Nikolay Kopnin and electronic structure of vortex state in superconductors: quasiclassical approach and beyond

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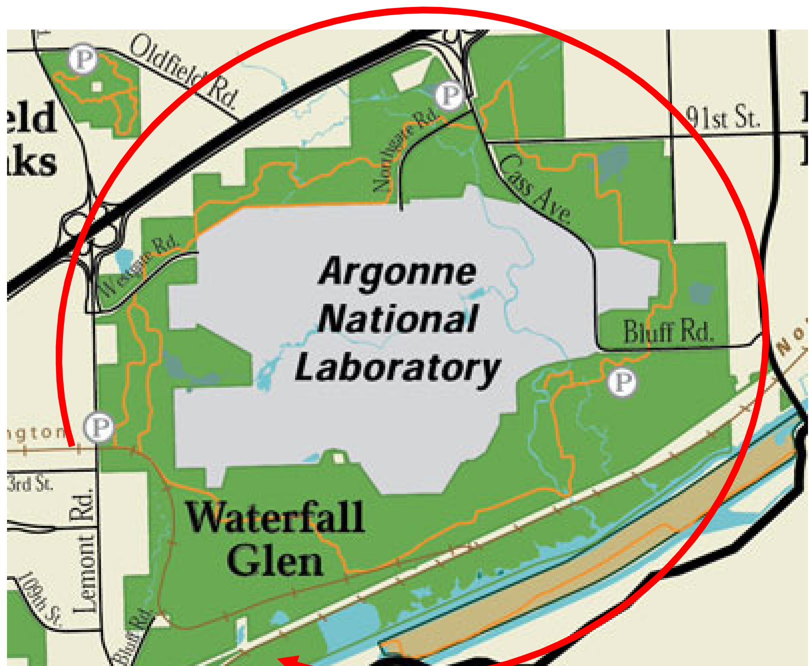


Lemont, Argonne, US

15km

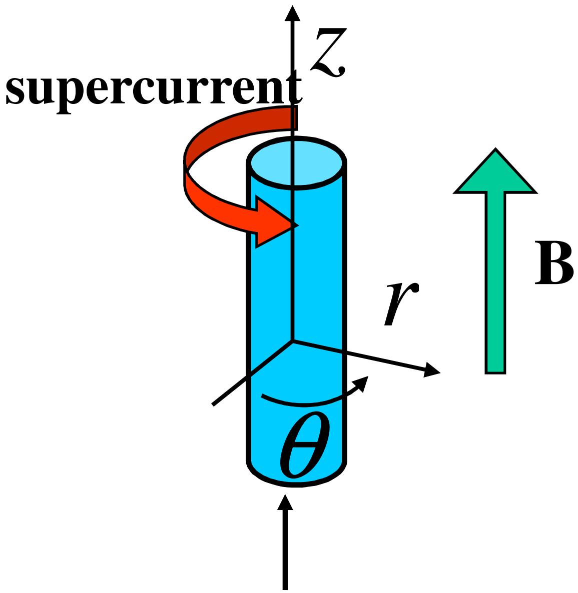
Outline

- Introduction.
- Electronic structure of vortex cores. Quasiclassical picture.
- Beyond the quasiclassical description.

  Thermal transport along vortex lines.
  Interplay of normal and Andreev reflection for quasiparticles in vortex cores.
  Quantized electron-hole levels inside the vortex core.
  Zero energy modes.
  Majorana states.

Vortex state of superconductors

Vortex line



$$\Delta = |\Delta(r)|\, e^{i\theta}$$

phenomenological theory of the mixed state

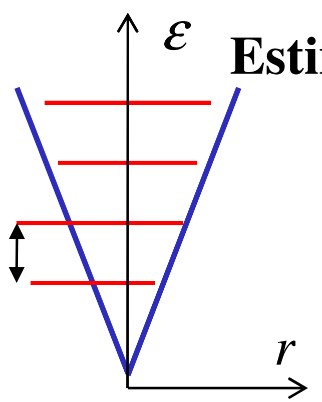
$$-\xi^2(T)\left(\nabla + \frac{2\pi i}{\Phi_0}\vec{A}(\vec{r})\right)^2 \Psi = \Psi - |\Psi|^2\,\Psi$$

ξ **Coherence length (core radius)**

Bound fermionic states in vortex core

Superconducting gap profile: potential well for electrons

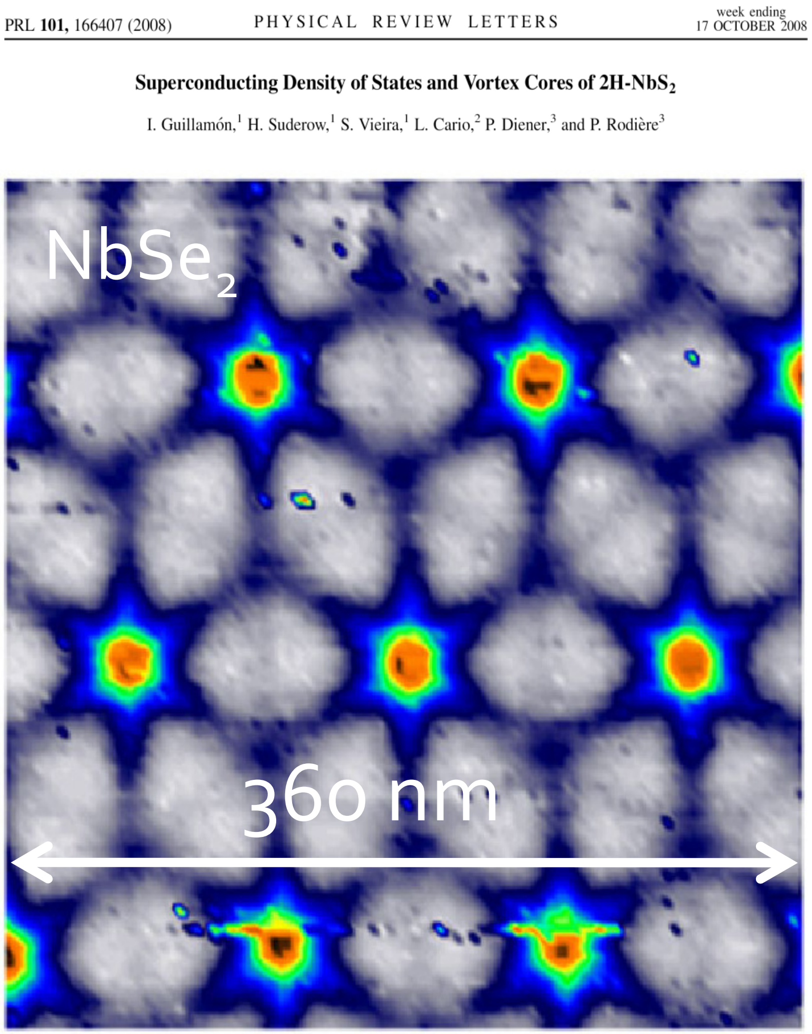
Estimate of minigap in excitation spectrum



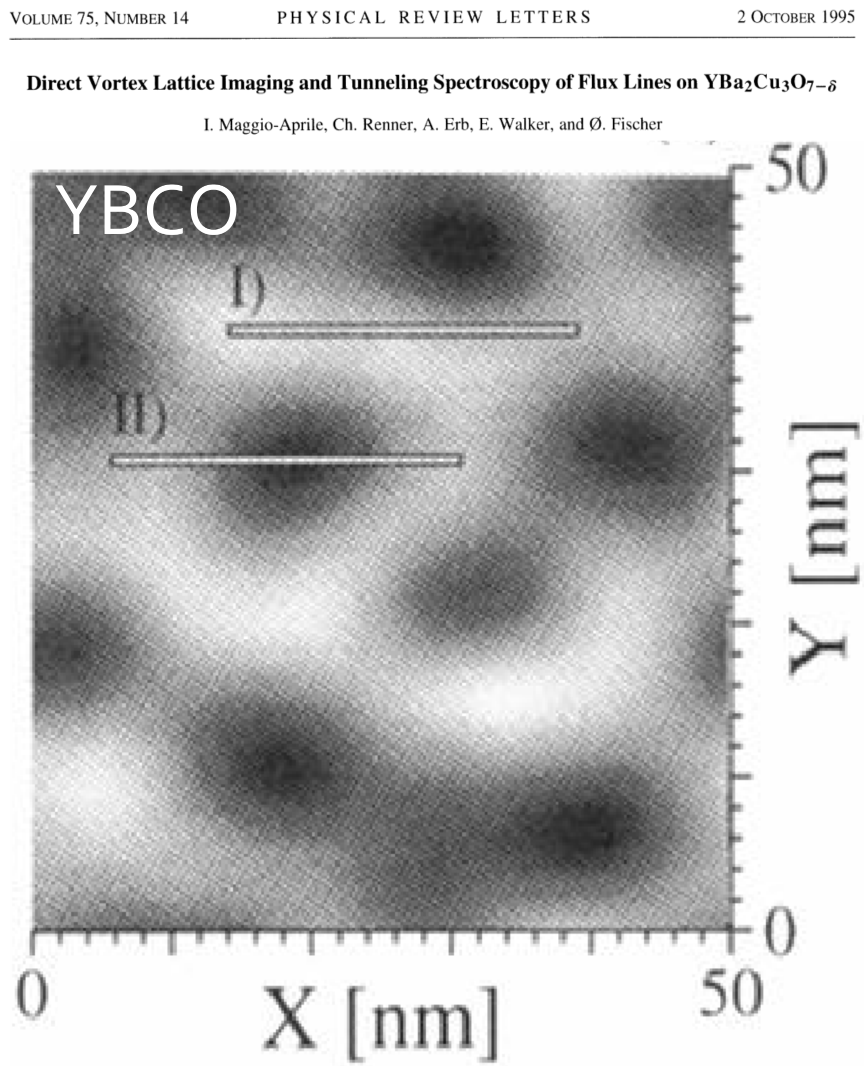
$$\frac{\Delta_0}{k_F \xi}$$

$$\varepsilon_{\min} \sim \frac{\hbar^2}{m\xi^2} \sim \frac{\hbar^2 \Delta_0}{m\hbar v_F \xi} \sim \frac{\Delta_0}{k_F \xi}$$

STM vortex images



PRL, ***101***, *166407 (2008)*



PRL, ***75***, *2754 (1995)*

Electrons inside the core

Convenient parametrization of a classical trajectory:

1. Orbital momentum

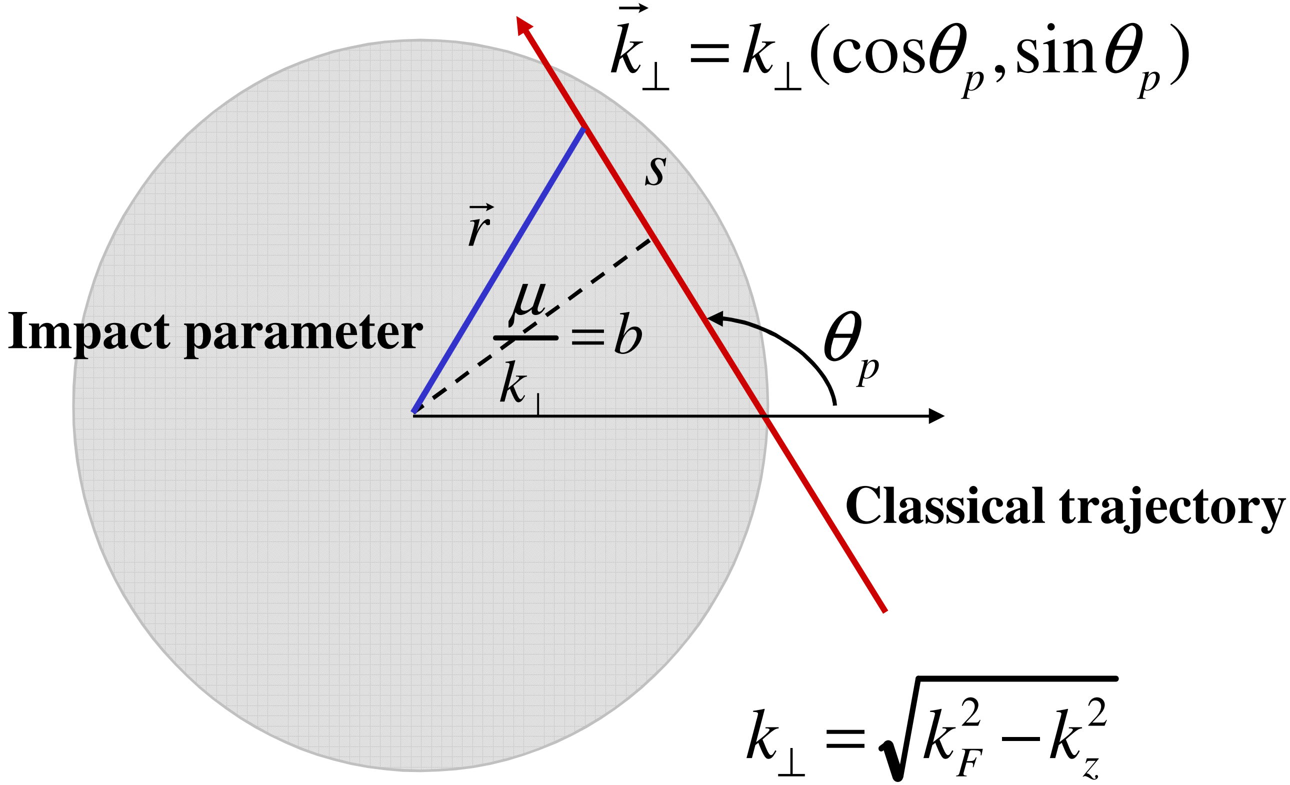
$$\mu = [\vec{r}, \vec{k}_\perp]\vec{z}_0 = k_\perp r \sin(\theta_p - \theta)$$

