Vortex in Coulomb blockaded granule:
parity effect suppression and electron pumping

To the memory of Nikolai Kopnin

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Russian academy of sciences

Collaboration with N.B. Kopnin

By the time when we met with N. B. Kopnin for the first time in Dec 2012 we had been coauthors for 3 years and have 2 common papers.

Then I had a great opportunity to work with him personally for a half of a year being his PostDoc. It was a great collaboration. The ideas discussed at that time actuate till now.

The main idea of this talk came up after the discussions with N.B. Kopnin.

Outline:

- **Electron pumping in SET:**

  Main ideas, ingredients, and results

- **Parity effect and its suppression by vortex penetration.**

- **New principle of electron pumping in SET**

  Parity effect tuning

- **Possible realization**

  Time-varying magnetic field

- **Summary**

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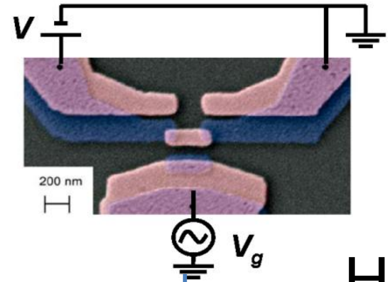
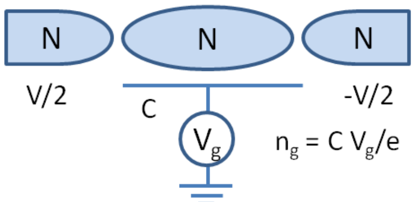
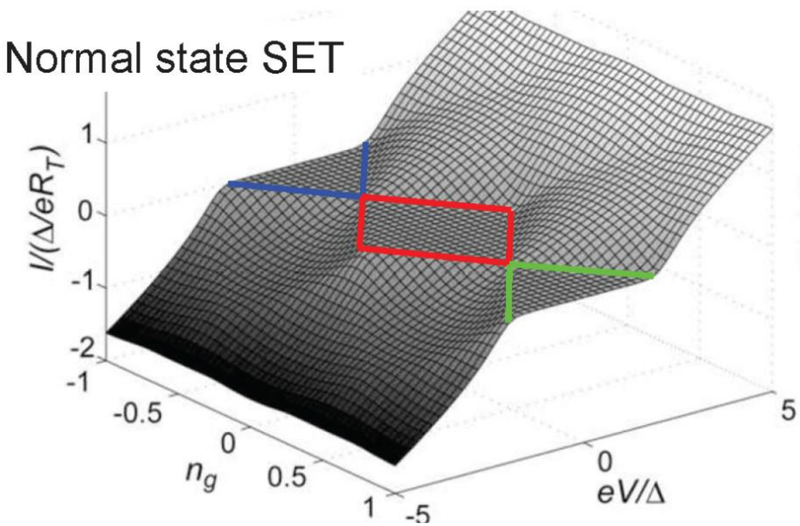
Single electron transistor (SET)



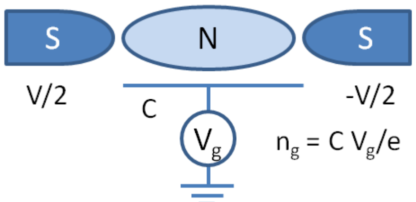
J.P. Pekola et al Nat. Phys. (2008)

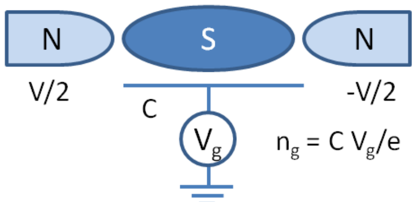
Normal state SET

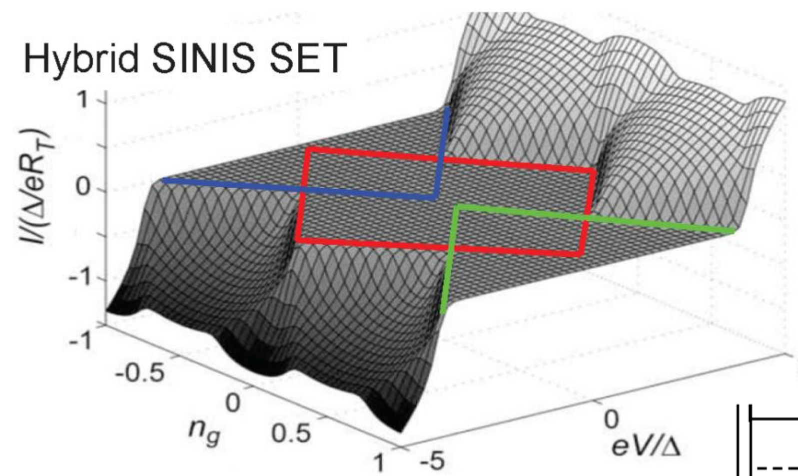


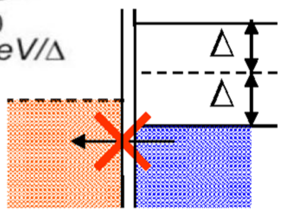


Hybrid SINIS or NISIN SET





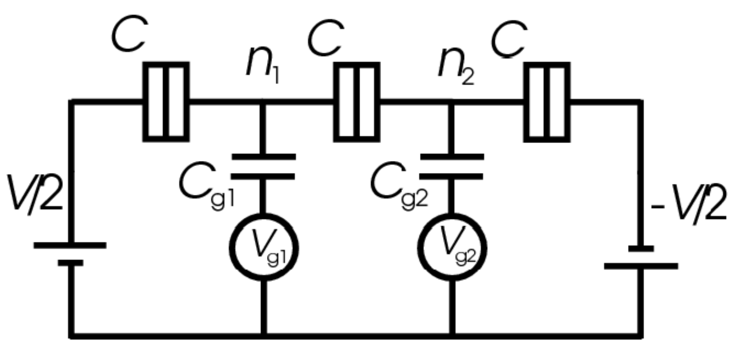


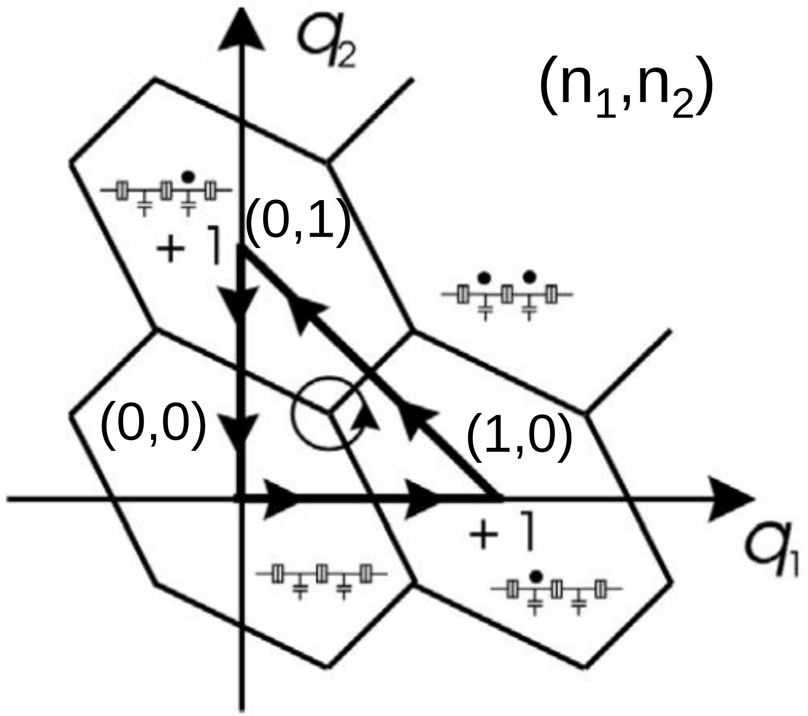


Averin, Likharev *Mesoscopic Phenomena in Solids,* p.173 (1991)
Averin, Nazarov *Single Charge Tunneling*, Vol. 294 (1992)

Single electron transistors as a turnstile

Cyclic operation (frequency *f*) of gates, $q_i = C_{gi}V_{gi}/e$, transports charge through the system



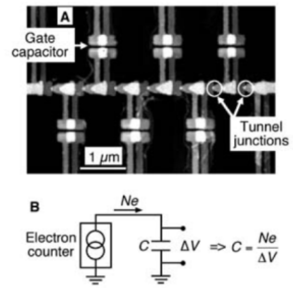


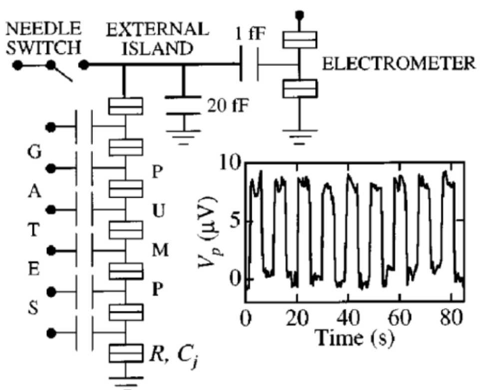
H. Pothier et al., EPL 17, 249 (1992)

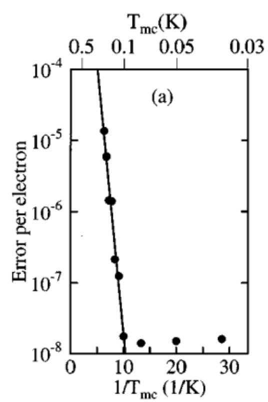
$I = ef$

Single electron sources

Towards frequency-to-current conversion







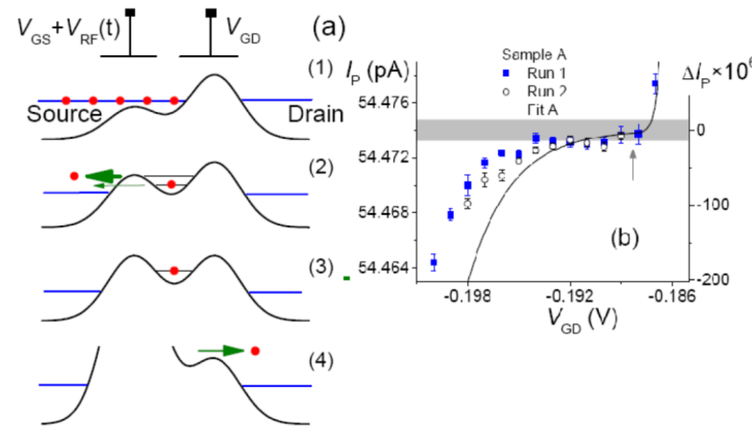
Normal single-electron pump: **$I = ef$**

