Kopnin force & chiral anomaly

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* *chiral anomaly in cond-matter* **(Kopnin force)**

* ***experiments on skyrmion lattice***

* **Kopnin number**

* **Kopnin mass** ***example of skyrmion***

two major universality classes of gapless topological matter

$\boxed{\text{Landau theory of Fermi liquid}}$

vacua with Fermi surface:
metals, normal ^{3}He

$\boxed{\text{universal properties of metals emerge from topological stability of } \textbf{Fermi surface}}$

$\boxed{\text{Standard Model + gravity}}$

vacua with Weyl, Dirac, Majorana points:
^{3}He-A, planar phase, Weyl semimetal,
vacuum of SM

$\boxed{\text{gravity \& SM emerge from topological stability of } \textbf{Fermi (Weyl) point}}$

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau \cdot \mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau \cdot \mathbf{W}_\nu) = 0}$$

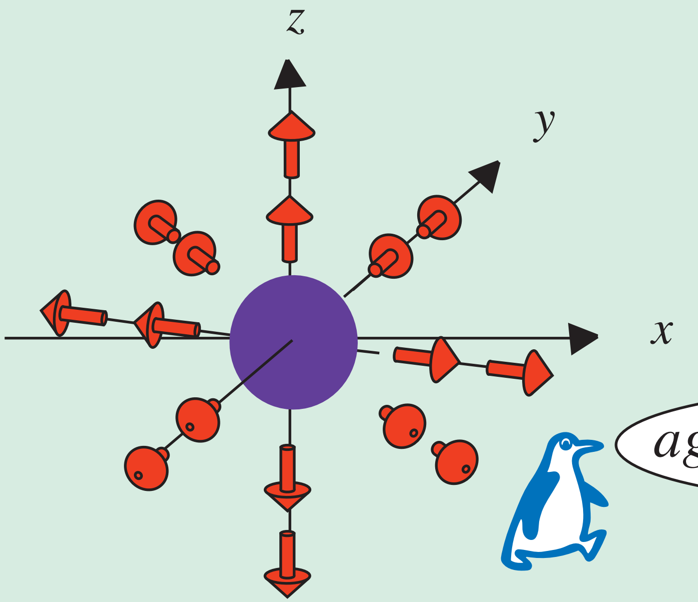
Theory of topological matter:
Nielsen, TKNN, Volkov, So, Ishikawa, Matsuyama, Haldane, Yakovenko, Kaplan, Read, Horava, Kitaev, Ludwig, Schnyder, Ryu, Furusaki, S-C Zhang, Kane, Liang Fu, ...

3. Weyl, Majorana & Dirac materials

quantum vacua of Fermi point universality classes

Superfluid ^{3}He-A, vacuum of Standard Model, topological semimetal, graphene, ...

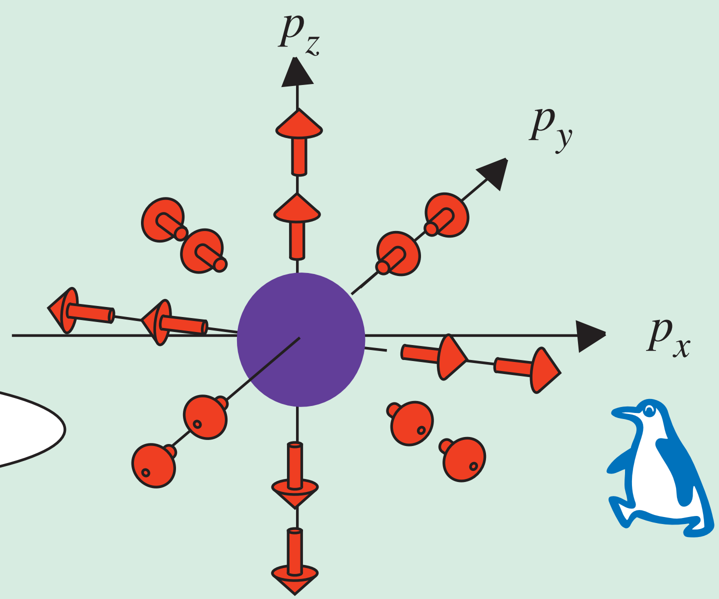
magnetic hedgehog vs right-handed Weyl electron



hedgehog in **r**-space

$$\sigma(\mathbf{r})=\hat{\mathbf{r}}$$

again no difference ?



hedgehog in **p**-space

$$\sigma(\mathbf{p}) = \hat{\mathbf{p}}$$

right-handed and left-handed massless quarks and leptons are building blocks of Standard Model

Landau CP symmetry is emergent

close to Fermi point

$$H = + c\,\sigma \cdot \mathbf{p}$$

right-handed electron =
hedgehog in **p**-space with spines = spins

effective relativistic gauge field in Weyl material

Atiyah-Bott-Shapiro construction near Weyl point

$$H = e_i^{\ k}\,\sigma^i \cdot (p_k - eA_k)$$

effective tetrad
emergent gravity

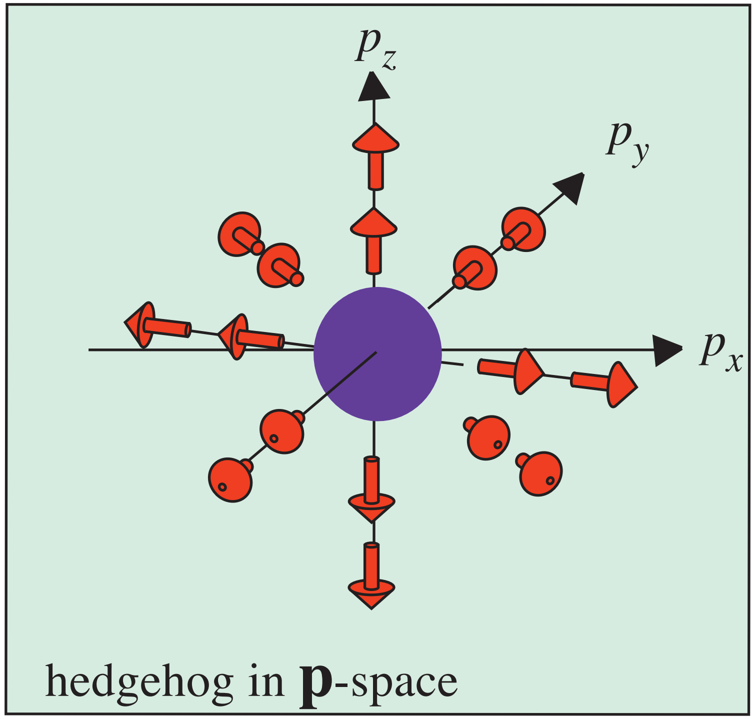
effective spin

effective
electromagnetic
field

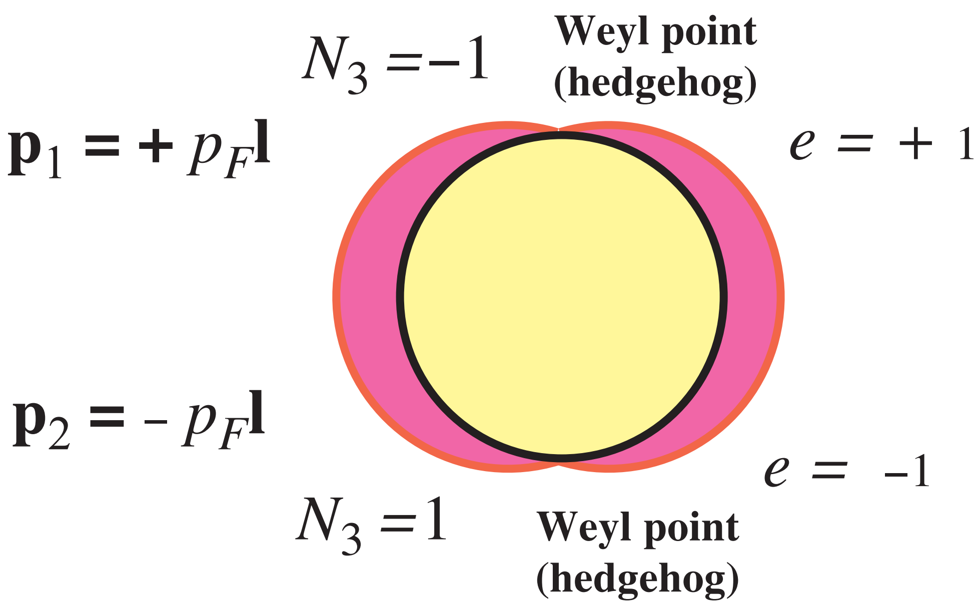
gauge field = position of Weyl point

gauge field in 3He-A

$$\boxed{\mathbf{A} = p_F \mathbf{l} \qquad \mathbf{B} = p_F \nabla \times \boldsymbol{l} \qquad \mathbf{E} = p_F \dot{\boldsymbol{l}} \qquad e = +1 \text{ or } -1}$$



