KOPNIN AND ROTATING SUPERFLUID ^{3}He

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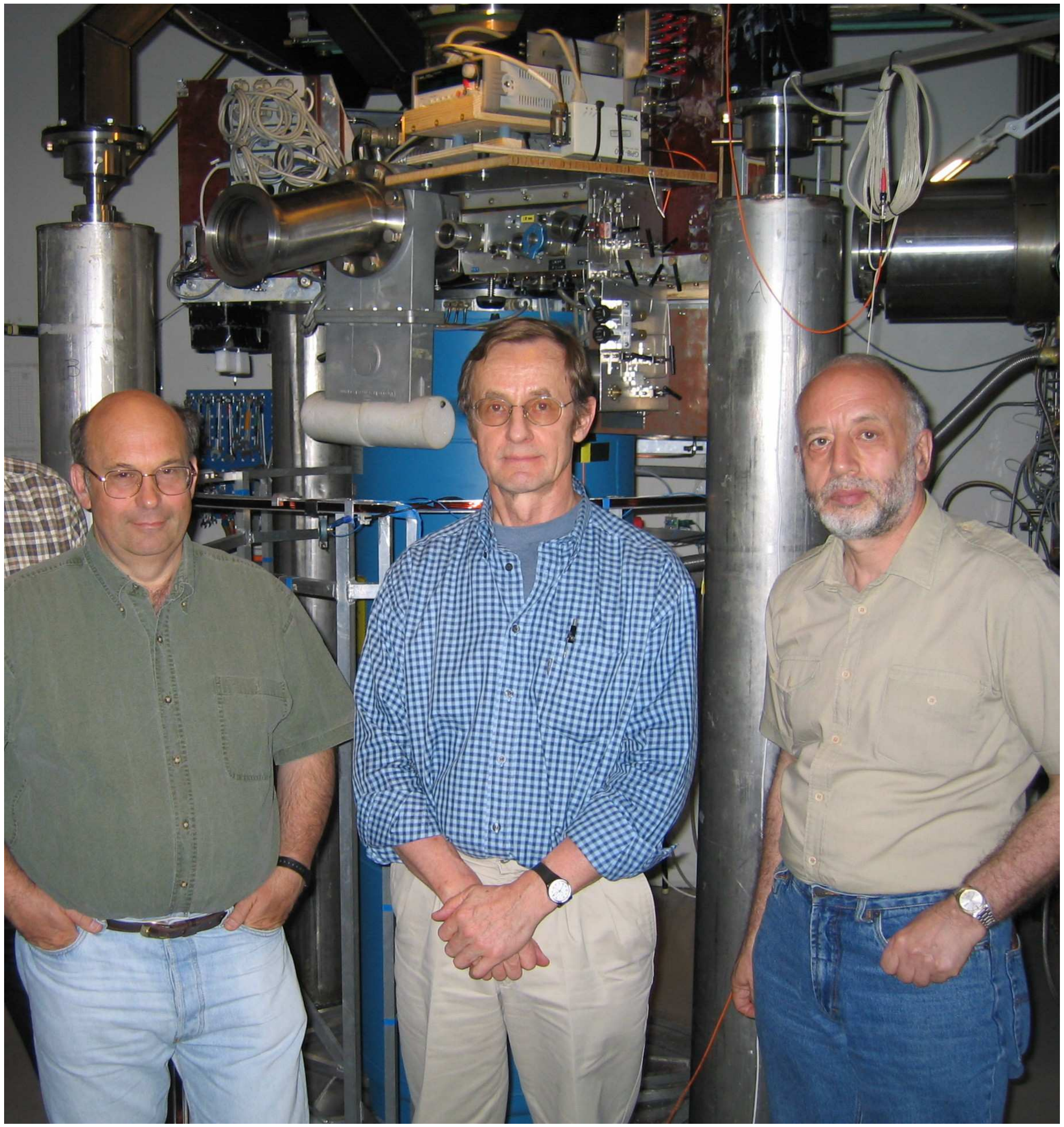


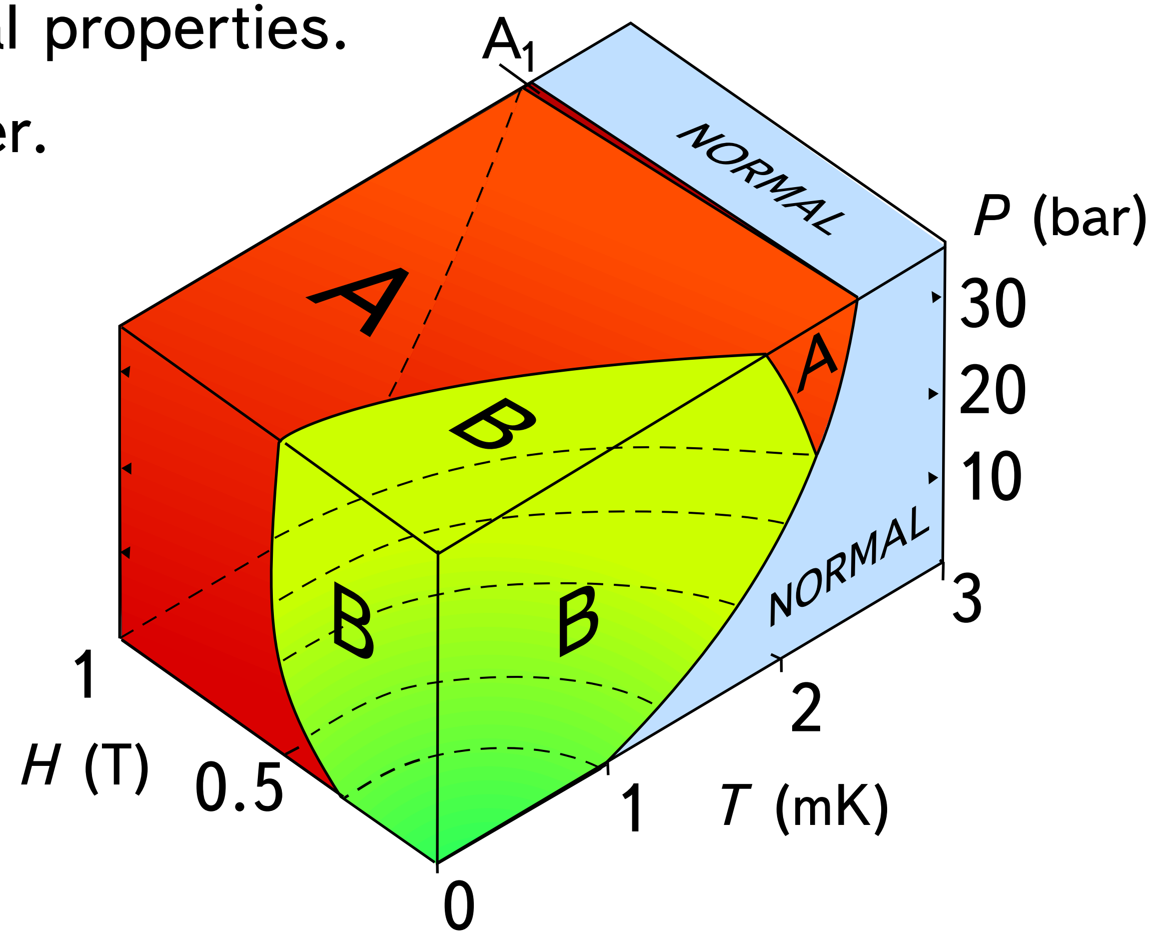
1. Introduction: Superfluid ^{3}He, rotation and NMR.
2. Historical:
   - Defect formation in non-equilibrium second-order phase transition.
   - Dynamic response of anisotropic superfluid: Vortex sheets.
3. Superfluid dynamics at low temperatures: Transition to turbulence and decoupling.
4. Probing vortex-core- and surface-bound fermion zero modes with magnon BEC.