2. Trajectory orientation angle θ_p

Quasiclassical approximation

$$\lambda_F << \xi$$

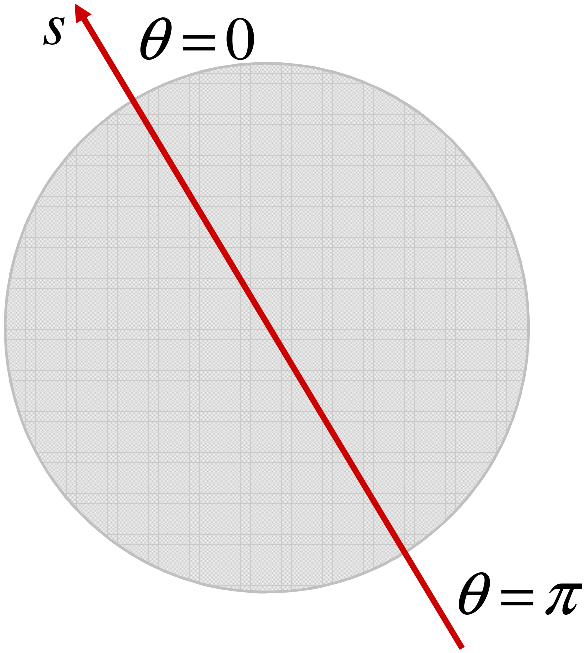
$$\vec{k}_\perp = k_\perp(\cos\theta_p, \sin\theta_p)$$



$$k_\perp = \sqrt{k_F^2 - k_z^2}$$

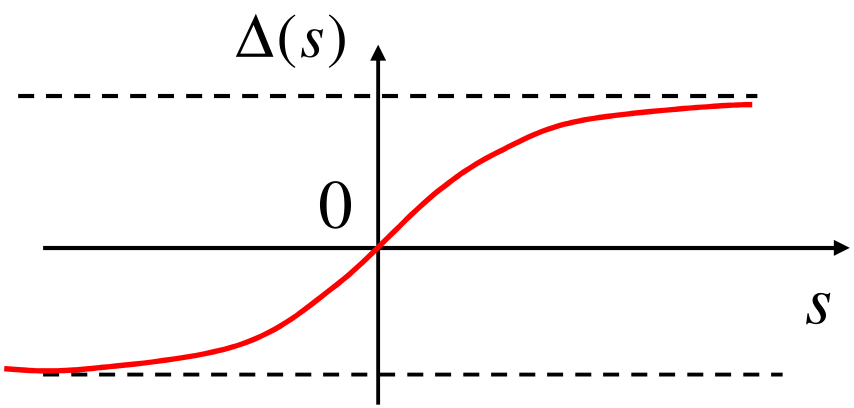
Zero energy modes: generic problem

Impact parameter=0



$$\hat{H} = -i\hbar V_\perp \hat{\sigma}_z \frac{\partial}{\partial s} + \hat{\sigma}_x \,\mathrm{Re}\Delta(s) - \hat{\sigma}_y \,\mathrm{Im}\Delta(s,b)$$

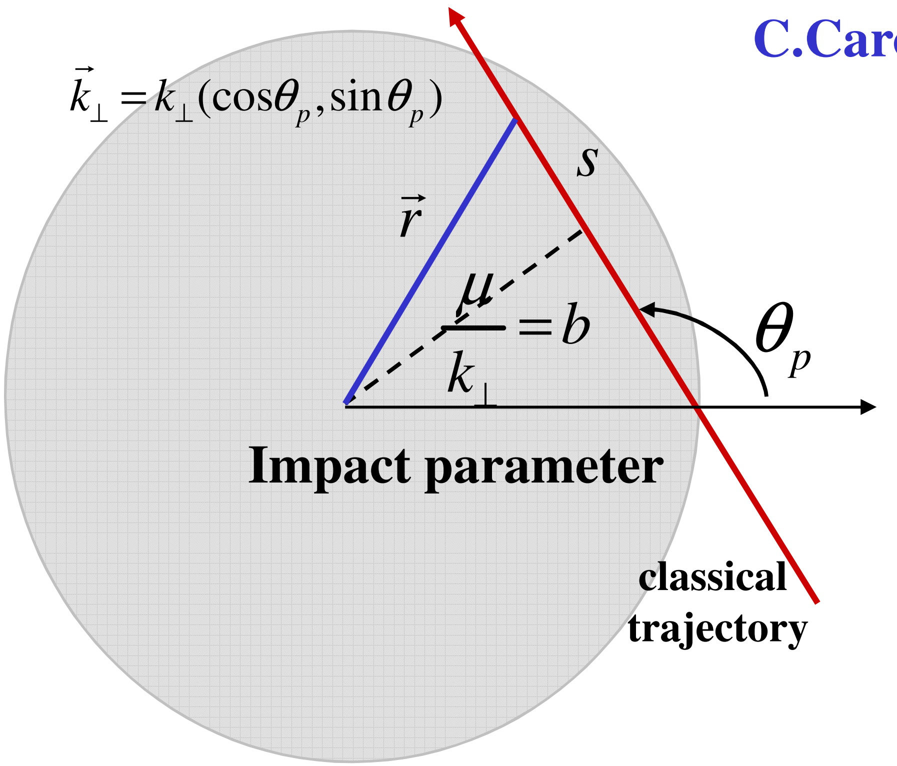
$$\hat{\Psi}_0 = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix} \exp\left(-\frac{1}{\hbar V_\perp} \int_0^s \mathrm{Re}\Delta(t)dt \right)$$



energy=0

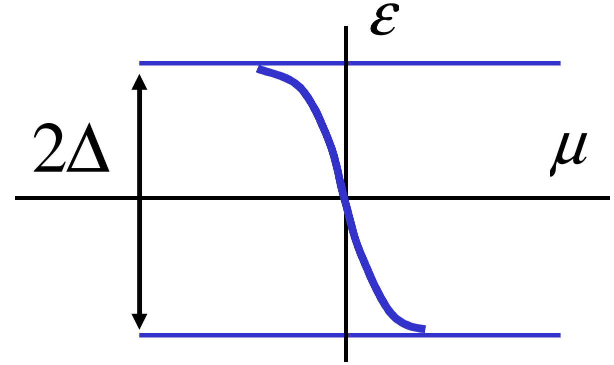
Bound quasiparticle states.

C.Caroli, P.G.de Gennes, J.Matricon (1964)



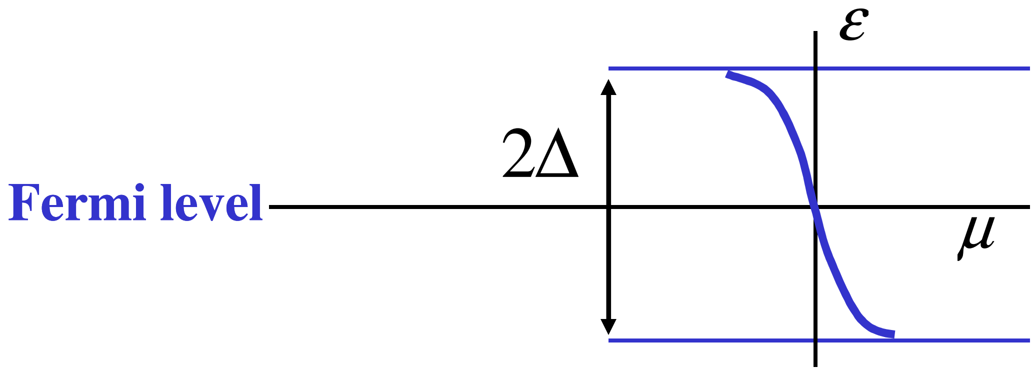
$$k_\perp = \sqrt{k_F^2 - k_z^2}$$

Anomalous spectral branch.



$$\varepsilon_\mu(k_\perp) = -\omega\mu \approx -\frac{\mu\Delta_0}{k_\perp \xi}$$

Anomalous spectral branch. Why is it important?



Strong dependence on the mean free path.
Difference between clean and dirty systems

Local DOS

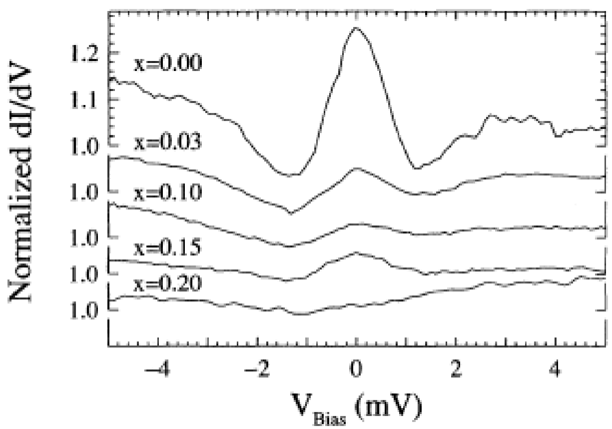
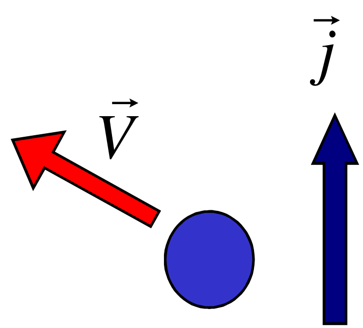


FIG. 3. Spectra taken at the center of a vortex core for various Ta substitutions at 1.3 K and 0.3 T. The spectra are normalized to the differential conductance at high bias.

Ch.Renner et al

Vortex dynamics



Thermal transport

Thermal conductivity along magnetic field:

$$\kappa(B) = n\kappa_v \propto \kappa_n \, B/H_{c2}$$

Experiment:

$$\kappa(B) << \kappa_n \, B/H_{c2}$$

Reference STM images of vortex cores.

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Scanning Tunneling Spectroscopy of a Vortex Core from the Clean to the Dirty Limit

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(Received 1 July 1991)

The local density of states of a superconducting vortex core has been measured as a function of disorder in the alloy system $Nb_{1-x}Ta_xSe_2$ using a low-temperature scanning tunneling microscope. The peak observed in the zero-bias conductance at a vortex center is found to be very sensitive to disorder. As the mean free path is decreased by substitutional alloying the peak gradually disappears and for $x=0.2$ the density of states in the vortex center is found to be equal to that in the normal state. The vortex-core spectra hence may provide a sensitive measure of the quasiparticle scattering time.

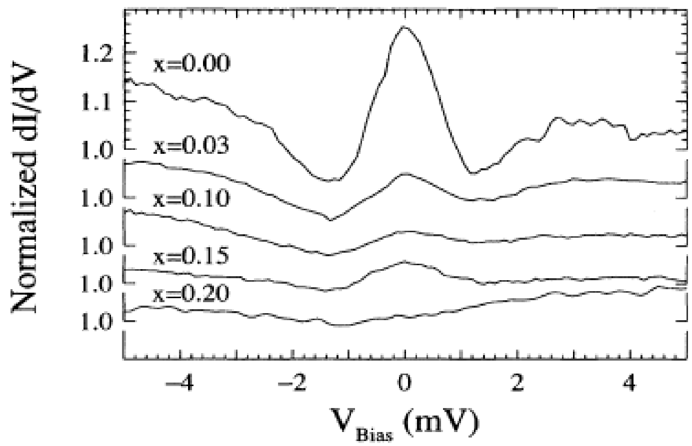


FIG. 3. Spectra taken at the center of a vortex core for various Ta substitutions at 1.3 K and 0.3 T. The spectra are normalized to the differential conductance at high bias.

Electronic structure of vortex cores beyond quasiclassical approach.
Are the nonquasiclassical effects important?

- **Quantization of spectrum. Minigap.**

- **Quasiparticle transport along the vortex lines.
Group velocity of the CdGM states.**

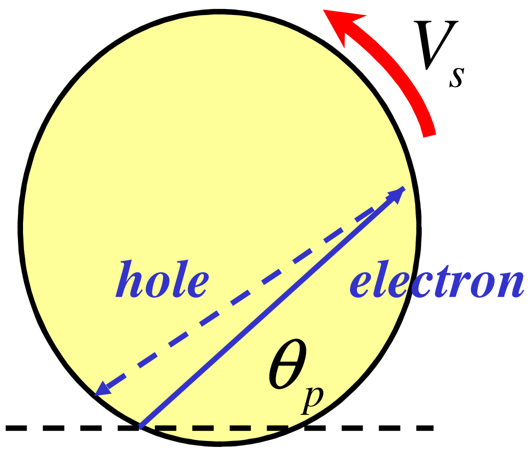
- **Normal scattering of quasiparticles inside the vortex cores.**

- **Tunneling of quasiparticles between vortices.**

- **Zero energy modes. Majorana states.**

Precession of classical trajectory

Deviation from exact backscattering during Andreev reflection in the core

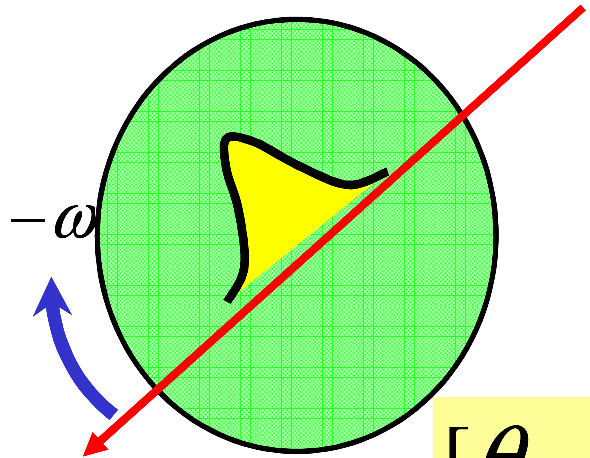


$$\delta\theta_p \sim \frac{V_s}{V_\perp} \qquad \delta t \sim \frac{\xi}{V_F}$$

$$\frac{\delta\theta_p}{\delta t} \sim \frac{V_s V_F}{\xi V_\perp} \sim \frac{V_s k_F}{\xi k_\perp} \sim \frac{\Delta_0}{k_\perp \xi \hbar} \sim \frac{\omega}{\hbar}$$

$$\hbar\frac{\partial\theta_p}{\partial t} = \frac{\partial\varepsilon}{\partial\mu} = -\omega$$

