

$$I_2 = n_+^2 + n_-^2 + \mathcal{N}^2 + 2(\rho_+^2 + \rho_-^2 + \mu^2) \quad \text{(unitarity)}$$

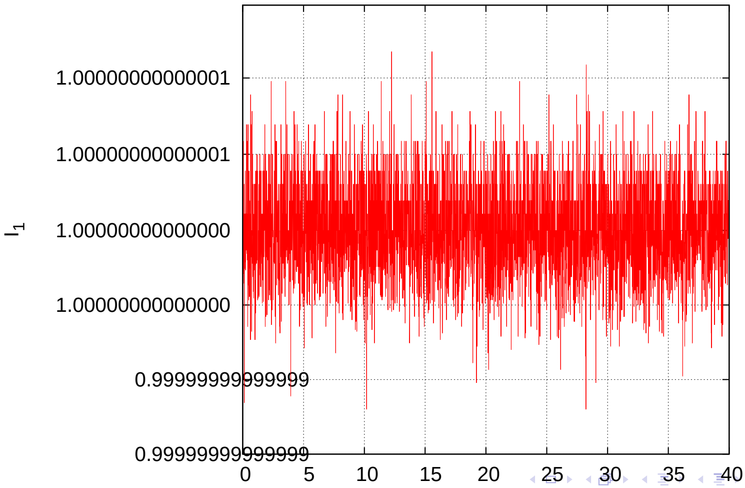
$$I_3 = n_+^3 + n_-^3 + \mathcal{N}^3 +$$

$$3[\mathcal{N}(\rho_+^2 + \rho_-^2) + n_+(\rho_+^2 + \mu^2) + n_-(\rho_-^2 + \mu^2) + \rho_+\rho_-^*\mu + \rho_+^*\rho_-\mu^*]$$

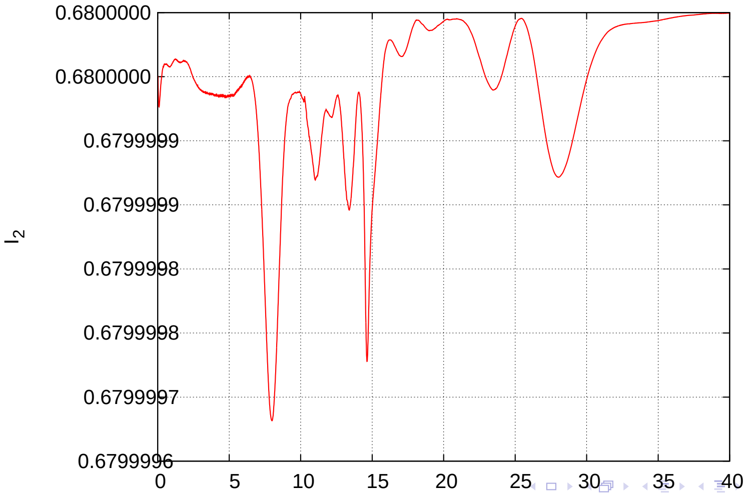
Numerical Scheme.



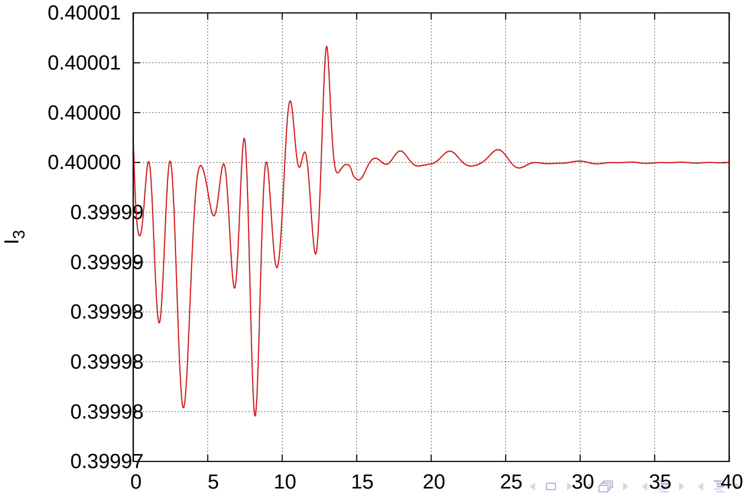
Number of particles.



Unitarity.


0.6800000
0.6800000
0.6799999
0.6799999
0.6799998
0.6799998
0.6799997
0.6799996
I_2
0
5
10
15
20
25
30
35
40

Third integral of motion.



Numerical Experiment.