SUPERFLUID ^{3}He

Fermi system with pairing in $L = 1$, $S = 1$ state.

Non-trivial topological properties.

3×3 order parameter.



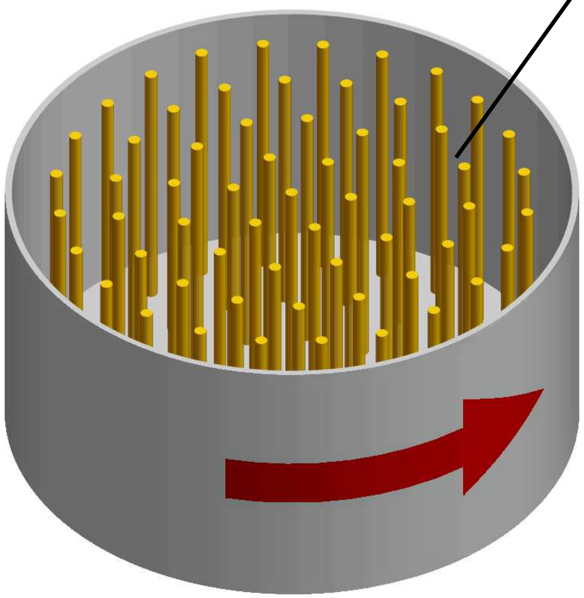
- Multiple superfluid phases: A and B in bulk (and A_1 in magnetic field) and others in the cores of topological defects and in restricted geometry.
- Topological defects of various dimensionality and structure.

ROTATING SUPERFLUID

Free energy of rotating superfluid

$$F'_{\mathrm{kin}} = F_{\mathrm{kin}} - \mathbf{\Omega}\cdot\mathbf{L} = \int \mathrm{d}^3r \cdot \frac{1}{2}\rho_{\mathrm{s}}(\mathbf{v}_{\mathrm{n}} - \mathbf{v}_{\mathrm{s}})^2 + F'_{\mathrm{kin,\ solid\ body}}$$

is at minimum when $\mathbf{v}_{\mathrm{n}} \approx \mathbf{v}_{\mathrm{s}}$



Vortex density: $n_{\mathrm{v}} = \dfrac{2\Omega}{\kappa}$

$\kappa = h/m-$ quantum of circulation

Number of vortices:

$N_{\mathrm{v}} = N_{\mathrm{eq}} = \pi R^2 n_{\mathrm{v}} \propto \Omega$

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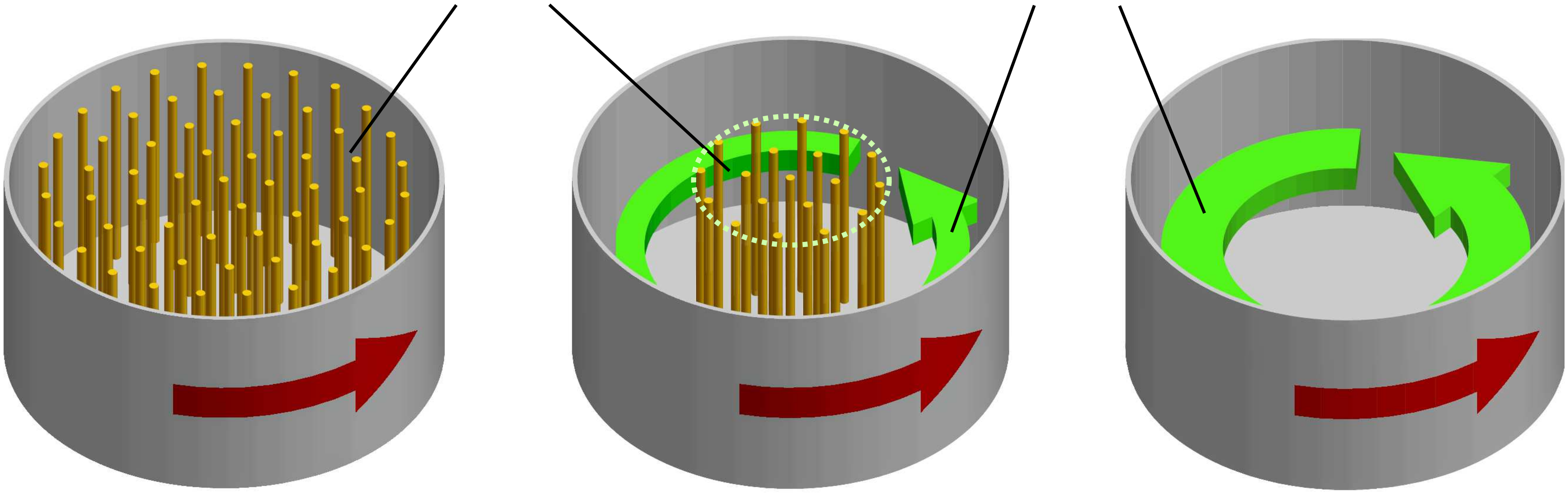