Geerligs et al. 1990, Pothier et al. 1992, Keller et al. 1996, Lotkhov et al. 2000

High accuracy but still slow: $I << 10$ pA

Semiconductor devices, travelling wave or quantum dots:

Shilton et al. 1996
Fujiwara et al. 2004
Blumenthal et al. 2007
Fève et al., 2007
Kaestner et al. 2007
Giblin et al., 2010



Fully superconducting devices:

Several versions

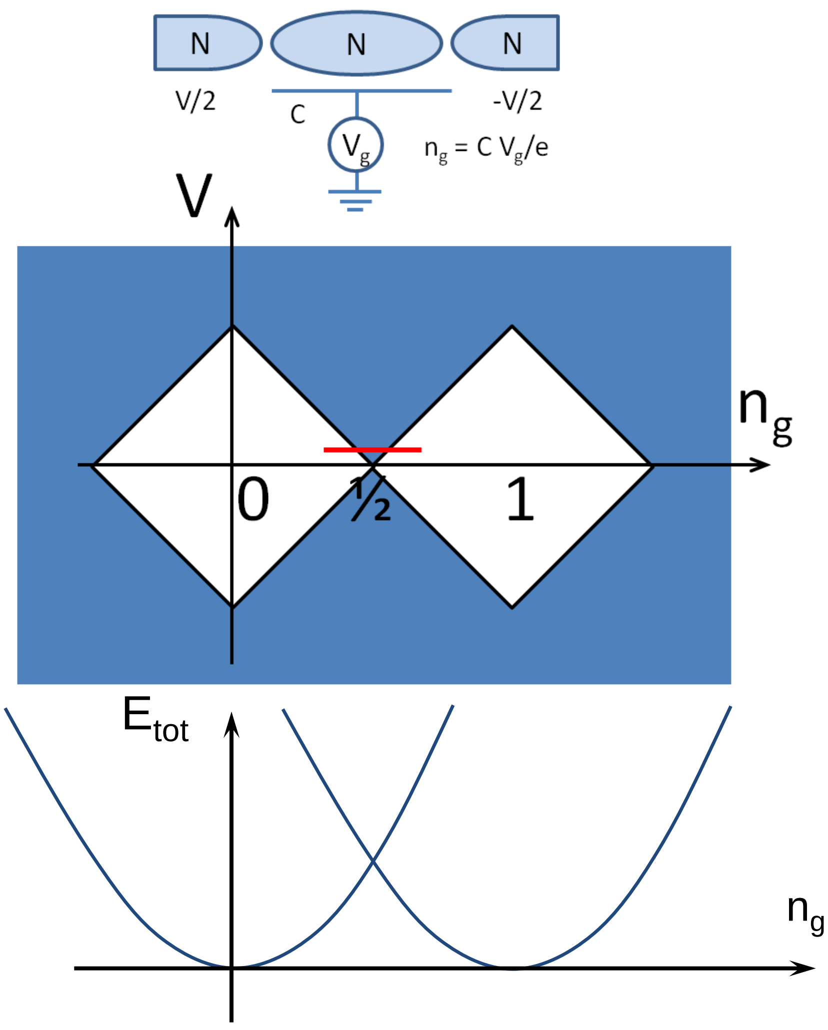
Fast, but difficult to suppress errors

Mechanical shuttles:

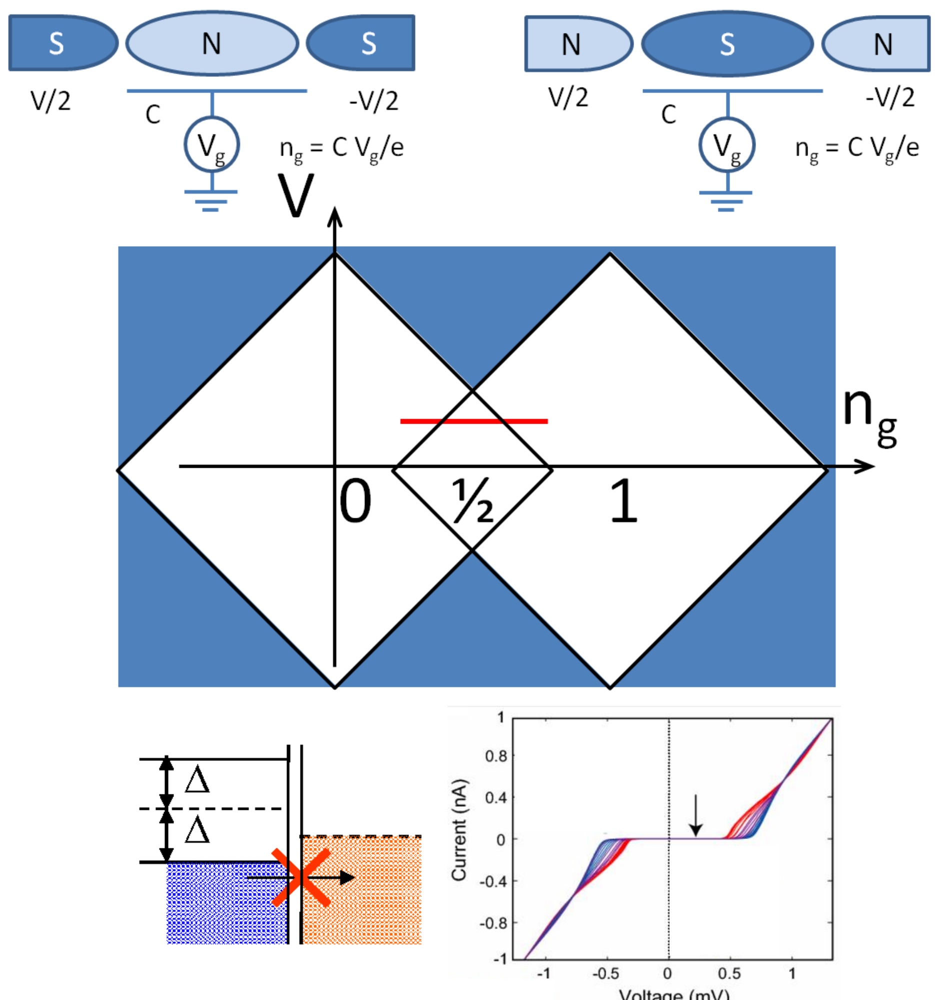
Konig et al. 2008

SET. Principle of electron pumping

Normal state SET



SINIS or NISIN SET



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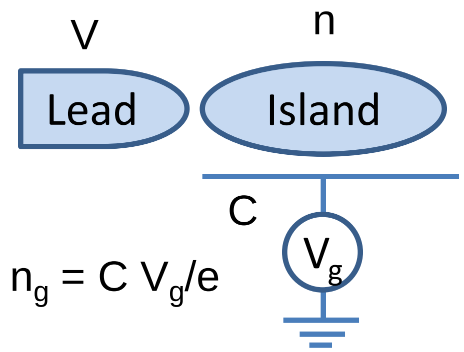
Master equation

$$\hat{H} = \hat{H}^0_{island} + U_C\left[\hat{N}\right] + H_{tunn} + H_{lead}$$

$$\frac{\partial}{\partial t}\hat{\rho} = -\frac{i}{\hbar}\left[\hat{H},\hat{\rho}\right]$$



$$U_C\left[\hat{N}\right] = E_C \cdot \left[\hat{N} - n_g(t)\right]^2$$

$$\frac{\partial}{\partial t} p_N = \sum_{N'}\left(\Gamma_{N'\to N} p_{N'} - \Gamma_{N\to N'} p_N\right)$$

$$E_C = e^2/2C_\Sigma$$

$$e\Gamma_{N\to N+1} = I_{L,I}(E_N^+) = \frac{G_T}{e}\int_{-\infty}^{\infty}\rho_L(\varepsilon)\rho_I(\varepsilon + E_N^+) f_L(\varepsilon)\left[1 - f_I(\varepsilon + E_N^+)\right]d\varepsilon$$

$$e\Gamma_{N\to N-1} = I_{I,L}(E_N^-) = \frac{G_T}{e}\int_{-\infty}^{\infty}\rho_L(\varepsilon)\rho_I(\varepsilon - E_N^-)\left[1 - f_L(\varepsilon)\right] f_I(\varepsilon - E_N^-)d\varepsilon$$

$$E_N^{\pm} = U_C[N] - U_C[N\pm 1] \mp eV \qquad G_T = 4\pi e^2 \nu_n^{(L)}\nu_n^{(I)}\left\langle\left|T_{kp}\right|^2\right\rangle \qquad \rho_k = \nu_k(E)/\nu_n^{(k)}$$

Master equation

Detailed balance

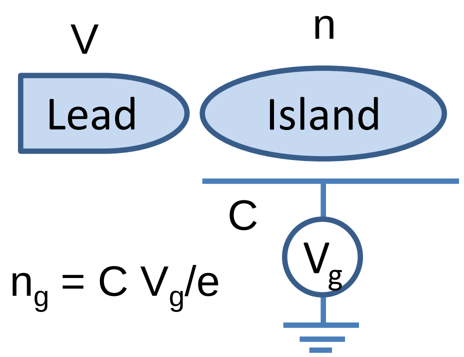
$$f_L(\varepsilon) = f_I(\varepsilon) = f_{Fermi}(\varepsilon, T)$$