- $\mu = 0.2$, $n_{+} = 0.6$, $n_{+} = 0.4$, $c = 1$.
- Gaussian pulse in $E_{+}(t)$ on the left boundary. Amplitude 2.0, width 1.0, delay 3.0.
- Supposed to get backfire in E_{-} on the left boundary.

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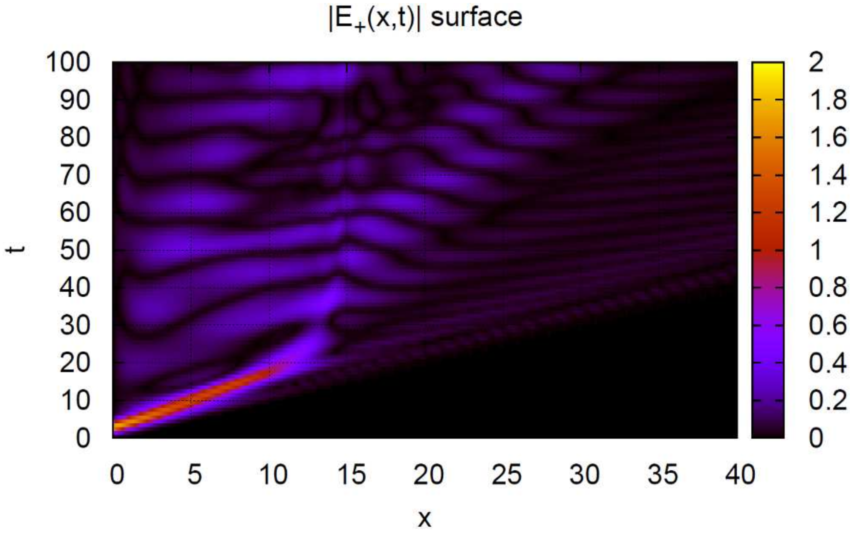
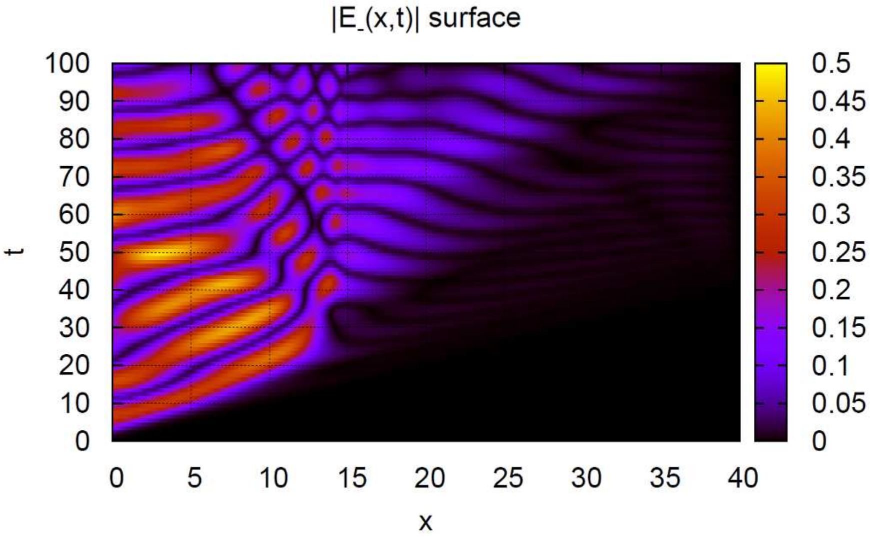