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Metastable rotation

$$0 \leqslant N_v < N_{eq}$$

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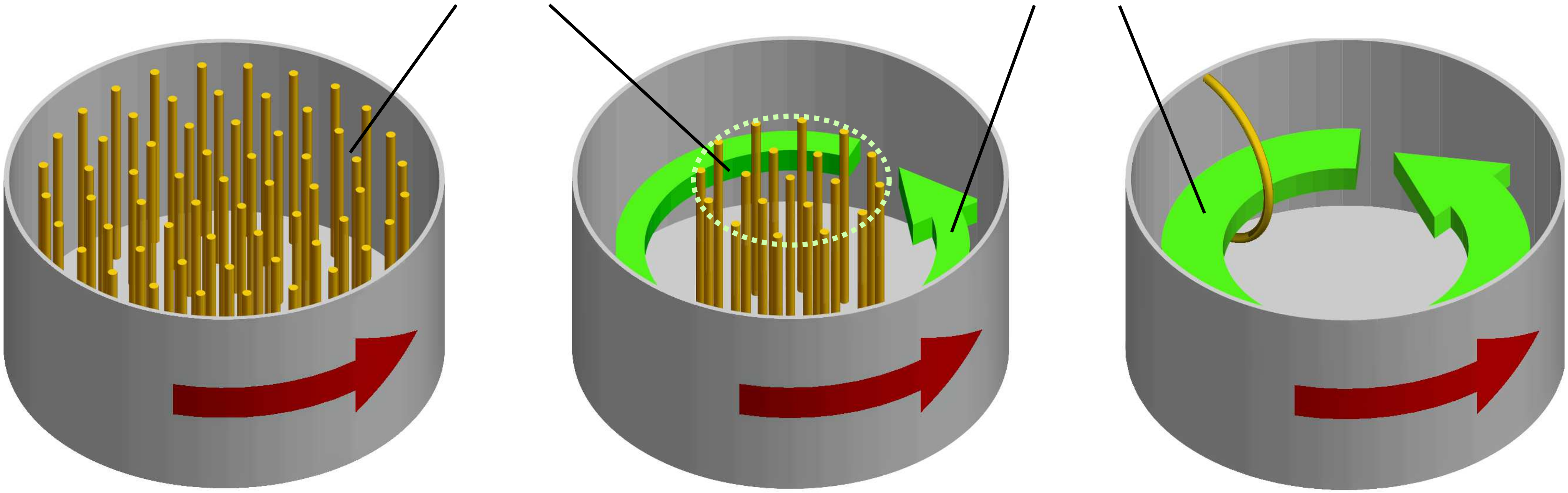

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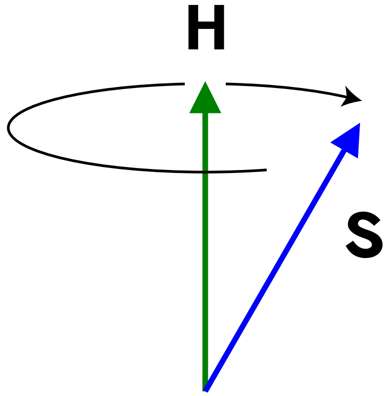
NMR: A TOOL TO SEE VORTICES



Spin-orbit interaction in Cooper pairs exerts additional torque on NMR precession:

$$\frac{\partial \mathbf{S}}{\partial t} = \gamma \mathbf{S} \times \mathbf{H} + \mathbf{R}_D$$

Flow pattern $\longrightarrow$ orbital part of the order parameter   frequency shift in NMR

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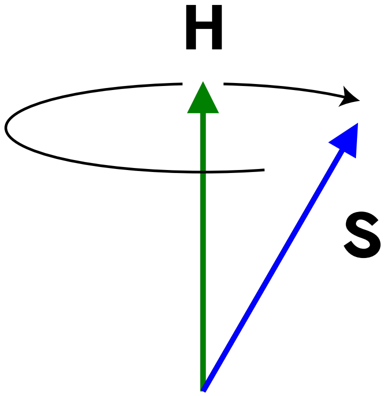