Symmetric DOS

$$\rho_k(-\varepsilon) = \rho_k(\varepsilon)$$

$$\Gamma_{N\to N\pm1} = \Gamma_{N\pm1\to N} e^{E_N^\pm / T}$$

$$\Gamma_{N\to N\pm1} = \Gamma(E_N^\pm)$$



$$U_C\left[\hat{N}\right] = E_C \cdot \left[\hat{N} - n_g(t)\right]^2$$

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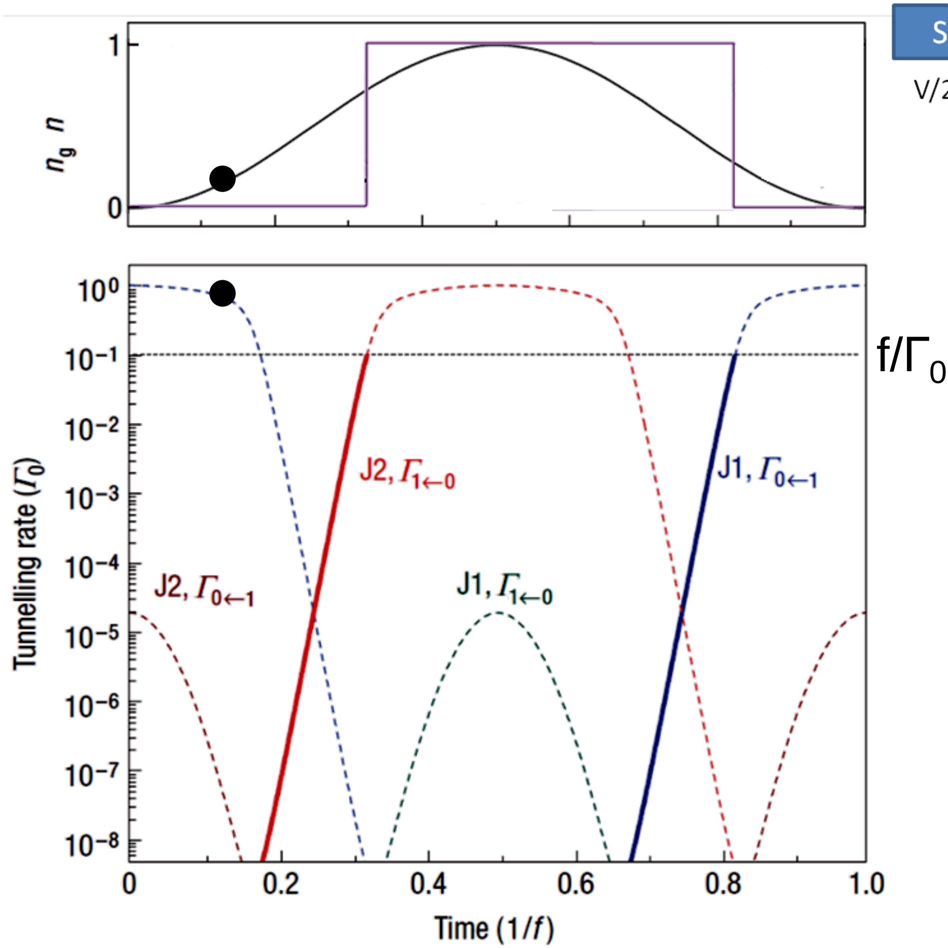
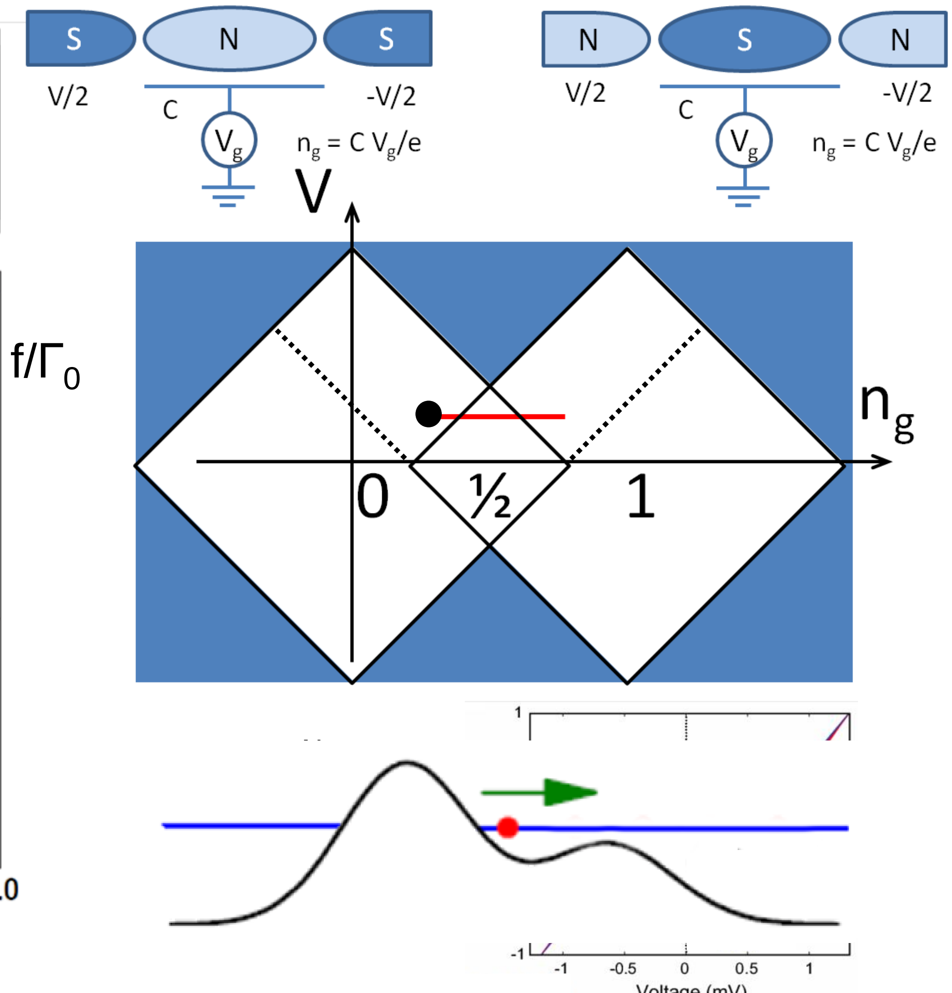
$$E_N^\pm = U_C[N] - U_C[N\pm1] \mp eV \qquad G_T = 4\pi e^2 \nu_n^{(L)} \nu_n^{(I)} \left\langle \left| T_{kp} \right|^2 \right\rangle \qquad \rho_k = \nu_k(E) / \nu_n^{(k)}$$

SET. Principle of electron pumping

SINIS or NISIN SET





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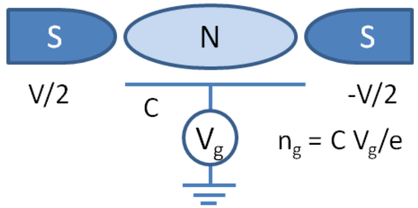
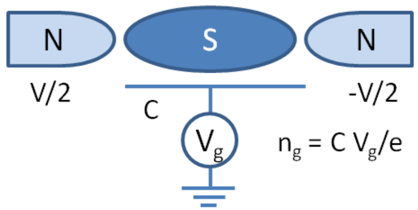
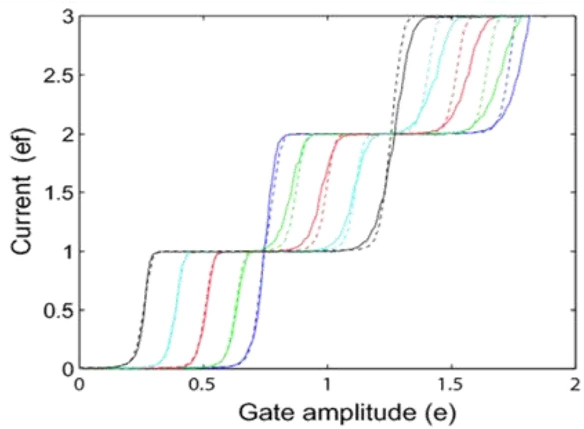
SINIS turnstile. Experimental results

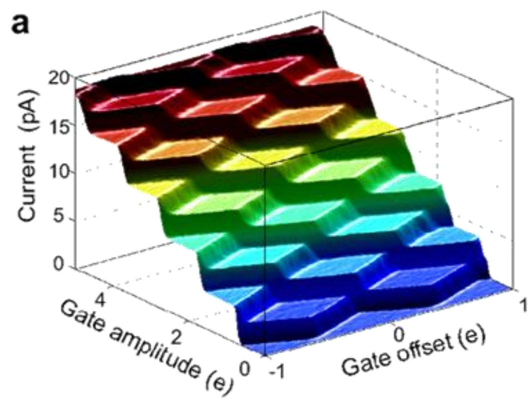
SINIS or NISIN SET

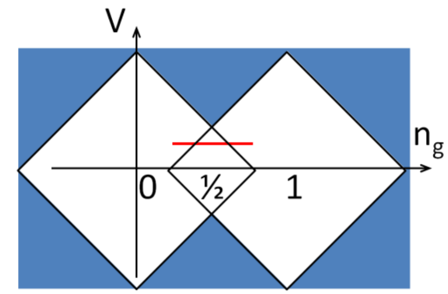
I = nef

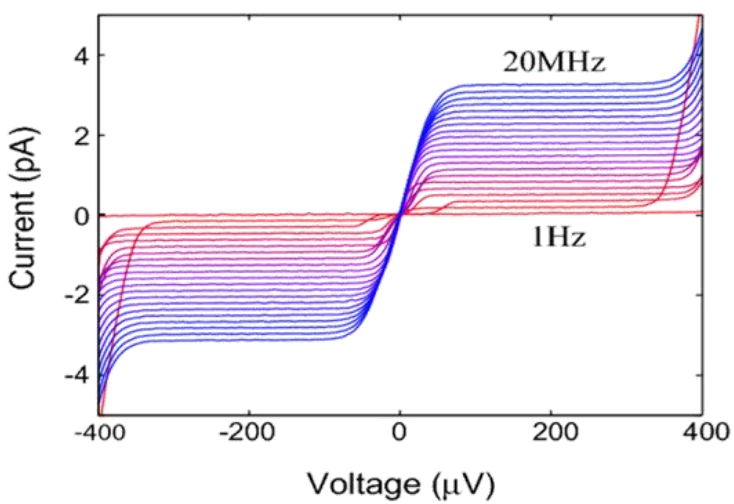


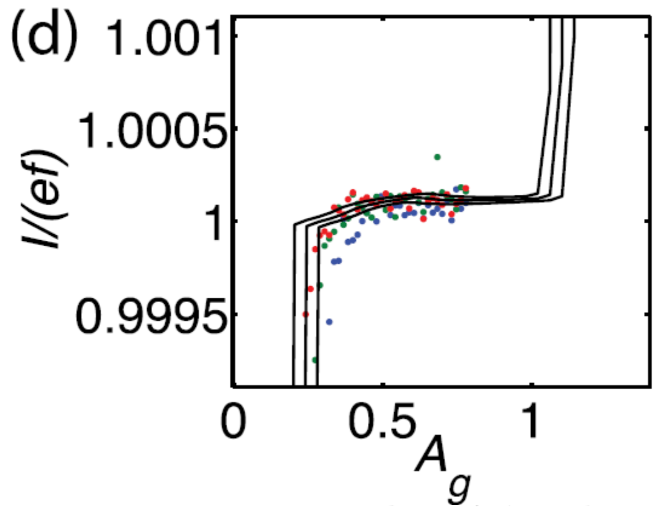












J.P. Pekola et al
Nature Phys. (2008);