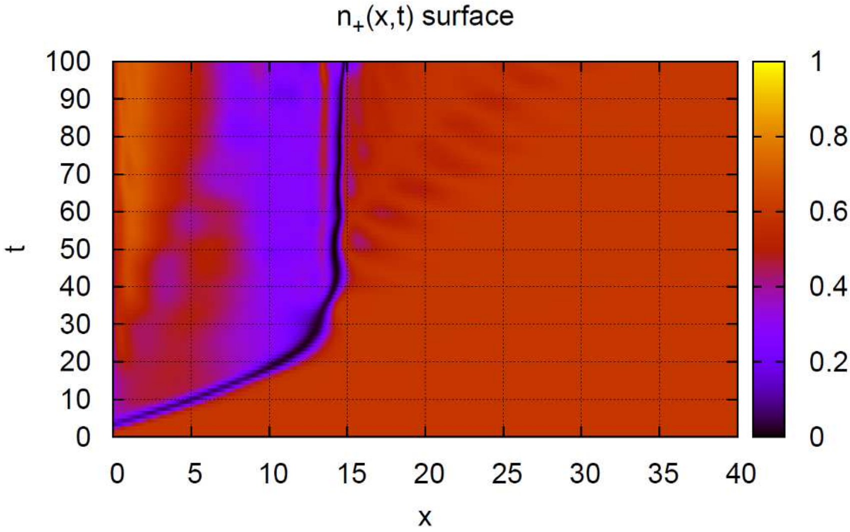
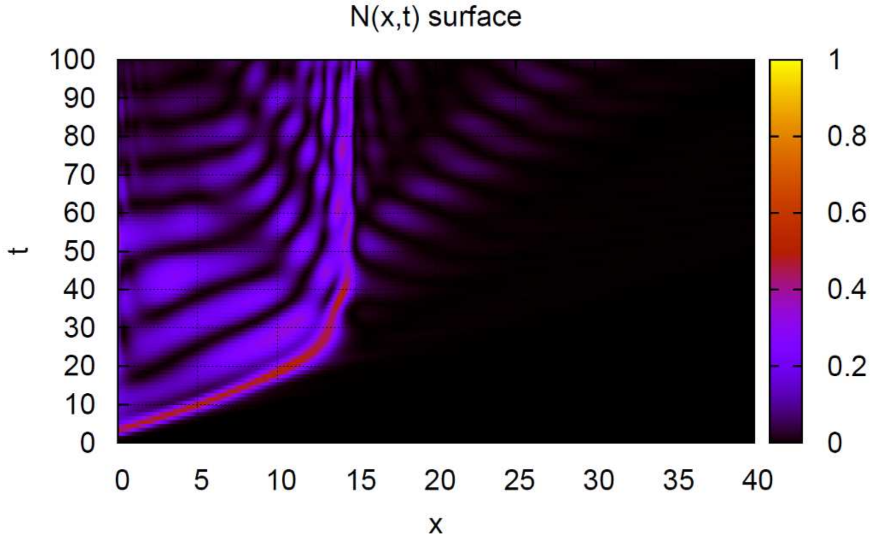
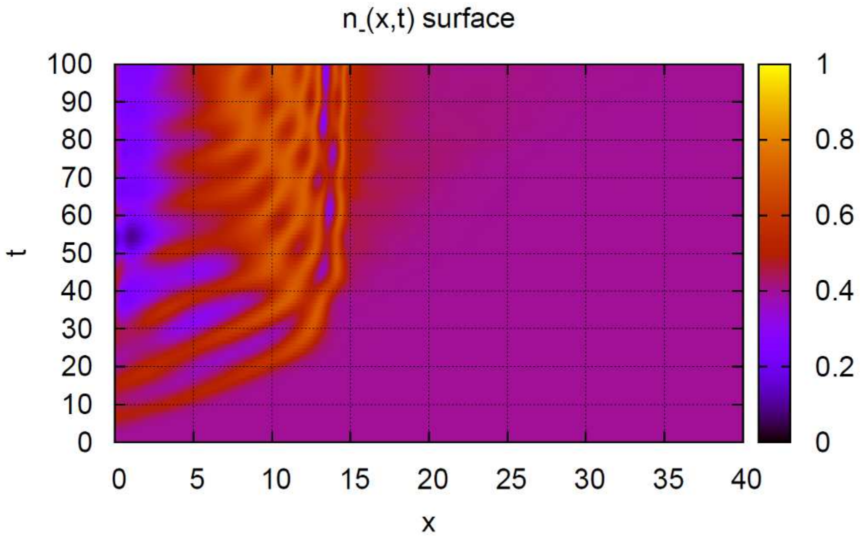
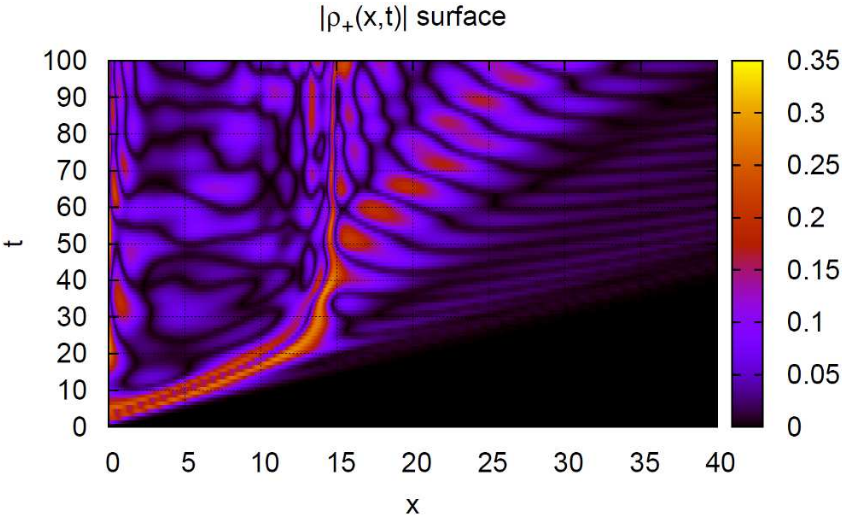

|E+(x,t)| surface
100
90
80
70
60
50
40
30
20
10
0
t
0
5
10
15
20
25
30
35
40
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0

