Caroli-de Gennes-Matricon states in 3He-B vortices: experimental status

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1. Fermion zero modes on vortices in topological materials

* Bulk-vortex correspondence in topological matter
* Majorana fermions in the vortex core of topological superfluid 3He-B

2. Vortex with asymmetric vortex core in 3He-B

* Broken symmetries in the vortex core
* Spectrum of fermion zero modes in asymmetric core

3. Experiment

* Magnon Bose condensate as a tool for studing core excitations
* Goldstone mode of broken rotational symmetry
* Relaxation of magnon BEC due to core fermions
* Observation of frequency comb
* Source of frequency comb is not clear

4. Conclusion

Zero energy states in the core of vortices in topological superfluids

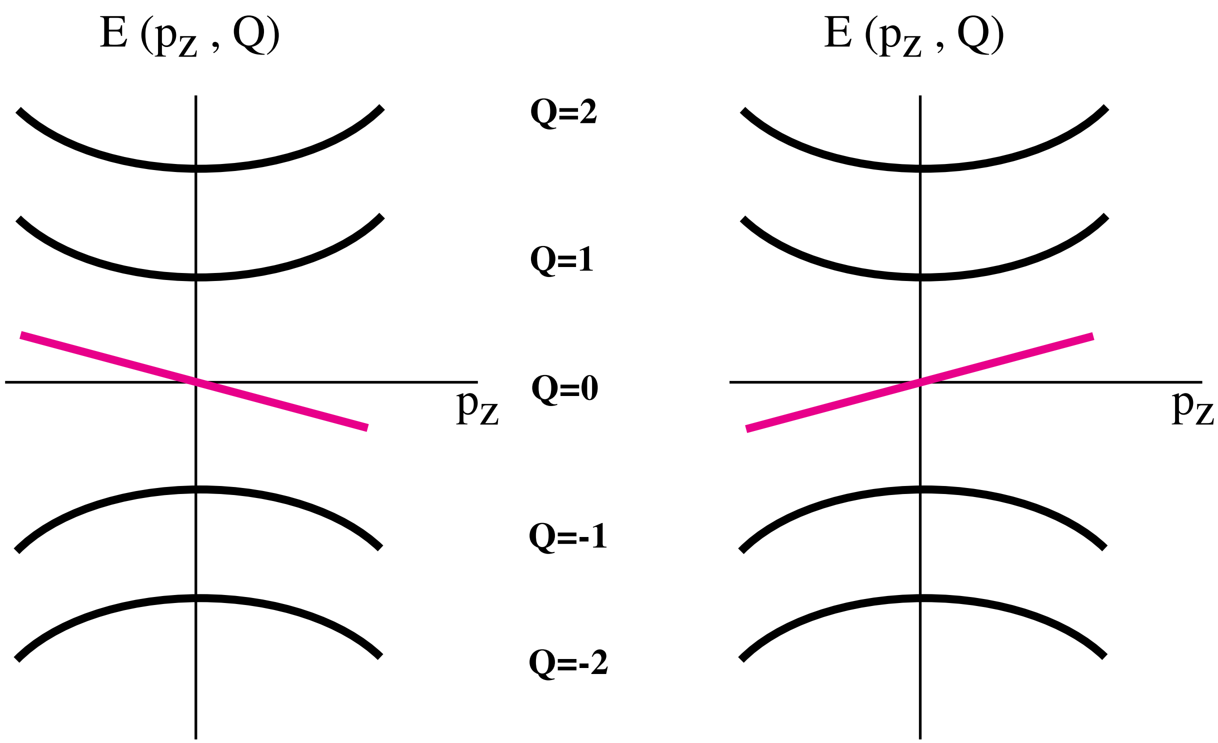
vortices in fully gapped 3+1 system

fermion zero modes in vortex core

Bound states of fermions on cosmic strings and vortices

Spectrum of quarks in core of electroweak cosmic string

quantum numbers: Q - angular momentum & p_z - linear momentum



$E(p_z) = - cp_z$ for d quarks $\quad$ $E(p_z) = cp_z$ for u quark

asymmetric branches cross zero energy

Index theorem:

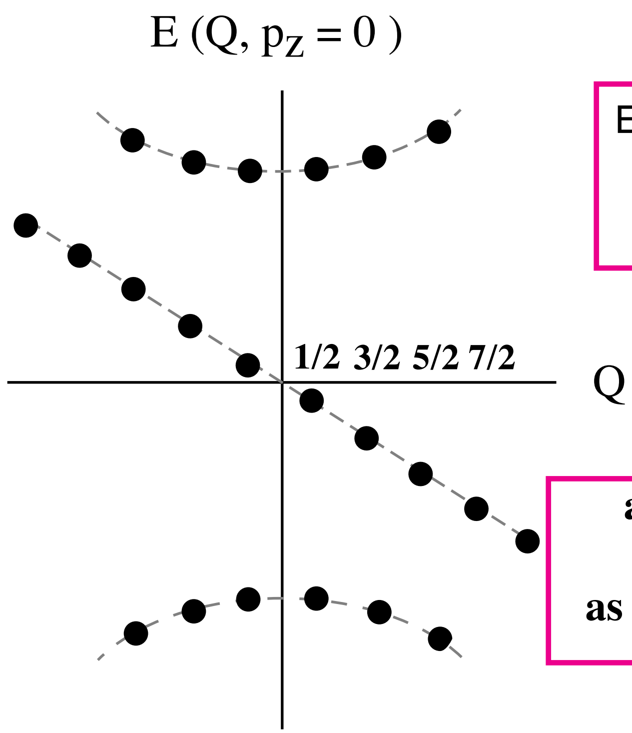
Number of asymmetric branches = N N is vortex winding number

Jackiw & Rossi
Nucl. Phys. B**190**, 681 (1981)

Bound states of fermions on vortex in s-wave superconductor

Caroli, de Gennes & J. Matricon, Phys. Lett. **9** (1964) 307

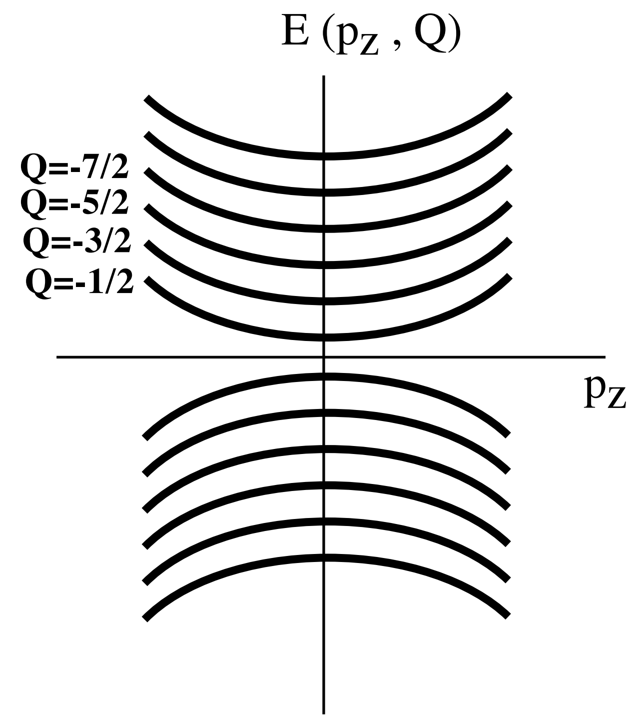
$N_K = 0$



$E(Q,p_z) = -Q\,\omega_0(p_z)$

$\omega_0 = \Delta^2/E_F \ll \Delta$

asymmetric branch as function of Q



Angular momentum Q is half-odd integer
in s-wave superconductor

no true fermion zero modes:
no asymmetric branch as function of p_z

Index theorem for approximate fermion zero modes:

Number of asymmetric Q-branches = 2N
N is vortex winding number

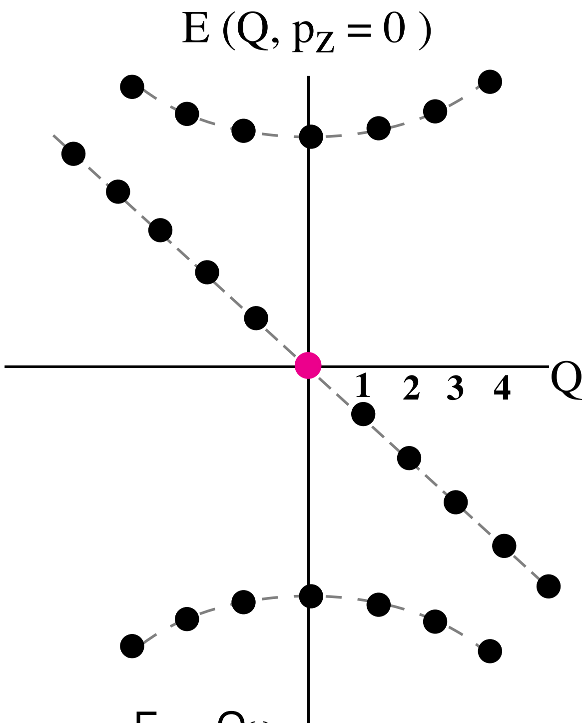
GV JETP Lett. **57**, 244 (1993)

Index theorem for true fermion zero modes?

is the existence of fermion zero modes
related to topology in bulk?

fermions zero modes on symmetric vortex in 3He-B

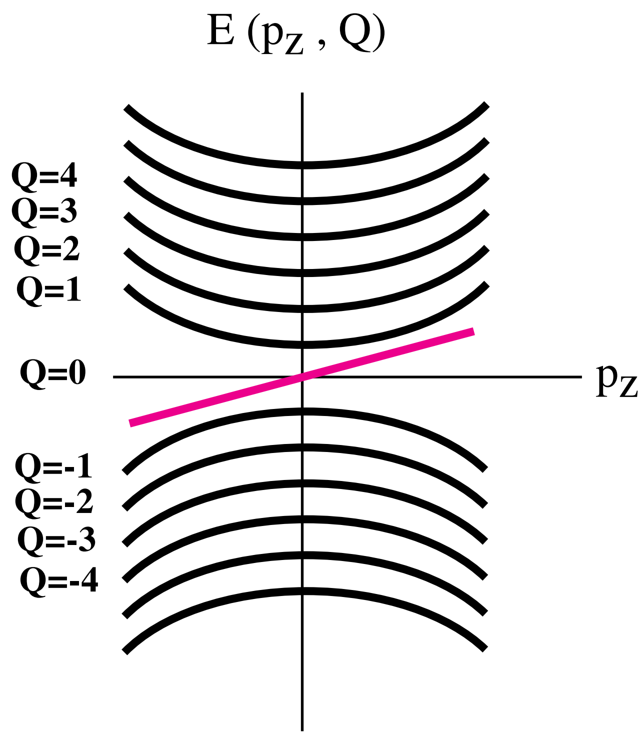
topological ^{3}He-B at $\mu > 0$: $N_K = 2$



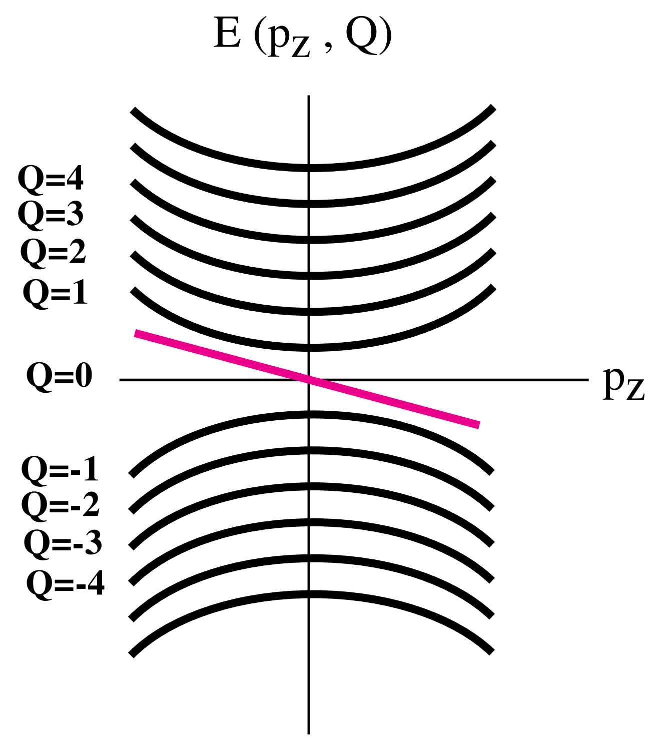
$E_Q = -Q\omega_0$

$\omega_0 = \Delta^2/E_F << \Delta$

Q is integer
for p-wave superfluid ^{3}He-B



helicity +



helicity –

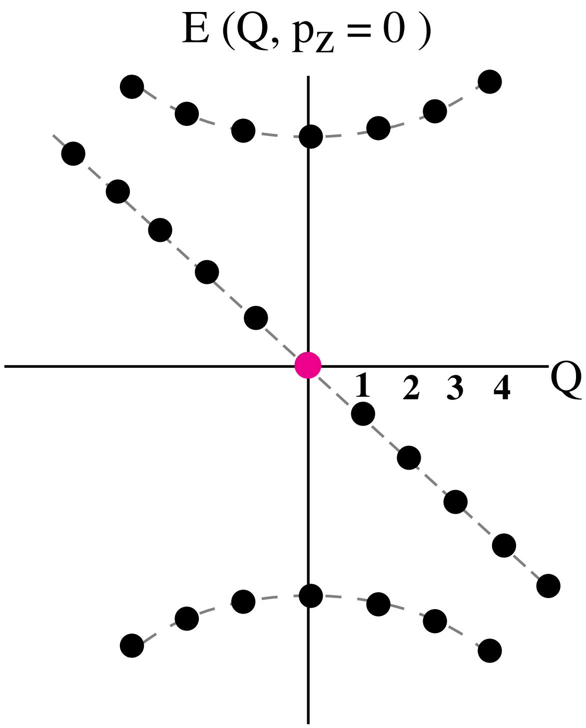
gapless fermions on Q=0 branch form

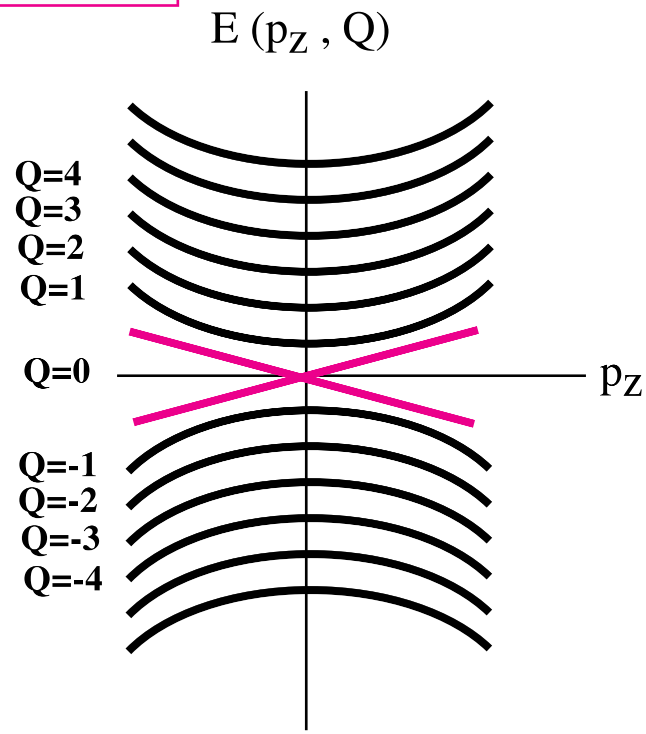
1D Fermi-liquid

Misirpashaev & GV
Fermion zero modes in symmetric vortices in superfluid 3He,
Physica B **210**, 338 (1995)