Knowles, Maisi, Pekola
APL (2012)

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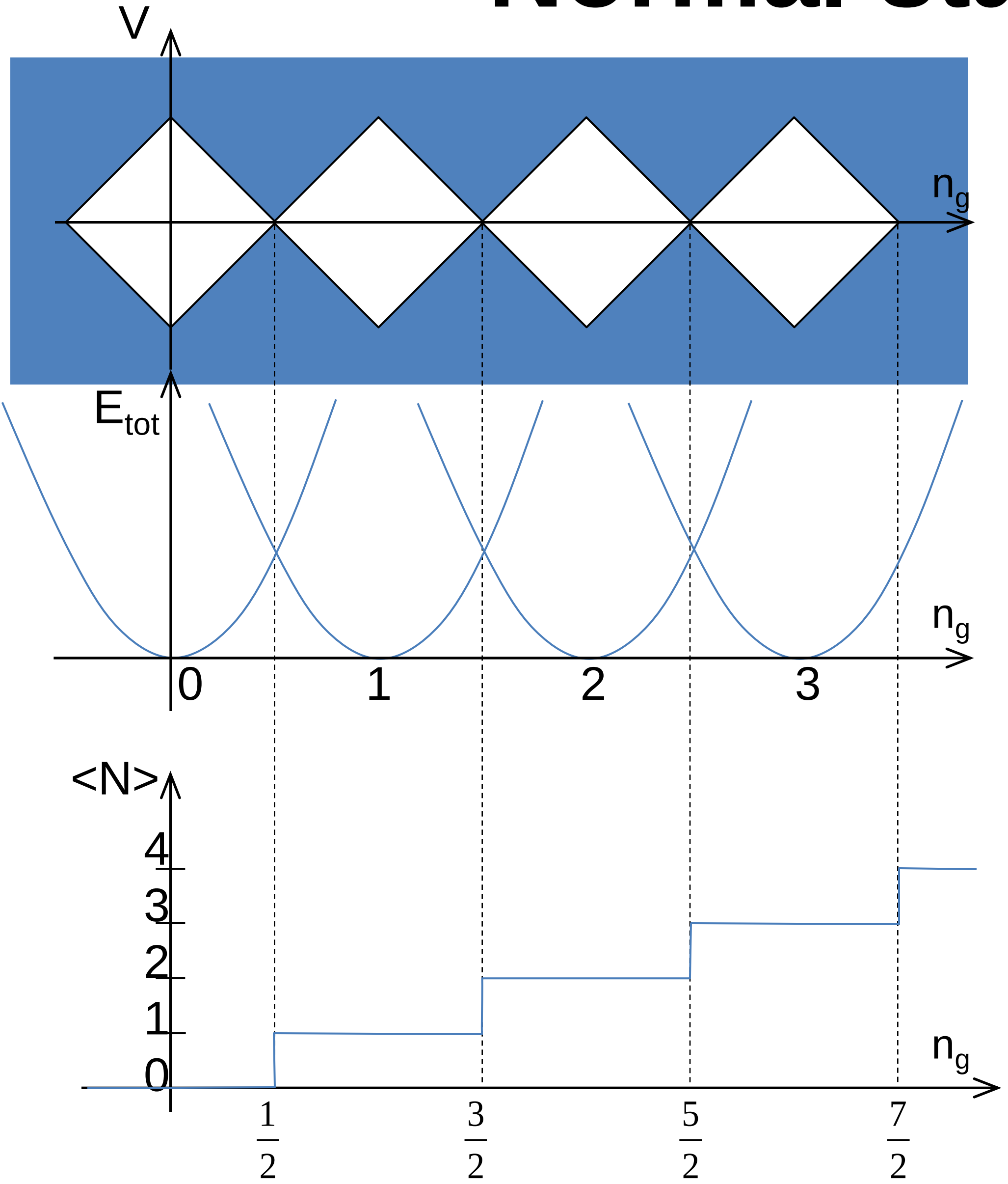

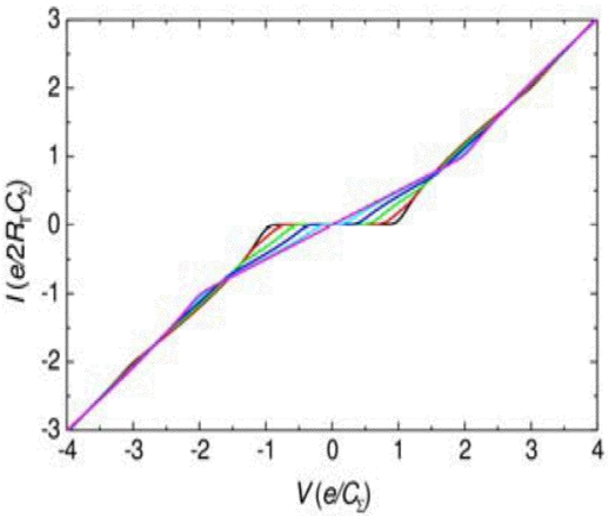

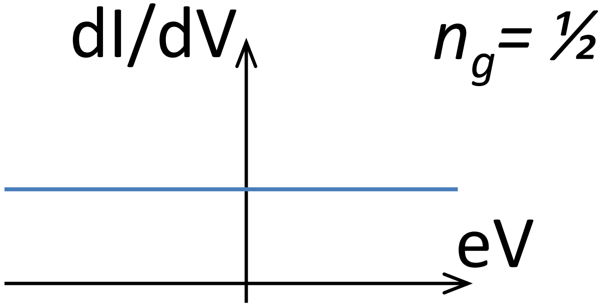

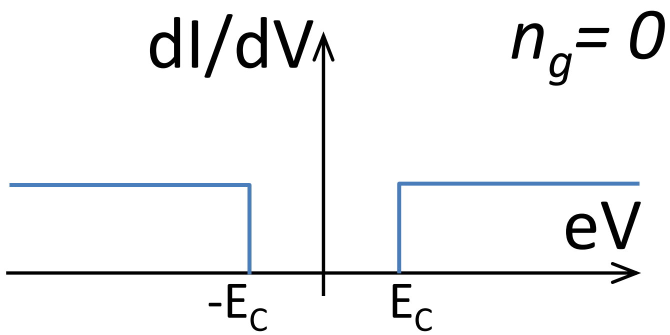

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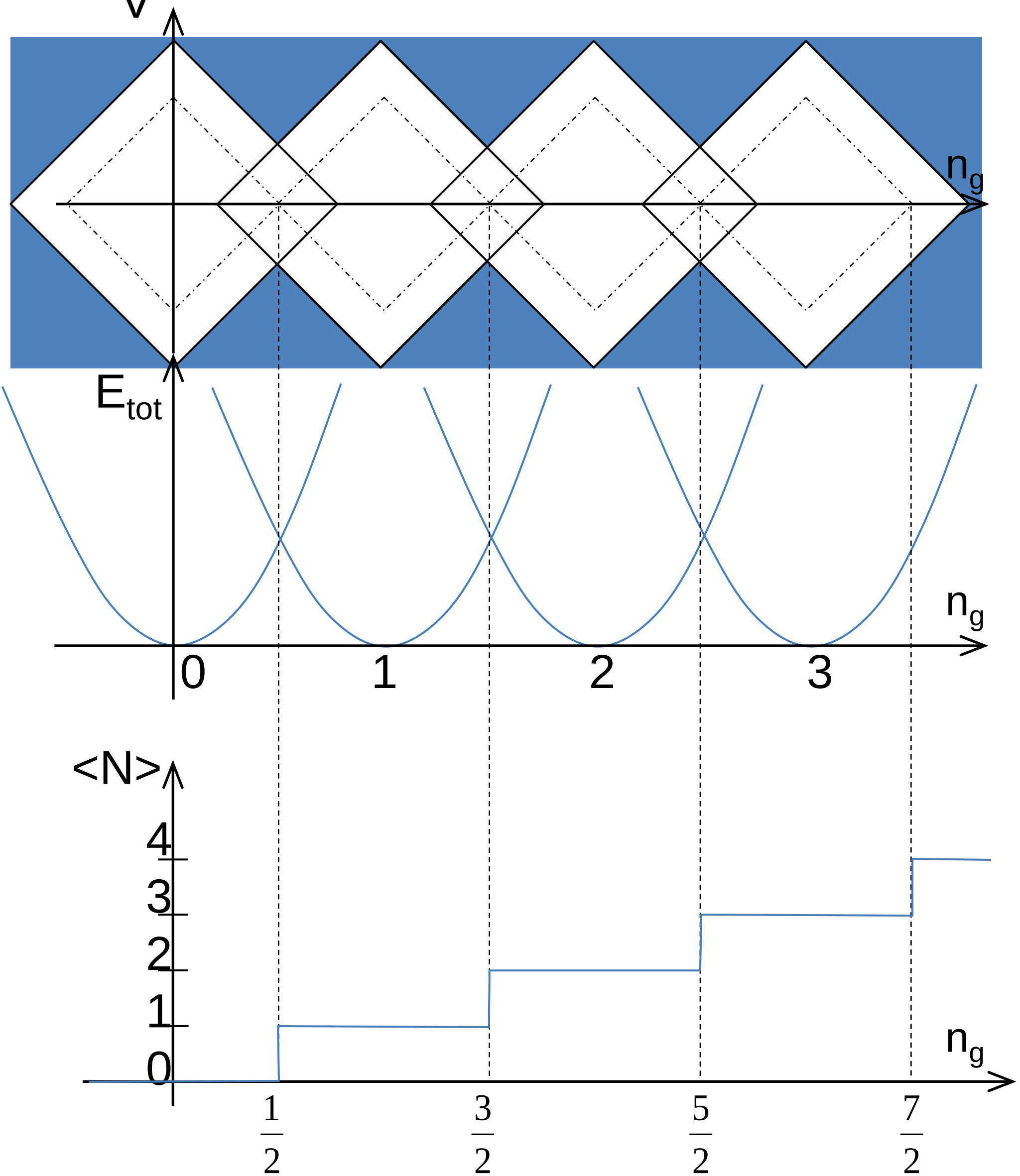
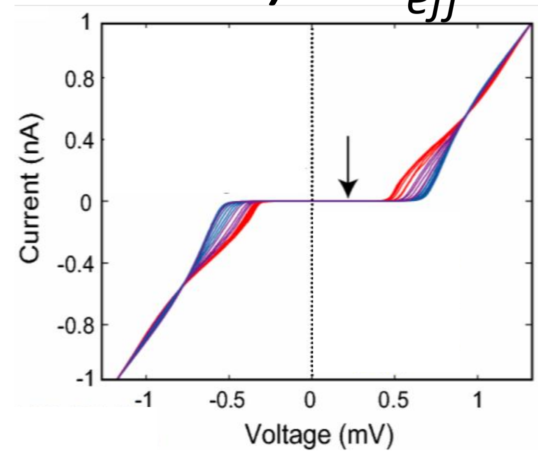
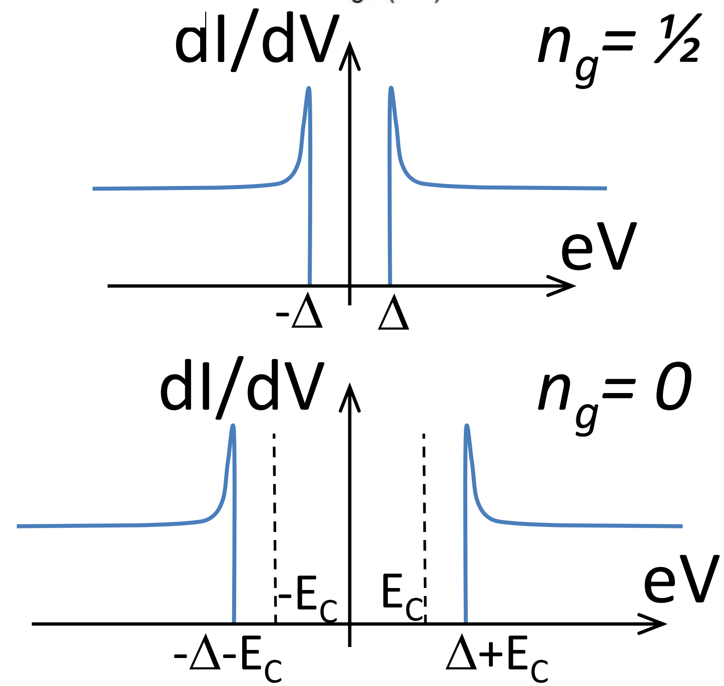