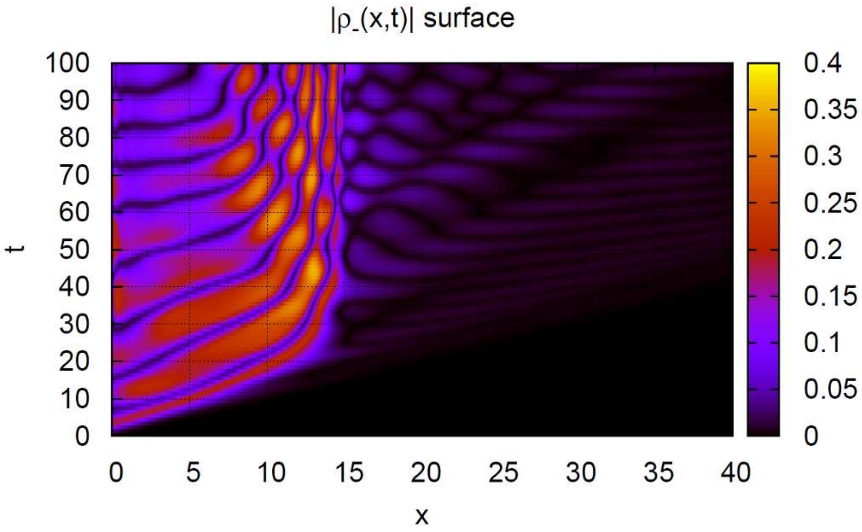

|E_(x,t)| surface
100
90
80
70
60
50
40
30
20
10
0
t
0
5
10
15
20
25
30
35
40
x
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

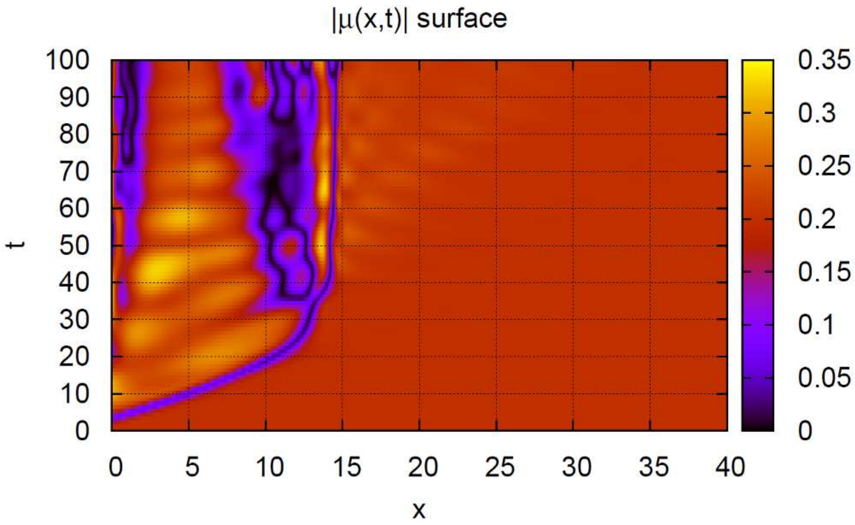

n+(x,t) surface
100
90
80
70
60
50
40
30
20
10
0
t
0
5
10
15
20
25
30
35
40
x
1
0.8
0.6
0.4
0.2
0


N(x,t) surface
t
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
x
1
0.8
0.6
0.4
0.2
0


n_(x,t) surface
100
90
80
70
60
50
40
30
20
10
0
t
0
5
10
15
20
25
30
35
40
x
1
0.8
0.6
0.4
0.2
0


|ρ+(x,t)| surface
100
90
80
70
60
50
40
30
20
10
0
t
0
5
10
15
20
25
30
35
40
x
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0


|ρ_(x,t)| surface
t
x
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0


|μ(x,t)| surface
t
x
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
30
35
40
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0

Summary

Conclusions

- The test for the negative refraction material is proposed.
- We created a code which simulates Maxwell-Bloch equations for Λ-configuration.
- We have found set of conditions when the proposed test works.
- Performed simulation of the backfiring pulse to confirm the proposed testing procedure.

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