fermions zero modes on symmetric vortex in 3He-B

topological ^{3}He-B at $\mu > 0$: $N_K = 2$





$E_Q = -Q\omega_0$

$\omega_0 = \Delta^2/E_F << \Delta$

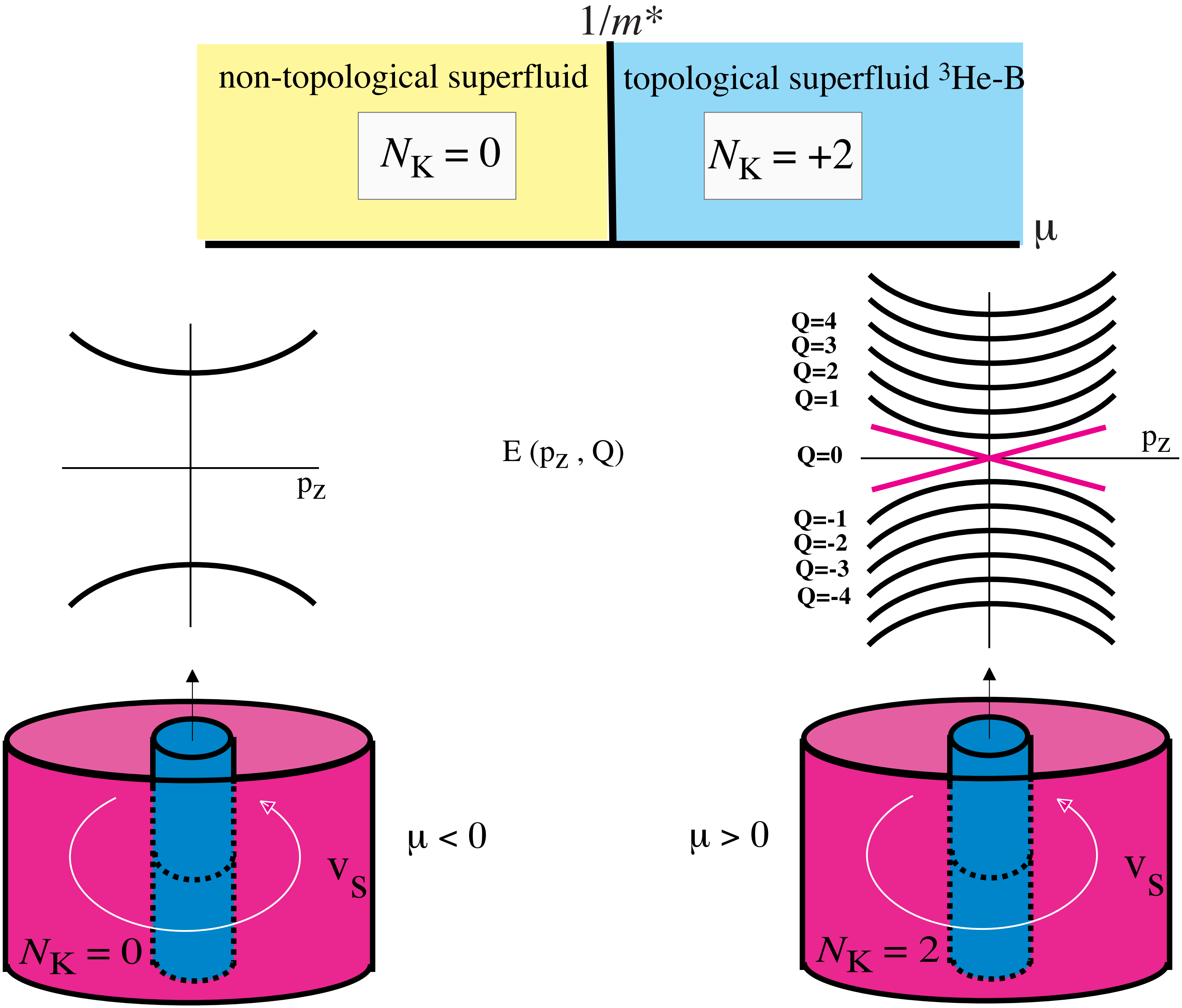
Q is integer
for p-wave superfluid ^{3}He-B

gapless fermions on Q=0 branch form

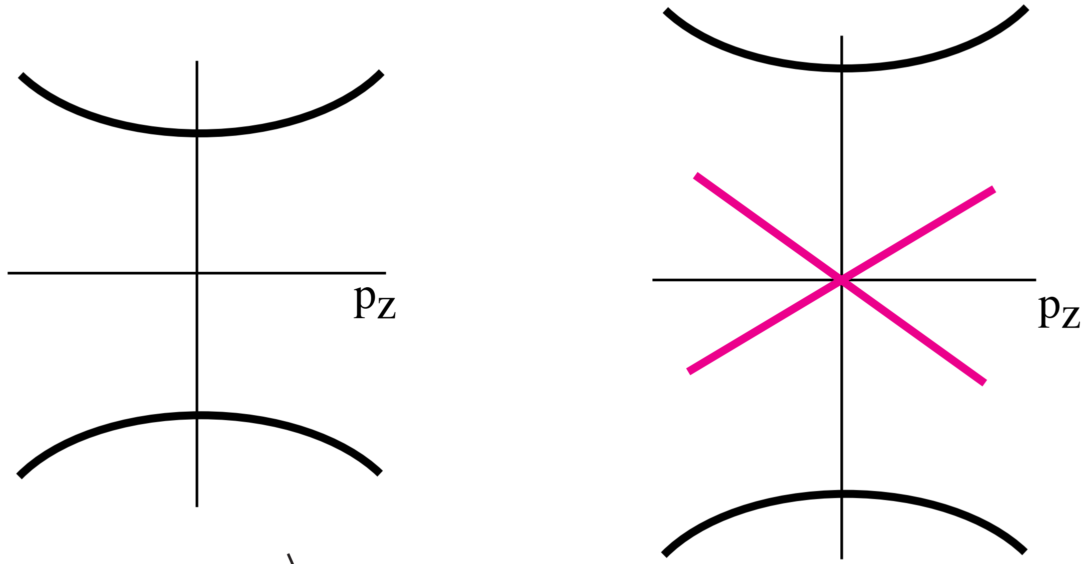
1D Fermi-liquid

Misirpashaev & GV
Fermion zero modes in symmetric vortices in superfluid 3He,
Physica B **210**, 338 (1995)

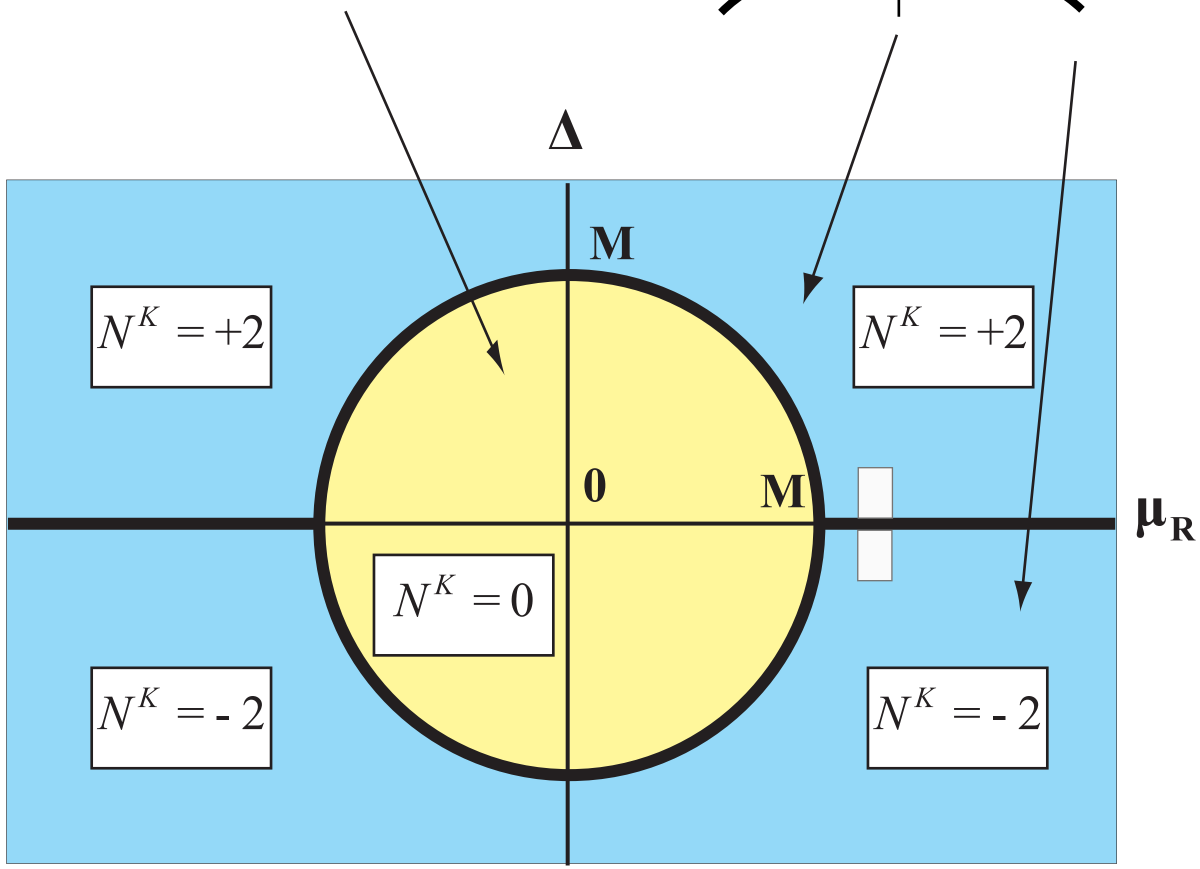
topological quantum phase transition in bulk & in vortex core



fermion zero modes in relativistic triplet superconductor



$$H = \begin{pmatrix} c\alpha\cdot\mathbf{p} + \beta M - \mu_R & \gamma_5\Delta \\ \gamma_5\Delta & -c\alpha\cdot\mathbf{p} - \beta M + \mu_R \end{pmatrix}$$



vortices in topological superconductors have fermion zero modes

generalized index theorem ?

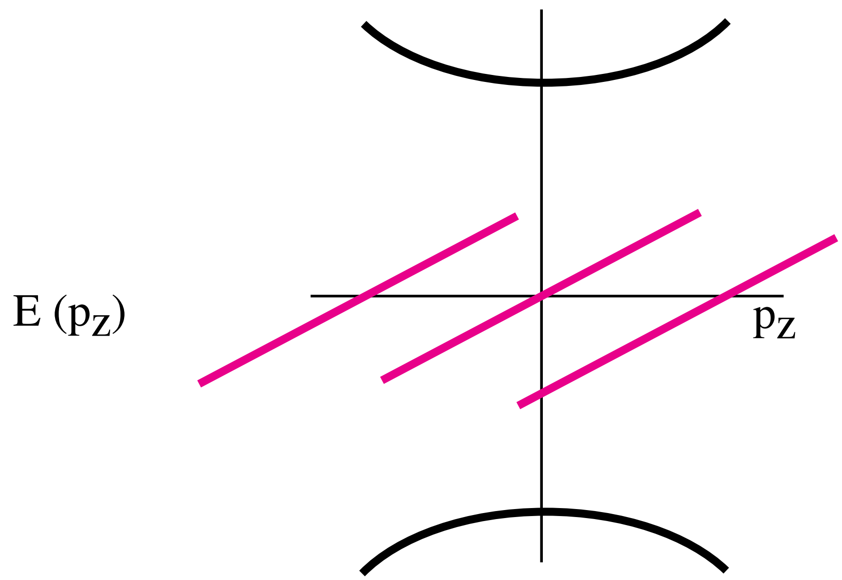
index theorem for fermion zero modes on vortices

(interplay of ***r***-space and ***p***-space topologies)

$$N_5 = \frac{1}{4\pi^3 i} \mathbf{tr} \left[\int d^3p \, d\omega \, d\phi \; \mathbf{G} \nabla_\omega \mathbf{G}^{-1} \mathbf{G} \nabla_\phi \mathbf{G}^{-1} \mathbf{G} \nabla_{p_x} \mathbf{G}^{-1} \mathbf{G} \nabla_{p_y} \mathbf{G}^{-1} \mathbf{G} \nabla_{p_z} \mathbf{G}^{-1} \right]$$

for vortices in Dirac vacuum

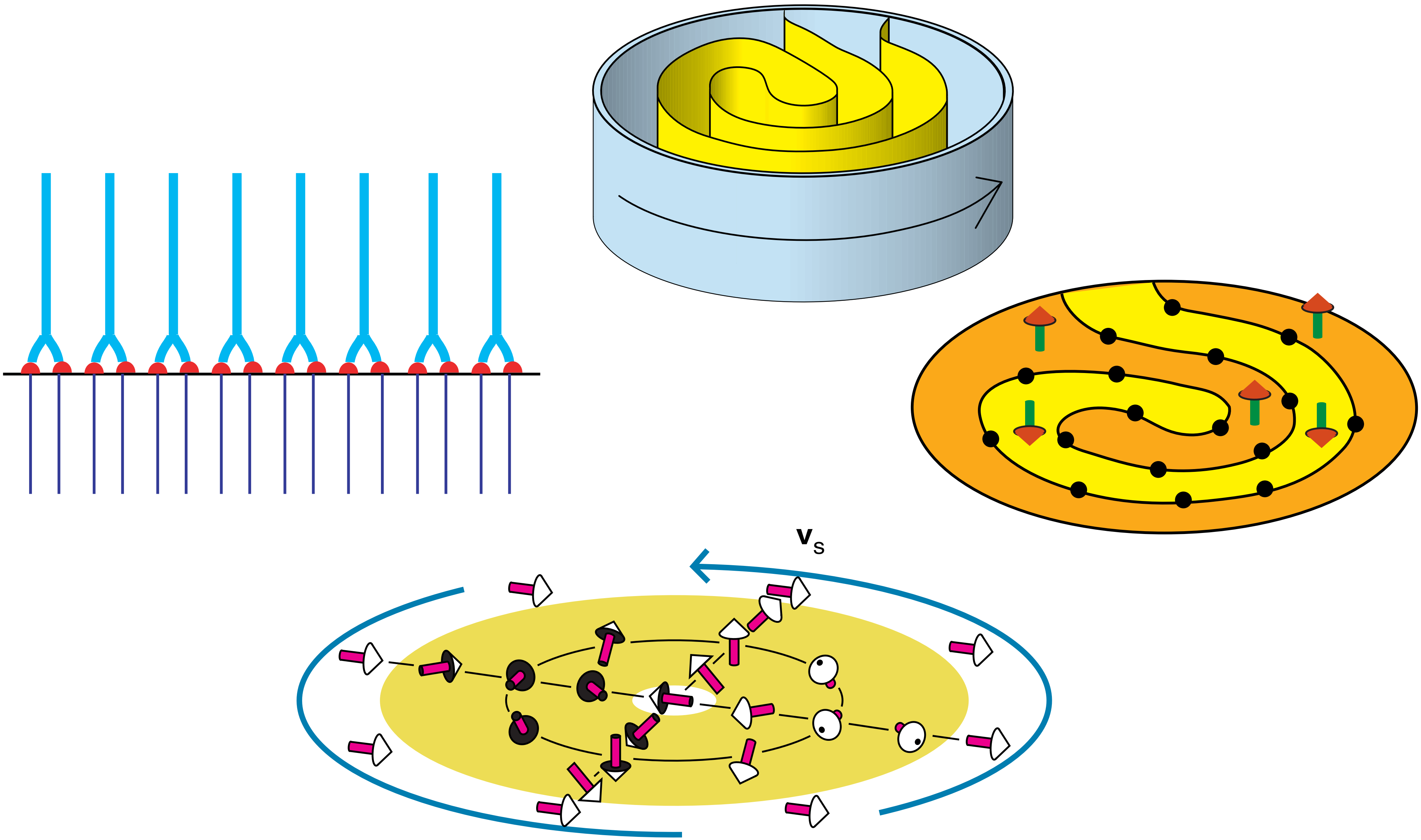
$N_5 = N$ **winding number**



N_5 invariant was introduced by Golterman, Jansen & Kaplan for lattice fermions
Phys. Lett. B **301** (1993) 219