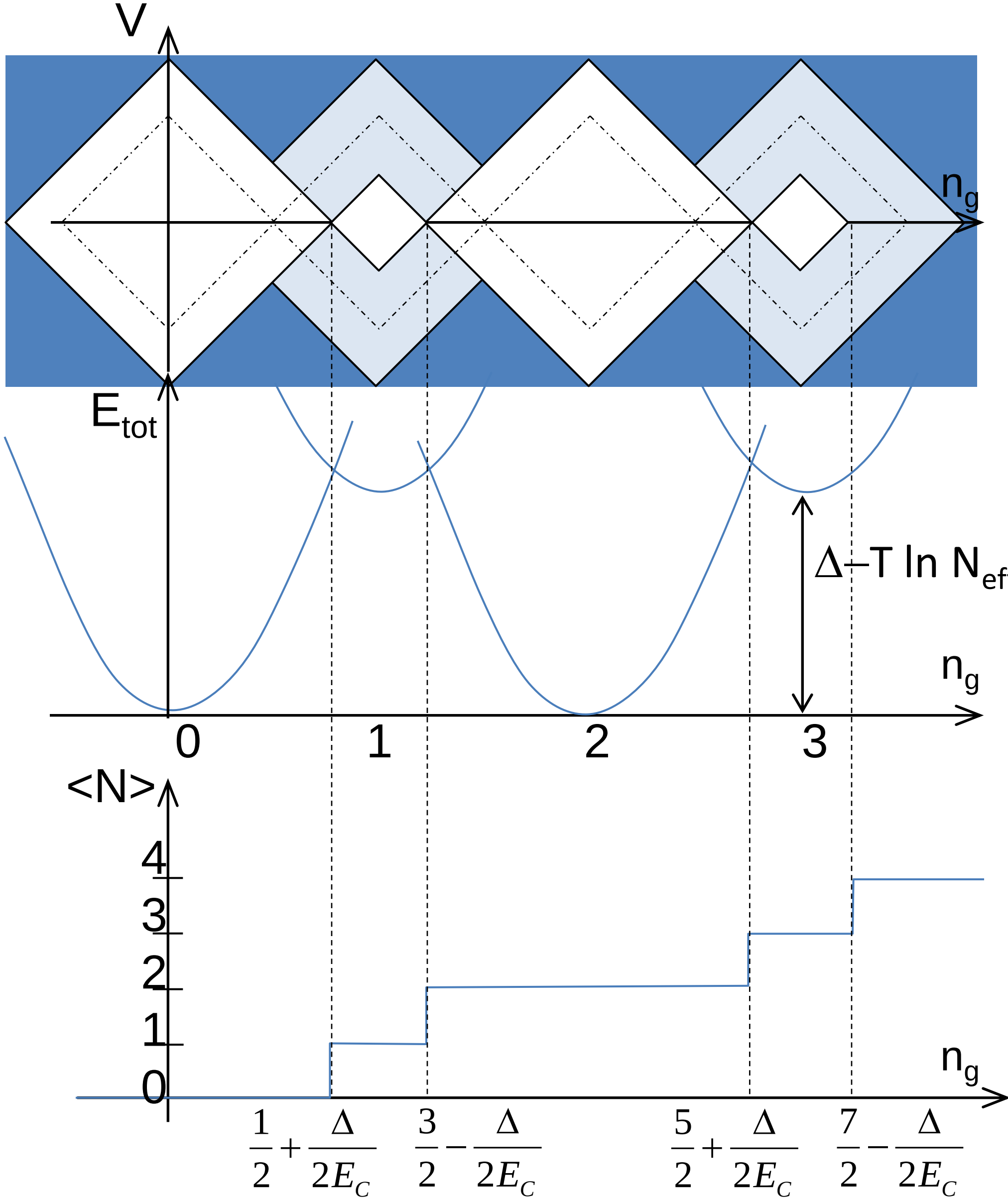
$T > \Delta / \ln N_{eff}$

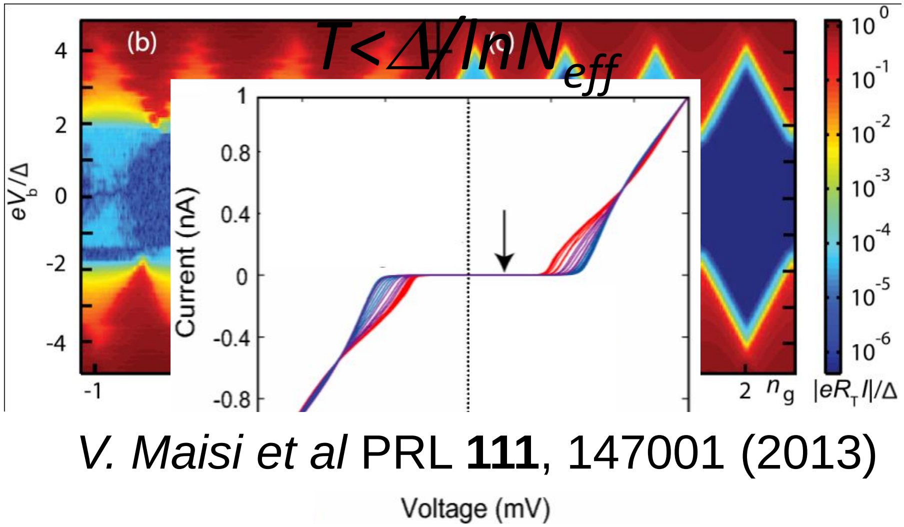




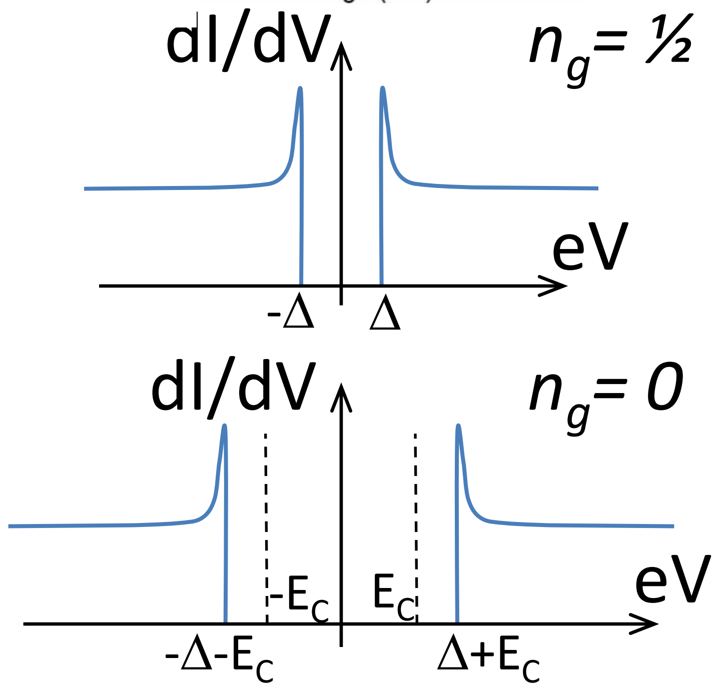
M.T. Tuominen et al PRL, **69**, 1997(1992); *Averin, Nazarov* Physica B **203**, 310 (1994)