hedgehog in **p**-space



chiral anomaly in topological Weyl vacua: Standard Model & 3He-A

electroweak baryogenesis in Standard Model of particle physics
baryon production from vacuum by hypermagnetic field in early Universe

chiral anomaly equation
(Adler, Bell, Jackiw)

$$\dot{B} = \frac{1}{4\pi^2} N_B \, \mathbf{B}_Y \cdot \mathbf{E}_Y$$

topological origin
of quantization of physical parameters

symmetry protected integer valued topological invariant

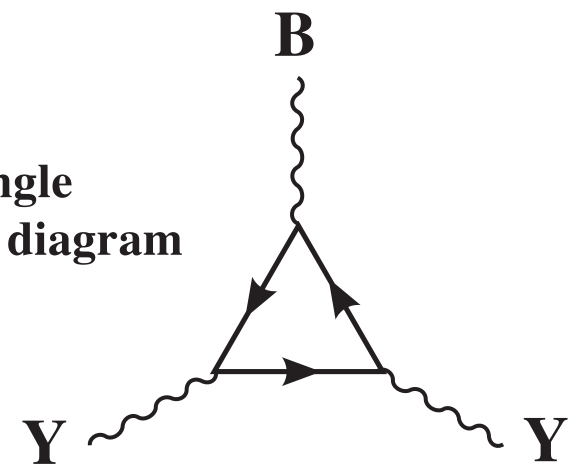
$$N_B = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \, \mathbf{tr} \int_{\text{over } S^3} dV \, \mathbf{B}\mathbf{Y}^2 \, \mathbf{G} \nabla^{\mu} \mathbf{G}^{-1} \, \mathbf{G} \nabla^{\nu} \mathbf{G}^{-1} \mathbf{G} \nabla^{\lambda} \mathbf{G}^{-1}$$

matrix of baryonic charge **matrix of hypecharge**

$$\dot{B} = \frac{1}{4\pi^2} \, \mathbf{B}_Y \cdot \mathbf{E}_Y \sum_a B_a C_a Y_a^2$$

B_a -- **baryonic charge**
Y_a -- **hypercharge**
C_a -- **chirality** $= +1$ **for right**
-1 **for left**

triangle
Feynman diagram



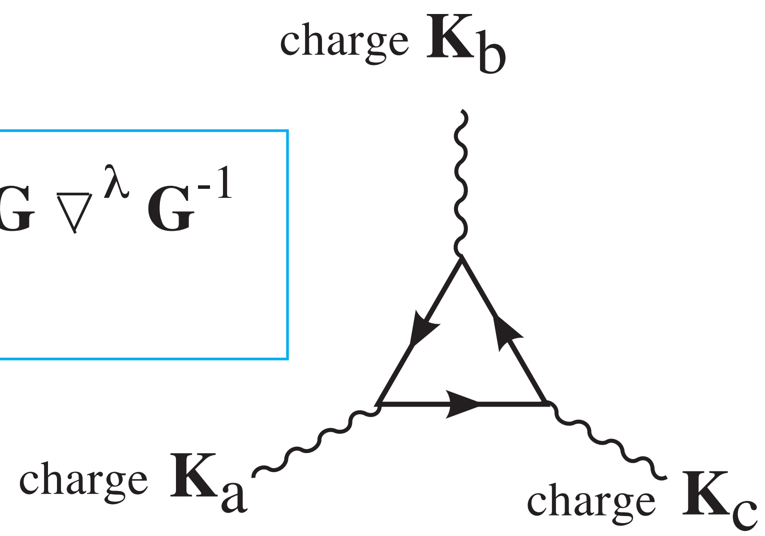
Symmetry protected p-space topological invariants & chiral anomaly

$$K_{abc} = \frac{1}{24\pi^2} e_{\mu\nu\lambda} \mathbf{tr} \int_{\text{over } S^3} dS\, \mathbf{K}_a \mathbf{K}_b \mathbf{K}_c\, \mathbf{G}\, \nabla^\mu \mathbf{G}^{-1}\, \mathbf{G} \nabla^\nu \mathbf{G}^{-1} \mathbf{G}\, \nabla^\lambda \mathbf{G}^{-1}$$

integral is around Weyl point

$\mathbf{G}(p_\mu)$ is Green's function matrix

$\mathbf{K}_c$ are charges of fields acting on fermions
(electric, weak, baryonic, hypercharge, etc.)



triangle
Feynman diagram

these invariants determine chiral anomaly effects due 3+1 Weyl points:
electroweak baryogenesis, chiral magnetic effect, Kopnin force, chiral vortical effect,
spin quantum Hall effect, ...

p-space invariants are prefactors of topological terms in ***r***-space action

topological origin
of quantization of physical parameters

experimental verification of chiral anomaly equation
measurement of *Kopnin force*

$$\mathbf{A} = p_F \mathbf{l} \qquad \mathbf{B} = p_F \nabla \times \boldsymbol{l}$$
$$\mathbf{E} = p_F \dot{\boldsymbol{l}} \qquad B_a = \mathbf{P}_a$$

translation from SM to language of 3He-A

momentum from vacuum of fermion zero modes

baryogenesis in early Universe

$$\dot{\mathbf{P}} = \sum \mathbf{P}_a \dot{n}_a$$

$\mathbf{P}_a$ -- momentum of Weyl point (fermionic charge)

e_a -- effective electric charge

$$\dot{\mathbf{P}} = (1/4\pi^2)\, \mathbf{B} \bullet \mathbf{E} \sum_a \mathbf{P}_a C_a e_a^2$$

applied to ^{3}He-A

$C_a = +1$ for right
$\quad\;\; -1$ for left

$$\dot{B} = \sum B_a \dot{n}_a$$

B_a -- baryonic charge

Y_a -- hypercharge

$$\dot{B} = (1/4\pi^2)\, \mathbf{B}_Y \bullet \mathbf{E}_Y \sum_a B_a C_a Y_a^2$$

applied to Standard Model

$C_a = +1$ for right
$\quad\;\; -1$ for left

chiral anomaly equation (Adler, Bell, Jackiw)

quasiparticles move from vacuum to the positive energy world,
where they are scattered by quasiparticles in bulk
and transfer momentum from vortex to normal component

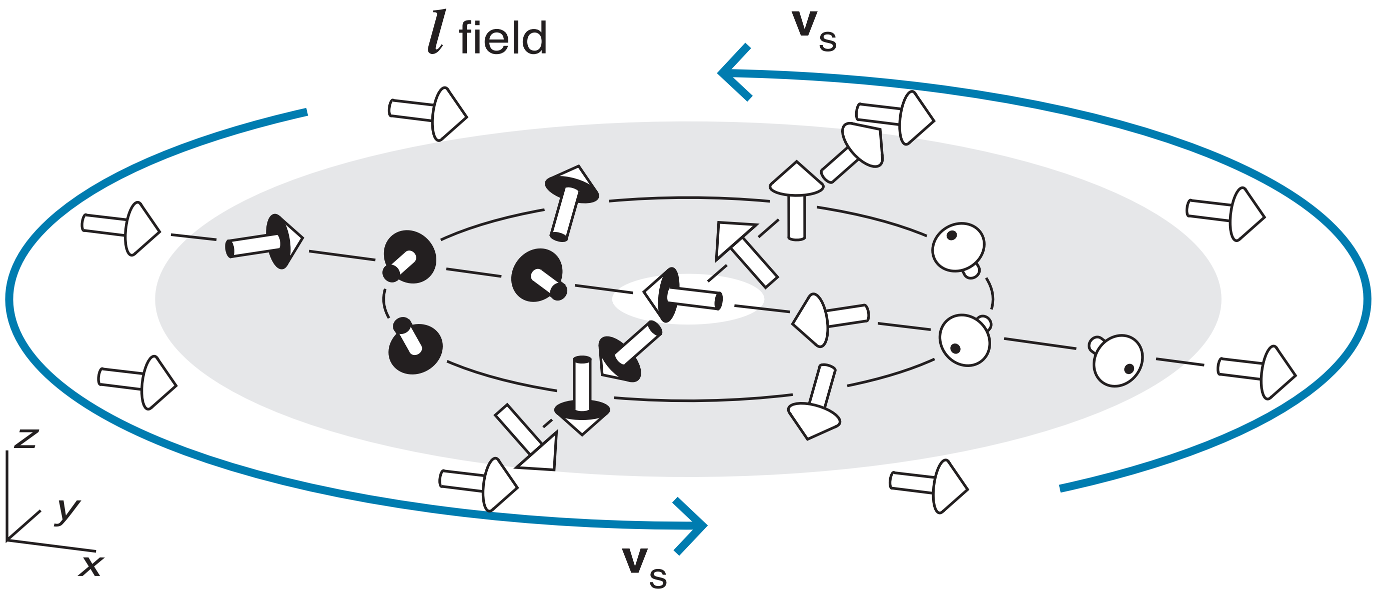
this is the source of Kopnin spectral flow force