Precession frequency

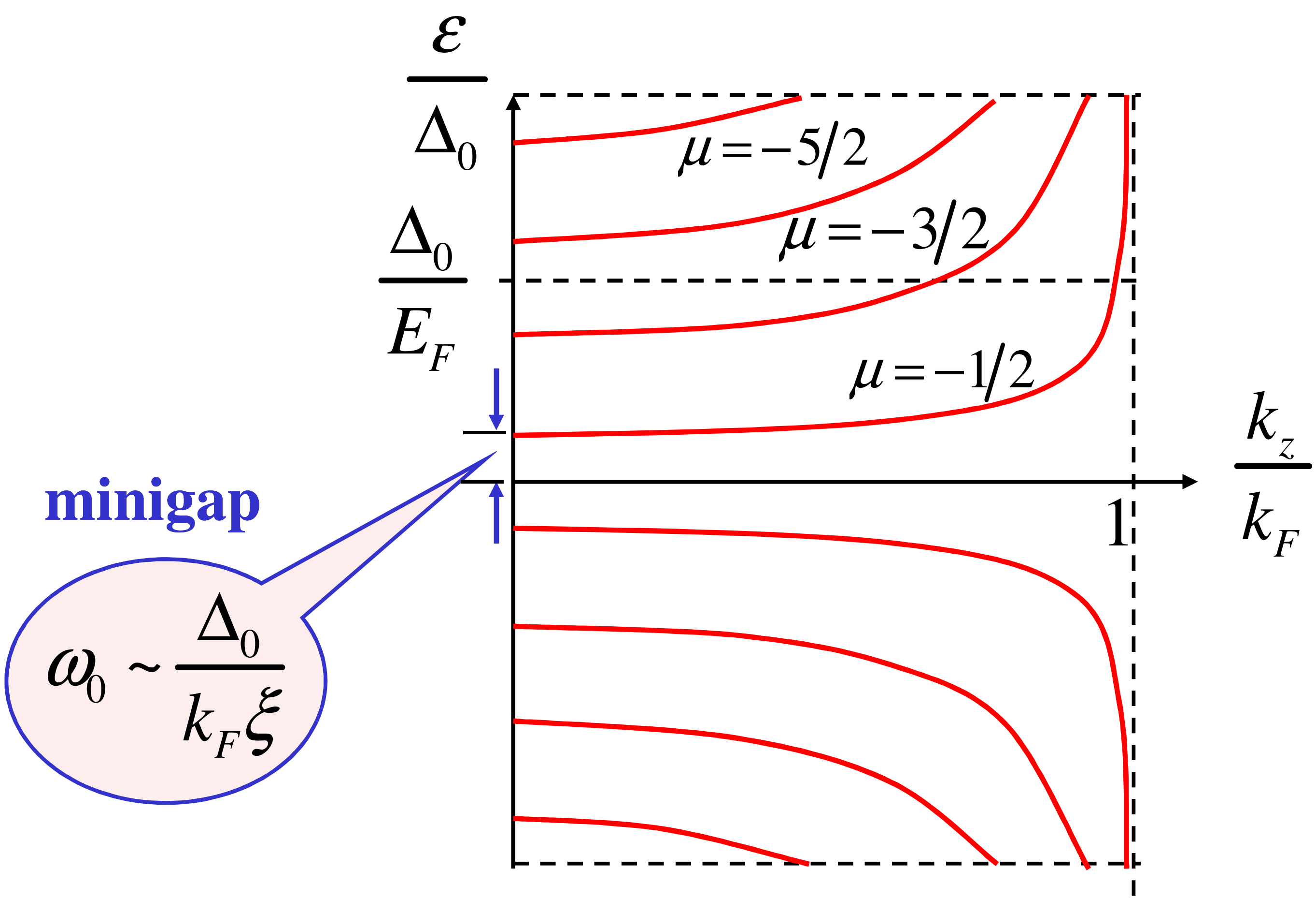


$$[\theta_p, \hat{\mu}] = i$$

Bohr-Sommerfeld quantization rule

$$\int_0^{2\pi n_\theta} \mu(\theta_p)\, d\theta_p = 2\pi(n+\beta)$$

Spectrum vs the momentum projection on the vortex axis



Thermal transport along vortex lines

Landauer approach

$$\kappa_v = \frac{\pi T}{3\hbar} N_{eff} = \kappa_0 N_{eff}$$

Number of transverse modes

Can we consider vortices as N wires?

$$N_{eff} \sim (k_F \xi)^2$$

$$\kappa(B) = n\kappa_v \propto \kappa_n \, B/H_{c2}$$

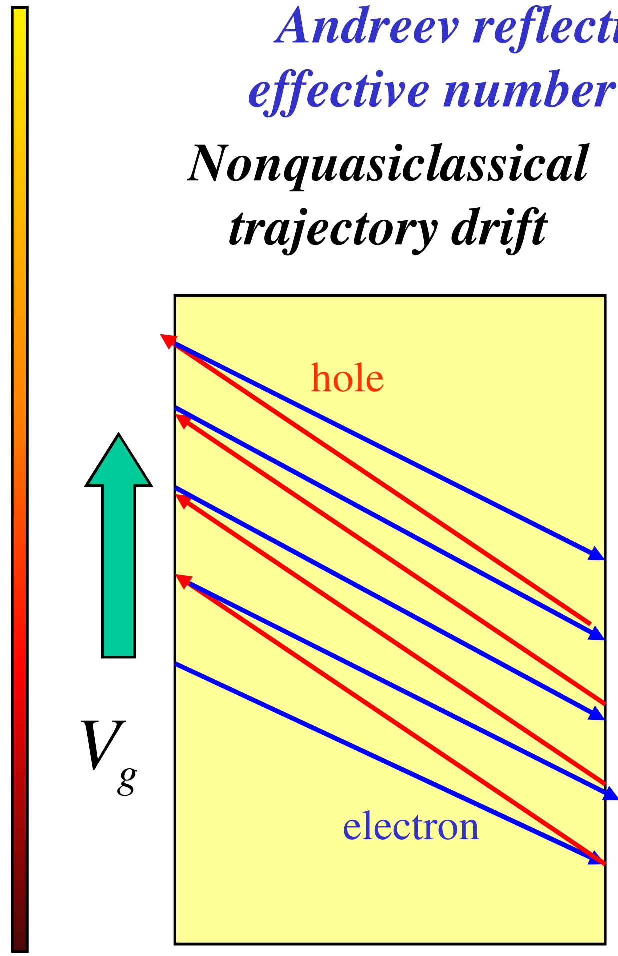
Experiment: J.Lowell and J.B.Sousa (1970); W. F. Vinen et al. (1971)

$$\kappa(B) << \kappa_n \, B/H_{c2}$$

?

Andreev reflection suppresses the effective number of transport modes

Nonquasiclassical trajectory drift



Group velocity along vortex axis

$$V_g = \frac{\partial \varepsilon}{\hbar \partial k_z} \sim V_F \frac{\varepsilon}{E_F}$$

$$\kappa \sim \frac{T}{\hbar}(k_F \xi)^2 \frac{V_g}{V_F}$$

$$\kappa \propto \frac{T^2}{\hbar \Delta} k_F \xi$$

Heat transport along vortices. Landauer approach.

$$\int \left[u^*_{\mu,k_z} \left(\hbar k_z - \frac{e}{c} A_z \right) u_{\mu,k_z} - v^*_{\mu,k_z} \left(\hbar k_z + \frac{e}{c} A_z \right) v_{\mu,k_z} \right] d^2 r =$$

$$= \frac{m}{\hbar} \frac{\partial \epsilon_{\mu,k_z}}{\partial k_z}$$

$$I_{\mathcal{E}} = \frac{1}{m} \int d^2 r \sum_\mu \int \frac{dk_z}{2\pi} \left[\epsilon_\mu \, u^*_\mu \left(\hbar k_z - \frac{e}{c} A_z \right) u_\mu n(\epsilon_\mu) \right.$$
$$\left. - \epsilon_\mu \, v^*_\mu \left(\hbar k_z + \frac{e}{c} A_z \right) v_\mu \left[1 - n(-\epsilon_\mu) \right] \right] =$$

$$= \sum_\mu \int_{p_z > 0} \epsilon_\mu \left[n_1(\epsilon_\mu) - n_2(\epsilon_\mu) \right] \left| \frac{\partial \epsilon_\mu}{\partial k_z} \right| \frac{dp_z}{2\pi \hbar}$$

$$\kappa = -\frac{1}{\pi \hbar T} \sum_\mu \int_0^{k_F} \varepsilon_\mu^2 \frac{dn(\varepsilon_\mu)}{d\varepsilon_\mu} \left| \frac{\partial \varepsilon_\mu}{\partial k_z} \right| dk_z$$

Impurity scattering. Standard diffusion.

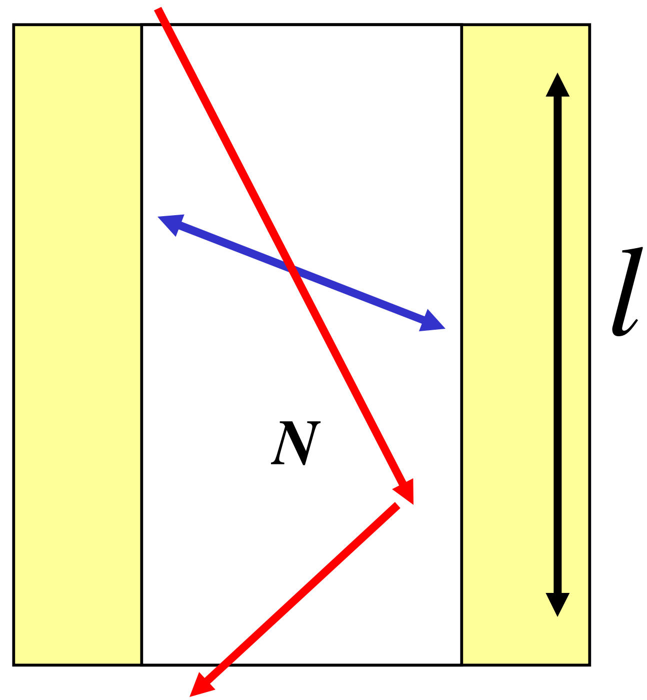
$$l < a \qquad \kappa \sim \frac{T}{\hbar}(k_F a)^2 \frac{l}{d}$$

Andreev diffusion (A.F.Andreev, 1964). Noneffective trajectories.

$$l > a$$

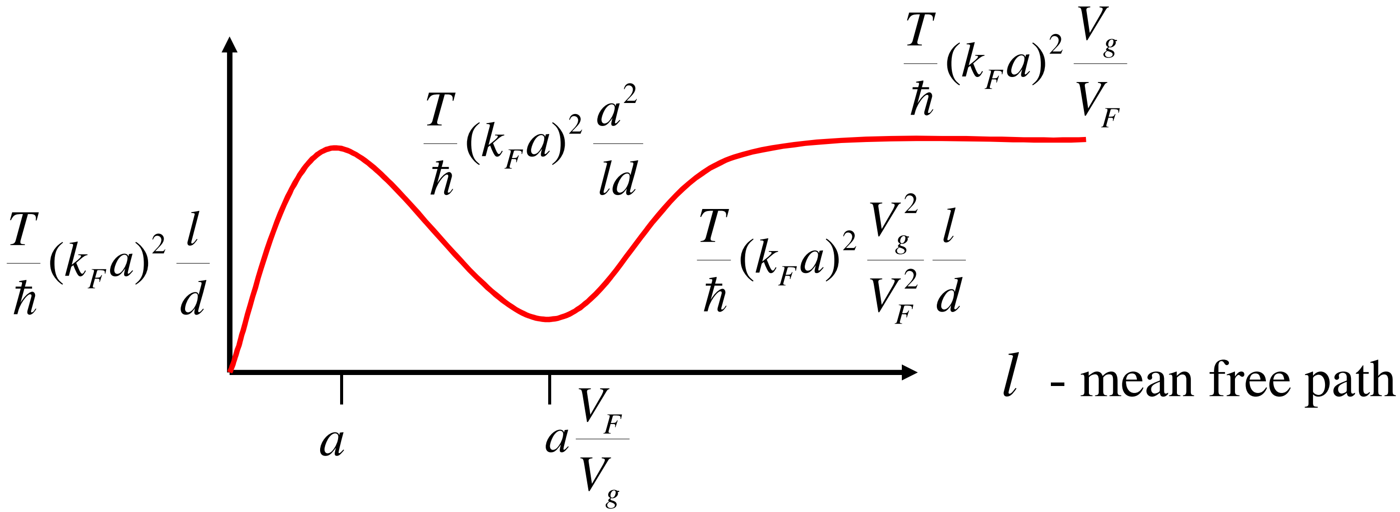
Part of effective trajectories

$$\kappa \sim \frac{T}{\hbar}(k_F a)^2 \frac{l}{d} \times \frac{a^2}{l^2} \sim \frac{T}{\hbar}(k_F a)^2 \frac{a^2}{ld}$$



Diffusive limit.
Qualitative picture for $d >> a\frac{V_F}{V_g}$

Thermal conductance



$$\kappa = \frac{T}{\hbar} N_{eff}$$

$$N_{eff} < 1$$

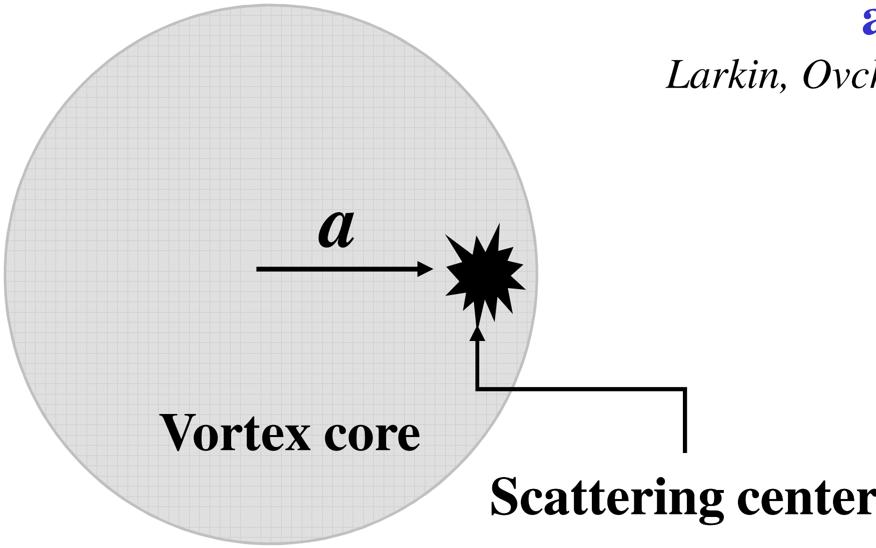
Thouless (1977)

$$\min N_{eff} \sim (k_F a)^2 \frac{a}{d}\frac{V_g}{V_F}$$

Re-entrant localization?