NISIN SET. Parity effect



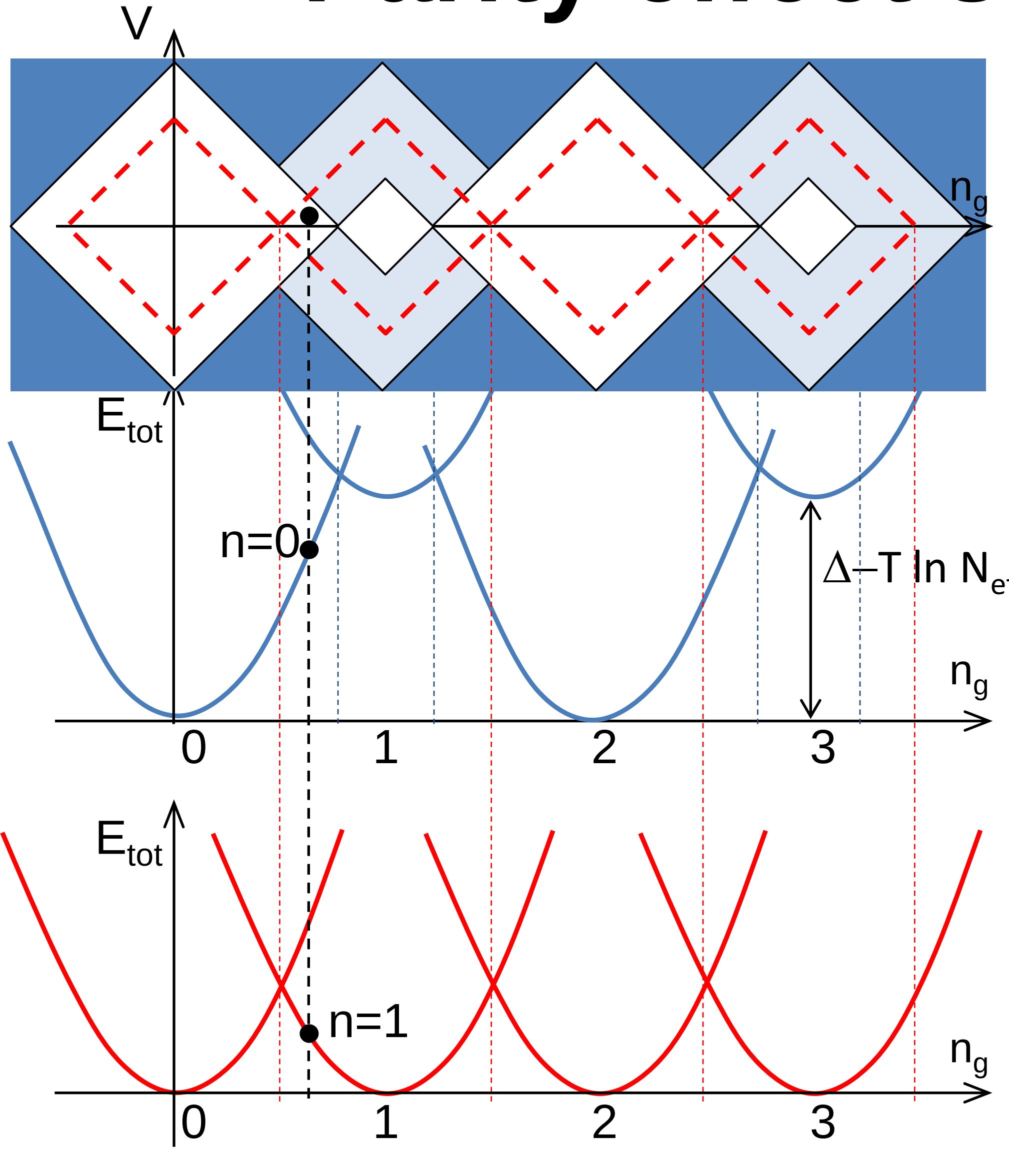


V. Maisi et al PRL **111**, 147001 (2013)

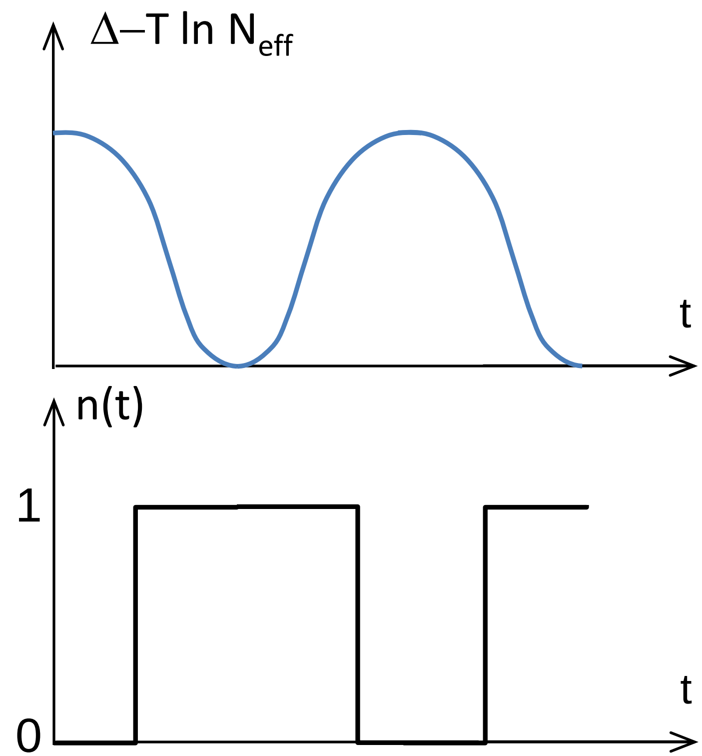


M.T. Tuominen et al PRL, **69**, 1997(1992); *Averin, Nazarov* Physica B **203**, 310 (1994)

Parity effect suppression

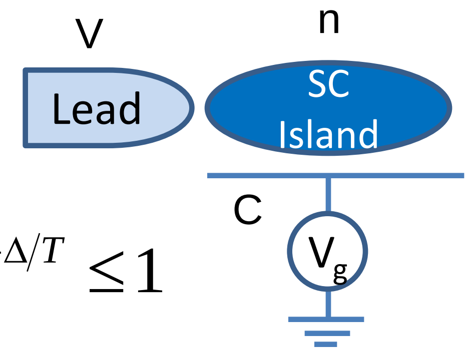


Δ(t) or T(t)



Master equation with parity effect

M.T. Tuominen et al PRL, **69**, 1997(1992)
Averin, Nazarov Physica B **203**, 310 (1994)



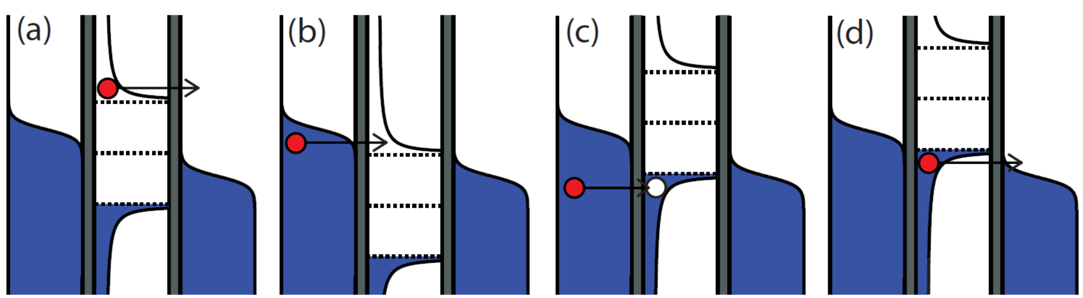
$$N_{qp} = 2V\nu_n \int_0^\infty \rho_S(\varepsilon) f_S(\varepsilon) d\varepsilon \approx N_{eff} e^{-\Delta/T} \leq 1$$

$$N_{eff} = \sqrt{2\pi\Delta T} / \delta >> 1$$

normal state interlevel distance $\delta = (V\nu_n)^{-1}$

$$\frac{\partial}{\partial t} p_{N,N_S} = \sum_{N',N_S'} \left(\Gamma_{N' \to N}^{N_S' \to N_S} p_{N',N_S'} - \Gamma_{N \to N'}^{N_S \to N_S'} p_{N,N_S} \right)$$

$\Gamma_{N \to N-1}^{N_S \to N_S - 1}$ $\Gamma_{N \to N+1}^{N_S \to N_S + 1}$ $\Gamma_{N \to N+1}^{N_S \to N_S - 1}$ $\Gamma_{N \to N-1}^{N_S \to N_S + 1}$

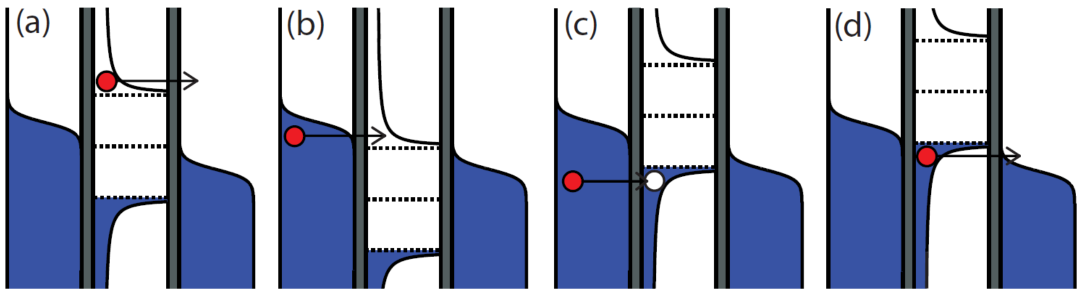


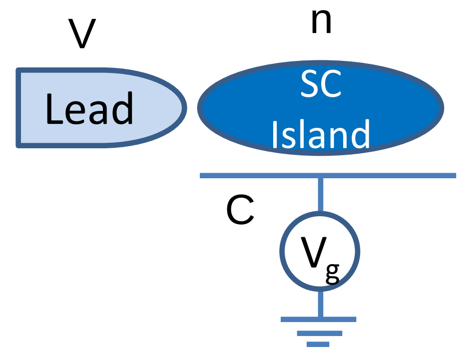
V. Maisi et al PRL **111**, 147001 (2013)

Master equation with parity effect

V. Maisi et al PRL **111**, 147001 (2013)

$\Gamma_{N\to N-1}^{N_S\to N_S-1}$ $\Gamma_{N\to N+1}^{N_S\to N_S+1}$ $\Gamma_{N\to N+1}^{N_S\to N_S-1}$ $\Gamma_{N\to N-1}^{N_S\to N_S+1}$





$$\Gamma_{N\to N+1}^{N_S\to N_S+1} = \frac{G_T}{e^2}\int_0^{\infty}\rho_S(\varepsilon) f_n(\varepsilon - E_N^+)\left[1 - f_S(\varepsilon, N_S)\right]d\varepsilon$$

$$\Gamma_{N\to N+1}^{N_S\to N_S-1} = \frac{G_T}{e^2}\int_{-\infty}^{0}\rho_S(\varepsilon) f_n(\varepsilon - E_N^+)\left[1 - f_S(\varepsilon, N_S)\right]d\varepsilon$$