Bevan, Manninen, Cook, Hook, Hall, Vachaspati & GV
Momentum creation by vortices in superfluid 3He as a model of primordial baryogenesis, Nature **386**, 689 (1997)

Kopnin force on vortex-skyrmion: chiral anomaly & momentogenesis

$$m = (1/4\pi) \iint dx\, dy\ \left(\boldsymbol{l} \cdot \left(\partial \boldsymbol{l} / \partial x \times \partial \boldsymbol{l} / \partial y \right) \right) = 1$$

vortex-skyrmion
with N=2m=2
circulation quanta



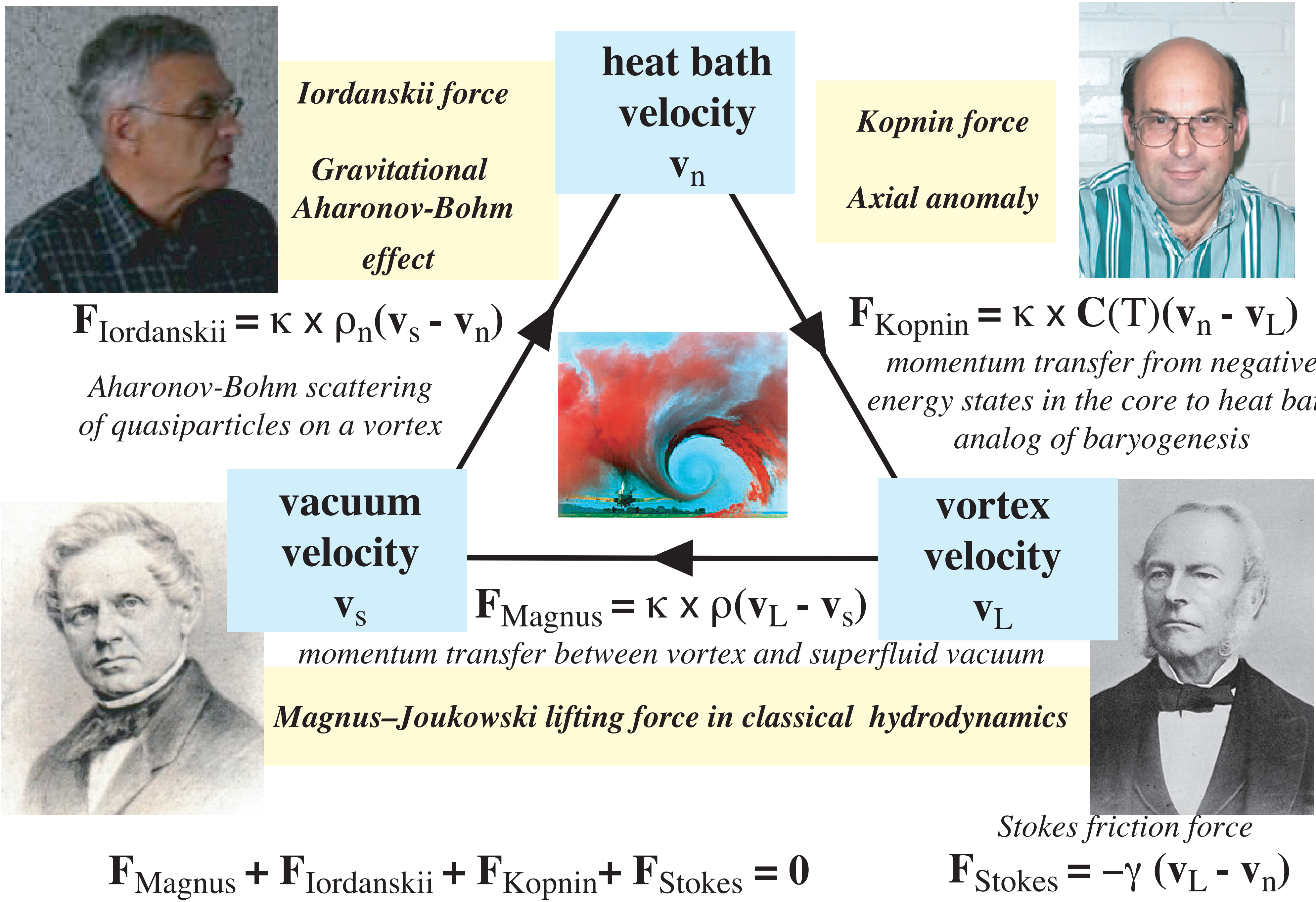
$$\boldsymbol{l} = \boldsymbol{l}(\mathbf{r}-\mathbf{v}t)$$

Momentum transfer from vacuum to the heat bath (matter)
gives extra topological force on skyrmion (spectral-flow Kopnin force)

$$\mathbf{F} = \int d^3r\ \dot{\mathbf{P}} = (1/2\pi^2) \int d^3r\ \ (\mathbf{B} \cdot \mathbf{E})\ p_F\, \mathbf{l} = (1/2\pi^2)\, \hbar\, p_F^{\,3} \int d^3r\ \ (\nabla \times \mathbf{l} \cdot\ d\mathbf{l}/dt)\ \mathbf{l}$$

$$= 2\pi\hbar\, (1/3\pi^2)\, p_F^{\,3}\ \hat{\mathbf{z}} \times (\mathbf{v}_n - \mathbf{v}_L)$$