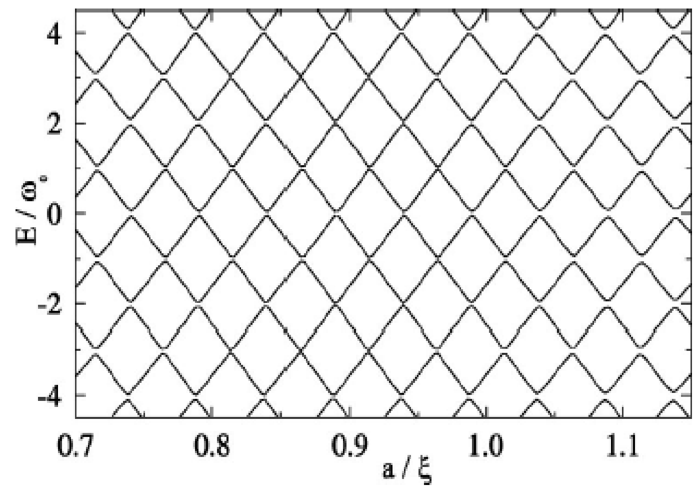
$$d > d_{loc} \sim a(k_F a)^2 \frac{V_g}{V_F}$$

Influence of scattering at boundaries and defects on the spectrum of localized core states?



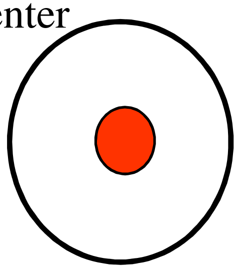
Quasiparticle spectrum in a vortex with an impurity atom in the core

Larkin, Ovchinnikov 1998; Koulakov, Larkin 1999 …..

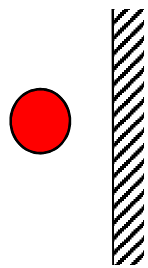


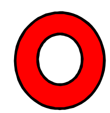
Examples illustrating the transformation of anomalous branches caused by the normal reflection at the boundaries:

Vortex is close to the cylinder center



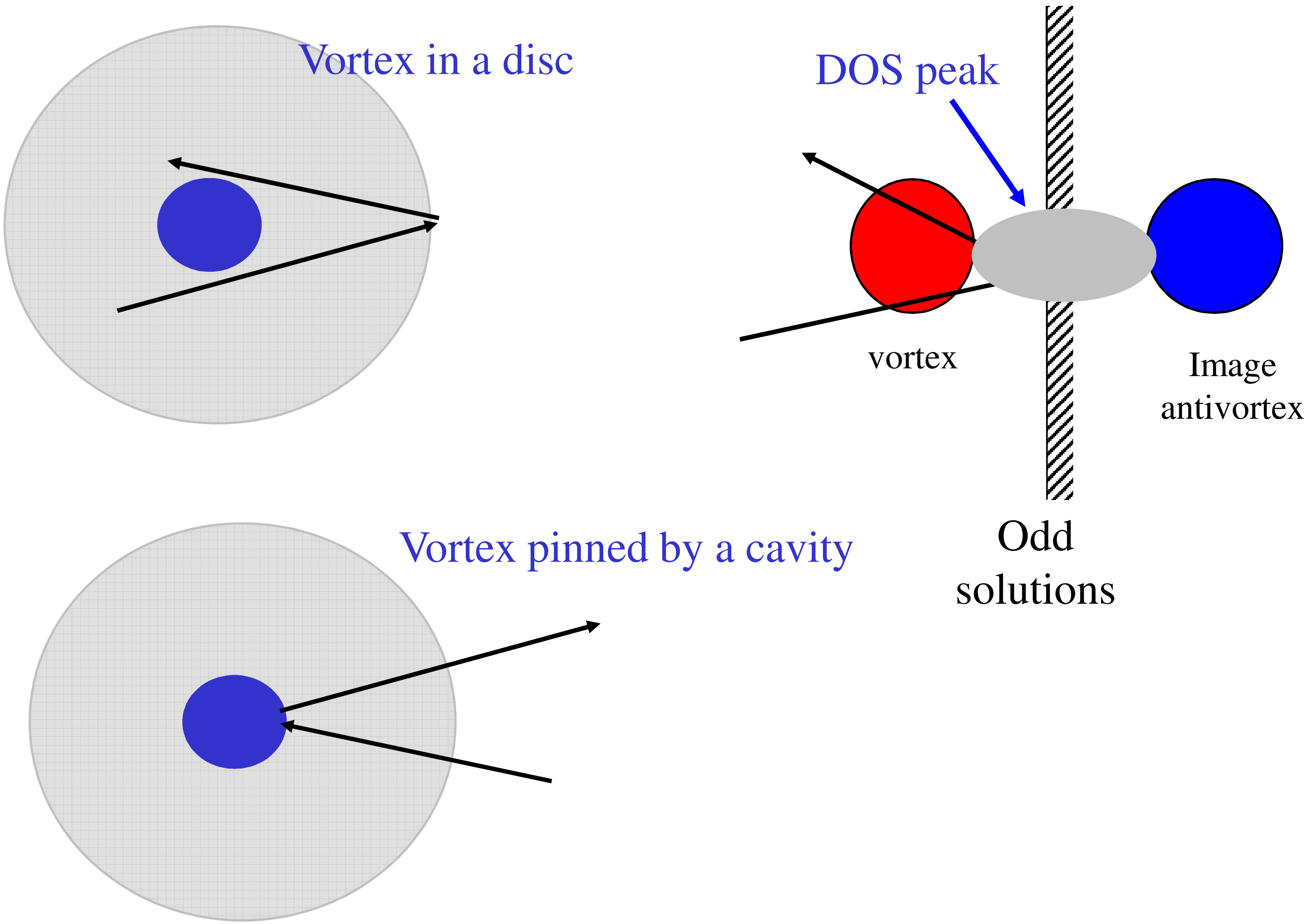
Vortex near the plane surface





Vortex pinned by a columnar defect

Examples illustrating the transformation of anomalous branches caused by the normal reflection at the boundaries:



Vortex in a cylinder: splitting of anomalous spectral branch

$$\varepsilon = -\omega(k_\perp)\mu + \frac{\Delta_0 e^{-D(R)}\cos(2k_\perp R - \pi\mu + \pi/2)}{\Lambda}$$

Spectrum oscillations

CdGM spectrum

$$\omega(k_\perp) = \frac{2\Delta_0}{\hbar^2 k_\perp V_\perp \Lambda}\int_0^\infty \frac{\Delta(r)e^{-D(r)}}{r}dr$$

$$\Lambda = \frac{2\Delta_0}{\hbar V_\perp}\int_0^\infty e^{-D(r)}dr$$

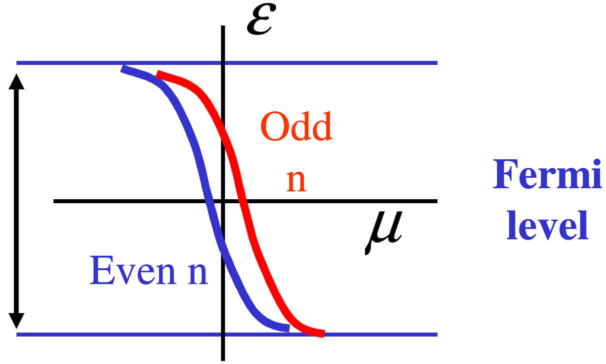
$$D(r) = \frac{2}{\hbar V_\perp}\int_0^r \Delta(s)ds$$

$$\omega(k_\perp) \approx \frac{k_F}{k_\perp}\omega_0 \qquad \omega_0 \sim \frac{\Delta_0^2}{E_F}$$

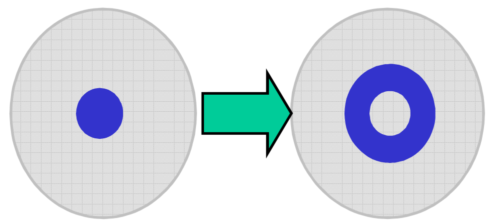
$$D(R) \approx \frac{2Rk_F}{\xi_0 k_\perp} \qquad \Lambda \sim 1$$