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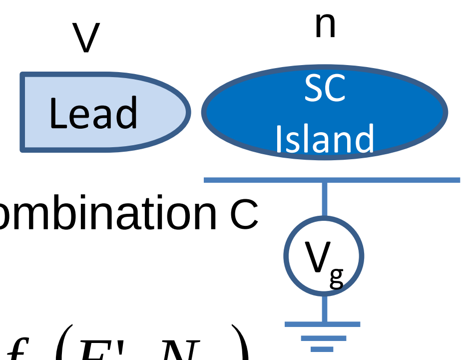
$E_N^{\pm} = E_C\cdot\left[\pm 2(n_g - N) - 1\right] \mp eV$ $\quad G_T = 4\pi e^2 \nu_n^{(S)}\nu_n^{(n)}\left\langle \left|T_{kp}\right|^2\right\rangle$ $\quad \rho_k = \nu_k(E)/\nu_n^{(k)}$

Master equation with parity effect

V. Maisi et al PRL **111**, 147001 (2013)



$$N_S = 2V\nu_n \int_0^\infty \rho_S(\varepsilon) f_S(\varepsilon, N_S) d\varepsilon$$

Electron-phonon interaction as a channel for quasiparticle recombination

$$Q_{rec} = V\int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\xi' (E+E')^3 \left(1+\tfrac{\Delta^2}{EE'}\right) f_S(E, N_S) f_S(E', N_S)$$

$$E = \sqrt{\xi^2 + \Delta^2}$$

$$\Gamma_{N\to N}^{N_S \to N_S - 2} \approx \frac{Q_{rec}}{2\Delta} \sim V\Delta^2 n_{qp}^2 = \frac{\Delta^2 N_S^2}{V}$$

If recombination rate is large compared with all tunneling rates, one can reduce the model assuming $N_S = N \bmod 2$

$$\frac{\partial}{\partial t} p_N = \sum_{N', N_S'} \left(\Gamma_{N'\to N}^{N_S'} p_{N'} - \Gamma_{N\to N'}^{N_S} p_N \right)$$

$$\Gamma_{N\to N\pm1}^{0} = \frac{G_T}{e^2}\int_0^\infty \rho_S(\varepsilon) f_n(\varepsilon - E_N^{\pm}) d\varepsilon \approx \frac{G_T\sqrt{2\pi\Delta T}}{e^2} e^{-(\Delta - E_N^{\pm})/T}$$

$$0 < E_N^{\pm} < \Delta$$

$$\Gamma_{N\to N\pm1}^{1} = \frac{G_T}{e^2}\int_{-\infty}^\infty \rho_S(\varepsilon) f_S(\varepsilon, 1)\left[1 - f_n(\varepsilon + E_N^{\pm})\right] d\varepsilon \approx \frac{G_T}{2e^2 V\nu_n}$$

NOTE! There is no detailed balance anymore

Outline:

- Electron pumping in SET:

  Main ideas, ingredients, and results

- Parity effect and its suppression by vortex penetration.
- New principle of electron pumping in SET

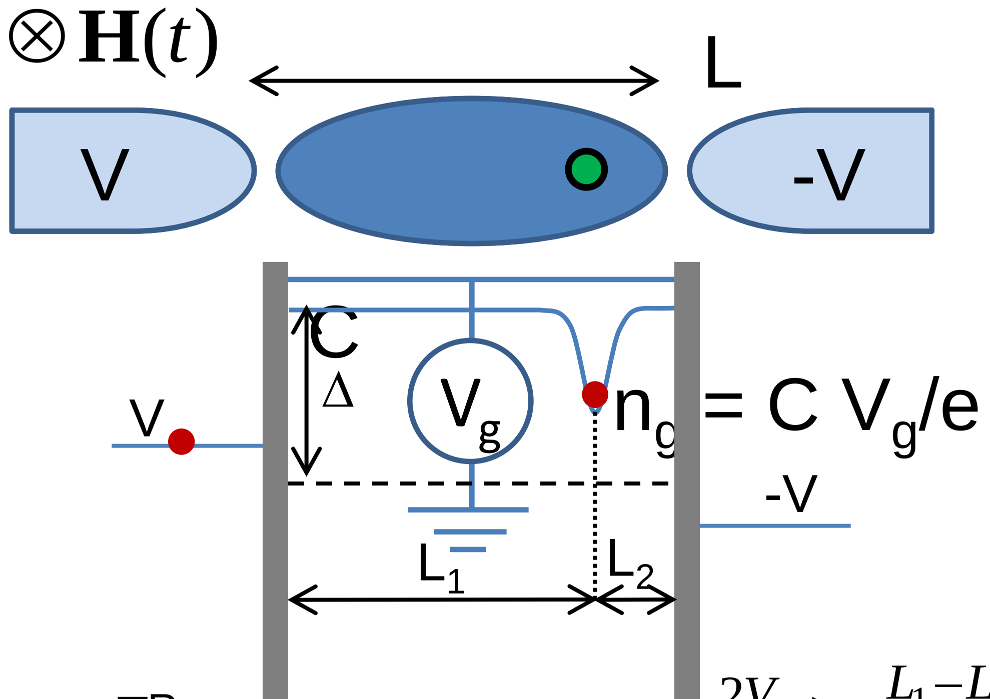
  Parity effect tuning

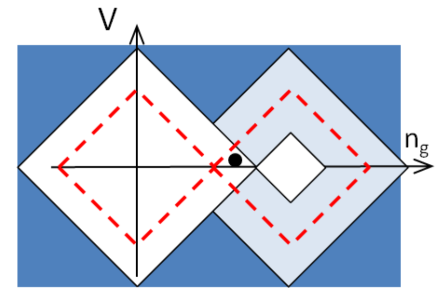
- **Possible realization**

  Time-varying magnetic field

- Summary

Possible realization.
Superconducting granule with vortex





Fixed
n_g and V

1 → 0

$L_2 < L_1 \Rightarrow \Gamma^L_{1,0} < \Gamma^R_{1,0}$

$$\Gamma^R_{1\to 0} \sim e^{-L_2/\xi}$$

$$\Gamma^L_{1\to 0} \sim e^{-L_1/\xi}$$

0 → 1

$\frac{2V}{T} > \frac{L_1 - L_2}{\xi} \Rightarrow \Gamma^L_{0,1} > \Gamma^R_{0,1}$

$$\Gamma^R_{0\to 1} \sim e^{-(\Delta_H + V)/T} e^{-L_2/\xi}$$

$$\Gamma^L_{0\to 1} \sim e^{-(\Delta_H - V)/T} e^{-L_1/\xi}$$

Master equation for inhomogeneous granule

$$N_{qp} = 2\nu_n \int_V d^3\mathbf{r} \int_0^\infty \rho_S(\varepsilon, \mathbf{r}) f_S(\varepsilon) d\varepsilon = N_{qp}^0(\Delta)\left[1 - \frac{\xi^2}{R^2}\right] + N_{qp}^0(\Delta_H)\frac{\xi^2}{R^2}$$

$$\rho_S(\varepsilon, \mathbf{r}) = \rho_S^0(\varepsilon, \Delta)\left[1 - e^{-|\mathbf{r}-\mathbf{r}_v|/\xi}\right] + \rho_S^0(\varepsilon, \Delta_H) e^{-|\mathbf{r}-\mathbf{r}_v|/\xi} \qquad N_{qp}^0(\Delta) = \sqrt{2\pi\Delta T} V \nu_n e^{-\Delta/T}$$

We assume quasiparticle recombination to be large enough to ensure N_S to be one of the closest integers to N_{qp}

$$N_S = \lfloor N_{qp} \rfloor + \left(N - \lfloor N_{qp} \rfloor\right) \bmod 2$$

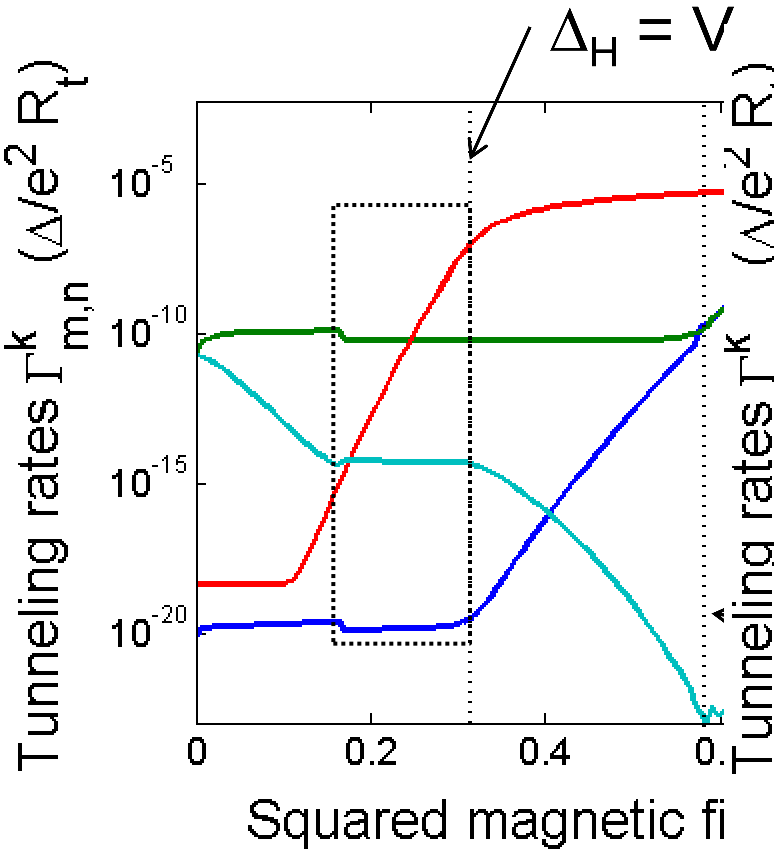
$$\frac{\partial}{\partial t} p_N = \sum_{N', N_S'} \left(\Gamma_{N' \to N}^{N_S'} p_{N'} - \Gamma_{N \to N'}^{N_S} p_N \right)$$