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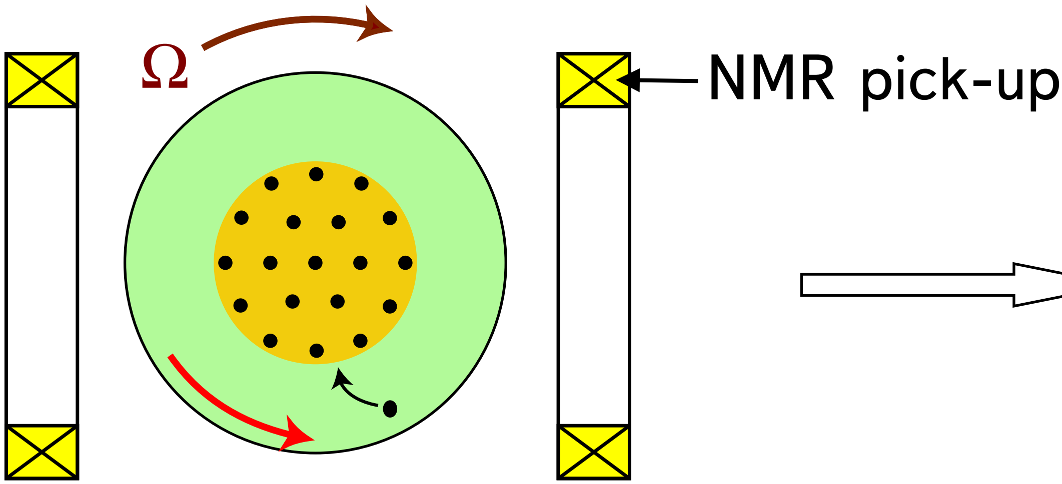
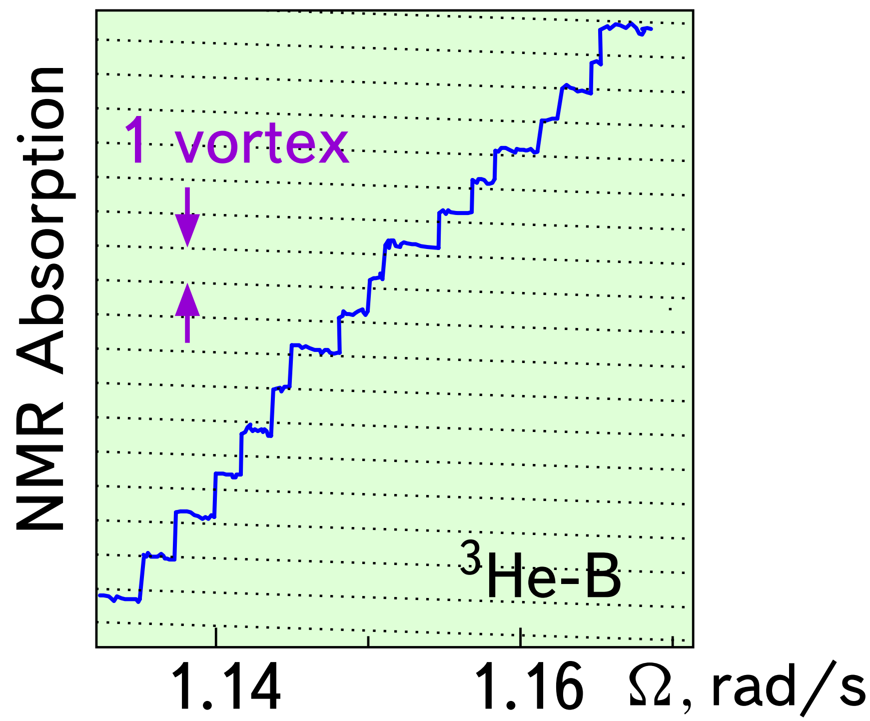
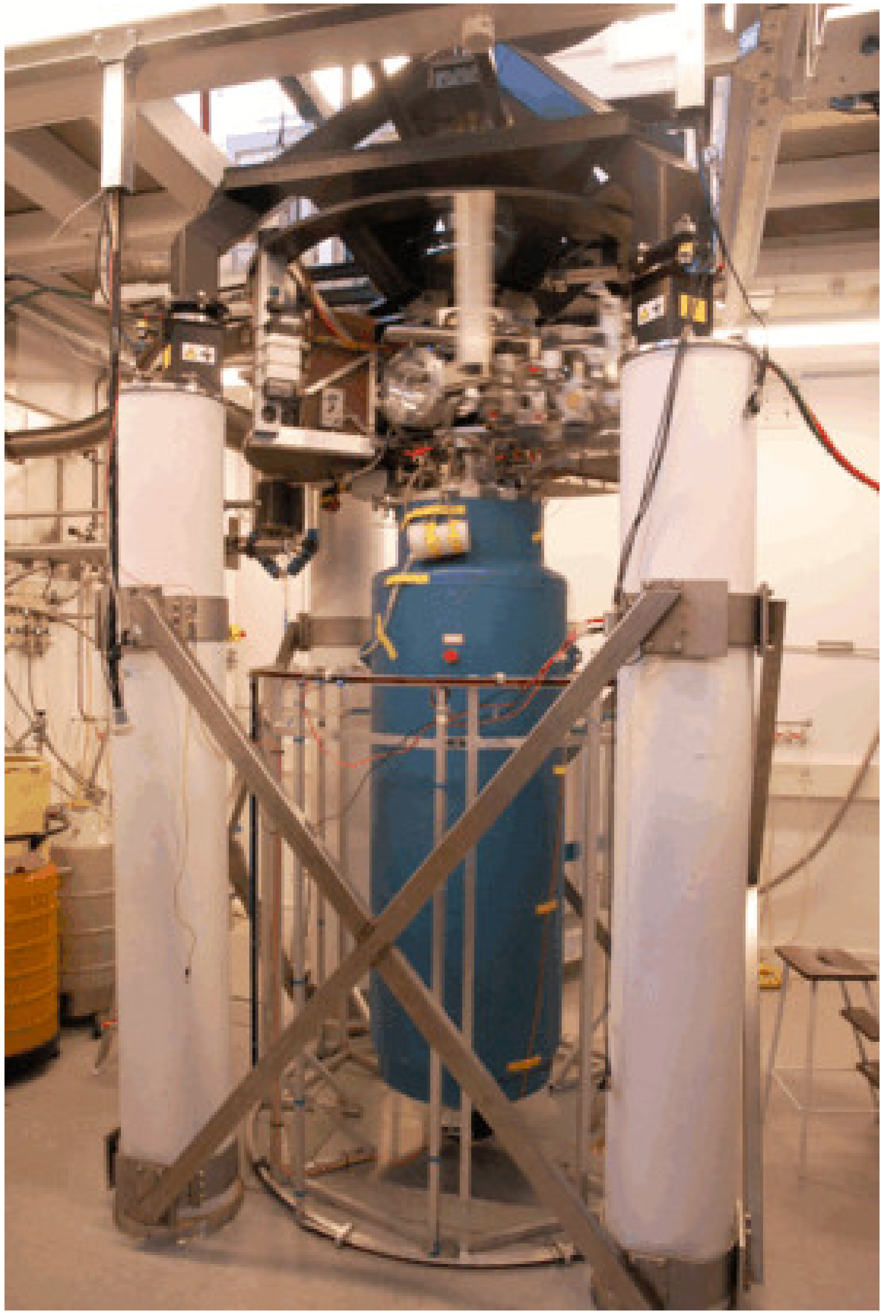


$v_s - v_n = v_c$

vortex formation



ROTATING SUBMILLIKELVIN CRYOSTAT



First rotating nuclear demagnetization cryostat built within Finnish-Soviet collaboration ROTA (operational 1981).

Upgraded in the last decade: Lowest temperature of superfluid ^{3}He in rotation.

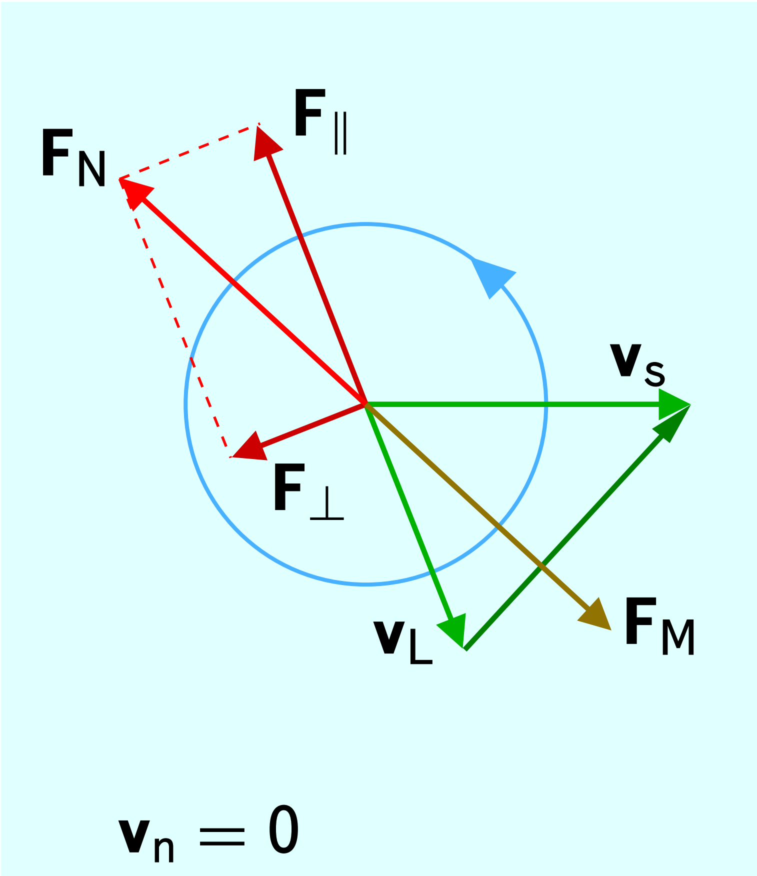
Rotation velocity: up to 3.5 rad/s.