$$\mu = n + \frac{1}{2}$$

2Δ

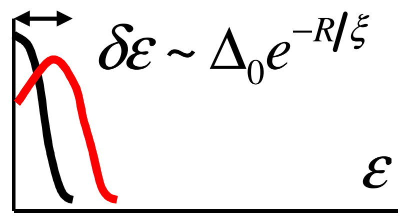


DOS profiles



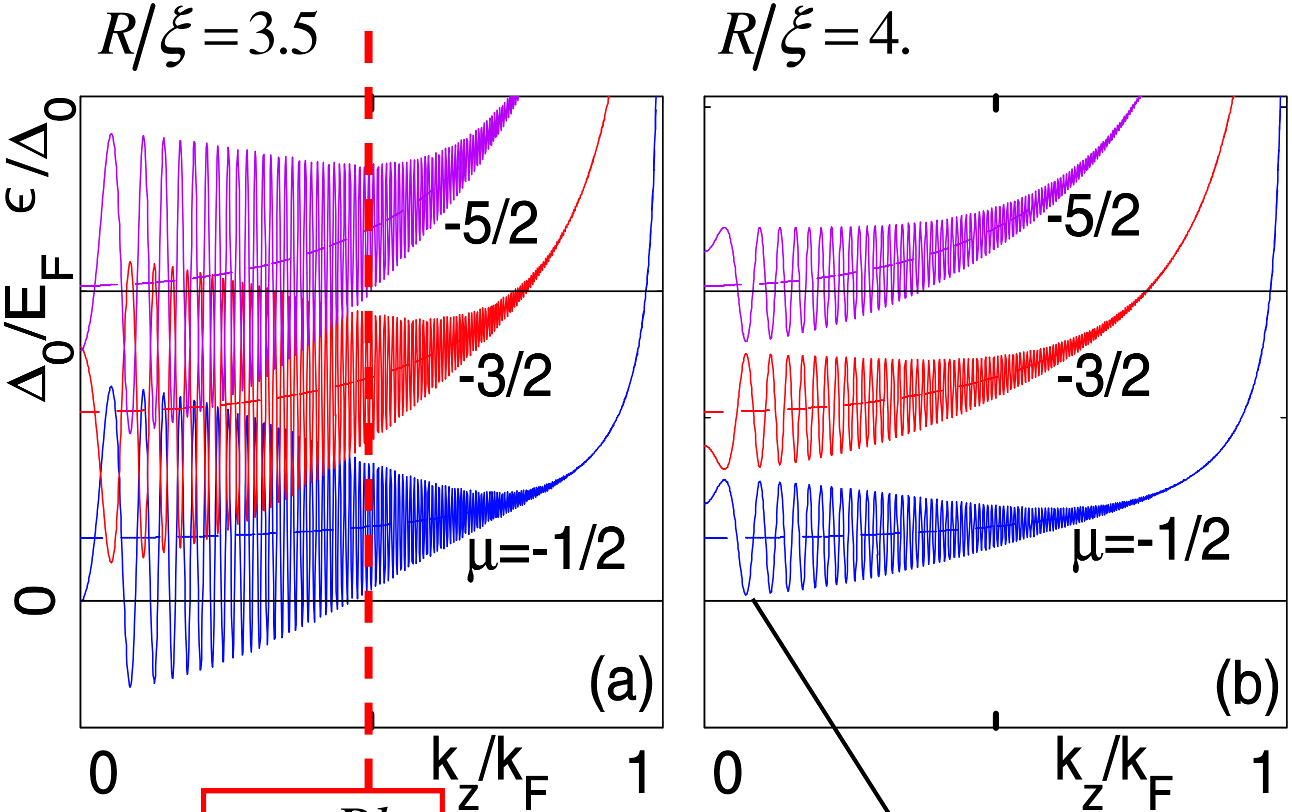
$$\delta \sim \xi e^{-R/\xi}$$

DOS in the vortex center



$$\delta\varepsilon \sim \Delta_0 e^{-R/\xi}$$

Vortex in a cylinder: spectrum



$$|\Delta(r)| = \frac{\Delta_0 r}{\sqrt{r^2 + \xi_v^2}}$$

$$\xi_v = \xi_0$$

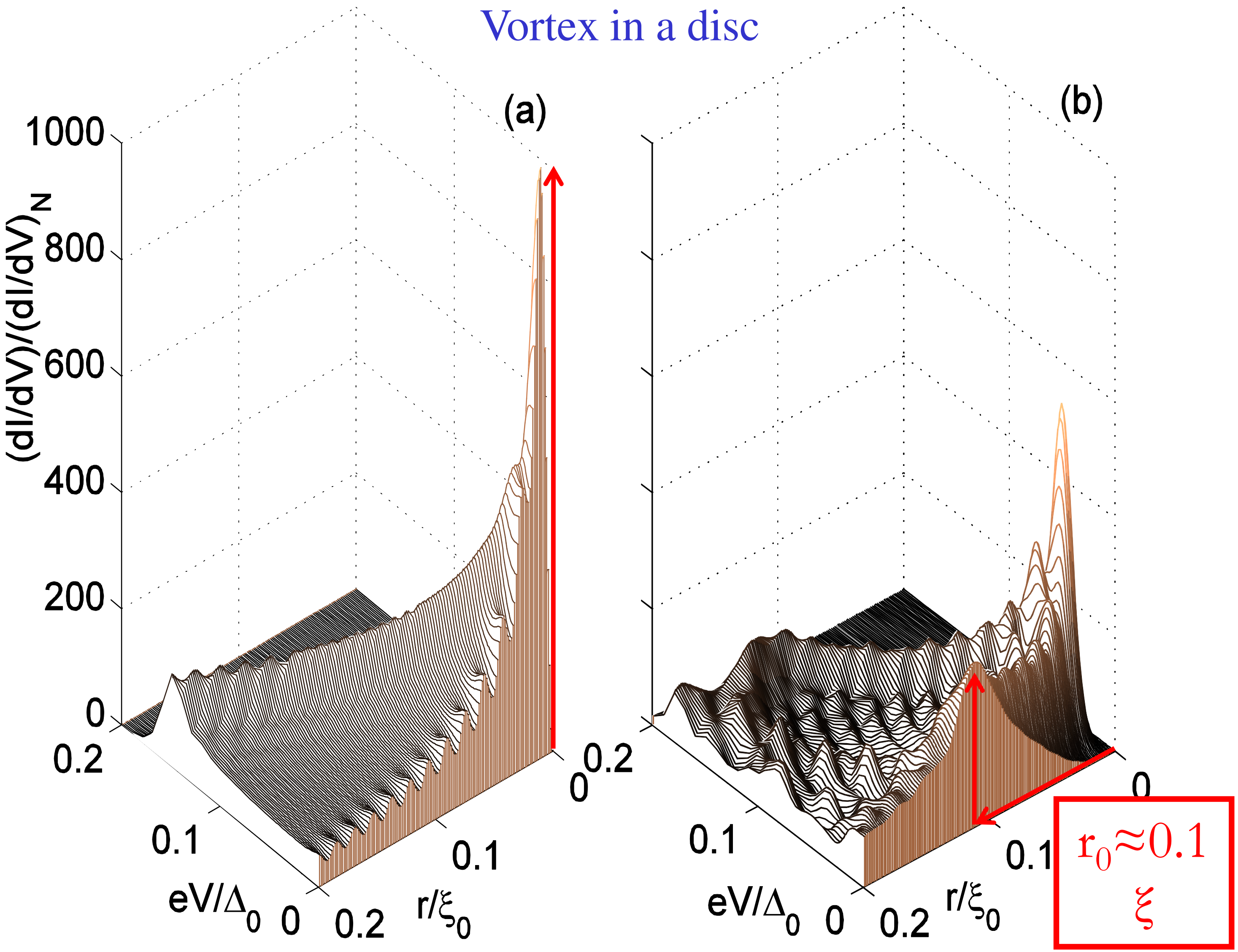
$$\Delta_0 / E_F = 0.01$$

$$k_\perp^* \approx \frac{R k_F}{R_c}$$

Critical radius $R_c \approx \frac{\xi_0}{2} \ln \frac{\Delta_0}{\omega_0}$

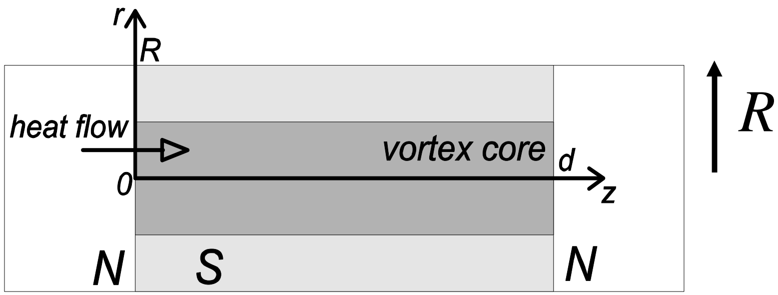
- **Strong increase in the number of transverse modes.**
- **Suppression of the minigap.**

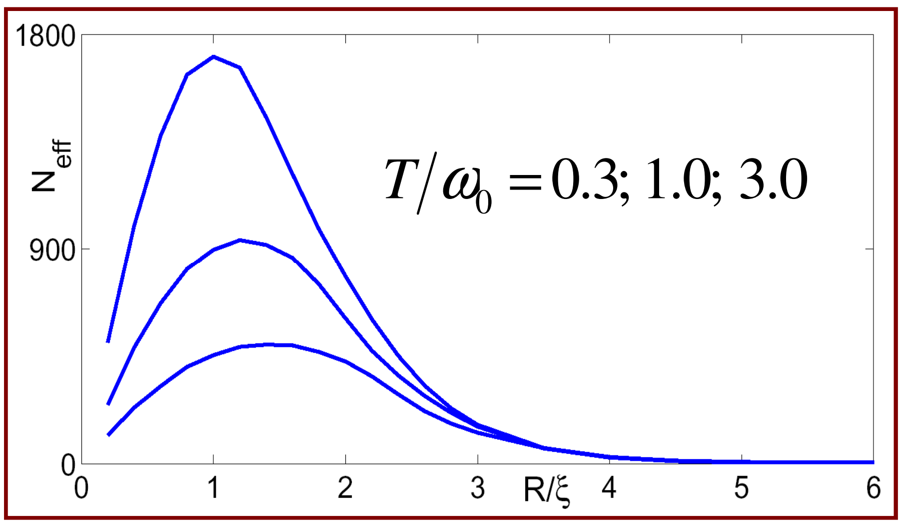
Differential tunneling conductance

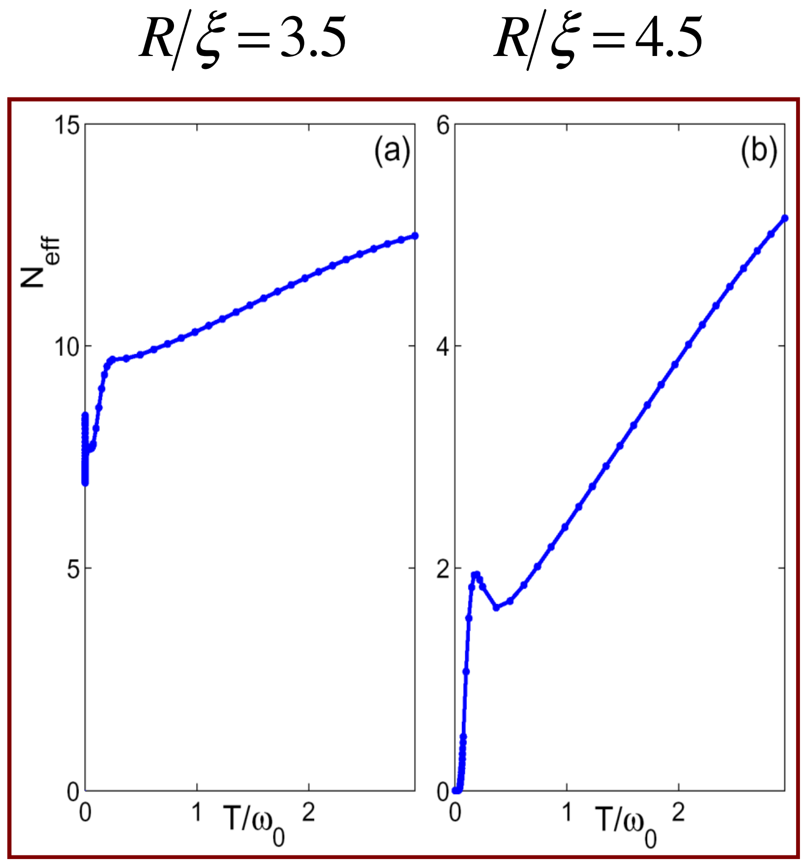


Heat transport along single vortex in a cylinder

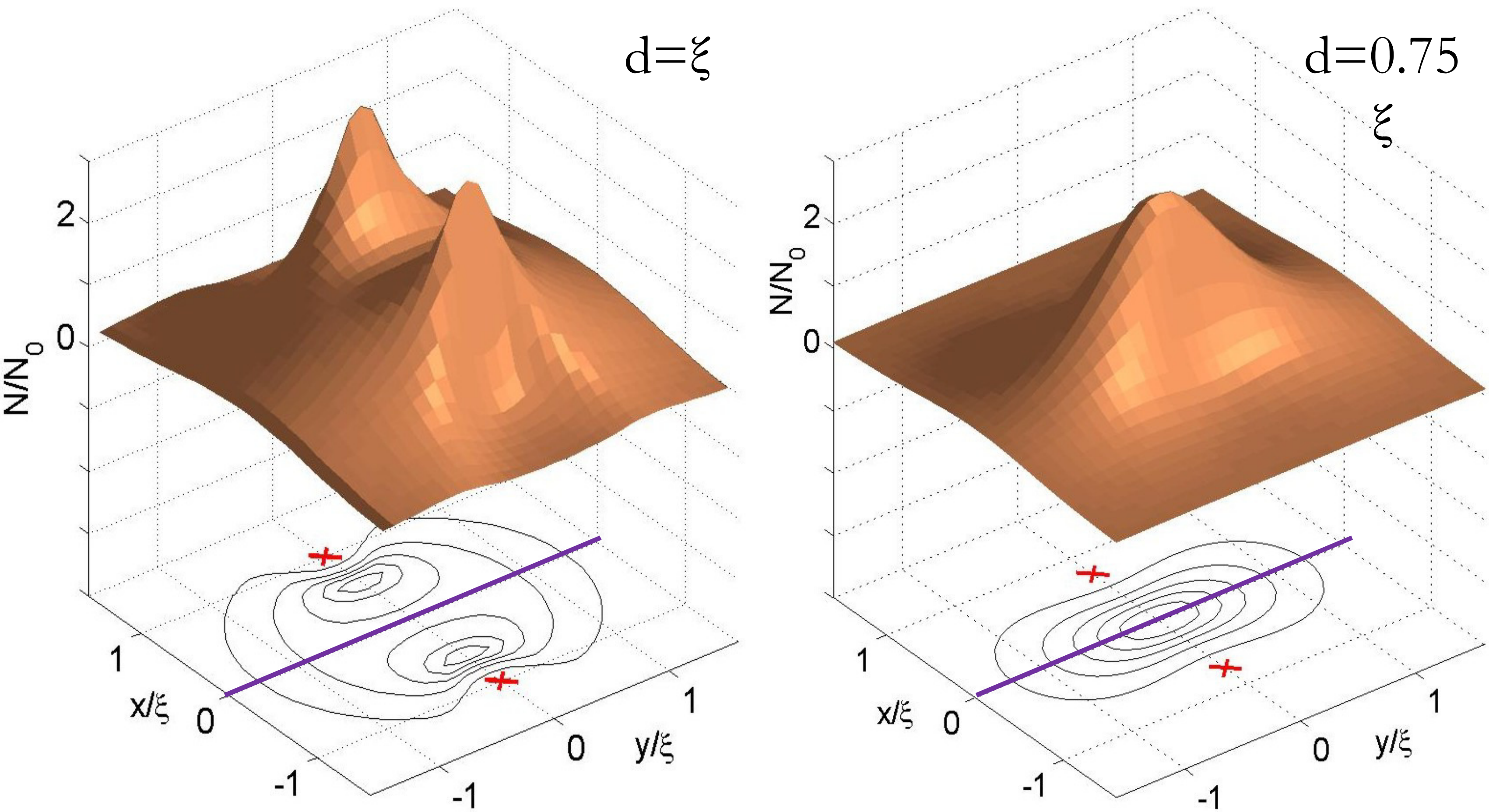
$$\kappa = -\frac{1}{\pi\hbar T}\sum_{\mu}\int_0^{k_F}\varepsilon_\mu^2 \frac{dn(\varepsilon_\mu)}{d\varepsilon_\mu}\left|\frac{\partial\varepsilon_\mu}{\partial k_z}\right| dk_z$$







Local DOS for a vortex positioned near the surface



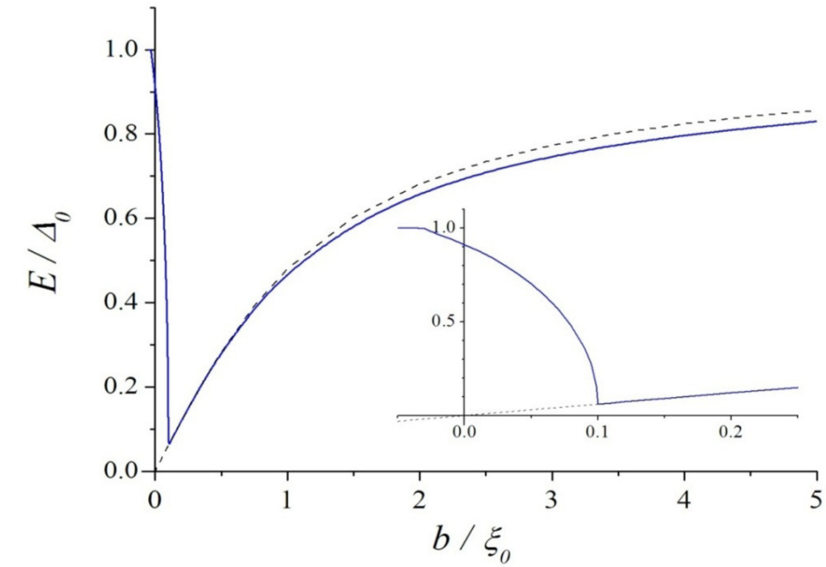
LDOS peak is shifted towards the boundary