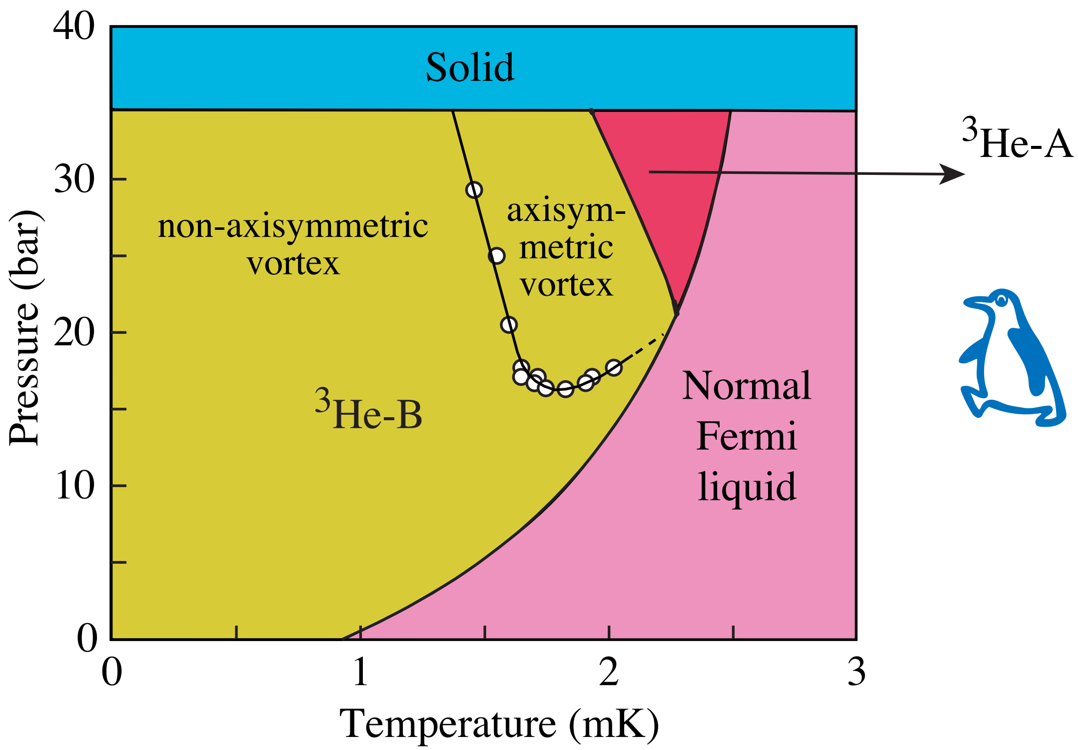
see also M.A. Zubkov & GV
Momentum space topological invariants for the 4D relativistic vacua with mass gap
Nucl. Phys. B **860** (2012) 295

r-space vortices

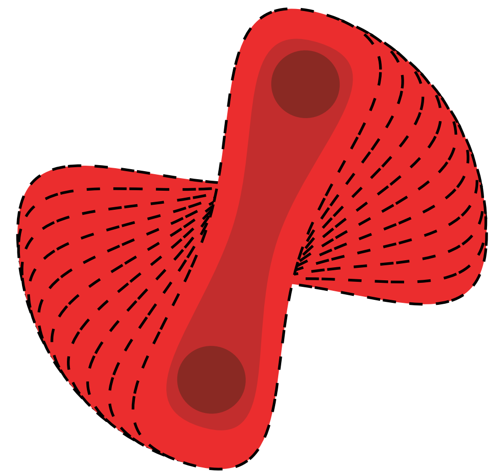


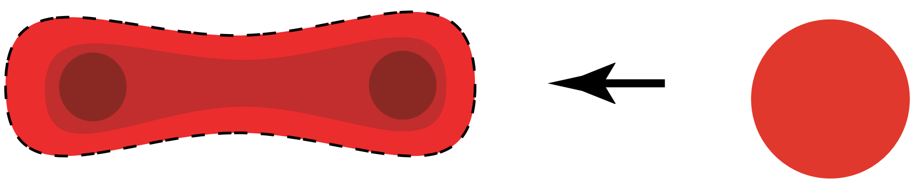
* experiment: **evidence of non-axisymmetric vortex core**

Kondo, Korhonen, Krusius, et al., PRL **67**, 81 (***1991***)