Heat leak to the sample: below 20 pW in rotation.

Temperature of ^{3}He: below 140 μK.

SUPERFLUID DYNAMICS AT LOW TEMPERATURES: TRANSITION TO TURBULENCE AND DECOUPLING

FORCES ACTING ON A VORTEX



Vortex motion: $\mathbf{F}_\mathrm{M} + \mathbf{F}_\mathrm{N} = 0$

- The Magnus force:

  $\mathbf{F}_\mathrm{M} = \kappa \rho_\mathrm{s} (\mathbf{v}_\mathrm{s} - \mathbf{v}_\mathrm{L}) \times \hat{\mathbf{z}}$

- Reaction of normal component:

  $\mathbf{F}_\mathrm{N} = D(\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{L})_\perp + D' \hat{\mathbf{z}} \times (\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{L})$

ρ_s – superfluid density

$\kappa = 2\pi\hbar/m$ – circulation quantum

$(m = m_4,\ 2m_3, \ldots)$

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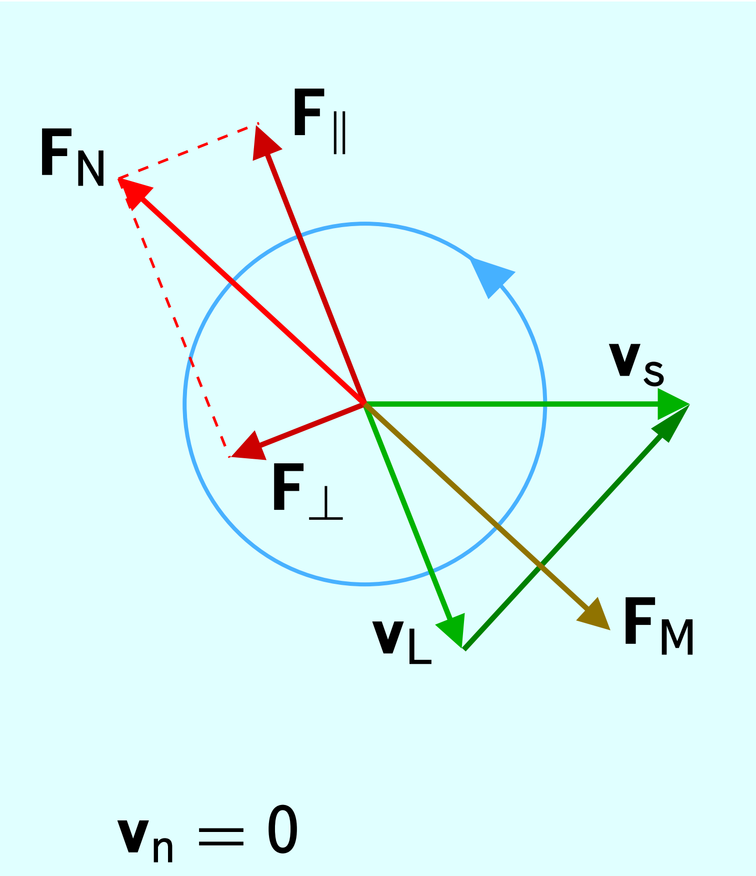

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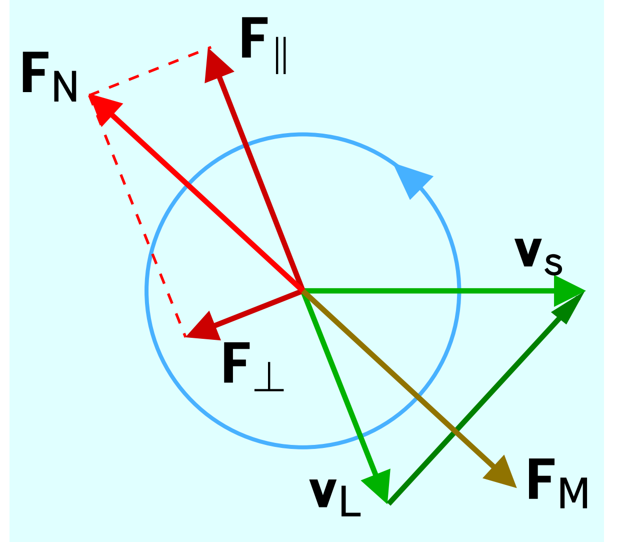
Universal contribution to D': $-\kappa\rho_{\mathrm{n}}$, the *Iordanskii* force.

In Fermi superfluids D and the rest of D': the *Kopnin* force.

FORCE BALANCE AND MUTUAL FRICTION

Equation for vortex motion: $\mathbf{F}_\mathrm{M} + \mathbf{F}_\mathrm{N} = 0$

$$\Rightarrow \quad \mathbf{v}_\mathrm{L} = \mathbf{v}_\mathrm{s} + \alpha'(\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{s})_\perp + \alpha \hat{\mathbf{z}} \times (\mathbf{v}_\mathrm{n} - \mathbf{v}_\mathrm{s}).$$



Mutual friction parameters:

$$\alpha = \frac{D/\kappa\rho_\mathrm{s}}{(D/\kappa\rho_\mathrm{s})^2 + (1 - D'/\kappa\rho_\mathrm{s})^2}\,, \qquad \alpha' = 1 - \frac{1 - D'/\kappa\rho_\mathrm{s}}{(D/\kappa\rho_\mathrm{s})^2 + (1 - D'/\kappa\rho_\mathrm{s})^2}$$

dissipative reactive

(Hall and Vinen 1956)

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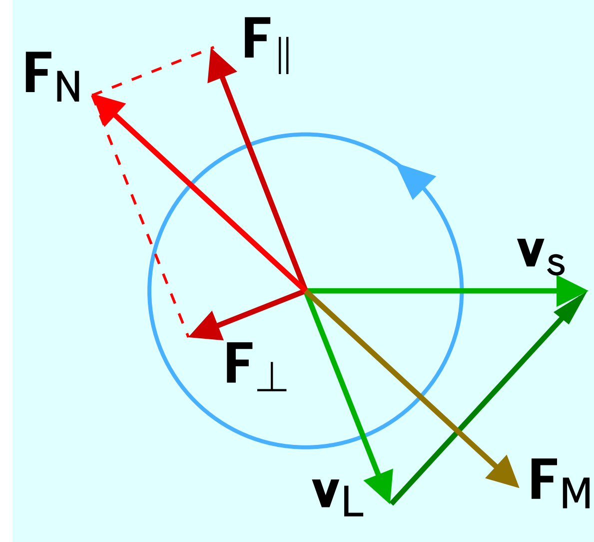