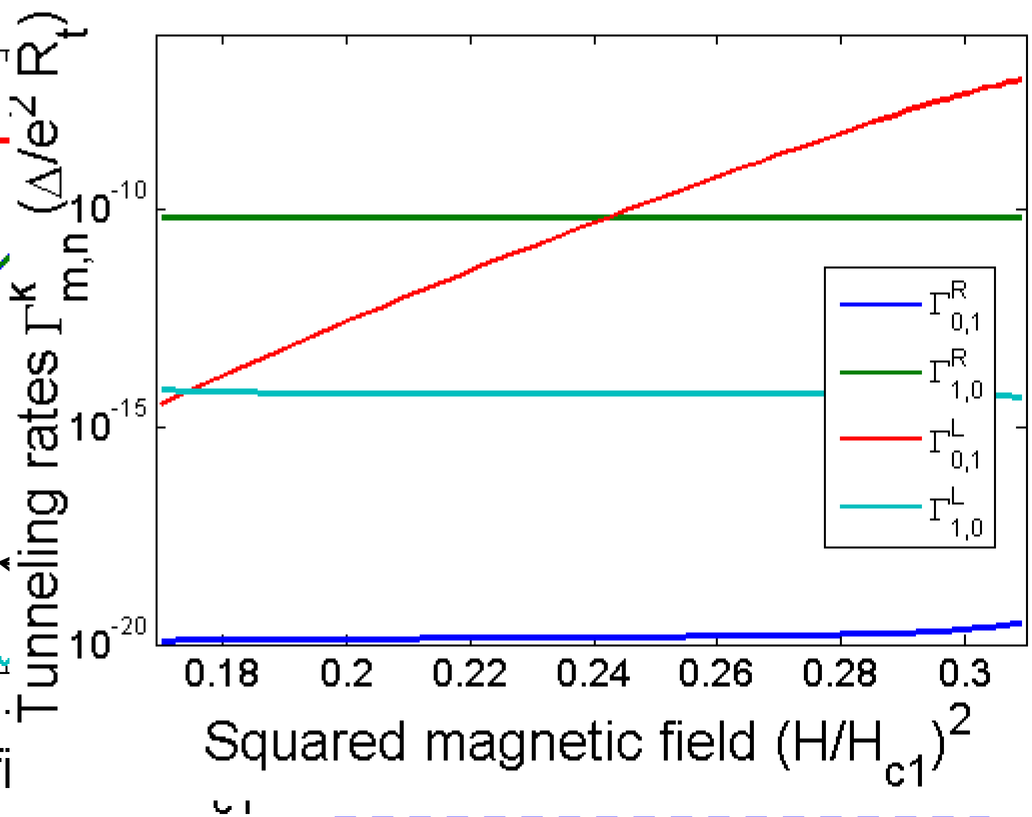
$$\Gamma_{N \to N\pm1}^0 = \frac{G_T}{e^2} \int_0^\infty \rho_S(\varepsilon, L) f_n(\varepsilon - E_N^\pm) d\varepsilon$$

$$\Gamma_{N \to N\pm1}^1 = \frac{G_T}{e^2} \int_{-\infty}^\infty \rho_S(\varepsilon, L) f_S(\varepsilon, 1)\left[1 - f_n(\varepsilon + E_N^\pm)\right] d\varepsilon$$

NOTE! There is no detailed balance anymore

Tunneling rates in NISIN with vortex





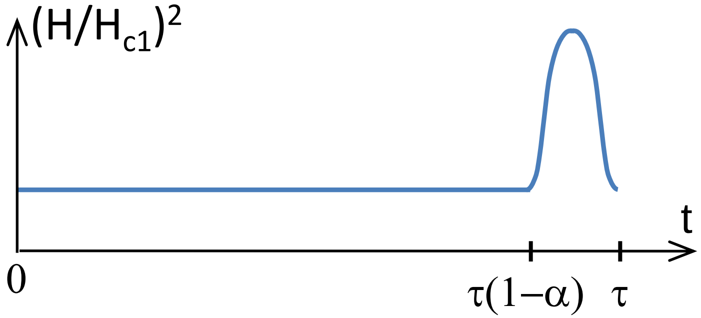
$$\Gamma^R_{1\to0} \sim e^{-L_2/\xi}$$

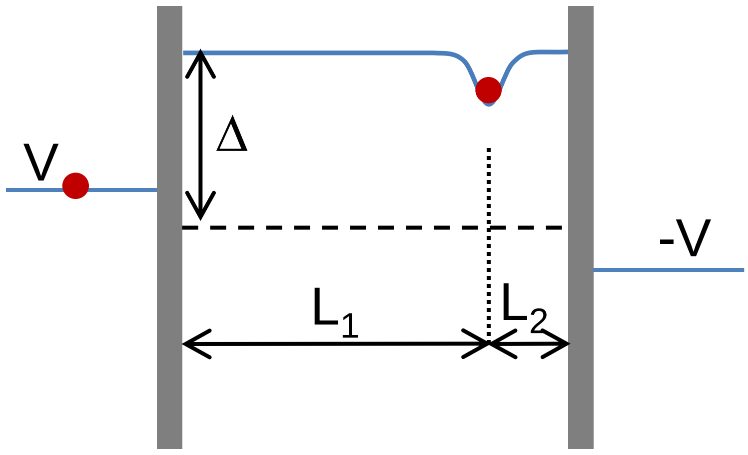
$$\Gamma^L_{1\to0} \sim e^{-L_1/\xi}$$

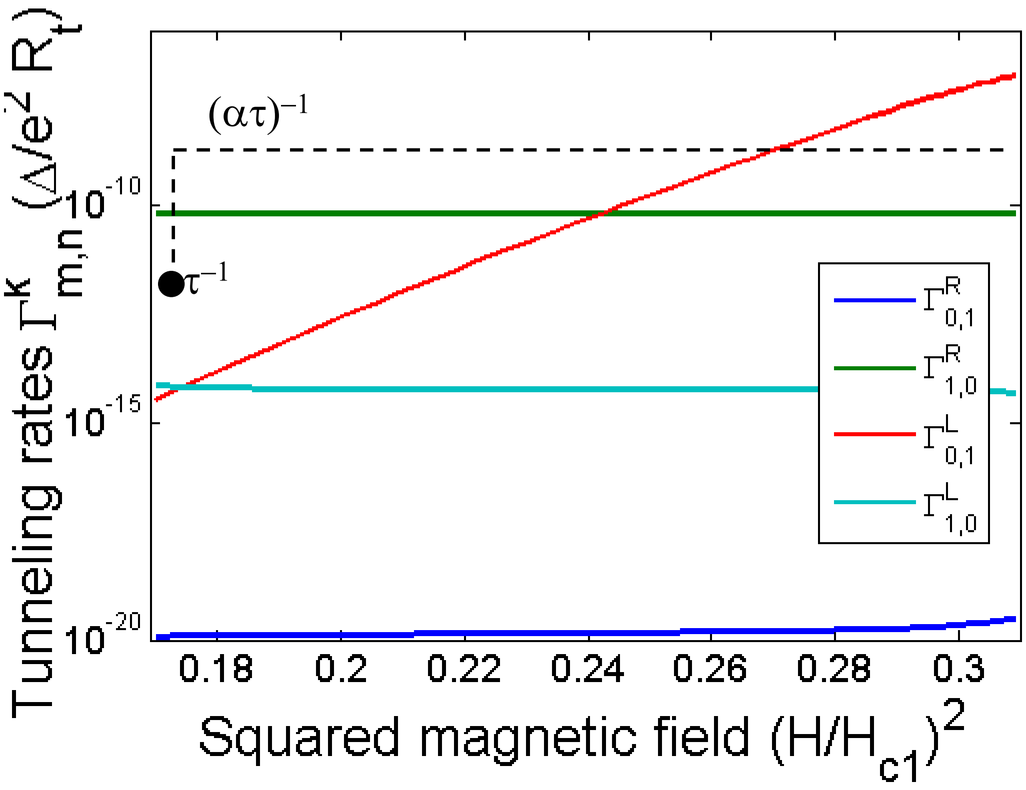
$$\Gamma^R_{0\to1} \sim e^{-(\Delta_H+V)/T} e^{-L_2/\xi}$$

$$\Gamma^L_{0\to1} \sim e^{-(\Delta_H-V)/T} e^{-L_1/\xi}$$

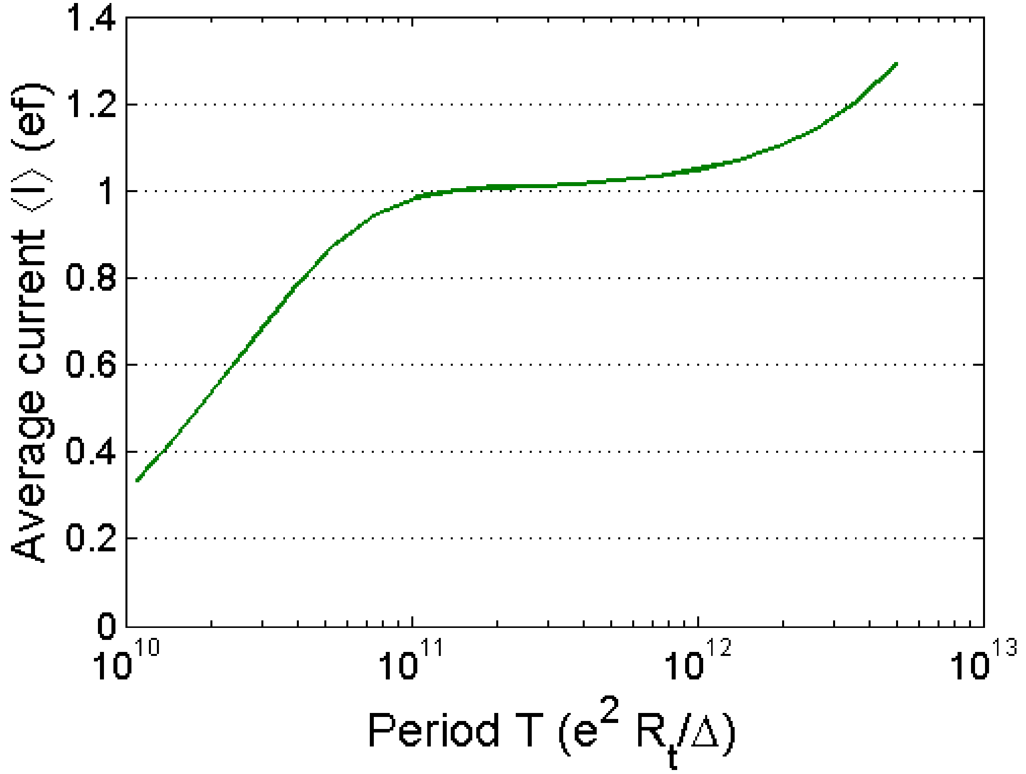
Magnetic field protocol in NISIN







Resulting e-pumping effect



Summary

A new principle of electron pumping based on supression of parity effect in NISIN single electron transistor is considered.

The time-variating magnetic field is considered as a possible realization of this principle.

Using the rate equation approach the pumping current is calculated as a function of period of the magnetic field cycling.