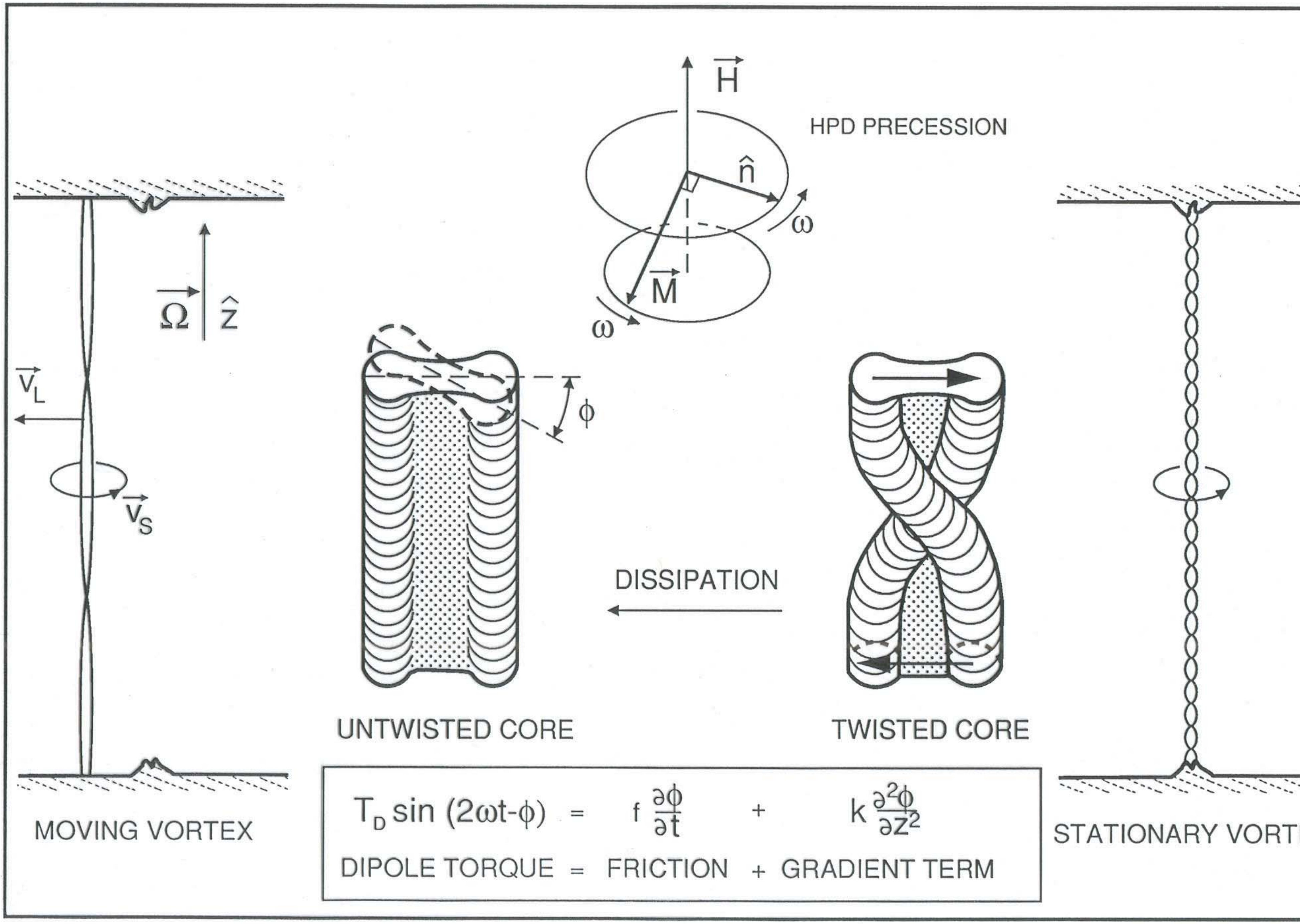


Goldstone boson (twist) is observed

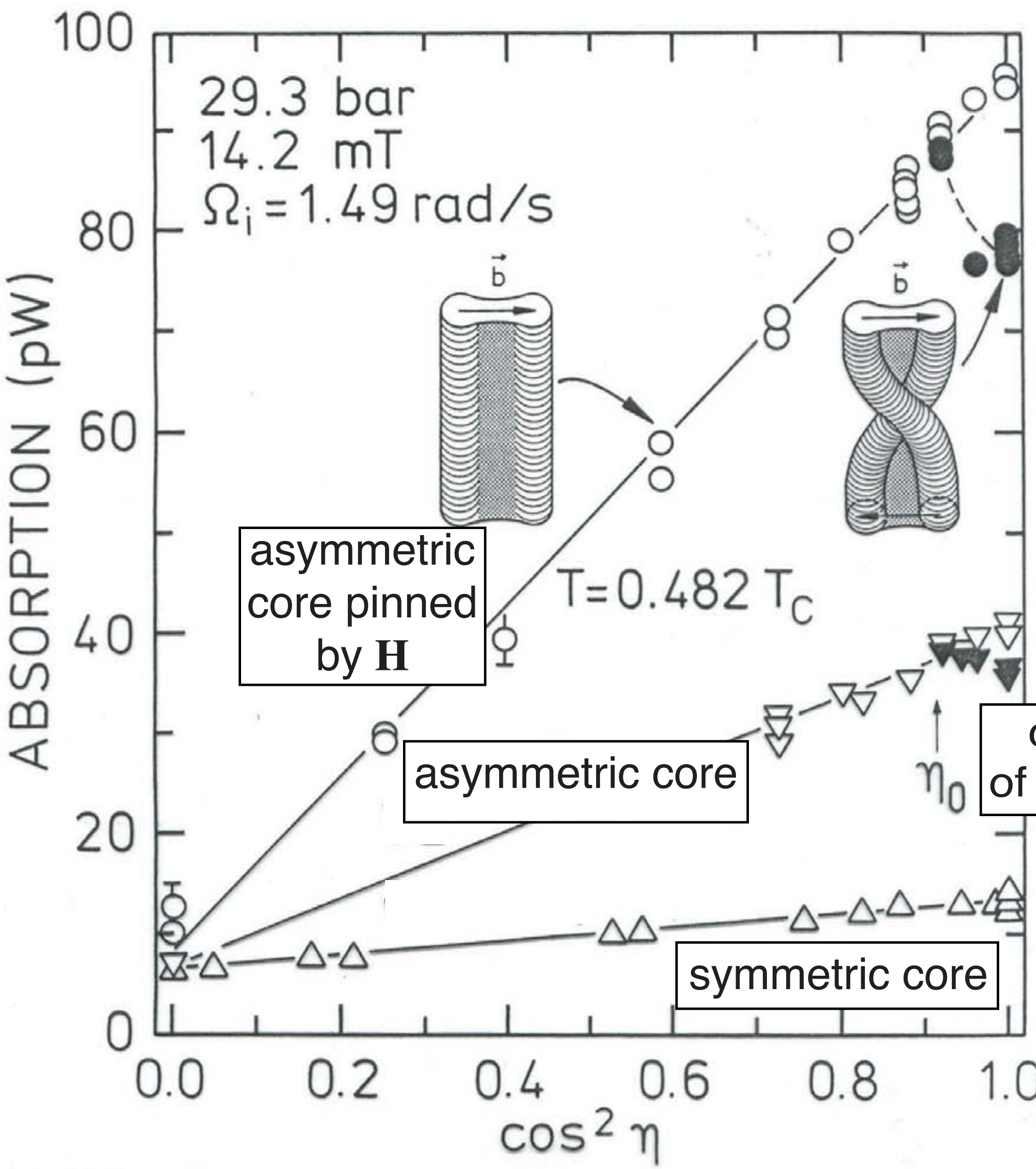




twisting the vortex core by HPD



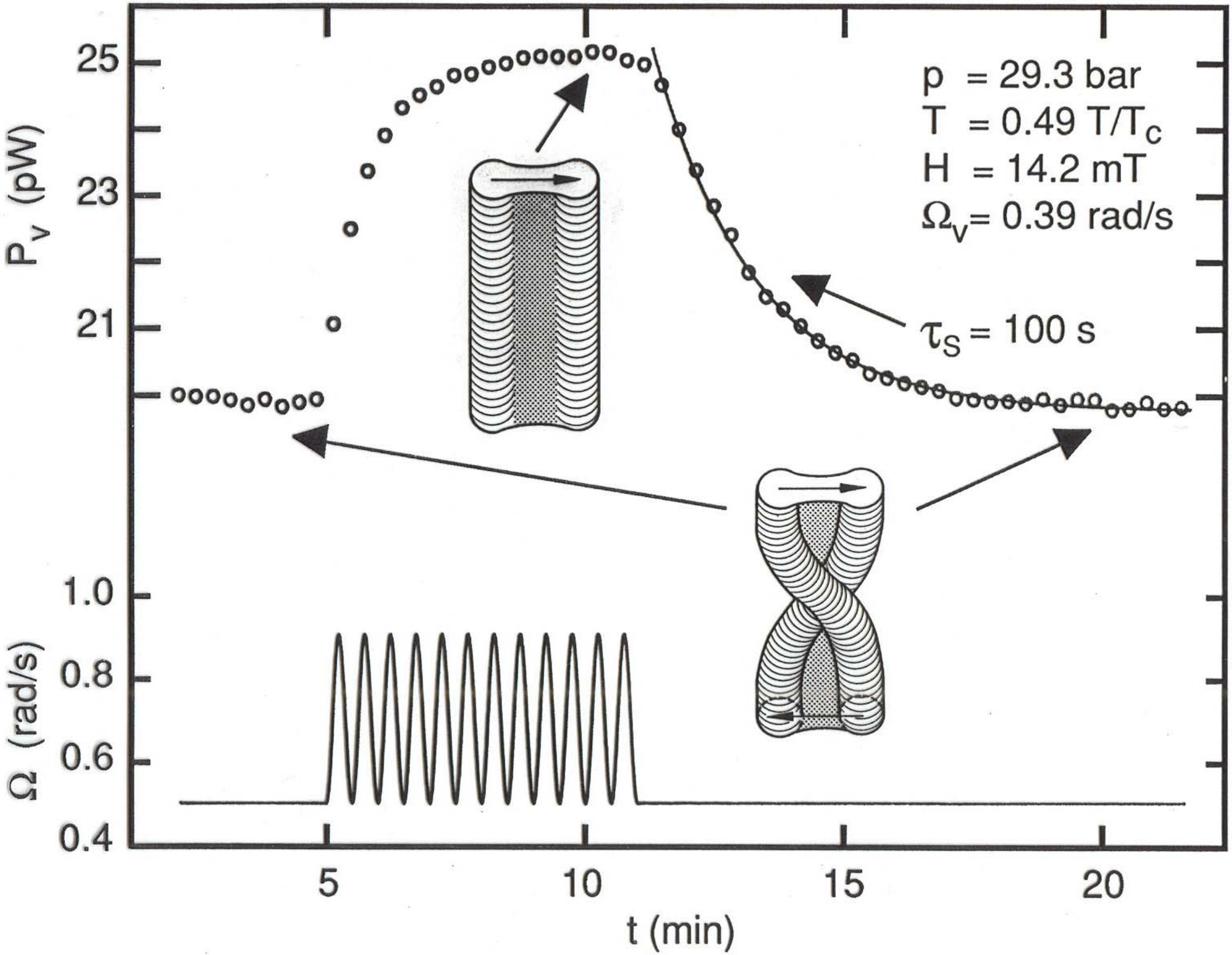
experimental identification of non-axi-symmetric vortex due to its Goldstone mode



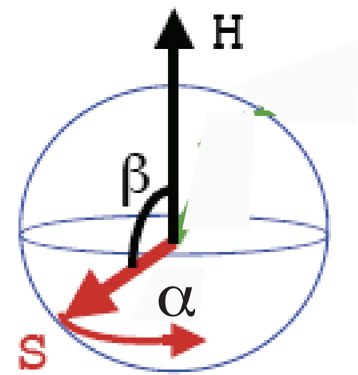
direction of magnetic field

tilted magnetic field pins the core
and prevents twisting if $\eta > \eta_0$ about 2

Untwisting the core by fast oscillations: faster than twisting time τ_S



Holstein-Primakoff transformation transforms coherent spin precession to magnon ODLRO



$$< S_x + iS_y > = S \sin\beta \, e^{i\alpha(t)}$$

$$\alpha(t) = \omega t + \alpha_0$$

angle of precession α ≡ condensate phase α

global frequency of precession ω ≡ chemical potential μ

$$\Psi = < a > = |\Psi| \, e^{i\alpha(t)}$$

$$n = |\Psi|^2 = \frac{S(1-\cos\beta)}{\hbar}$$

magnon destruction operator

rotational symmetry ≡ $U(1)$ symmetry

conservation of S_z ≡ conservation of N

$$\mathbf{v}_s = \frac{\hbar}{m} \nabla\alpha$$

phase coherent precession ≡ magnon BEC

off-diagonal long-range order as in superfluids & superconductors

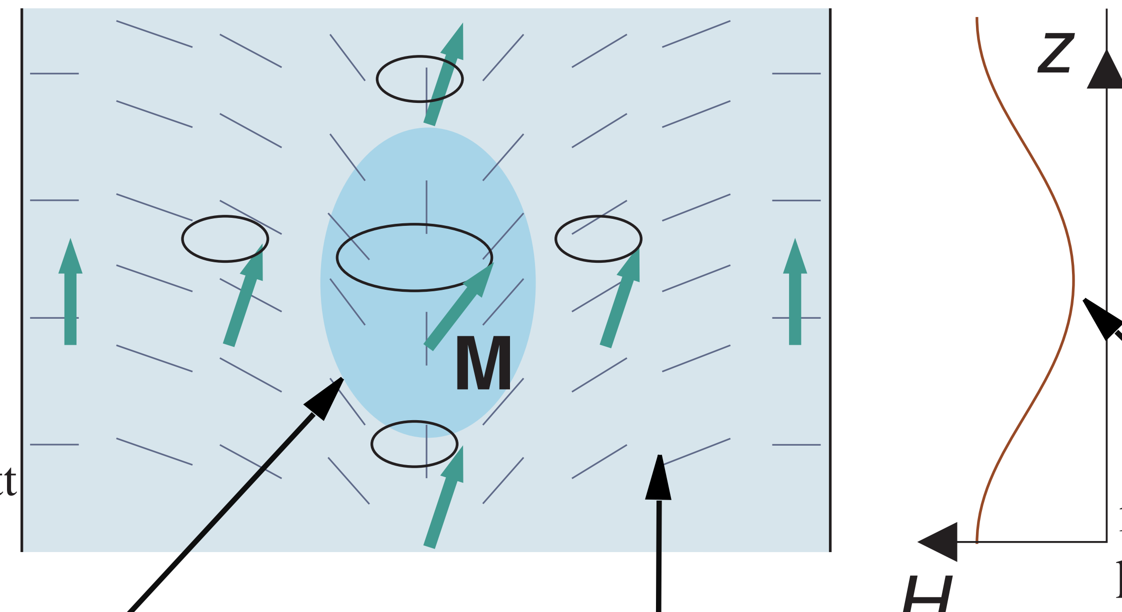
magnon BEC in magneto-textural trap

magnon BEC in bulk
experiences Suhl instability
at low T

no analog in atomic BEC

magnon BEC survives
in magnetic traps

Bunkov, Fisher, Guénault & Pickett
PRL **69**, 3092 (1992);
Bunkov & GV
PRL **98**, 265302 (2007)
Magnon condensation
into a Q-ball in 3He-B



axial
magnetic
harmonic
trap

$\beta_L(r)$ texture forms
radial harmonic trap

engineered magnetic textural 3D trap
Autti, *et al*. arXiv:1002.1674

$$F_{GL} = \frac{|\nabla\Psi|^2}{2m} + (U(\mathbf{r}) - \mu)\,|\Psi|^2 + \lambda(r)\,|\Psi|^4$$

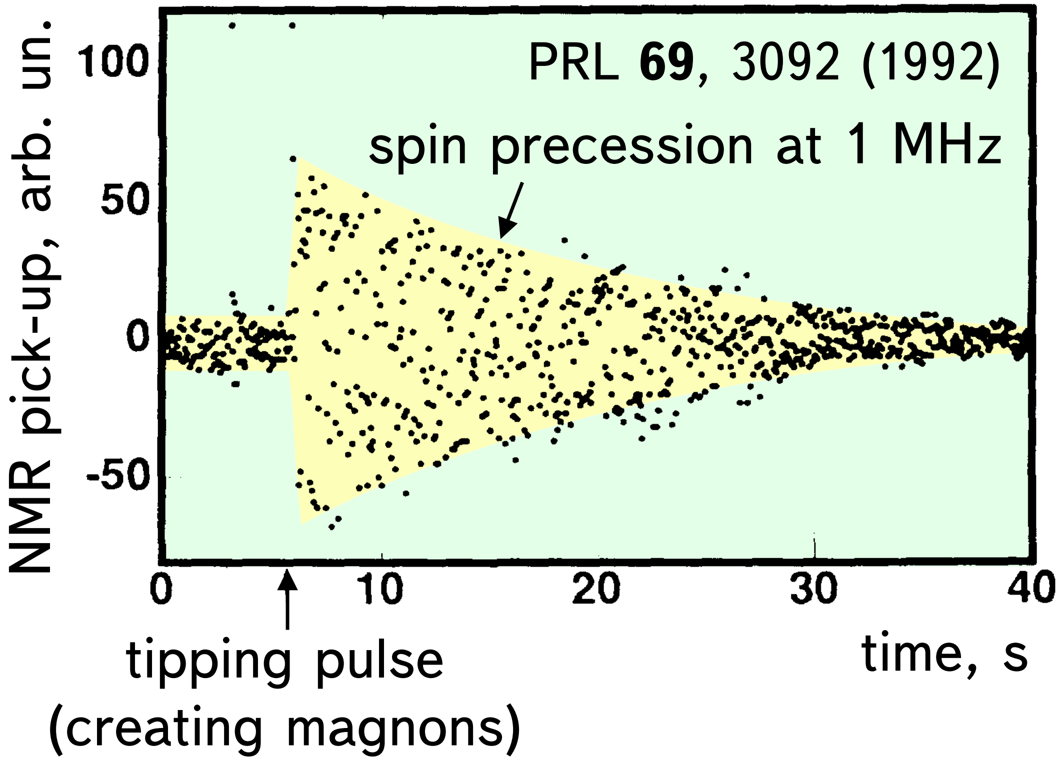
$$U(\mathbf{r}) = \omega_L(z) + \kappa \sin^2\beta_L(r)/2$$

nonlinearity comes from
other source:
self trapping by bubble formation

4-th order term
is negligible

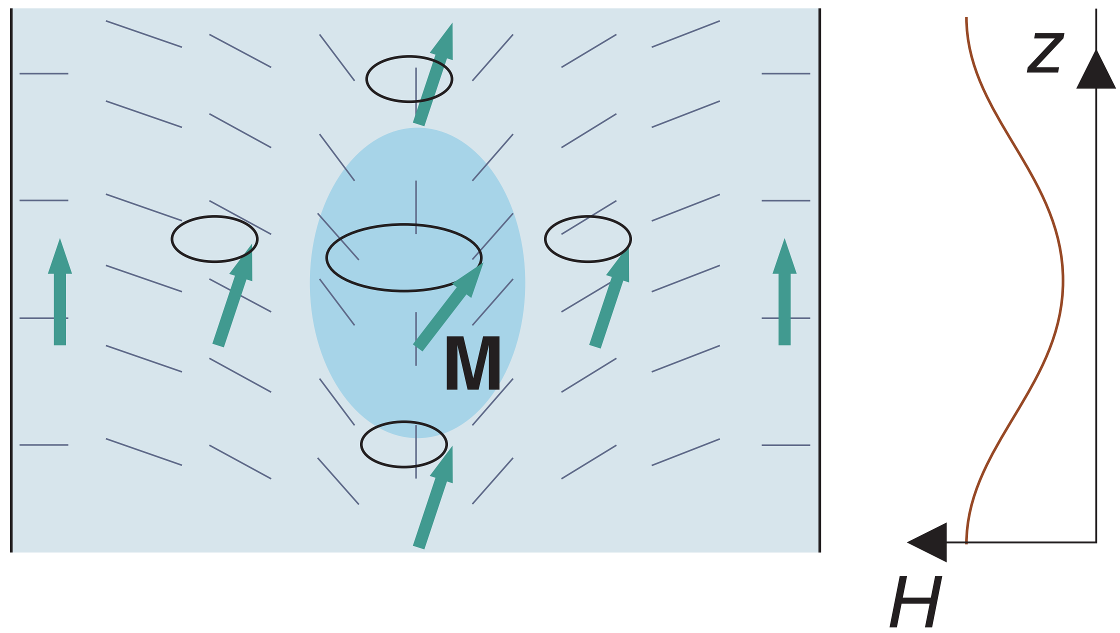
"PERSISTENT" PRECESSION AT LOW TEMPERATURES

Discovered in Lancaster in pulsed NMR experiments at $T < 0.2T_c$



- Relaxation times up to $\sim 10^3$ s.
- Precession frequency increases during relaxation.
- Off-resonance excitation (even with noise) at higher frequencies.

initial harmonic magneto-textural trap



$$F_{GL} = \frac{|\nabla\Psi|^2}{2m} + \left(U(\mathbf{r}) - \mu\right) |\Psi|^2$$

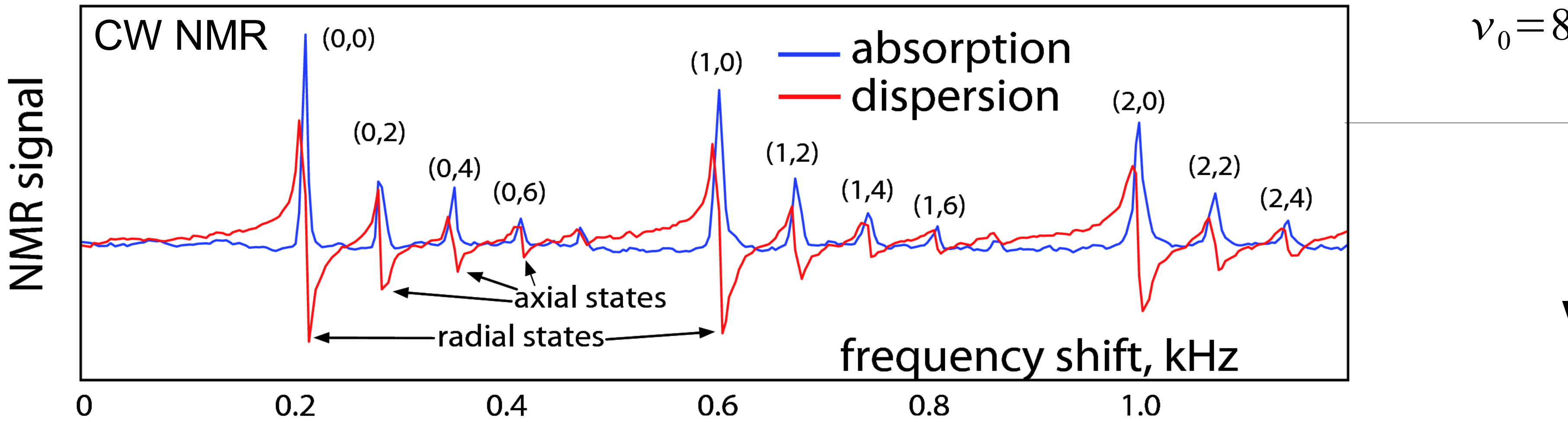
for small magnon number
the original trap is harmonic:

$$U(\mathbf{r}) = \omega_L + m\left(\omega_r^{\,2} r^2 + \omega_z^{\,2} z^2\right)/2$$

$$\omega_{nk} = \omega_L + \omega_r(n+1) + \omega_z(k+1/2)$$

Spinwave spectrum

$P = 0.5\,bar$
$T = 0.15\,T_c$
$\Omega = 0\,rad/s$
$\nu_0 = 826\,kHz$



Spinwave Schrödinger eq.