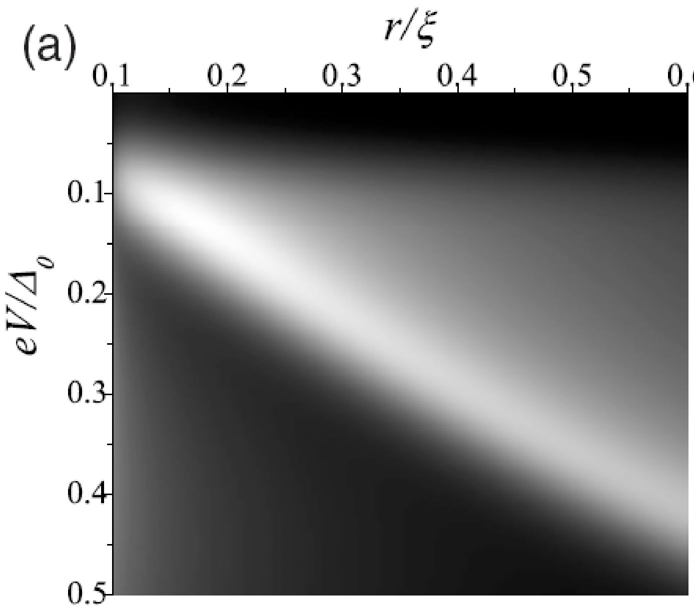
Several boundaries: splitting of LDOS peak becomes possible

Vortex (M=1) pinned at the defect. Spectrum and DOS.

$$|\Delta(r)| = \Delta_0 \left(\frac{r}{\sqrt{r^2 + \xi_v^2}} \right)^m$$

$$\xi_v = \xi_0$$





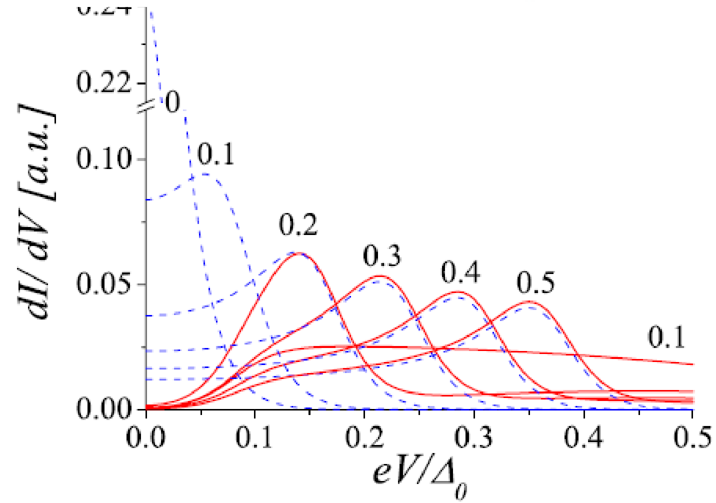
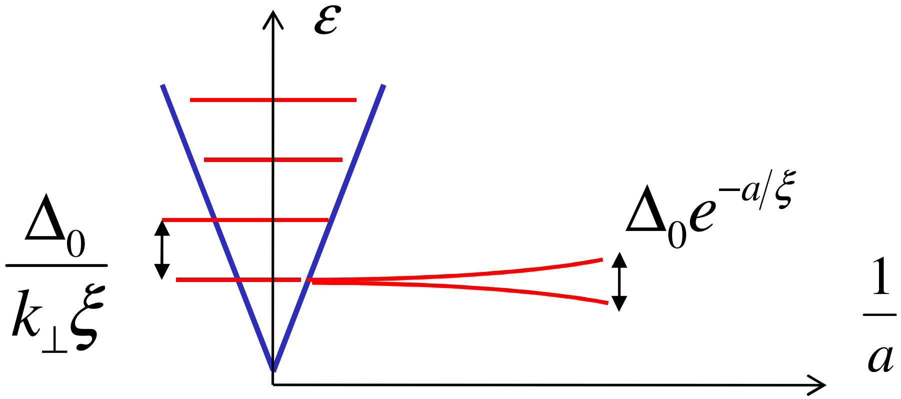


FIG. 5. (Color online) (a) Distribution of the local differential conductance dI/dV as a function of voltage (eV) and distance from the cylinder axis (r). (b) Local differential conductance dI/dV versus bias voltage (eV) at different distances r from the cylinder axis are shown by red solid lines. For reference blue dash lines show local dI/dV at different distances r from the Abrikosov vortex axis when a columnar defect is absent. The numbers near the curves denote the corresponding values of distance r in the units of the coherence length ξ. We put here $R/\xi=0.1$ and $T/\Delta_0=0.02$.

Intervortex tunneling.
Critical intervortex distance:
minigap = energy level splitting due to tunneling



$$a_c \approx \frac{\xi}{2} \ln(k_F \xi)$$

Typical intervortex distance $a \sim \sqrt{\frac{\phi_0}{H}}$

$$k_F \xi \sim 10^2 \div 10^3$$

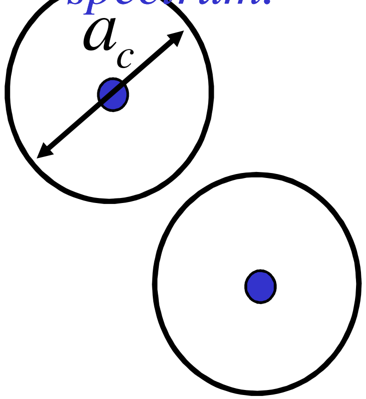
$$\frac{a_c}{\xi} \approx 2 \div 3$$

$$H^* \sim \frac{\phi_0}{a_c^2} \sim \frac{H_{c2}}{[\ln(k_F \xi)]^2}$$

$$a_c \approx \frac{\xi}{2}\ln(k_F\xi)$$

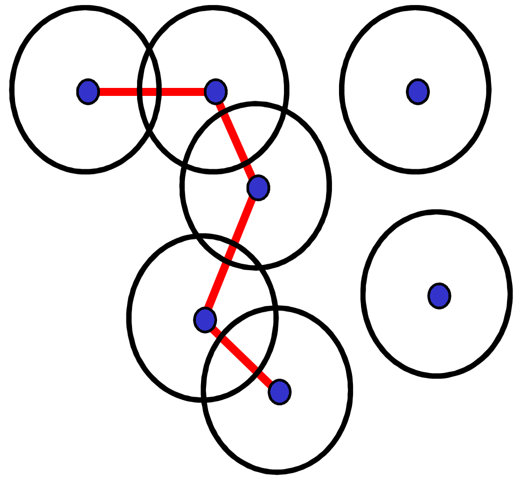
$$a_{ij} > a_c$$

Intervortex tunneling is negligible. Degenerate CdGM spectrum.



$$a_{ij} < a_c$$

Vortices are strongly coupled by tunneling.



Vortex cluster in a disordered flux line array: Spectrum is similar to the one in m-quanta vortex

Cluster size~ cyclotron orbit radius. Can we restore Landau quantization?

Heat transport along two parallel vortex lines

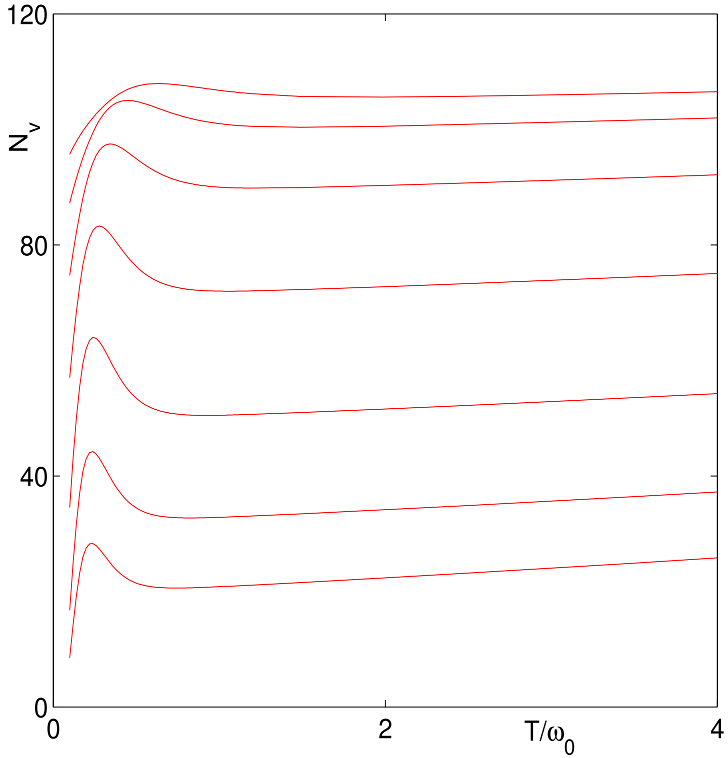


FIG. 11: Temperature dependence of the number of conducting modes N_v for a two–vortex system. Curves are plotted for $a = 2\xi$ to $a = 5\xi$ with the step 0.5ξ (from top to bottom). The vortex core profile for a single vortex is approximated by Eq. (37) with $\xi_v = \xi$, $k_F\xi = 200$.

$$\max N_0 \sim k_F \xi$$

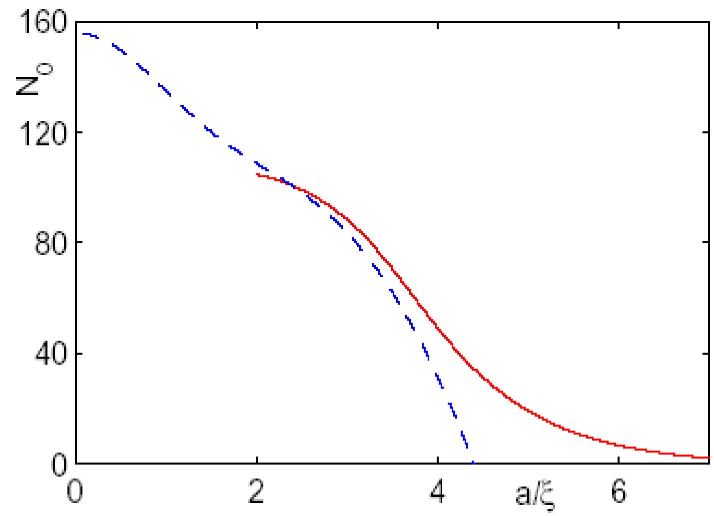


FIG. 12: Residual number of modes as a function of the intervortex distance. Solid line shows the result of the exact calculation based on Eq. (88), while dash line is obtained from the analytical approximate expressions (93) and (44).

Fermionic zero modes. Majorana states.

$$(H-\mu)u+\int \Delta(r,r')v(r')d^3r' = \varepsilon u$$

$$\int \Delta^{+}(r',r)u(r')d^3r' + (\mu - H^{*})v = \varepsilon v$$

Singlet pairing

$$\Delta(r,r') = i\sigma_y D(r,r')$$

$$D(r,r') = D(r',r)$$

$$\varepsilon \to -\varepsilon$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \to \begin{pmatrix} -v^{*} \\ u^{*} \end{pmatrix}$$

Triplet pairing

$$\Delta(r,r') = i\sigma_y \vec{\sigma}\vec{D}(r,r')$$

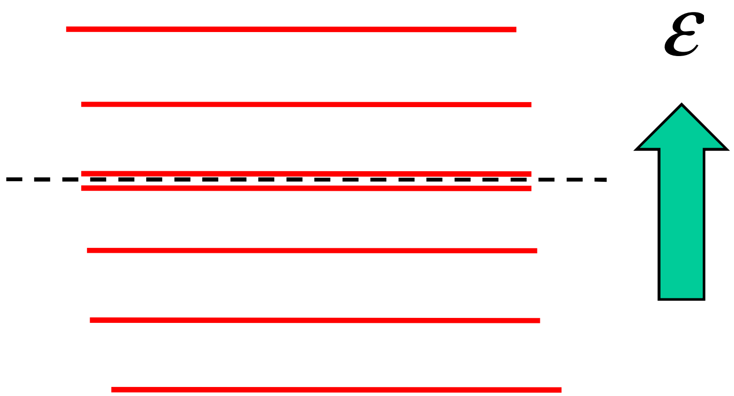
$$\vec{D}(r,r') = -\vec{D}(r',r)$$

$$\varepsilon \to -\varepsilon$$

$$\begin{pmatrix} u \\ v \end{pmatrix} \to \begin{pmatrix} v^{*} \\ u^{*} \end{pmatrix}$$

Fermionic zero modes. Majorana states.