Kopnin force on singular vortex & chiral anomaly

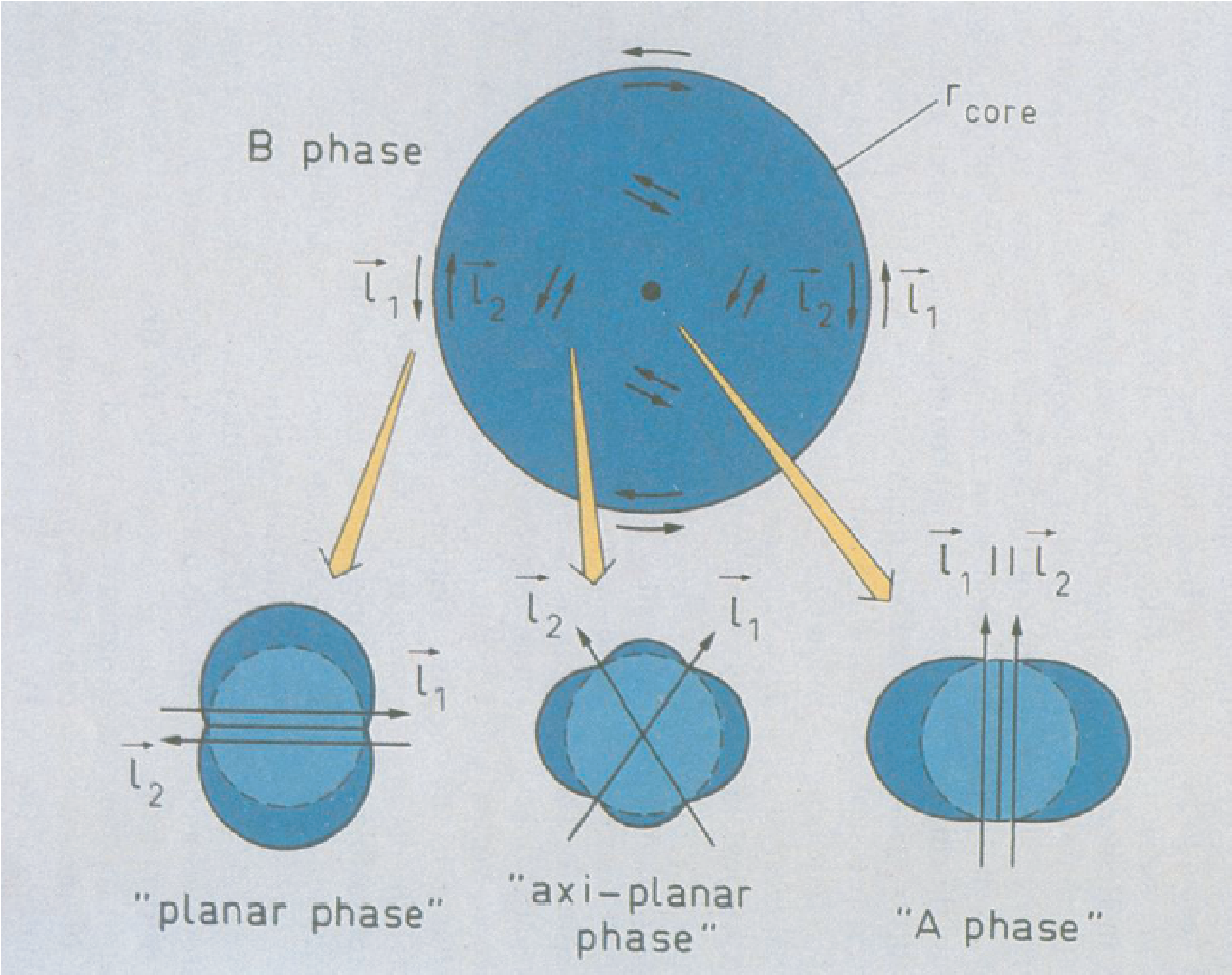


Iordanskii force
Gravitational Aharonov-Bohm effect
heat bath velocity $\mathbf{v}_n$
Kopnin force
Axial anomaly
$\mathbf{F}_{\text{Iordanskii}} = \kappa \times \rho_n(\mathbf{v}_s - \mathbf{v}_n)$
Aharonov-Bohm scattering of quasiparticles on a vortex
$\mathbf{F}_{\text{Kopnin}} = \kappa \times \mathbf{C}(T)(\mathbf{v}_n - \mathbf{v}_L)$
momentum transfer from negative energy states in the core to heat bath
analog of baryogenesis
vacuum velocity $\mathbf{v}_s$
vortex velocity $\mathbf{v}_L$
$\mathbf{F}_{\text{Magnus}} = \kappa \times \rho(\mathbf{v}_L - \mathbf{v}_s)$
momentum transfer between vortex and superfluid vacuum
Magnus–Joukowski lifting force in classical hydrodynamics
Stokes friction force
$\mathbf{F}_{\text{Stokes}} = -\gamma\,(\mathbf{v}_L - \mathbf{v}_n)$
$\mathbf{F}_{\text{Magnus}} + \mathbf{F}_{\text{Iordanskii}} + \mathbf{F}_{\text{Kopnin}} + \mathbf{F}_{\text{Stokes}} = \mathbf{0}$

Kopnin force on singular vortex & chiral anomaly

extended core of B-phase singular vortex, $R_{core} >> \xi$

four Weyl points in axi-planar phase

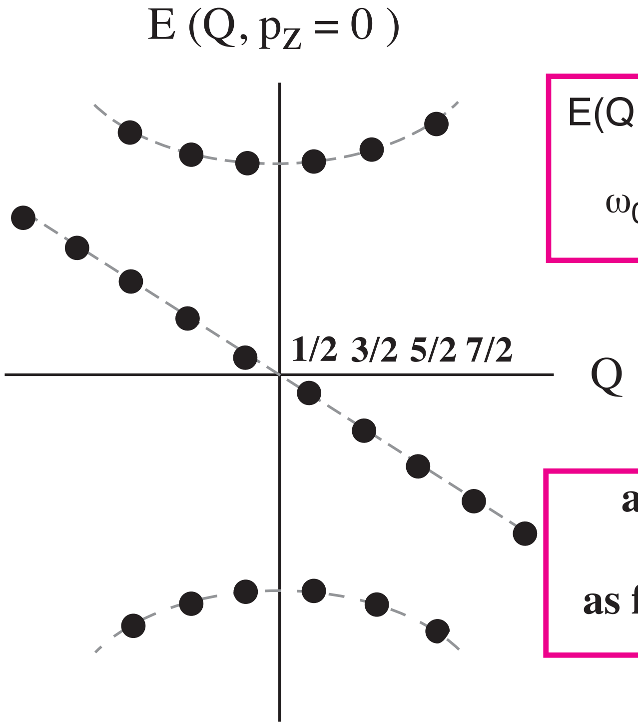


chiral anomaly equation for 4 species of Weyl fermions
gives again Kopnin force with $C(T) = \rho$