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Averaging $\mathbf{F}_{\mathrm{N}} = -\mathbf{F}_{\mathrm{M}}(\alpha, \alpha', \mathbf{v}_{\mathrm{n}}, \mathbf{v}_{\mathrm{s}})$ over vortex lines and putting to Euler equation gives

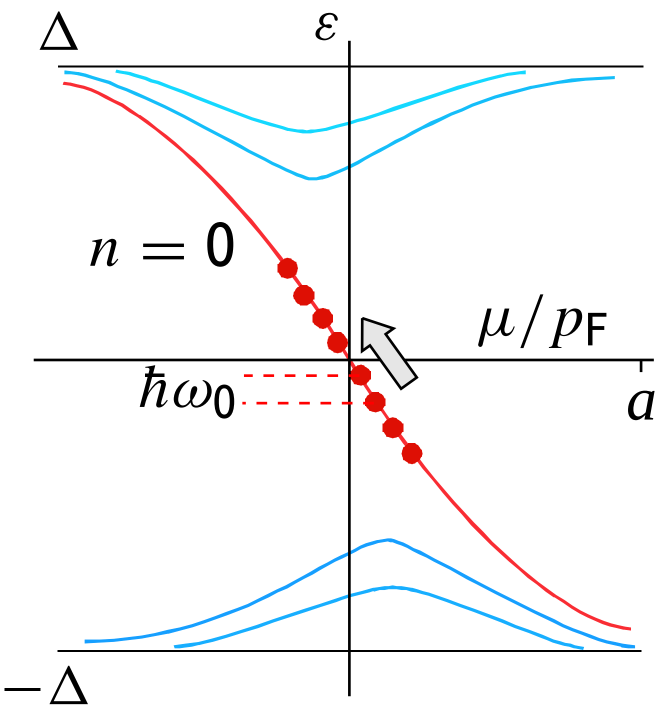
$$\frac{\partial \boldsymbol{\omega}}{\partial t} = (1 - \alpha')\nabla \times [\mathbf{v}_{\mathrm{s}} \times \boldsymbol{\omega}] + \alpha\nabla \times [\hat{\boldsymbol{\omega}} \times (\boldsymbol{\omega} \times \mathbf{v}_{\mathrm{s}})]$$

$(\boldsymbol{\omega} = \langle \nabla \times \mathbf{v}_{\mathrm{s}} \rangle$ and $\mathbf{v}_{\mathrm{n}} = 0)$

$$\text{Inertial} \sim (1 - \alpha')\frac{U\omega}{R}, \ \text{viscous} \sim \alpha\frac{U\omega}{R} \ \Rightarrow \ \mathrm{Re}_{\alpha} = \frac{\text{inertial}}{\text{viscous}} = \frac{1 - \alpha'}{\alpha}$$

(Volovik 2003)

KOPNIN FORCE FROM CORE-BOUND FERMIONS



Core-bound states

Vortex motion leads to pumping of q.p. along anomalous branch.

Relaxation (τ) towards equilibrium distribution via interaction with bulk q.p. results in

$$D = \rho\kappa \frac{\omega_0 \tau}{1+\omega_0^2\tau^2} \tanh \frac{\Delta(T)}{2T}$$

Kopnin force

$$D' = \rho\kappa \left[1 - \frac{\omega_0^2\tau^2}{1+\omega_0^2\tau^2} \tanh \frac{\Delta(T)}{2T} \right] - \rho_n \kappa$$

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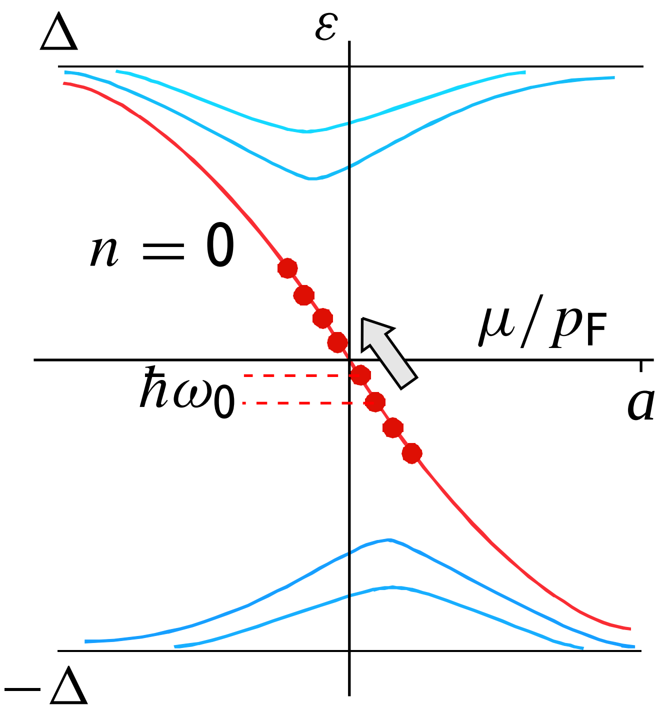

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Kopnin number

$$T \to T_c \;:\; \omega_0 \sim \Delta^2/E_F \to 0 \quad \Rightarrow \mathrm{Re}_\alpha \to 0$$

$$T \to 0 \;:\; \tau \sim \tau_n \exp\frac{\Delta}{T} \to \infty \Rightarrow \mathrm{Re}_\alpha \to \infty$$

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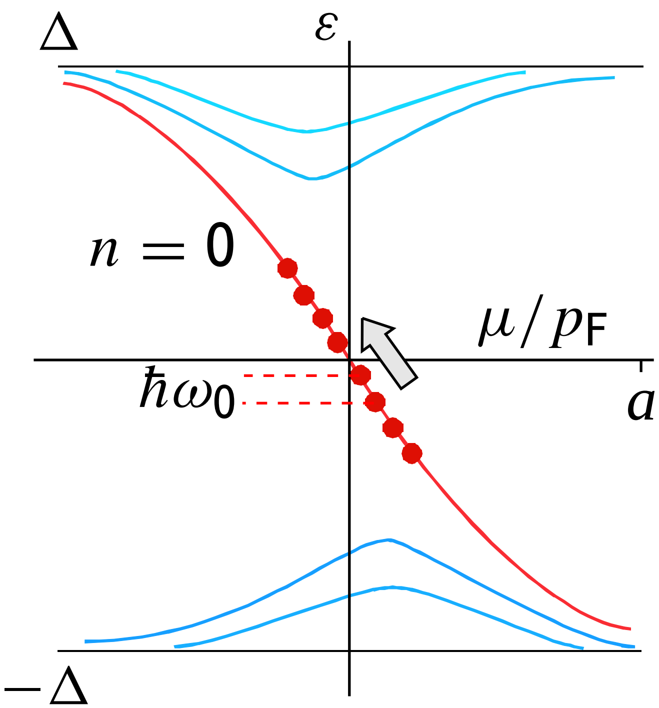