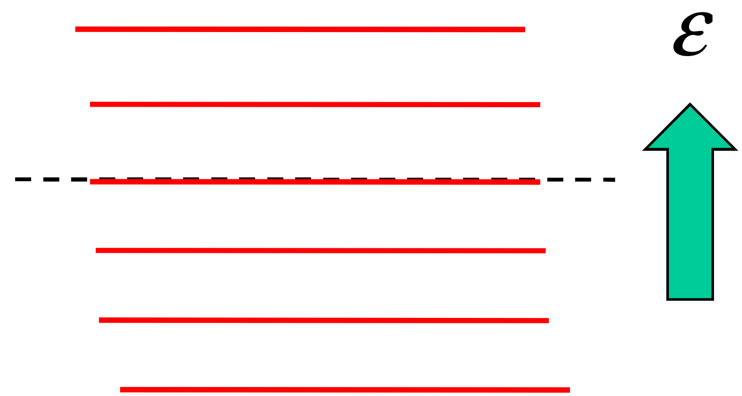
Singlet pairing



$$\gamma^{+} \neq \gamma$$

Standard fermions
(with usual
commutation rules)

Triplet pairing



$$\gamma^{+} = \gamma$$

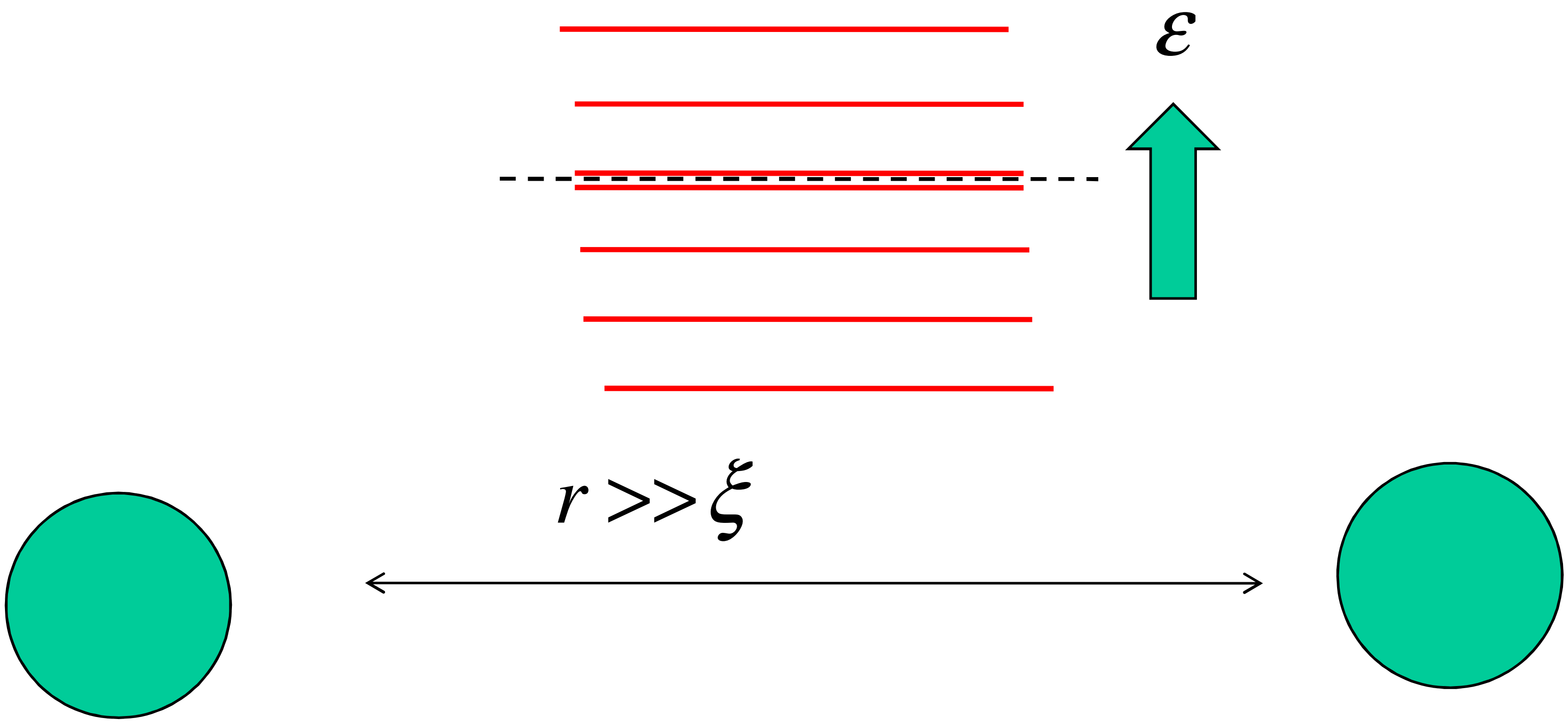
Majorana fermions
(not fermions at all)

Obvious contradiction:
We can not change statistics using
canonical Bogolubov tranformation

$$\gamma^{+}\gamma + \gamma\gamma^{+} = 1$$

$$\gamma\gamma + \gamma\gamma = 0$$

A standard way to overcome the problem:
We introduce 2 Majorana fermions
Far away from each other



Examples: vortices in p-wave superconductors (G.E.Volovik, 1997)
Edge states (Kitaev 1D p-wave superconductor)
Systems with induced superconductivity

Self – consistency equation for the gap function

$$\Delta(r,r') \propto \sum_n u_n(r) v_n^*(r') \tanh\frac{\varepsilon_n}{2T}$$

$$\delta\Delta_M(r,r') \propto u_M(r) v_M^*(r') \tanh\frac{\varepsilon_M}{2T} \propto u_M(r) u_M(r') \tanh\frac{\varepsilon_M}{2T}$$

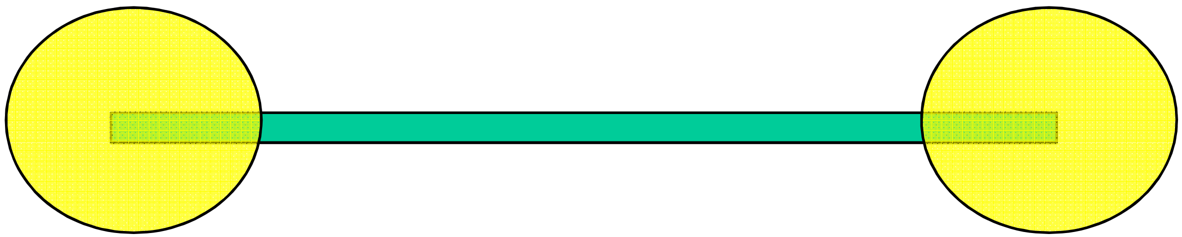
Hybridized Majorana states give an even contribution to the odd gap function

Superconductor with Majorana states can not be stable

Nonzero divergence of supercurrent!

$$div\vec{j}_s \neq 0$$

$$div\vec{j}_s \neq 0$$



Example of instability scenario:
Vortex attraction in p-wave superconductors

$$\vec{j}(r) = \frac{e}{m}\sum_n \left(u_n^*(r)\left(-i\nabla - \frac{e}{c}\vec{A}\right)u_n(r) - u_n(r)\left(-i\nabla + \frac{e}{c}\vec{A}\right)u_n^*(r)\right) f\left(\frac{\varepsilon_n}{T}\right)$$

$$\varepsilon_n = \varepsilon_n^0 + \vec{p}_F \vec{V}_s$$

$$\vec{j}(r\sim\xi) \propto -\frac{2ne^2}{m}\vec{A} + \frac{2ne^2}{m}\frac{\omega_0}{T}\vec{A}$$

$$T \to 0$$

Paramagnetic Meissner efffect

$$\vec{j}(r\sim\xi) \propto -\frac{2ne^2}{m}\vec{A} + \frac{2ne^2}{m}\frac{\omega_0}{\delta\varepsilon}\vec{A}$$

$$\delta\varepsilon \sim \Delta e^{-r/\xi} < \omega_0 \qquad r > \xi \ln(k_F \xi)$$

$$\vec{F} \propto \left[\vec{j}, \vec{n}\right]$$

Paramagnetic Meissner effect and the FFLO instability

$$\vec{j} = -\frac{1}{4\pi}\lambda^{-2}\vec{A} = -\frac{\delta F_A}{\delta \vec{A}} \quad \Longrightarrow \quad F_A = \frac{1}{8\pi}\int \lambda^{-2}\vec{A}^2 dV$$

$$\lambda^{-2} = \frac{4\pi e^2 n_s}{m}$$

$$F_A = \int \Lambda \left(\vec{A} - \frac{\Phi_0}{2\pi}\nabla\varphi \right)^2 dV$$

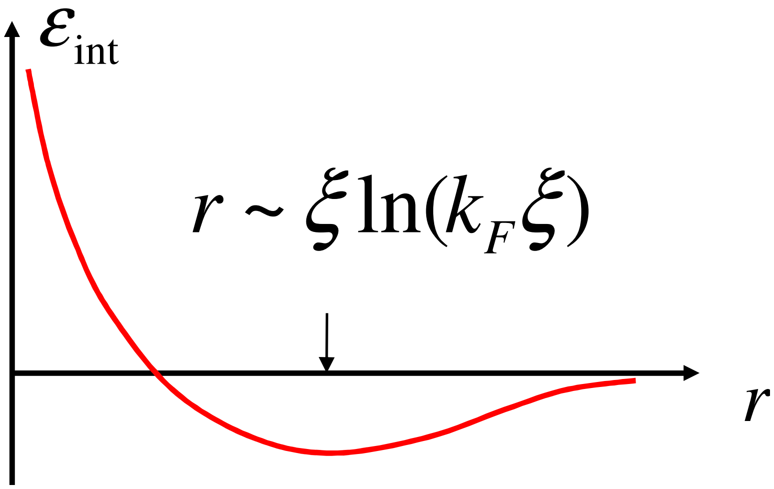
$$\Big\Downarrow \quad \vec{A} = 0$$

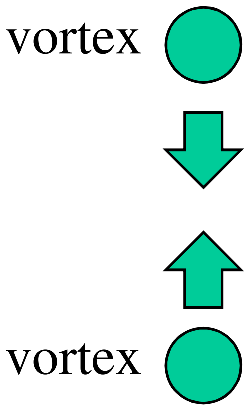
$$F_A = \int \Lambda \left(\frac{\Phi_0}{2\pi}\nabla\varphi \right)^2 dV$$

$$\Lambda < 0?$$

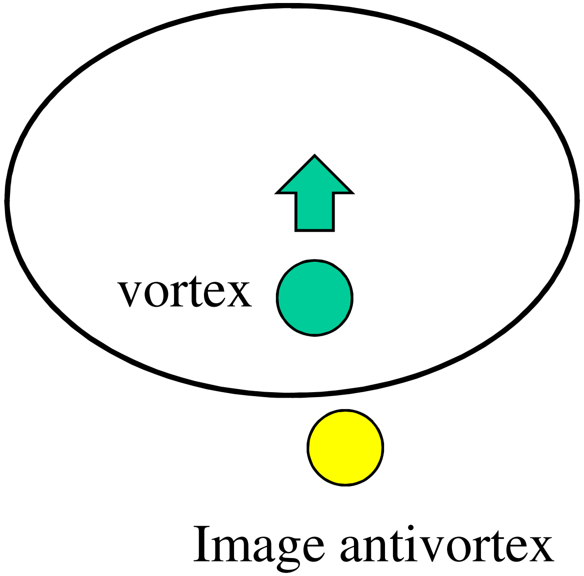
The uniform ground state can be unstable!

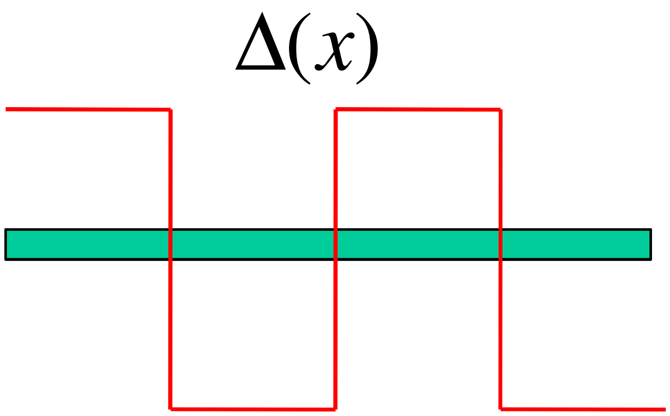
Vortex attraction in p-wave superconductors

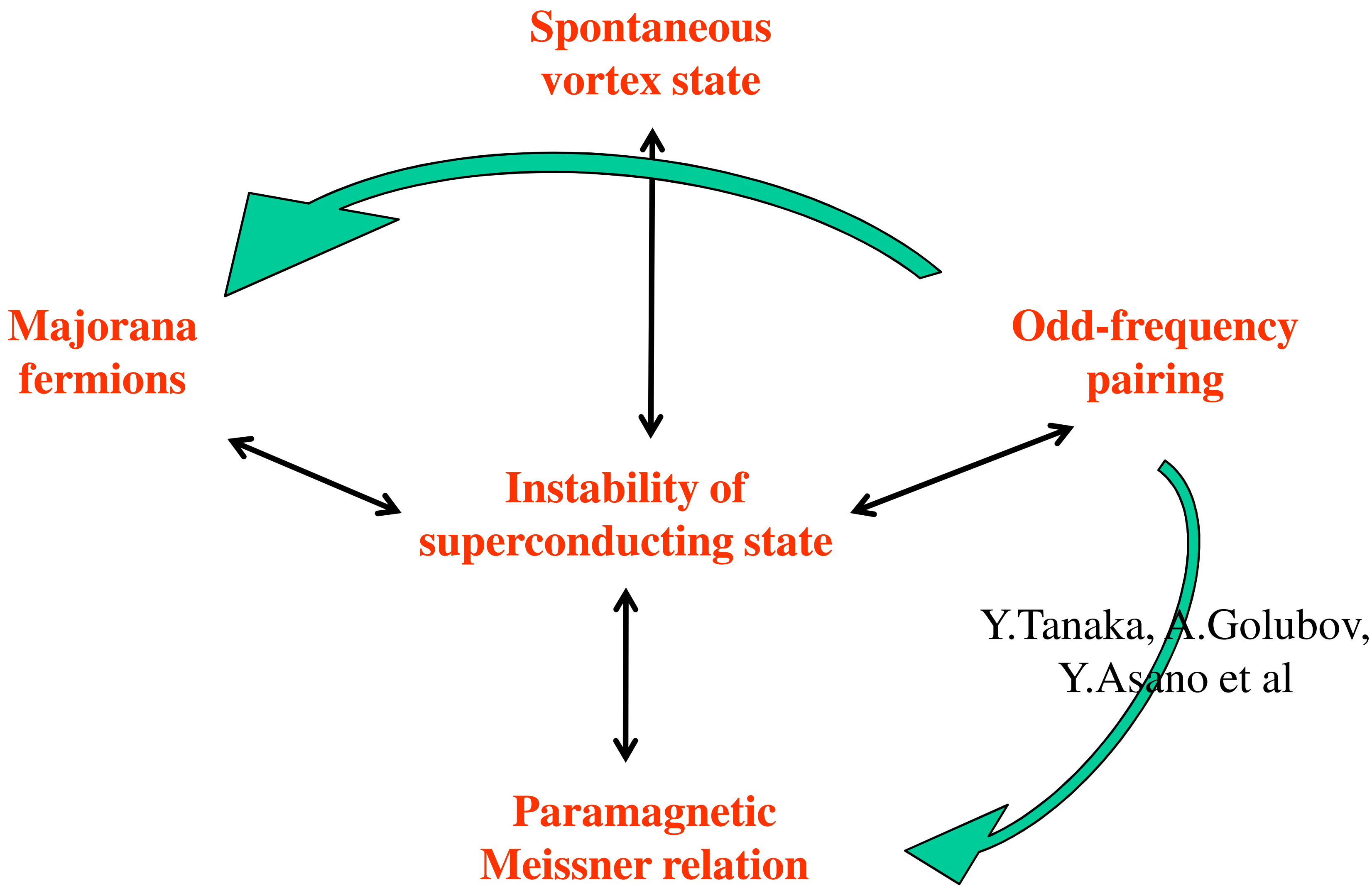




Sspontaneous vortex state






Spontaneous
vortex state
Majorana
fermions
Odd-frequency
pairing
Instability of
superconducting state
Y.Tanaka, A.Golubov,
Y.Asano et al
Paramagnetic
Meissner relation