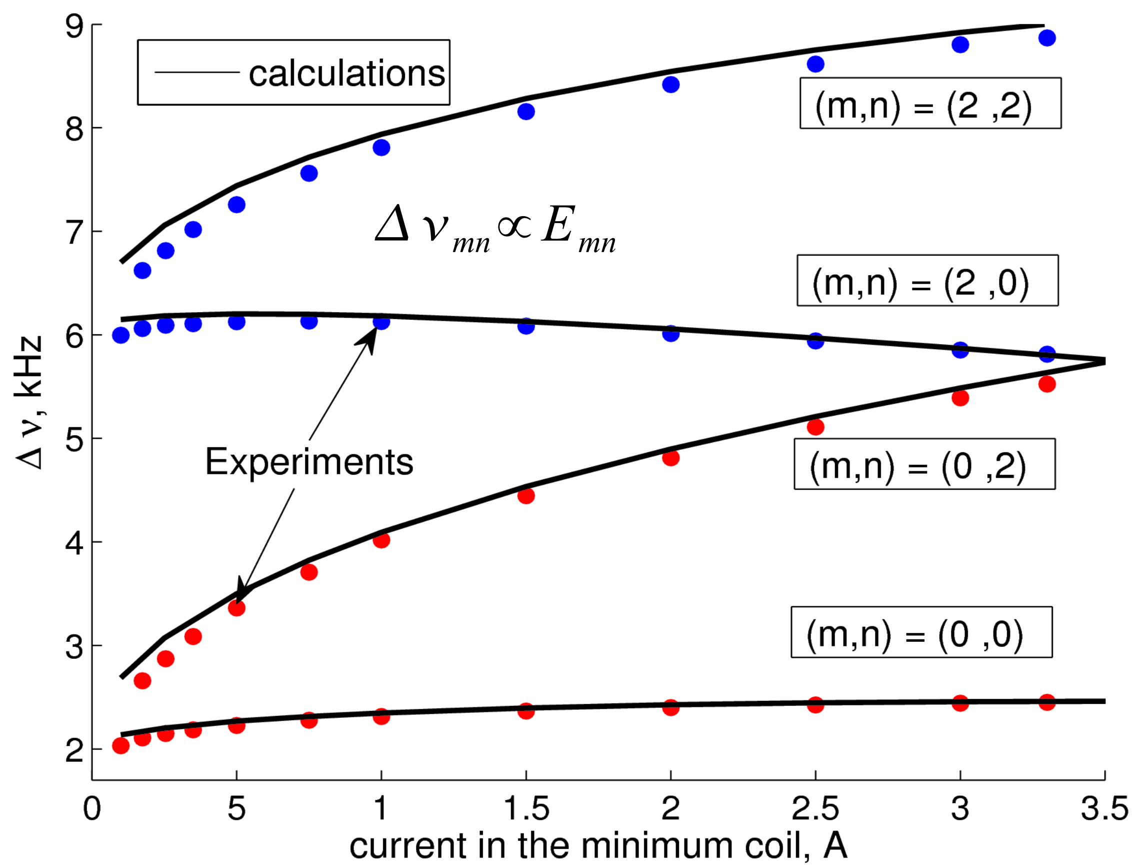
$$\left[-\frac{48}{65}\xi_D \nabla^2 + V\right]\Psi = E\,\Psi$$

Potential

$$V = \frac{4\,\Omega_L}{5\,\omega_L}\sin^2(\beta_L/2) + \gamma\,H$$

Harmonic approximation

$$\Delta\nu_{mn} = \nu_0 + \nu_r(m+1) + \nu_z(n+1/2)$$

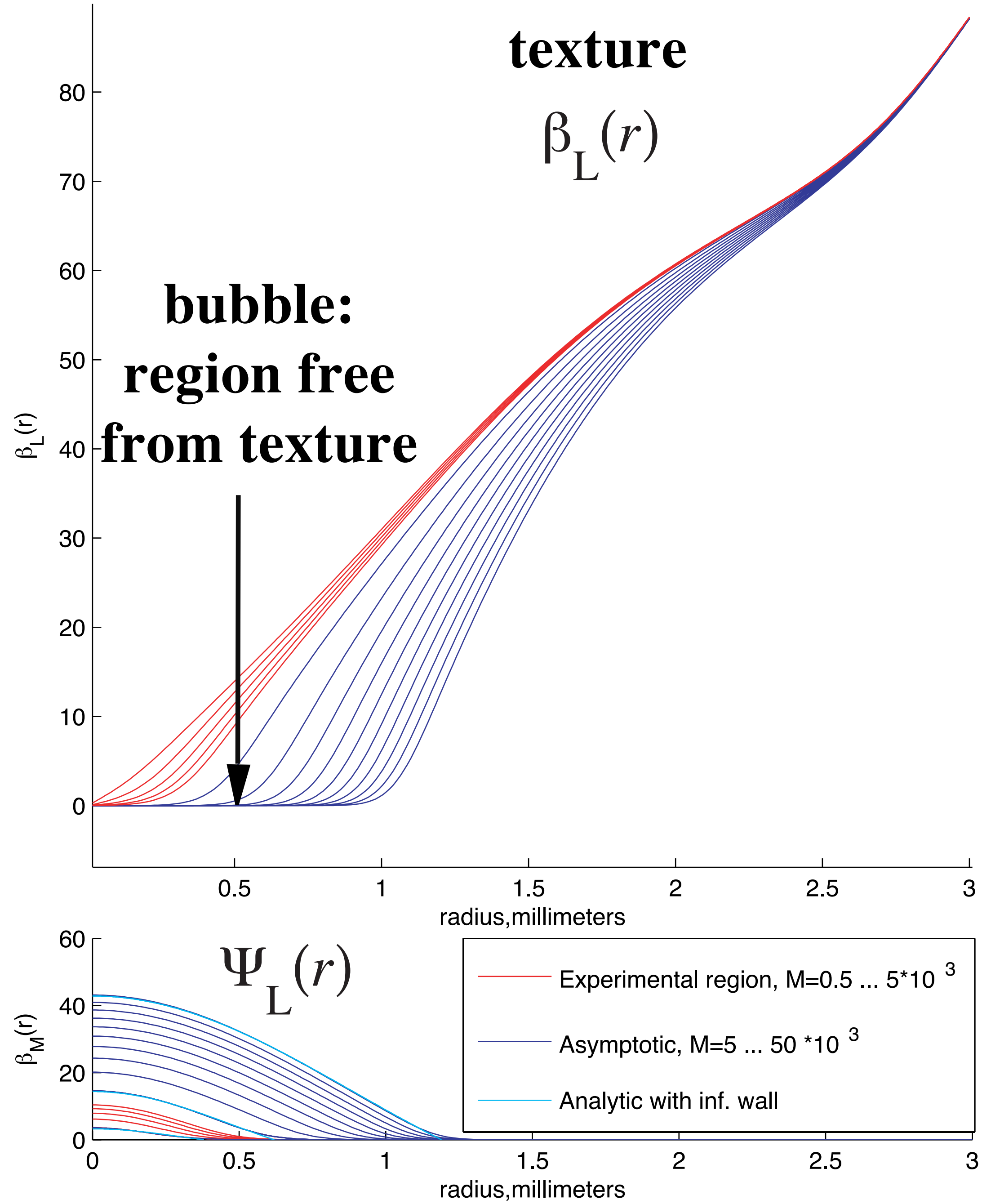


Formation of bubble with magnon BEC in ground state

When the number of magnons increases,
they via spin-orbit interaction modify
flexible orbital texture

self-localization of magnons is bosonic analog
of formation of electron bubble in helium,
(injected electron opens a cavity
whose size is determined
by a balance between the zero-point energy
of electron & surface energy of the bubble)

$$F_D \sim \sin^2\beta_L(\mathbf{r})/2\ |\Psi|^2$$



condensate
self-localized
in the bubble

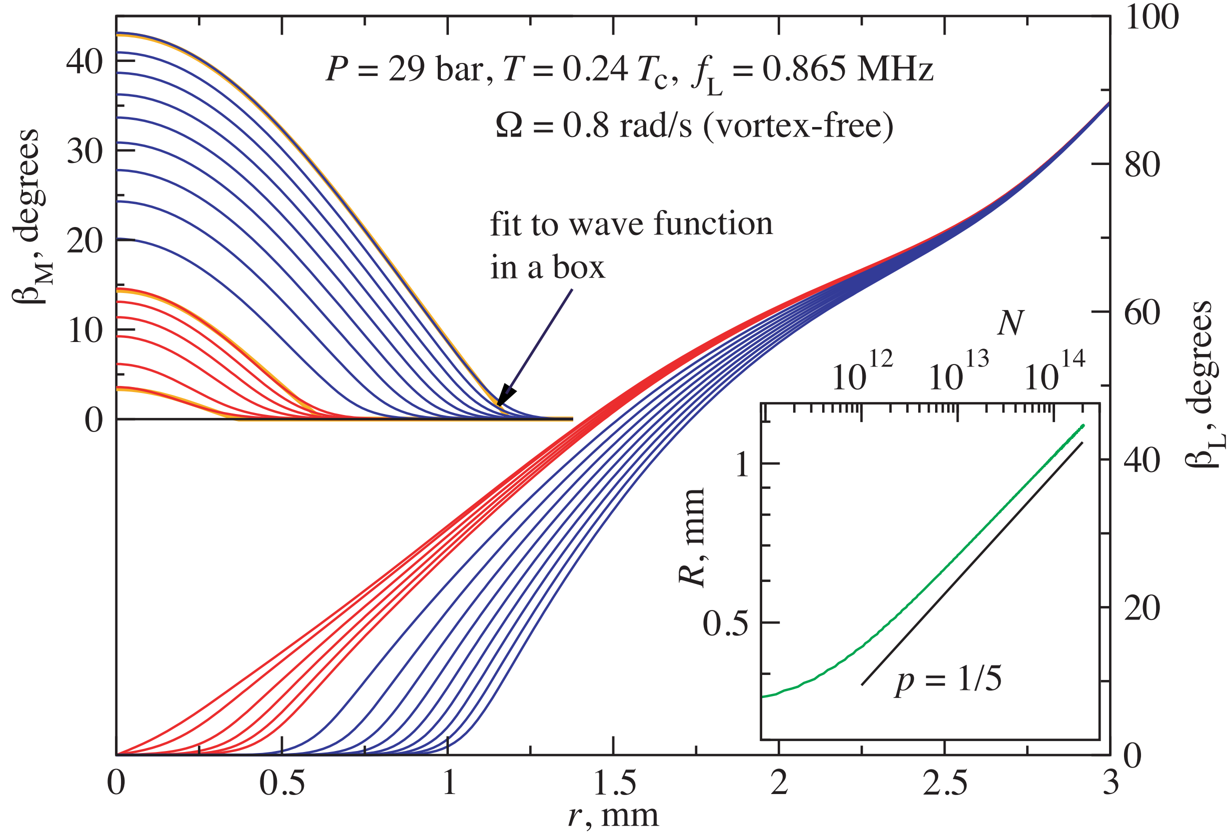
S. Autti, Yu.M. Bunkov, V.B. Eltsov, P.J. Heikkinen, J.J. Hosio, P. Hunger, M. Krusius, G.E. Volovik

Self-trapping of magnon Bose-Einstein condensates in the ground state & on excited levels: From harmonic to box confinement

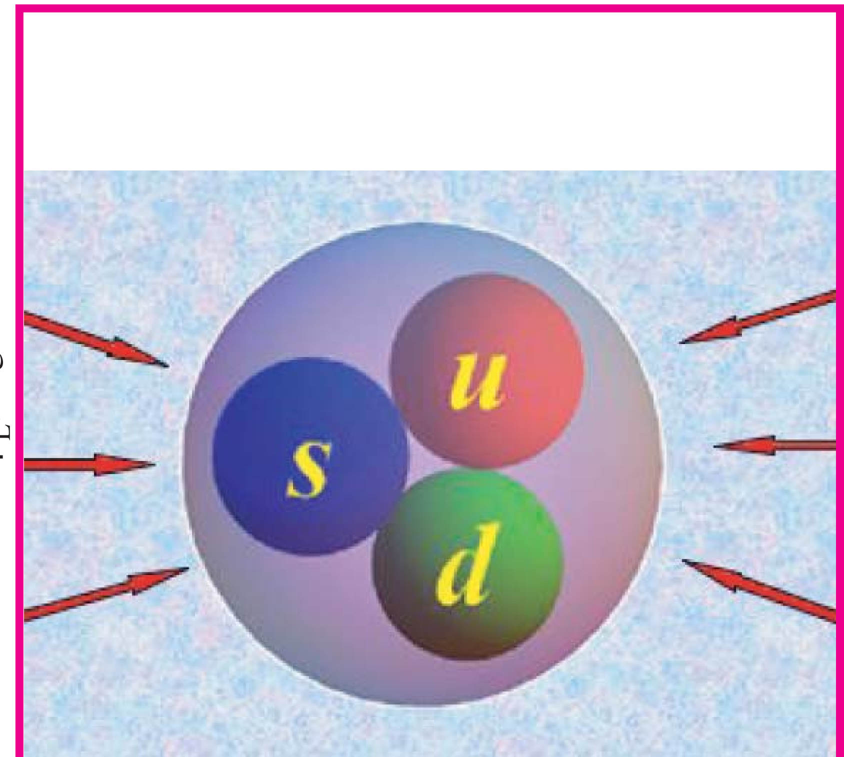
PRL 108, 145303 (2012)

Bose analog of bag model of hadrons

magnons repel the texture and form bubble free of texture within which magnon BEC is confined

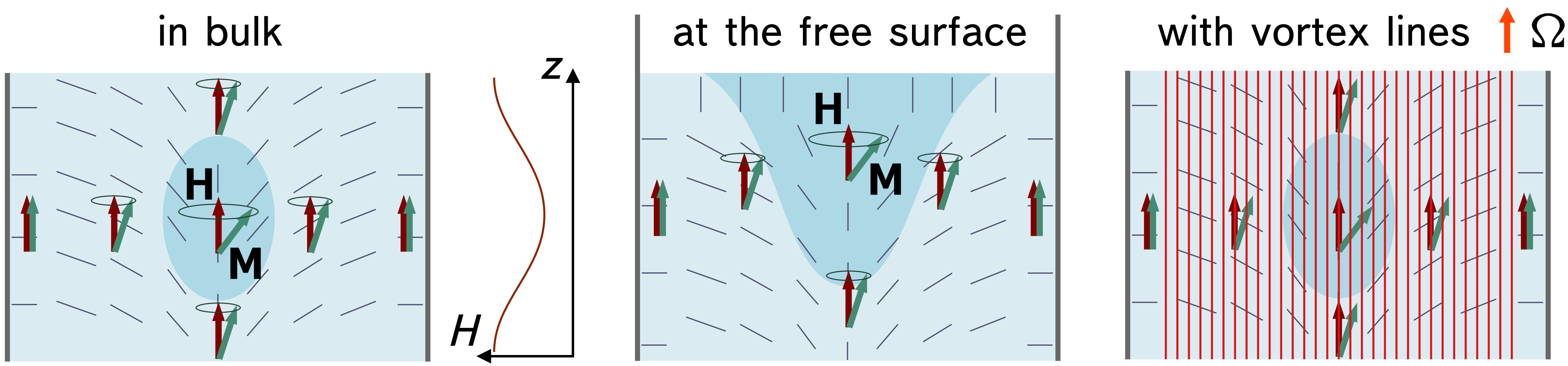


MIT bag model: free quarks repel QCD vacuum and form bubble of false vacuum within which quarks are confined