fermion bound states on vortex in topologically trivial superconductor

Caroli, de Gennes & J. Matricon, Phys. Lett. **9** (1964) 307

Angular momentum Q is half-odd integer
in s-wave superconductor

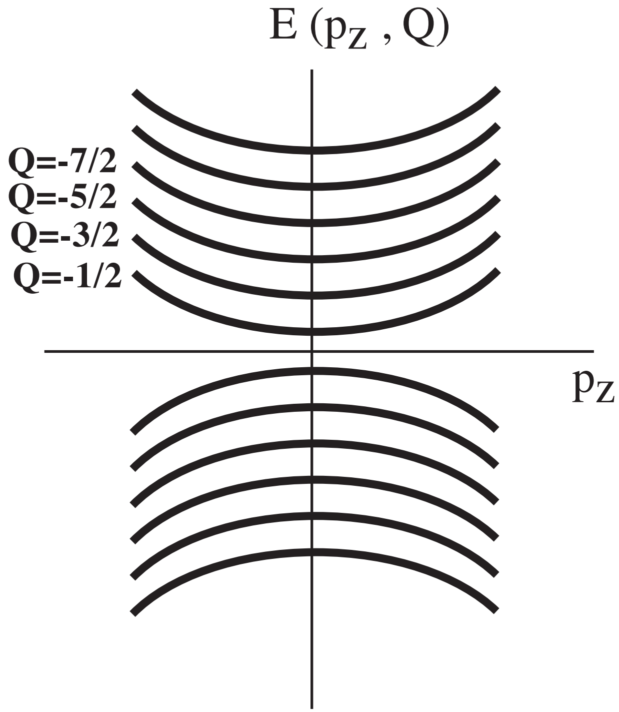


$E(Q,p_z) = -Q\,\omega_0\,(p_z)$

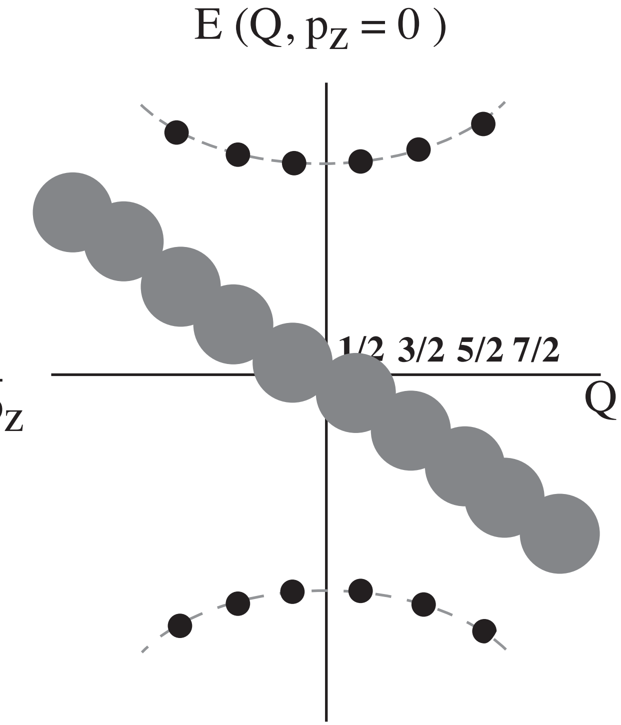
$\omega_0 = \Delta^2/ E_F << \Delta$

minigap

asymmetric branch as function of Q



level width $1/\tau$ due to collisions



$\omega_0\tau >> 1$

no spectral flow between discrete levels

no true fermion zero modes:
no asymmetric branch as function of p_Z

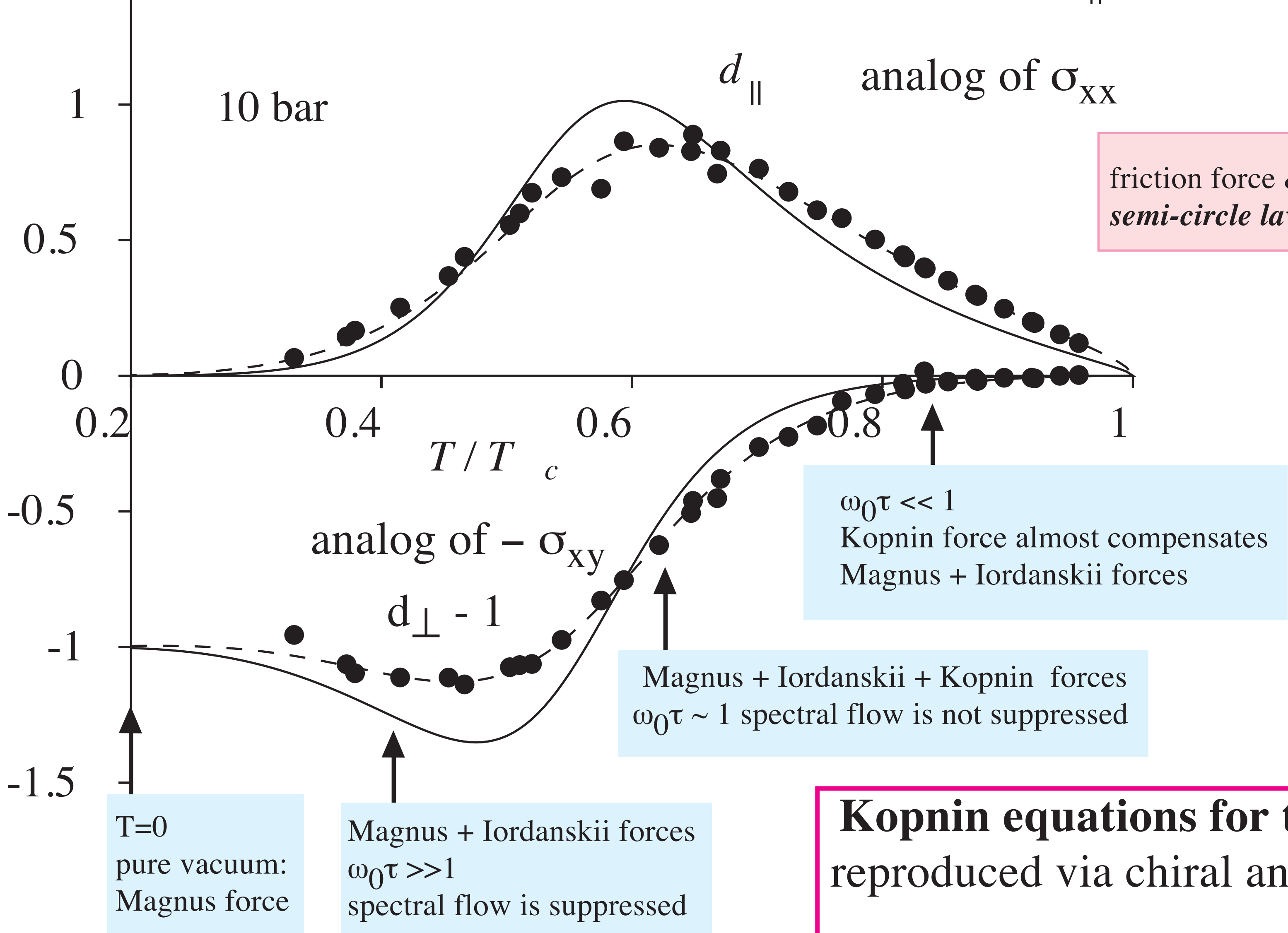
$\omega_0\tau << 1$

maximal spectral flow

spectral flow along asymmetric branch
is the source of Kopnin force on singular vortex
efficiency of spectral flow is determined by Kopnin number $\omega_0\tau$
spectral flow is maximal when minigap is smaller than level width, $\omega_0\tau << 1$
this limit corresponds to axial anomaly equation for extended core

Observation of Kopnin force in Manchester experiments on ^{3}He-B vortices

$$\hat{z} \times (\mathbf{v}_L - \mathbf{v}_s) + d_\perp \hat{z} \times (\mathbf{v}_n - \mathbf{v}_L) + d_\parallel (\mathbf{v}_n - \mathbf{v}_L) = 0$$



friction force & nondissipative forces obey ***semi-circle law*** as σ_{xx} and σ_{xy} in QHE

$$\alpha + i(1 - \alpha') = 1/(d_\parallel - i(1 - d_\perp))$$

Kopnin equations for transport parameters

reproduced via chiral anomaly by Stone (1996)

$$1 - d_\perp = (\rho / \rho_s) \tan(\Delta/2T)\, (\omega_0\tau)^2 / [1+(\omega_0\tau)^2]$$

$$d_\parallel = (\rho / \rho_s) \tan(\Delta/2T)\, \omega_0\tau / [1+(\omega_0\tau)^2]$$

Kopnin number for superfluid hydrodynamics

equations of superfluid hydrodynamics with vorticity

$$\frac{\partial \mathbf{v}_\mathrm{s}}{\partial t} + \nabla\mu - \mathbf{v}_\mathrm{s} \times (\nabla \times \mathbf{v}_\mathrm{s}) = \mathbf{F}$$

$$\mathbf{F} = -\alpha'(\mathbf{v}_\mathrm{s} - \mathbf{v}_\mathrm{n}) \times (\nabla \times \mathbf{v}_\mathrm{s}) - \alpha\, \hat{\mathbf{n}} \times ((\mathbf{v}_\mathrm{s} - \mathbf{v}_\mathrm{n}) \times (\nabla \times \mathbf{v}_\mathrm{s}))$$

two dimensionless parameters in superfluid hydrodynamics:
non-dissipative $1-\alpha'$ & frictional α
their ratio is analog of Reynolds number for superfluid

$$\boxed{\mathrm{Ko} = (1-\alpha')/\alpha}$$

$$\mathrm{Ko} \sim \omega_0\tau$$

Kopnin number for superfluid turbulence

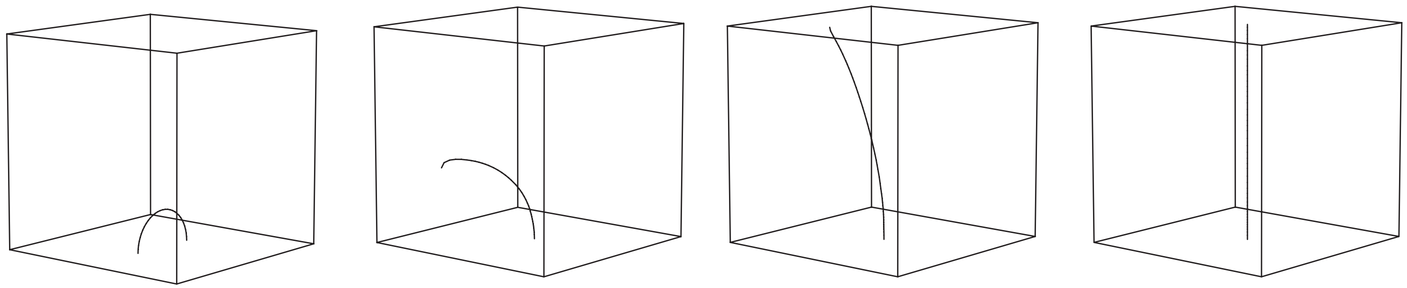
two dimensionless parameters in superfluid hydrodynamics:

non-dissipative $1-\alpha'$ & frictional α

their ratio is analog of Reynolds number for superfluid

$$\mathrm{Ko} = (1-\alpha')/\alpha$$

$$\mathrm{Ko} \sim \omega_0\tau$$



0 | 4 s | 8 s | 24 s

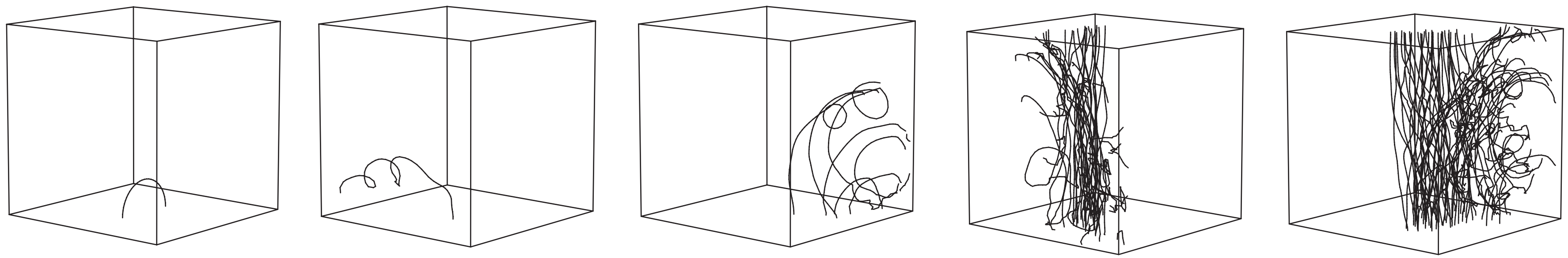
Ω

T=0.8 T_c

Ko < 1 $\quad \omega_0\tau < 1$

spectral flow supresses turbulence

transition to superfluid turbulence occurs at Ko ~ 1



0 | 44 s | 68 s | 128 s | 160 s

T=0.4 T_c

Ko > 1 $\quad \omega_0\tau > 1$

phase diagram of turbulent flow in Fermi superfluids (^{3}He-B)

3 Reynolds numbers in 2-fluid hydrodynamics

$$\boxed{Ko = (1 - \alpha')/\alpha}$$

Kopnin number $Ko \sim \omega_0 \tau$

$$\boxed{Re_\nu = UR/\nu_n} \ll 1$$

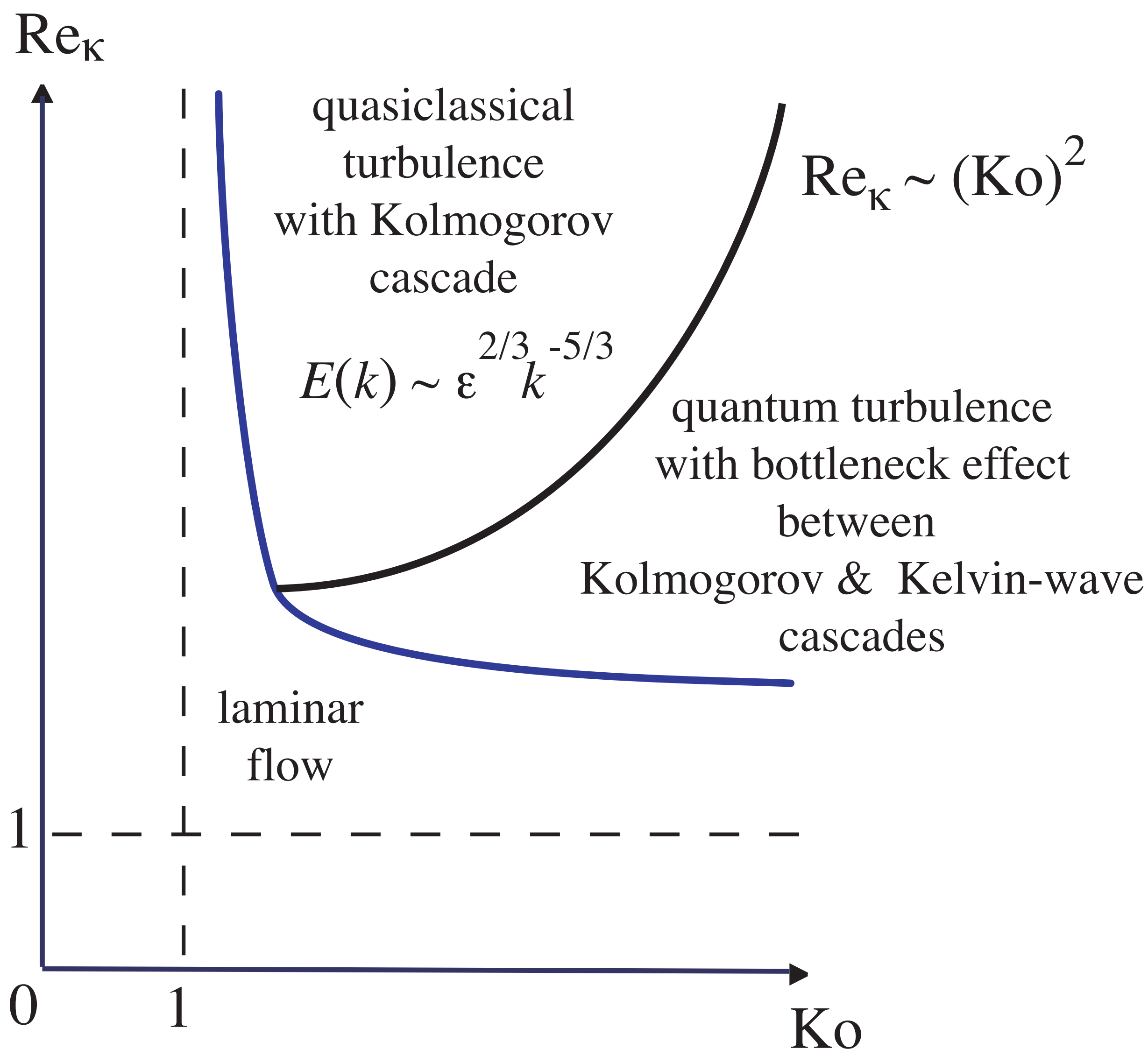
conventional
Reynolds number

ν_n – viscosity
of normal component

$$\boxed{Re_\kappa = UR/\kappa}$$

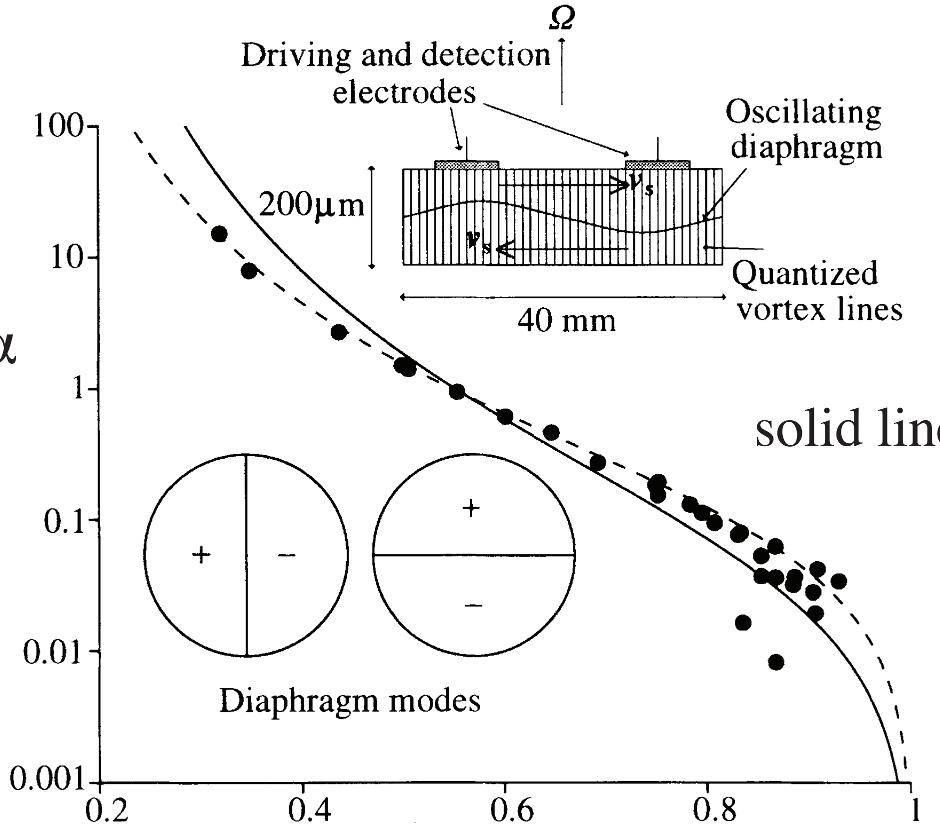
vorticity
Reynolds number

κ – circulation quantum



measured temperature dependence of Kopnin number in 3He-B (Manchester)