CONCLUSION

- **Nonquasiclassical effects are small but can be important for**

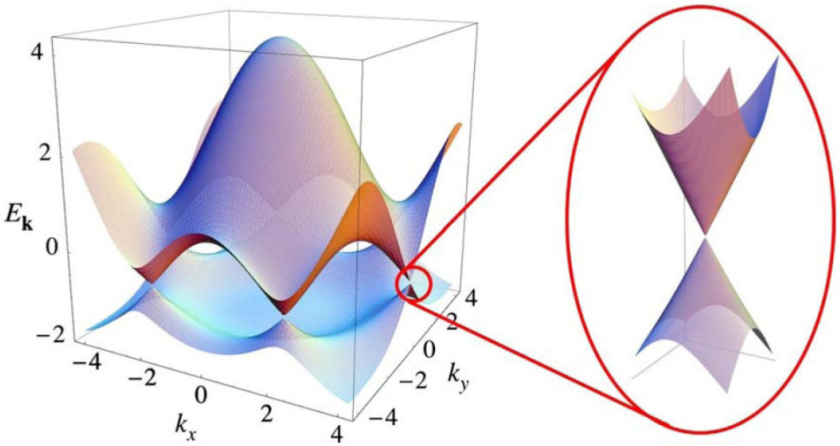
thermal transport

level quantization

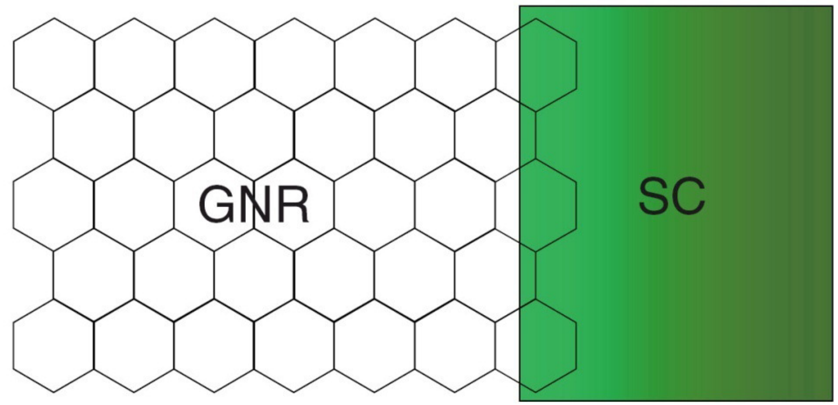
interplay of normal and Andreev scattering

- **Majorana states indicate the system instability**

Induced superconductivity in graphene, topological insulators



Graphene spectrum



Superconductor – graphene junction

Dirac – Bogolubov – de Gennes equations.

$$v_F \hat{\sigma} \cdot \left(\check{\mathbf{p}} - \frac{e}{c}\mathbf{A} \right) \hat{u} + \Delta \hat{v} = (E + \mu)\hat{u},$$

$$-v_F \hat{\sigma} \cdot \left(\check{\mathbf{p}} + \frac{e}{c}\mathbf{A} \right) \hat{v} + \Delta^* \hat{u} = (E - \mu)\hat{v},$$

PHYSICAL REVIEW VOLUME 175, NUMBER 2 10 NOVEMBER 1968

Tunneling Model of the Superconducting Proximity Effect

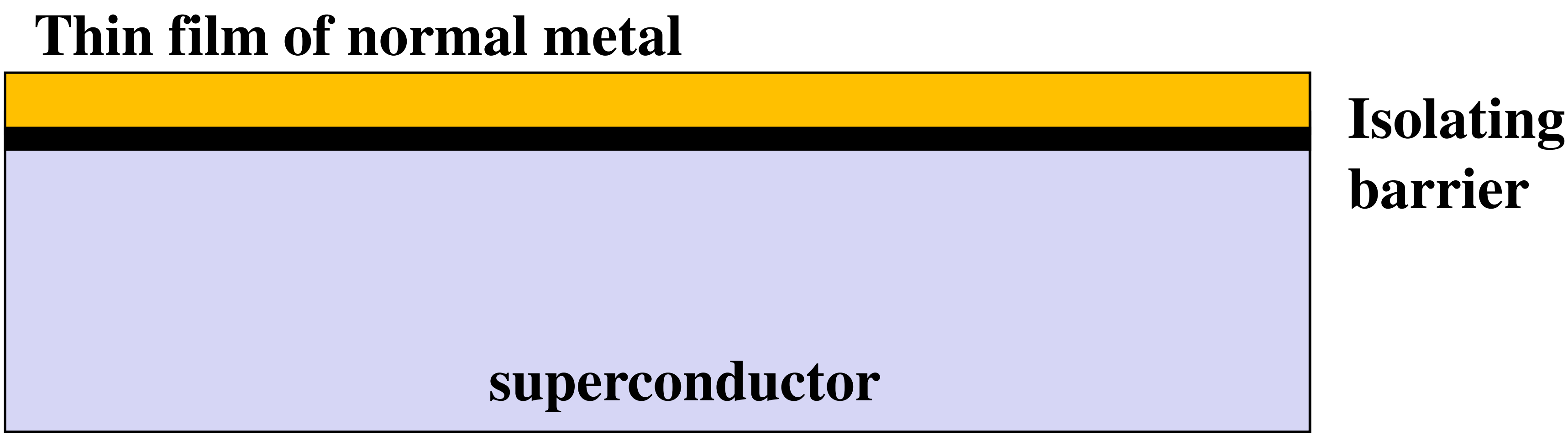
W. L. McMillan*

Bell Telephone Laboratories, Murray Hill, New Jersey and *Cavendish Laboratory, University of Cambridge, Cambridge, England*

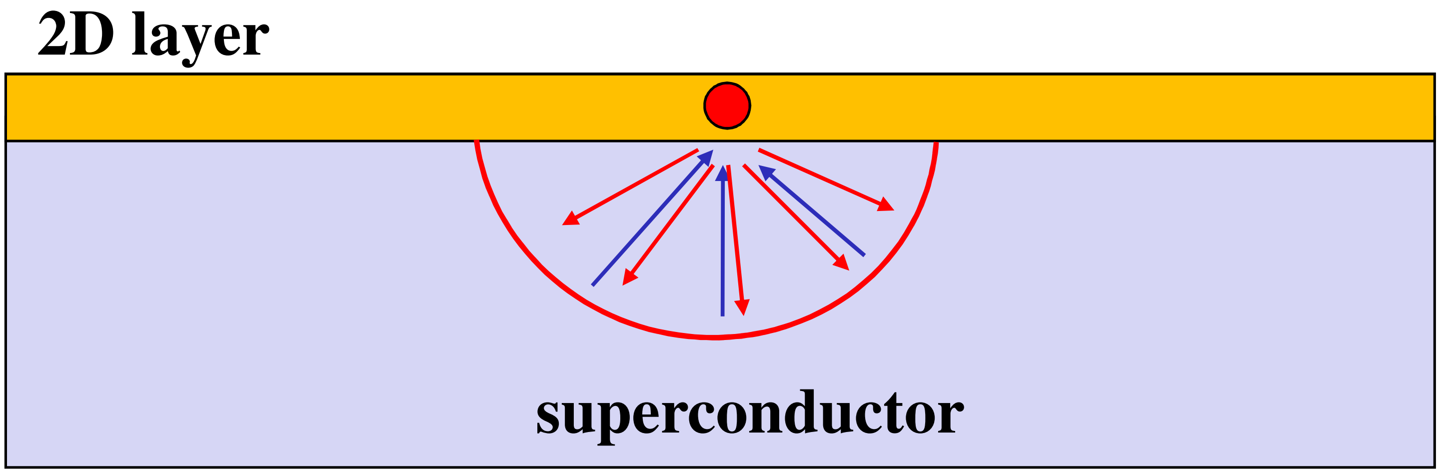
Proximity and Josephson effects in superconductor–two-dimensional electron gas planar junctions

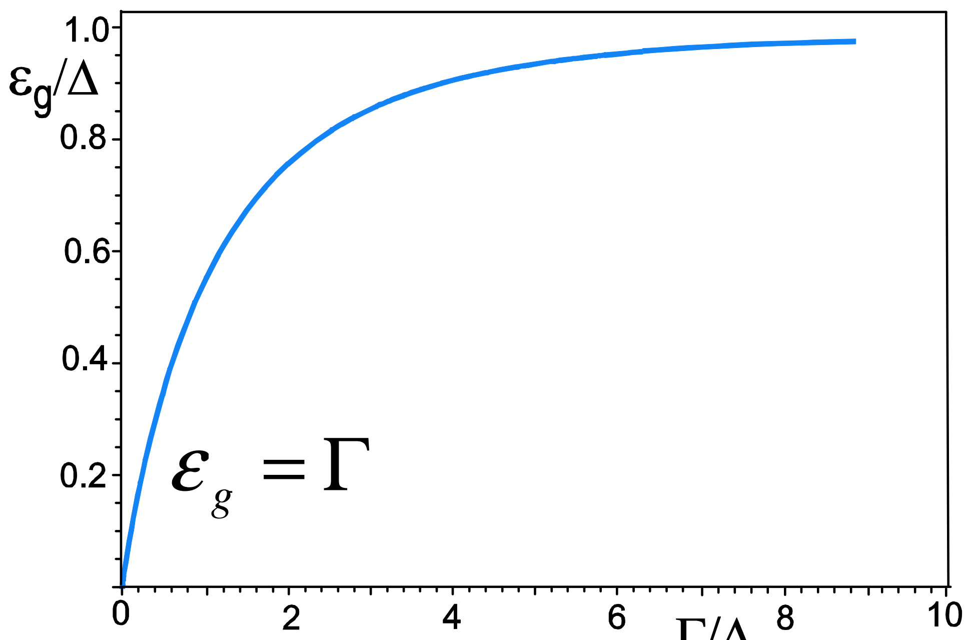
A.F. Volkov [a], P.H.C. Magnée [b,*], B.J. van Wees [b], T.M. Klapwijk [b]

Physica C 242 (1995) 261–266



Induced superconducting gap



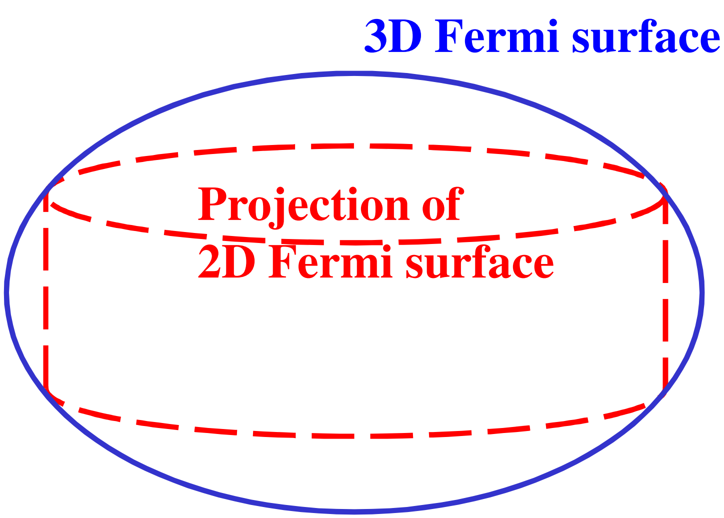


$$\Gamma = \frac{t^2}{\varepsilon_F}$$

Different models of tunneling

Coherent tunneling (conservation of quasiparticle momentum)

$$\check{\Sigma}(\mathbf{p},\mathbf{r}) = \frac{i\Gamma}{2}\left[\check{g}_S(\mathbf{p},p_{3z};\mathbf{r},0) + \check{g}_S(\mathbf{p},-p_{3z};\mathbf{r},0)\right]$$



Incoherent tunneling (no conservation of quasiparticle momentum)

$$\langle t(\mathbf{r}_1)t(\mathbf{r}_2)\rangle = t^2 s_0 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Sigma_T(\mathbf{R}_j,\mathbf{R}_l) = \Sigma(\mathbf{R}_j)\delta(\mathbf{R}_j - \mathbf{R}_l).$$

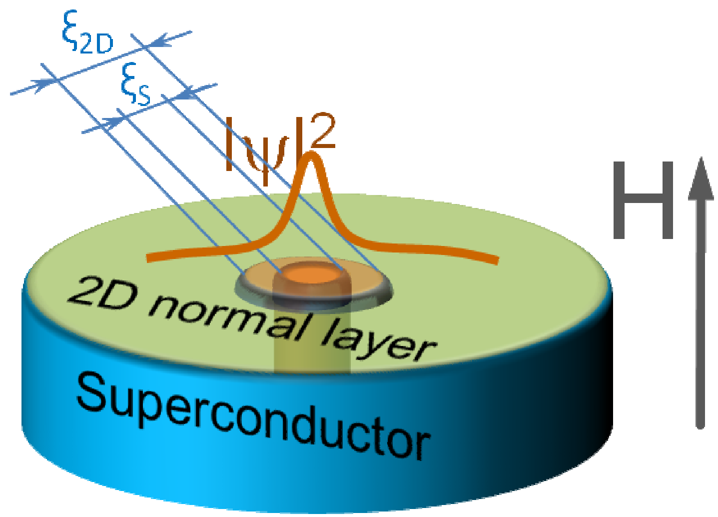
Tunneling rate

$$\check{\Sigma}(\mathbf{R}) = i\Gamma\langle\check{g}_S(\mathbf{R};0)\rangle.$$

Quasiclassical Green's function averaged over trajectories

Vortex states in a system with induced superconducting order.

Multiple vortex core.



New coherence length $$\xi_{2D} = \frac{\hbar V_F}{\Gamma}$$