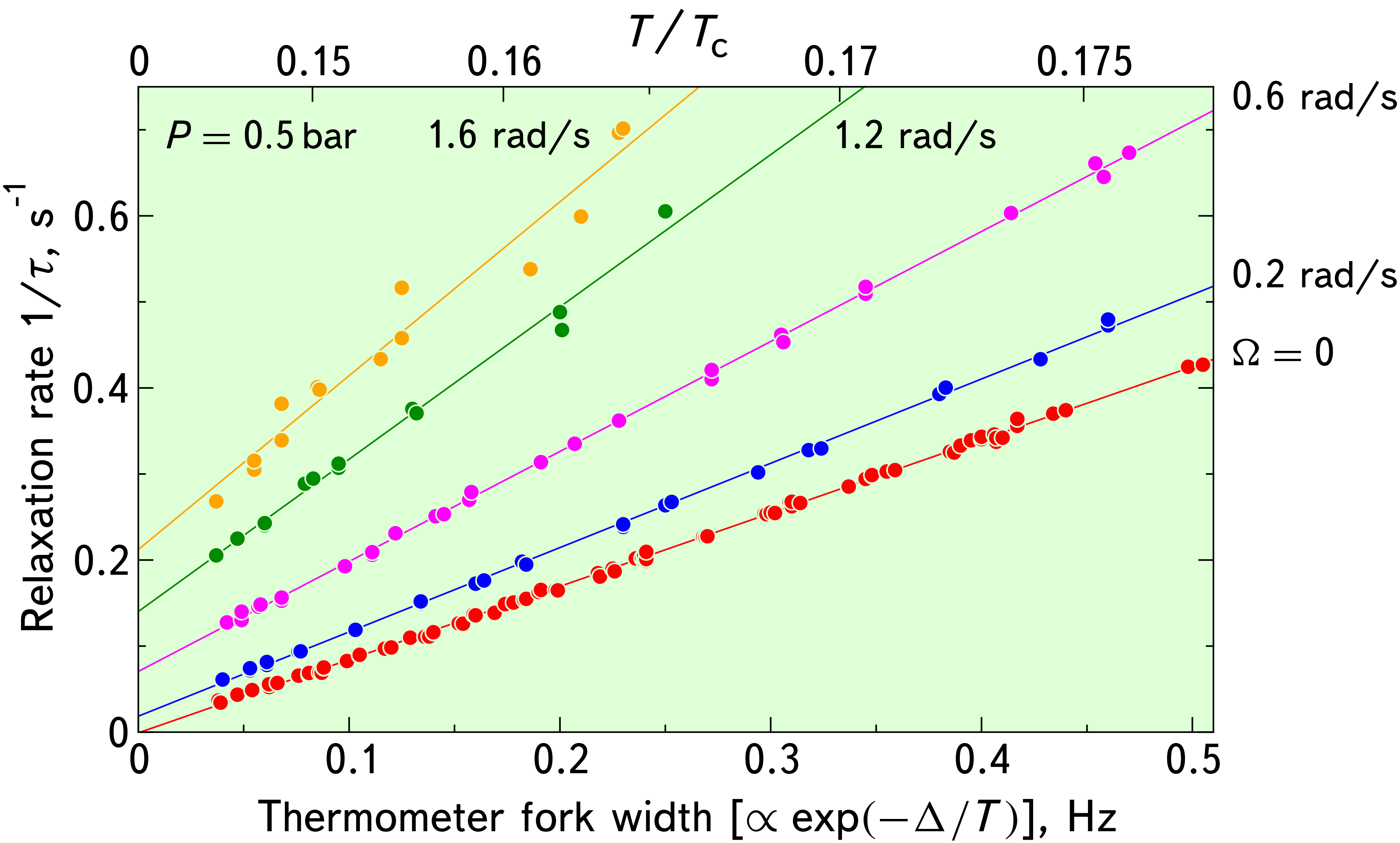
MOTIVATION FOR RELAXATION STUDIES

Long life time of the magnon BEC in the $T \to 0$ limit (exceeding the life time of atomic condenstates) makes them a sensitive probe for extra relaxation sources.



We hope to find the contribution from the Majorana fermion zero modes bound to the surface of cores of quatized vortices by comparing relaxation of magnon condensates in different trap configurations.

RELAXATION IN THE VORTEX STATE

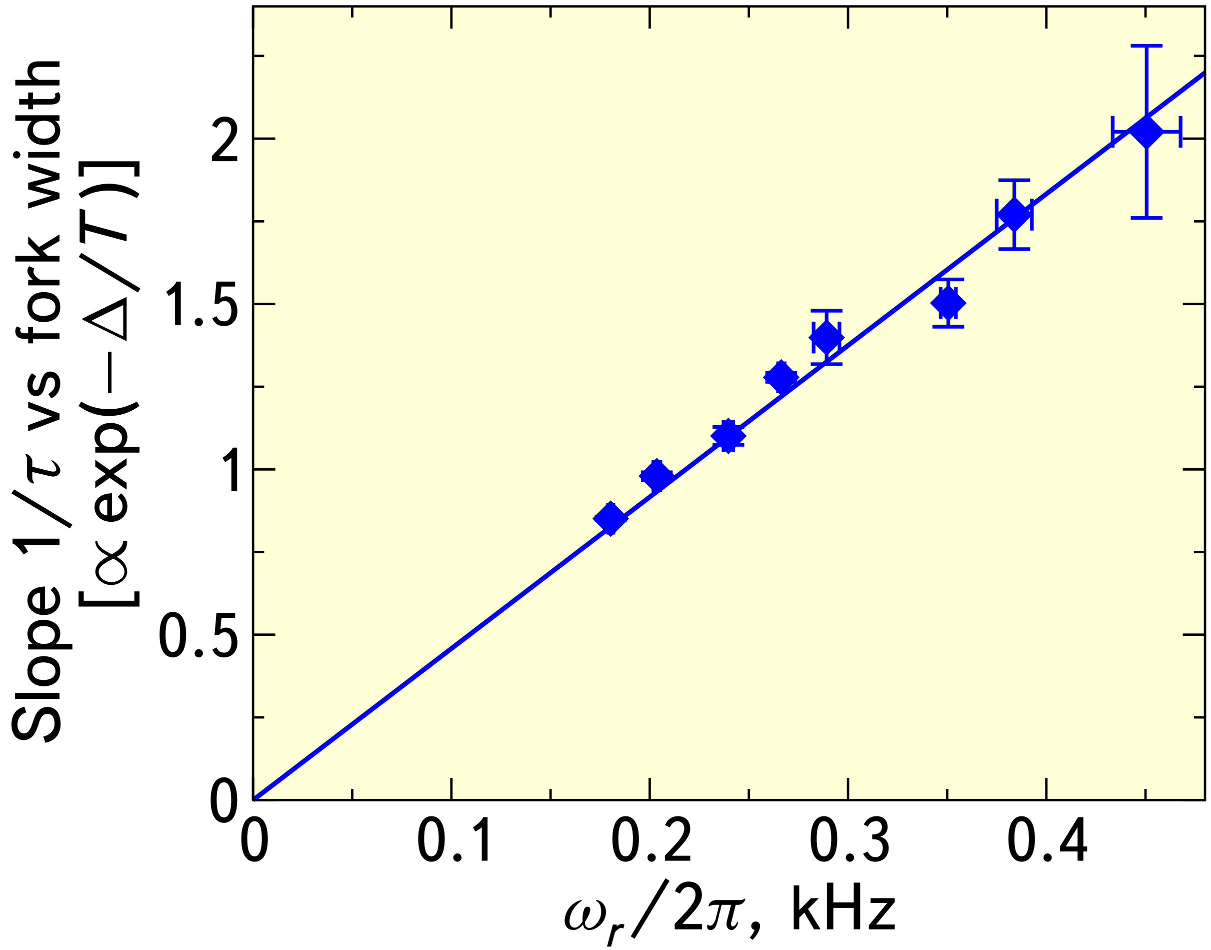


Relaxation rate: $1/\tau = 1/\tau_0(\Omega) + C(\Omega)\ \exp(-\Delta/T)$

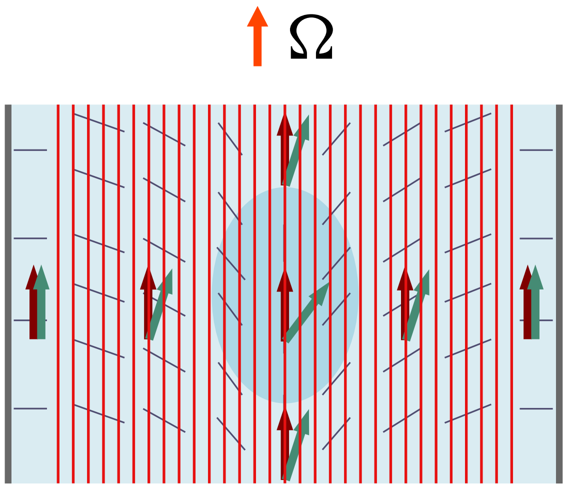
TEMPERATURE DEPENDENCE OF RELAXATION

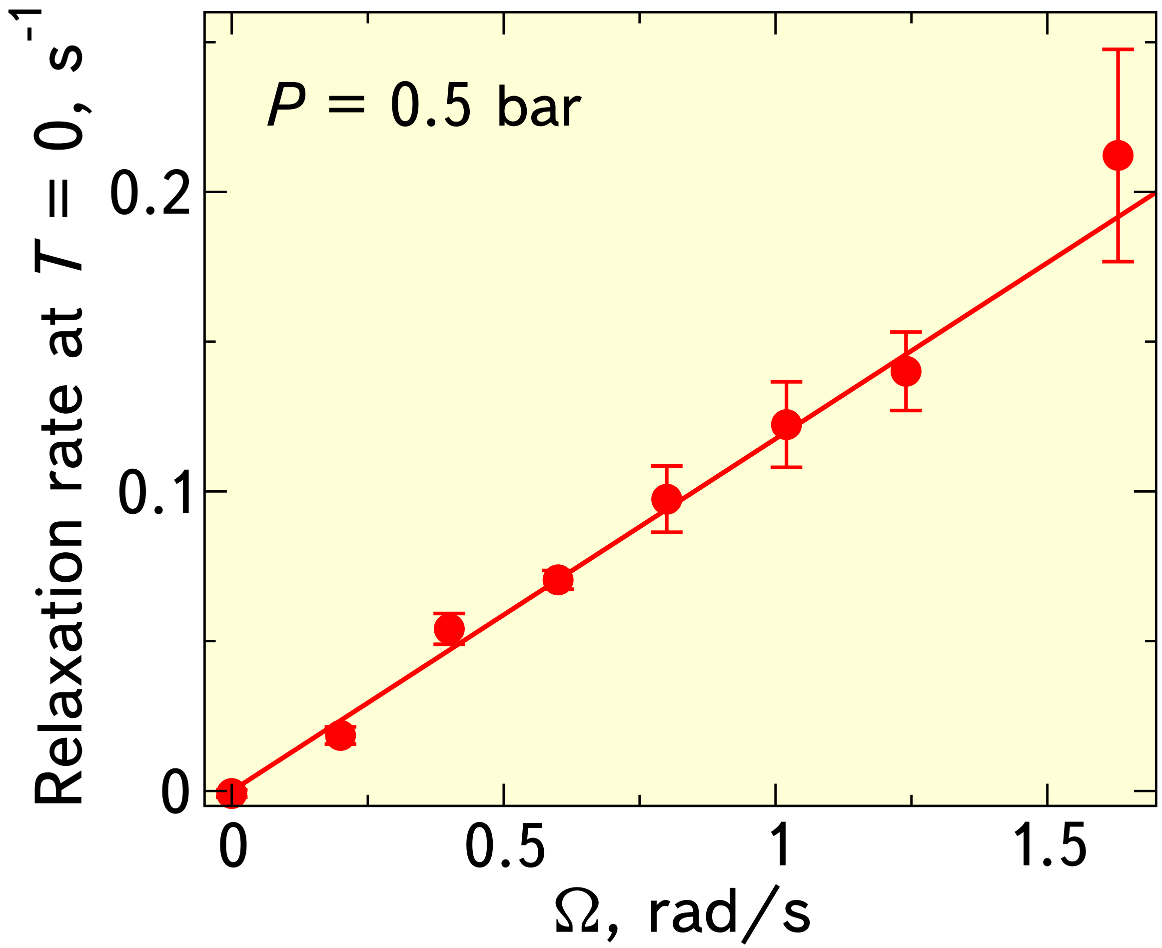
Spin diffusion via normal component (bulk thermal quasiparticles):

$$1/\tau \propto \rho_n |\nabla\Psi|^2 \propto \rho_n R_b^{-2} \propto \exp(-\Delta/T)\,\omega_r(\Omega)$$