$Ko = (1- \alpha')/ \alpha$



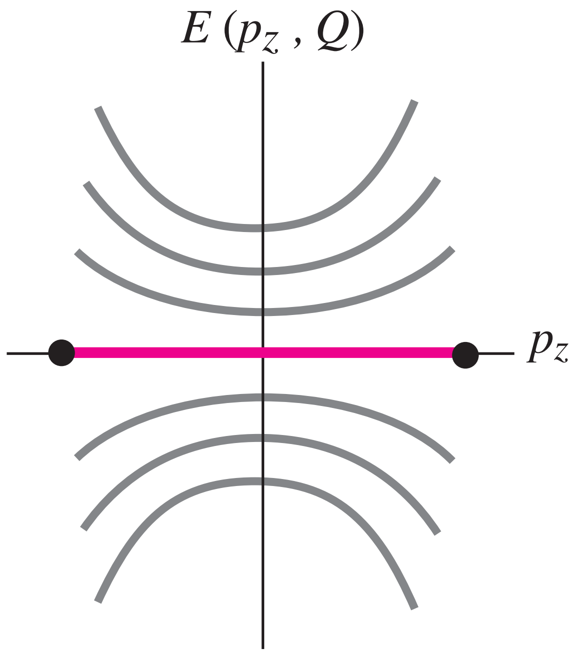
solid line - from Kopnin theory

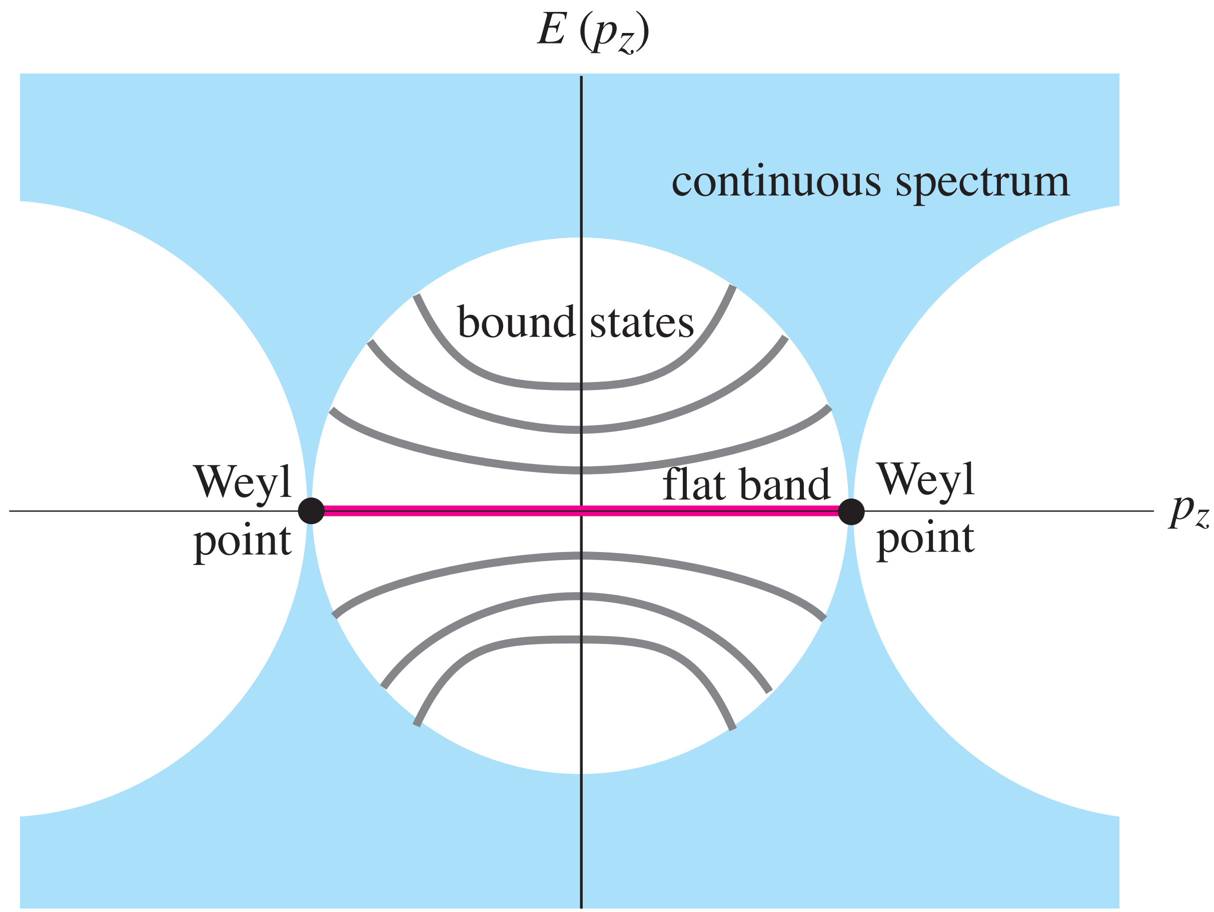
T/T_c

Start of Majorana cond-mat physics + start of topologically protected flat bands

topologically protected Majorana flat band
in vortex core of superfluids with Weyl points

(Kopnin-Salomaa 1991)