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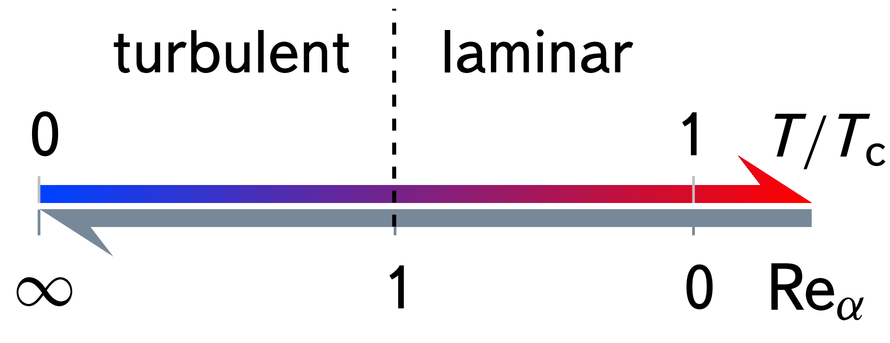
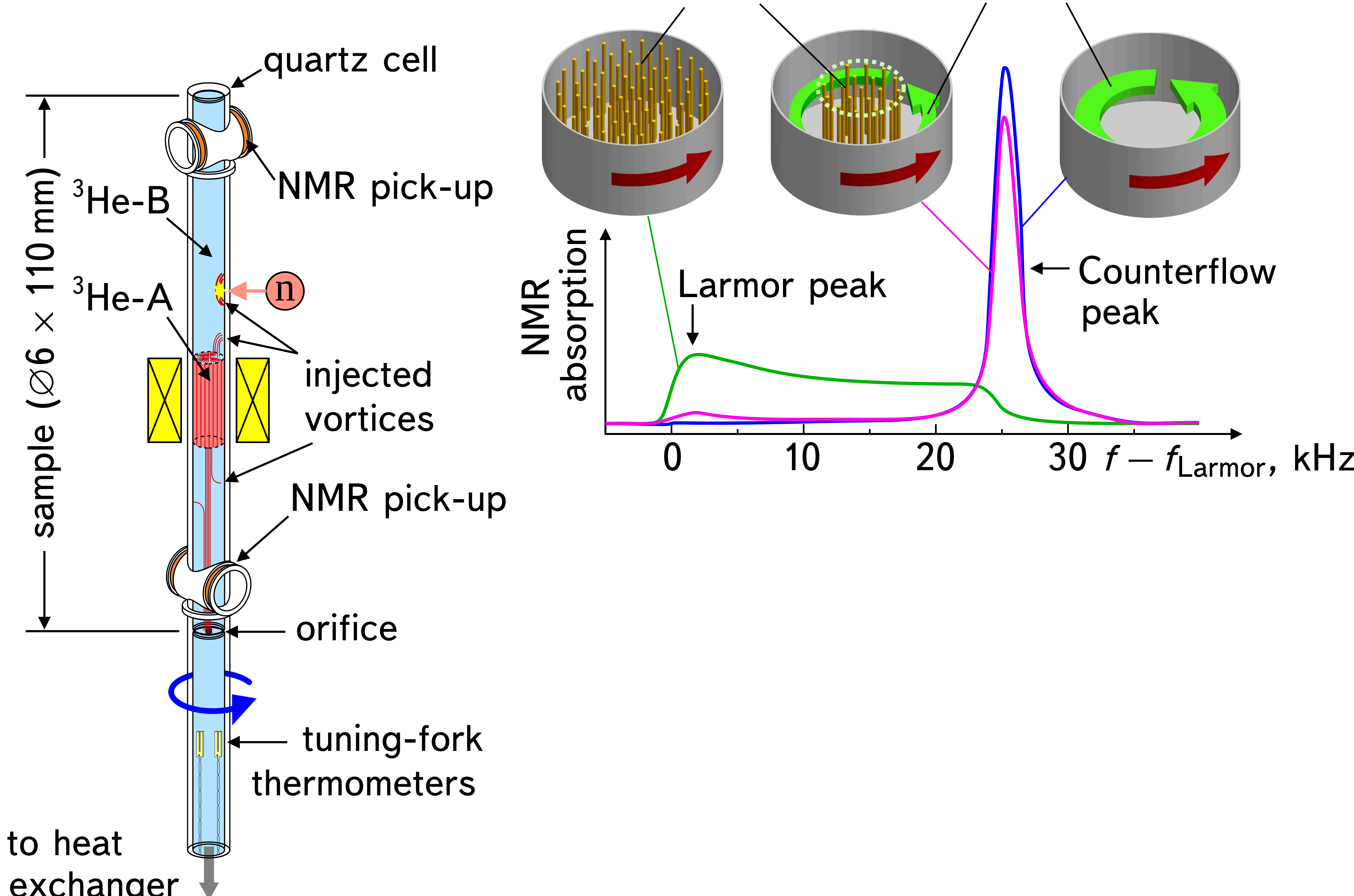
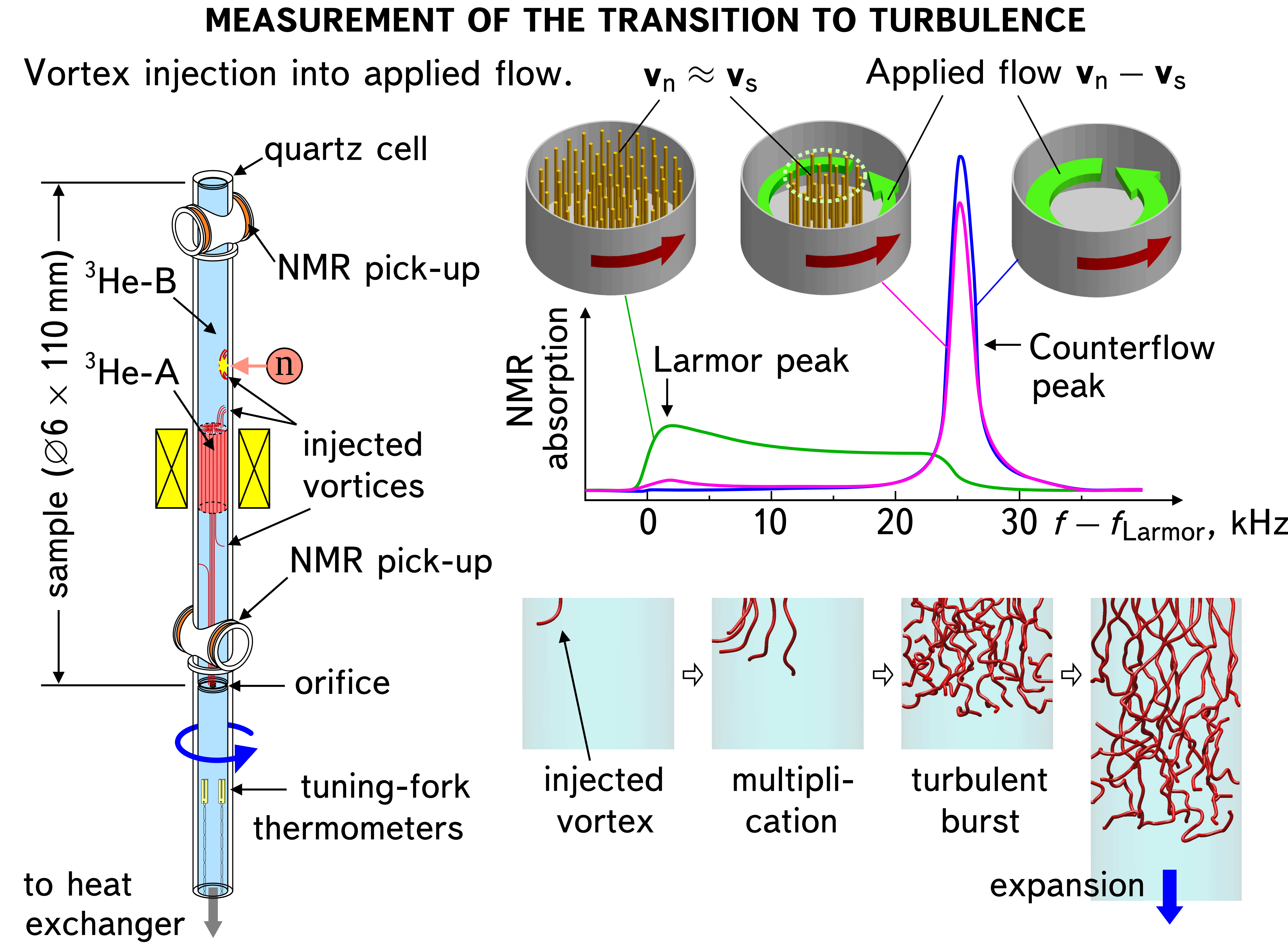