DEPENDENCE OF RELAXATION ON VORTEX DENSITY

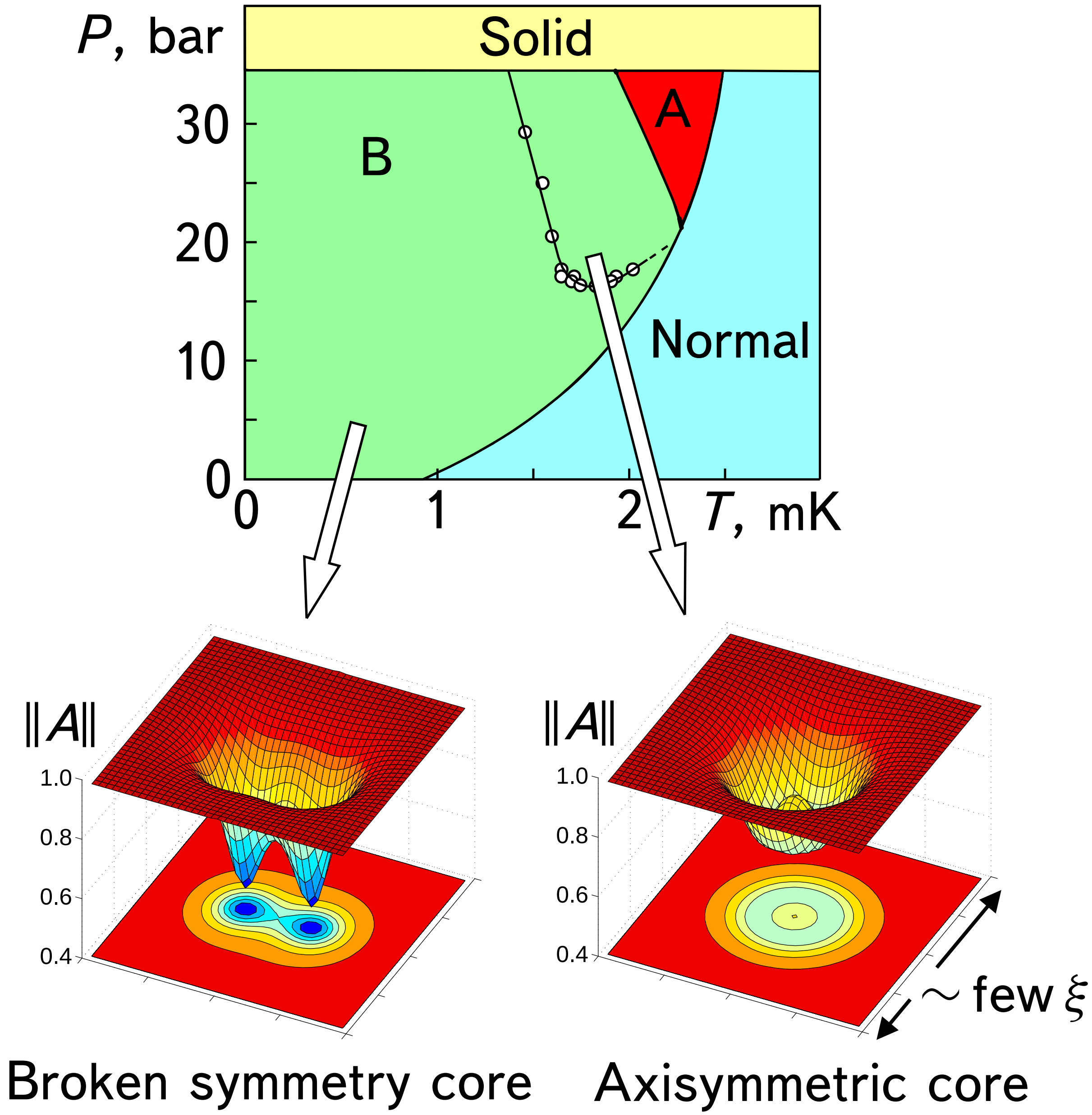




Vortices definitely contribute to the relaxation of magnon condensates.

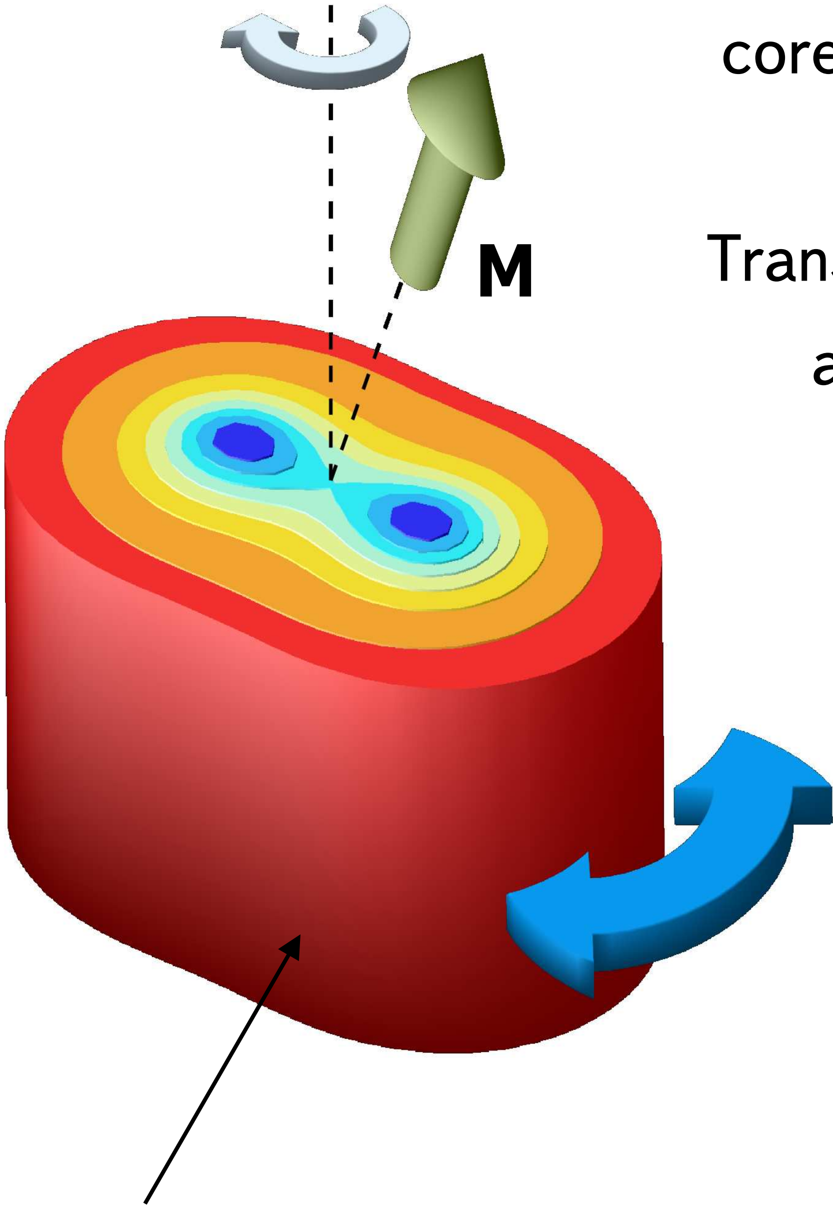
The most probable source is the fermions bound to vortex cores.

BROKEN SYMMETRY OF VORTEX CORES IN ^{3}He-B



Ikkala, Hakonen, Bunkov, Krusius et al 1982-
Salomaa, Volovik, Thuneberg et al

DAMPING OF SPIN PRECESSION VIA VORTEX CORES



Torque from precessing magnetic moment puts vortex core in twisting motion (oscillations / precession)

⇓

Transitions between the core-bound fermion states are triggered and the core gets overheated

⇓

Dissipation

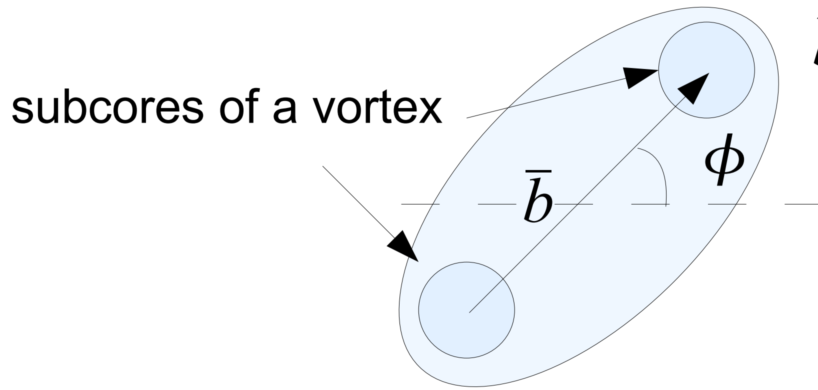
(Kopnin and Volovik, 1998)

Core of the non-axisymmetric vortex

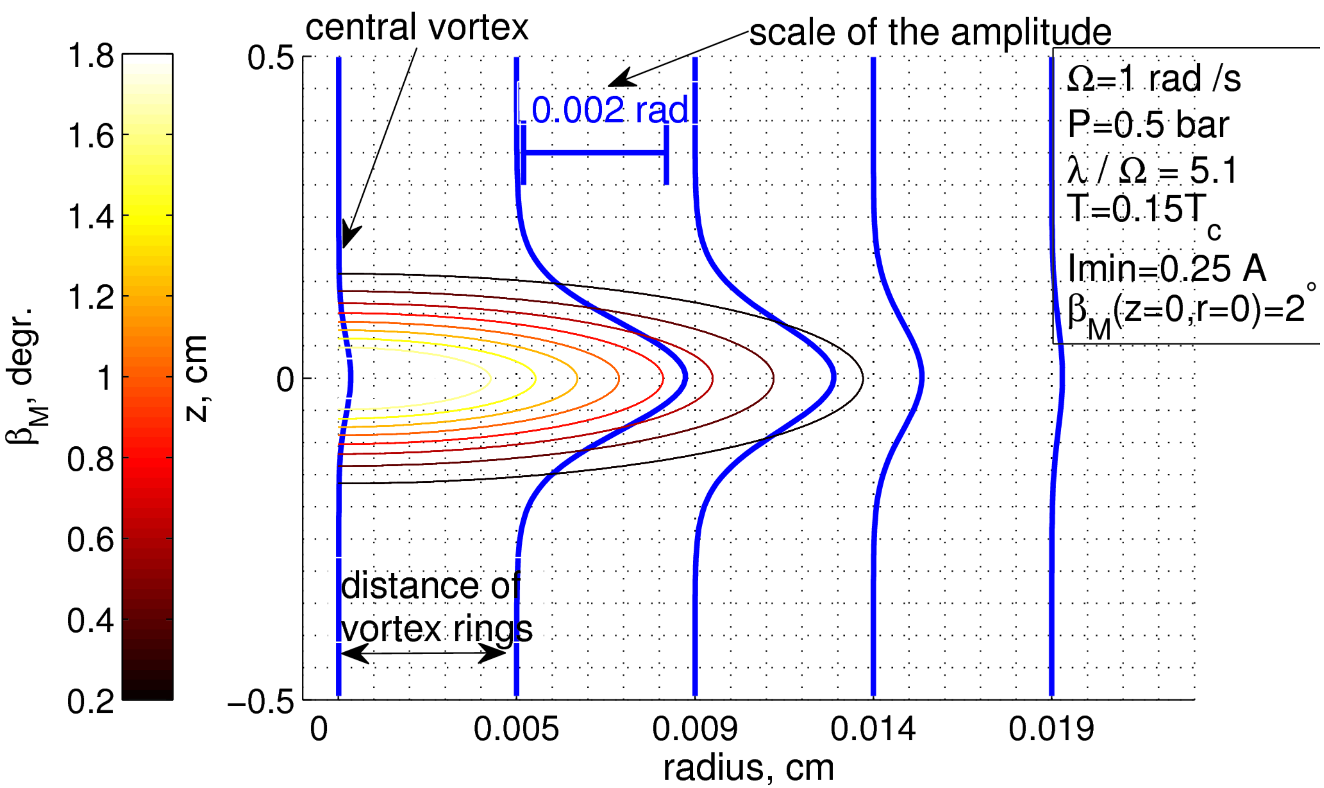
Magnon BEC and vortices

Precessing magnetization

→ precessing n-vector → drive



$$\hat{b}=\frac{\bar{b}}{|\bar{b}|}$$



friction (core-bound excitations)

stiffness term

$$f\frac{\partial \phi}{\partial t}=\frac{\partial F_d}{\partial \phi}+K\frac{\partial^2 \phi}{\partial z^2}$$

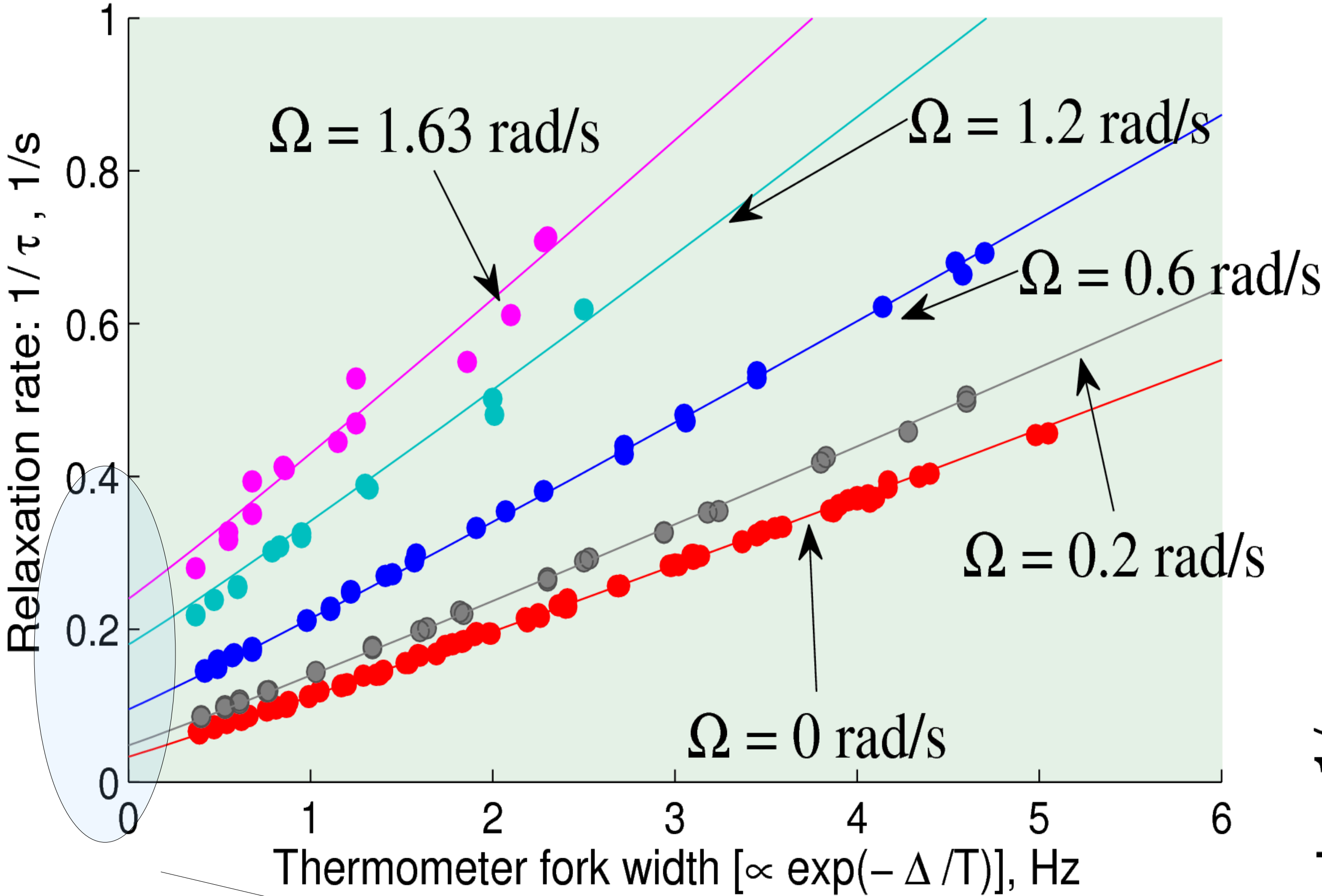
$$F_d=T_d(\hat{b}\cdot\hat{n})^2+T_H(\hat{b}\cdot\hat{l})$$