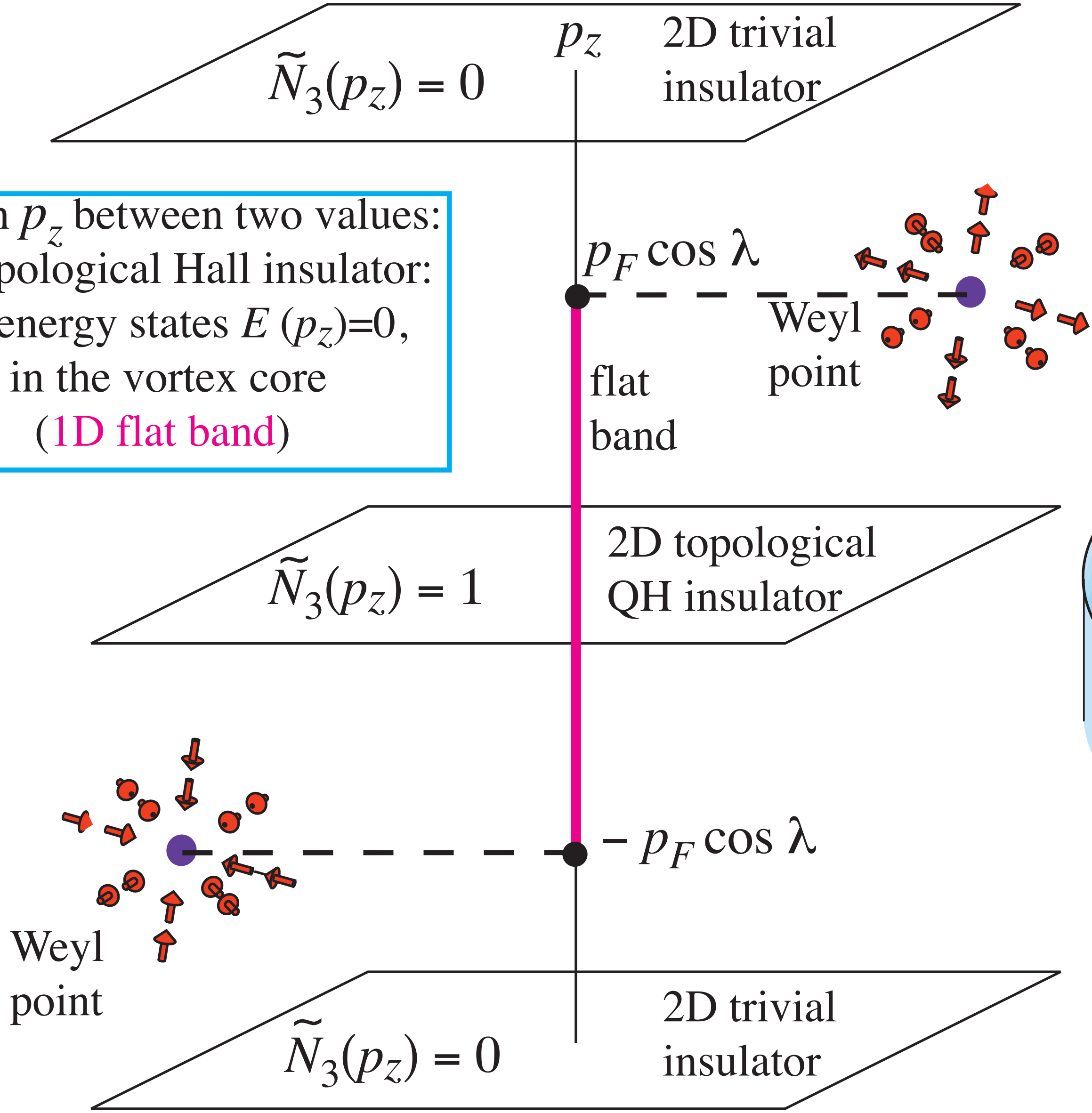


flat band of bound states
terminates on zeroes
of continuous spectrum
(i.e on Weyl points)

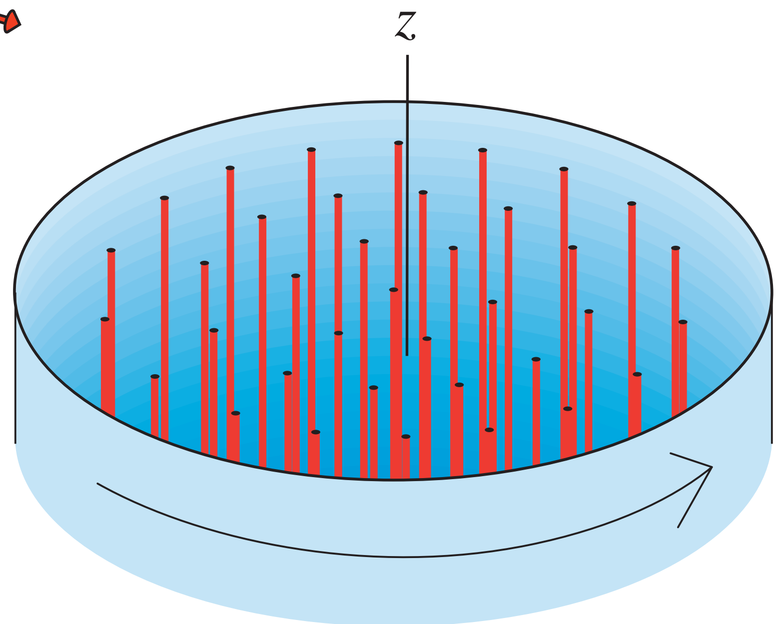
Topologically protected flat band of Majorana modes in vortex core of Weyl superfluid

Kopnin & Salomaa
Mutual friction in superfluid 3He:
effects of bound states in the vortex core
PRB **44**, 9667 (1991)

at each p_z between two values:
2D topological Hall insulator:
zero energy states $E\,(p_z)$=0,
in the vortex core
(1D flat band)



vortices in **r**-space



$$\tilde{N}_3(p_z) = \frac{1}{4\pi^2}\,\mathbf{tr}\int dp_x\, dp_y d\omega\; \mathbf{G}\, \nabla_\omega\, \mathbf{G}^{-1}\, \mathbf{G}\, \nabla_{p_x}\, \mathbf{G}^{-1} \mathbf{G}\, \nabla_{p_y}\, \mathbf{G}^{-1}$$

Chern number
for interacting systems
(So, Ishikawa, ...)
GV & Yakovenko
(1989)

Kopnin mass

Vortex mass in Bose superfluid (4He)

$$M_{\rm vortex} = E_{\rm vortex}/c^2$$

speed of sound

$$M_{\rm vortex} \sim \rho a^2 \ln (R/\xi)$$

coherence length

$a \sim h/mc$

~ interatomic space

intervortex space

Kopnin vortex mass in Fermi superfluid (3He-B & superconductor)

$$M_{\rm vortex} \sim \rho \xi^2 >> \rho a^2$$

coherence length

what is the origin of Kopnin mass ?

Kopnin mass of skyrmion

Kopnin vortex mass in Fermi superfluid
(skyrmion in 3He-A)

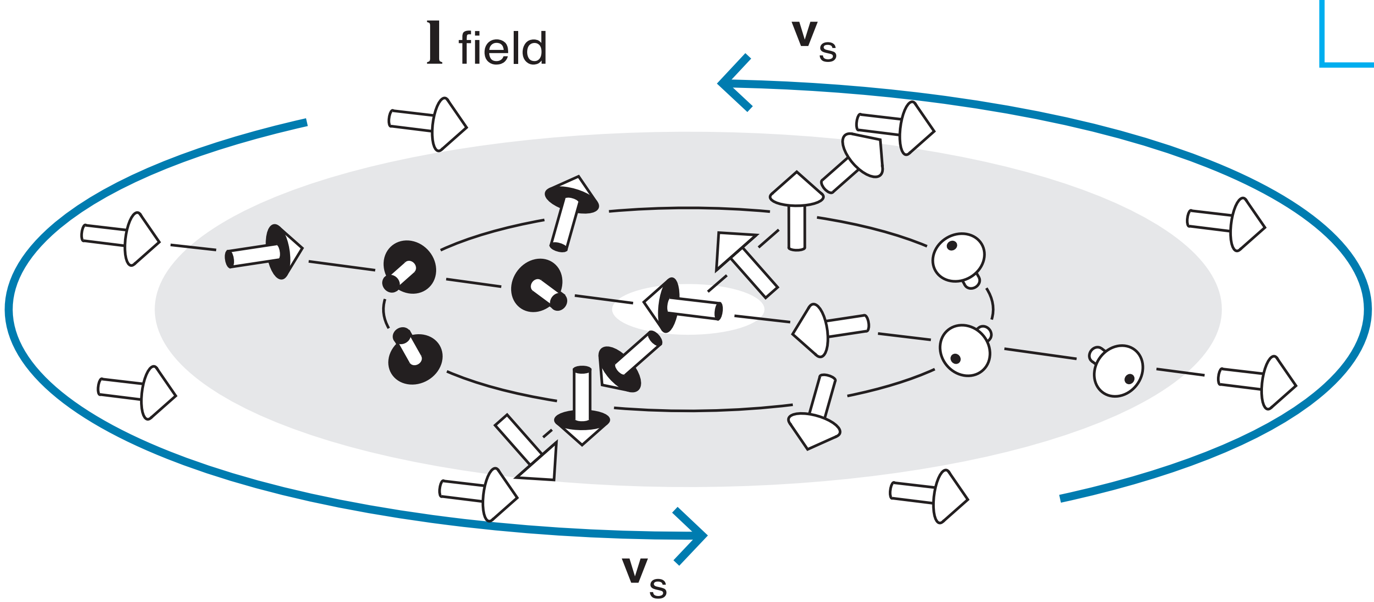
$$M_{\text{skyrmion}} \sim \rho\, \xi\, \underbrace{R_{\text{core}}}_{\text{core radius}} >> \rho \underbrace{\xi}_{\text{coherence length}}{}^2$$

what is the origin of Kopnin mass ?

Kopnin mass of skyrmion from effective magnetic field

Kopnin vortex mass in Fermi superfluid
(skyrmion in 3He-A & extended core in 3He-B)

$$\mathbf{A} = p_F \boldsymbol{l} \qquad \mathbf{B} = p_F \nabla \times \boldsymbol{l}$$



DoS in effective magnetic field
leads to normal component density at T=0

GV & Mineev, JETP **54**, 524 (1981)

$$\rho_n \sim |\mathbf{B}| \sim \rho\, \xi / R_{\rm core}$$

$$M_{\rm vortex} \sim \rho_n R^2_{\rm core} \sim \rho\, \xi\, R_{\rm core}$$

Kopnin mass comes from excitations
localized in the vortex core