drive due to BEC

Pinning due to L-texture

Vortex damping

$P = 0.5\,bar$
$T = 0.15\,T_c$
$\nu_0 = 826\,kHz$
$I_{min} = 0.25\,A$

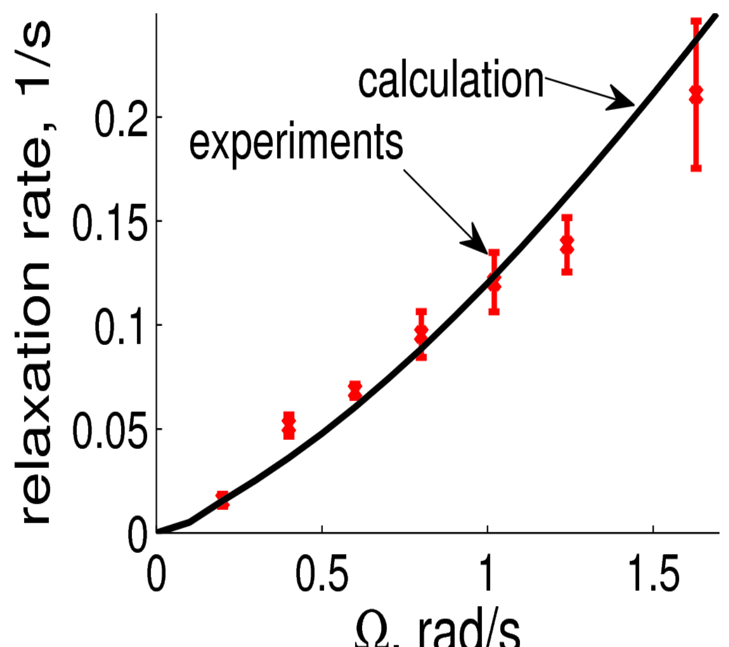


Zeeman energy

$$E_z = \frac{\omega^2 \chi_B}{2\gamma^2} \int dV\, sin^2(\beta_M)$$

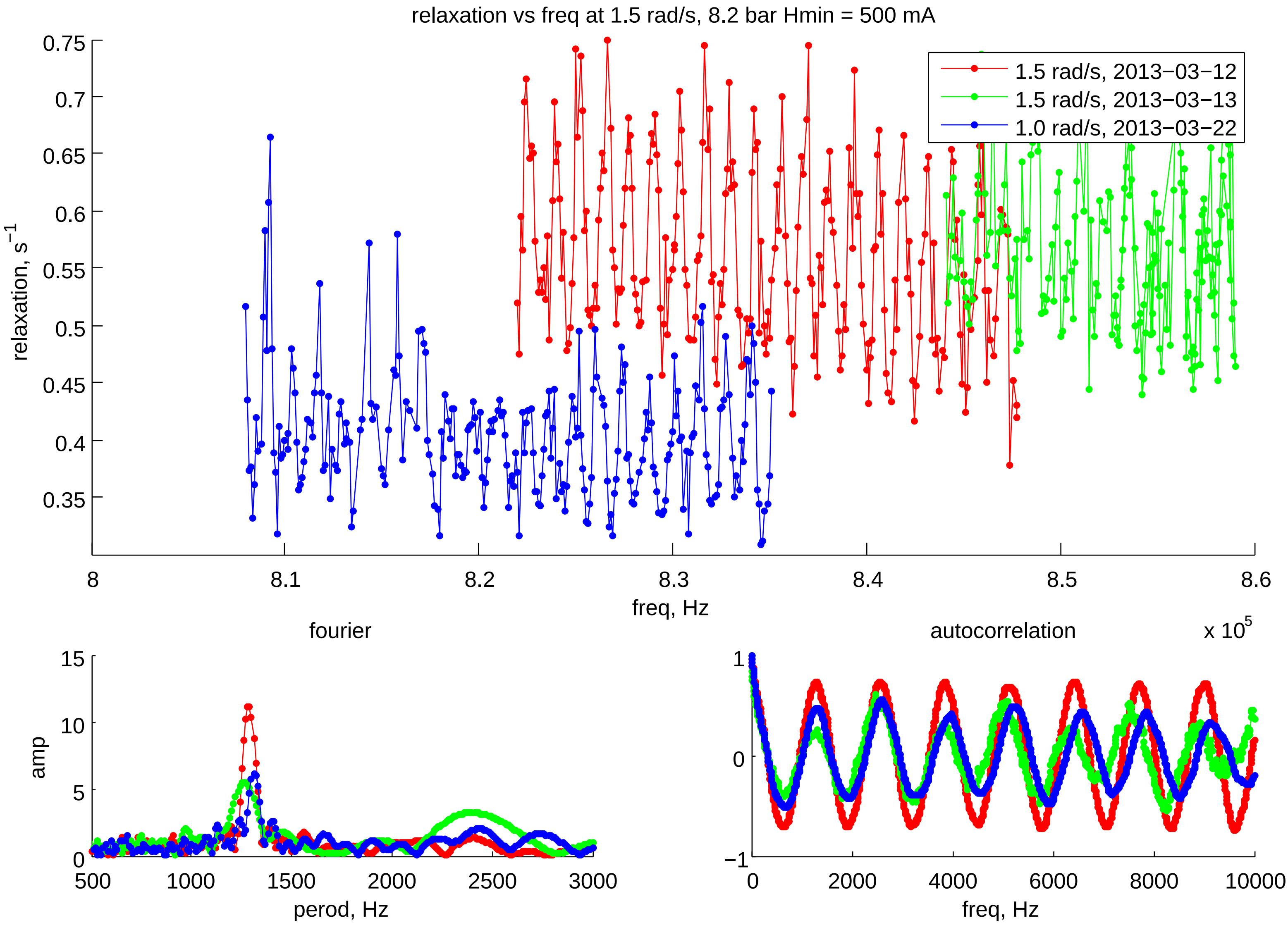
Power (friction, core bound excitatons)

$$P = -\sum_{vortices} f\left(\frac{\partial \phi}{\partial t}\right)^2$$



Fit parameters → order-of-magnitude agreement with estimates derived from 29bar HPD measurements [3]

[3] Y. Kondo et al: PRL **67**, 81 (1991)


relaxation vs freq at 1.5 rad/s, 8.2 bar Hmin = 500 mA
1.5 rad/s, 2013−03−12
1.5 rad/s, 2013−03−13
1.0 rad/s, 2013−03−22
relaxation, s−1
freq, Hz
fourier
amp
perod, Hz
autocorrelation
x 10^5
freq, Hz

resonance absorption & frequency comb

friction force due to resonance absorption $$f(\omega) = \omega \int dp_z \, |Q_{0n}|^2 \, \delta(\omega - n\omega_0(p_z))$$

$$f(\omega) = \omega \, |Q_{0n}|^2 \, / \, (d\omega_0 / dp_z) \qquad \omega = n\,\omega_0(p_z)$$

$$d\omega_0 / dp_z = \omega_0 \, p_z \, / p_F^2$$

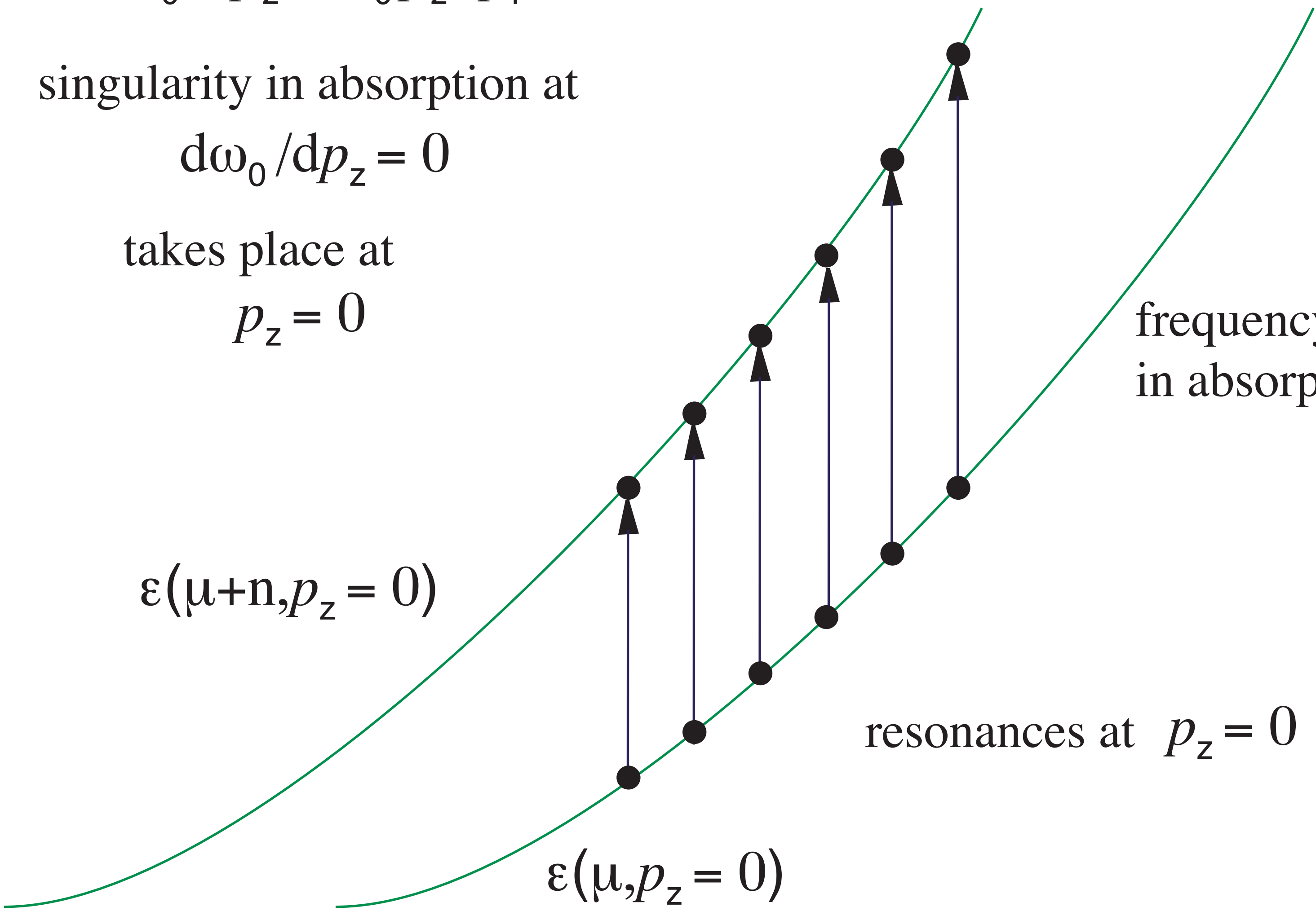
singularity in absorption at

$$d\omega_0 / dp_z = 0$$

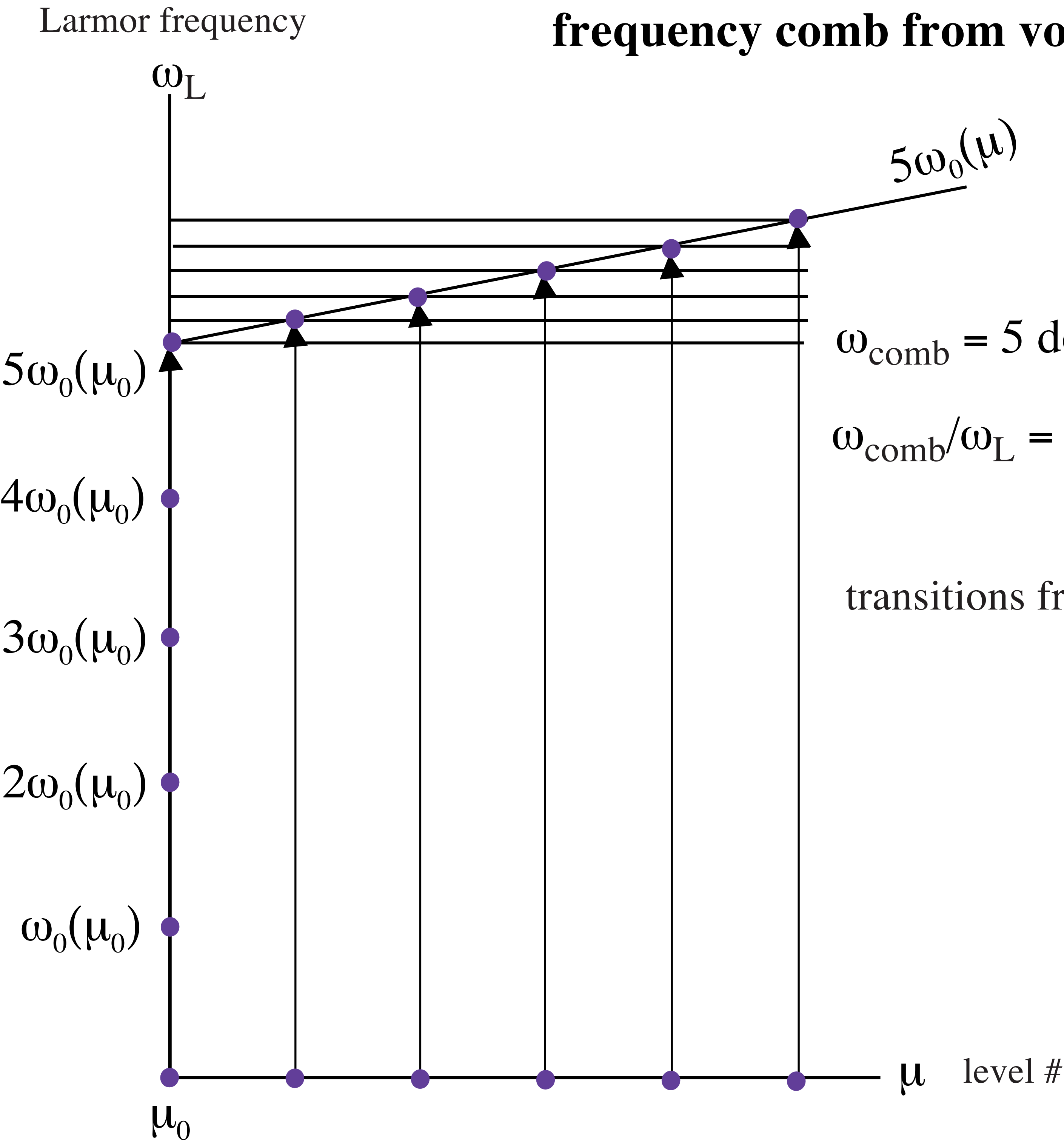
takes place at

$$p_z = 0$$

frequency comb is due to singularity in absorption at $p_z = 0$



frequency comb from vortex core fermions



$\omega_{comb} = 5\, d\omega_0/d\mu$

$\omega_{comb}/\omega_L = (d\omega_0/d\mu)/\omega_0$

transitions from μ to $\mu+5$

Caroli-de Gennes-Matricon states in 3He-B vortices: experimental status

Conclusion

* magnon Bose condensate in magnetic-textural trap is sensitive tool for studing core excitations

* relaxation of magnon BEC at T=0 is due to resonance absorption on core fermions

* observed frequency comb is most probably due to singularities in DoS in core fermion spectrum

* details are not clear and require full calculation of energy spectrum in asymmetric core