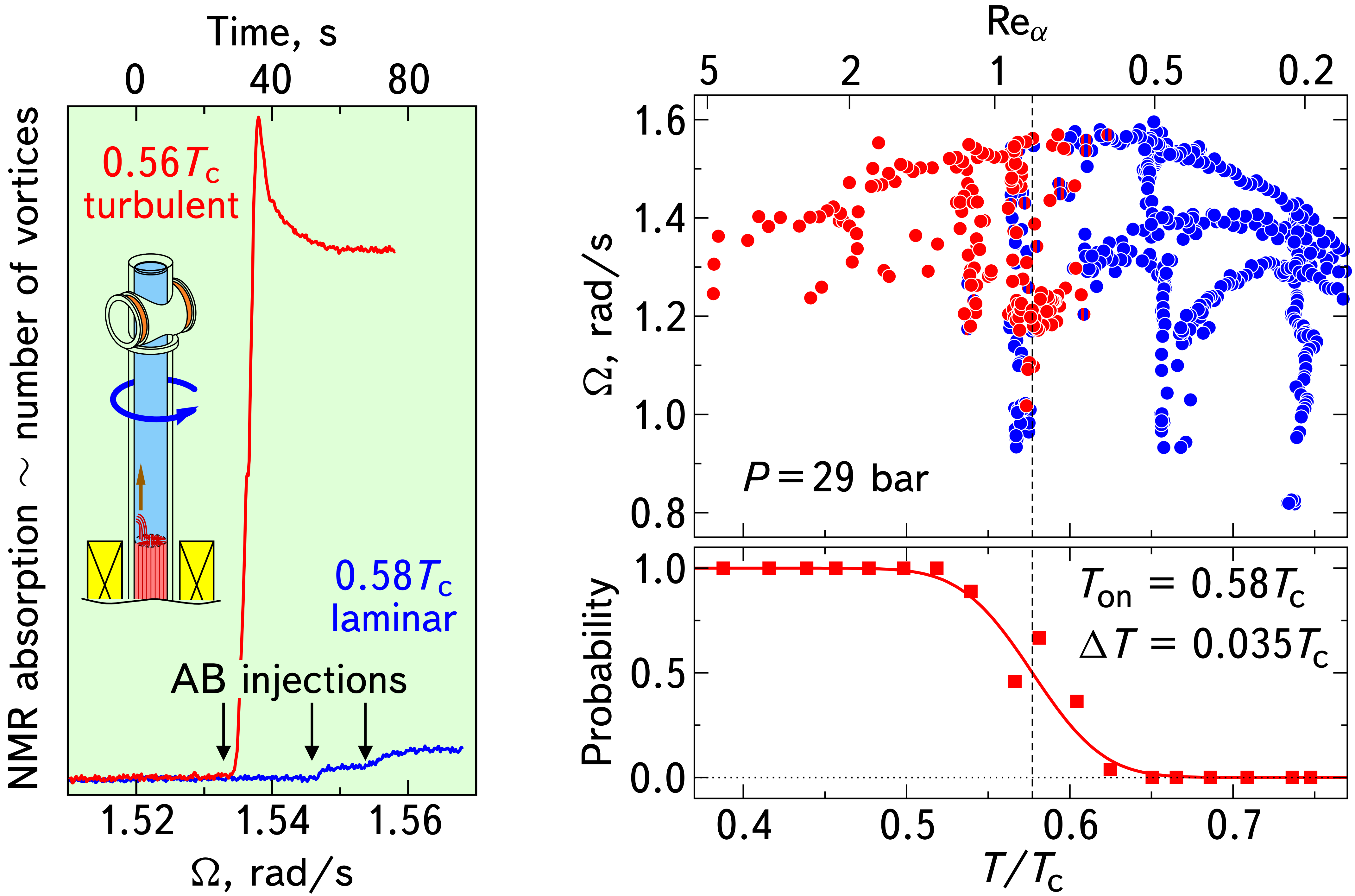
MEASUREMENT OF THE TRANSITION TO TURBULENCE

Vortex injection into applied flow.




MEASUREMENT OF THE TRANSITION TO TURBULENCE
Vortex injection into applied flow.
quartz cell
NMR pick-up
^{3}He-B
^{3}He-A
n
injected vortices
sample ($\varnothing 6 \times 110$ mm)
NMR pick-up
orifice
tuning-fork thermometers
to heat exchanger
$\mathbf{v}_n \approx \mathbf{v}_s$
Applied flow $\mathbf{v}_n - \mathbf{v}_s$
NMR absorption
Larmor peak
Counterflow peak
0
10
20
30
$f - f_{\mathrm{Larmor}}$, kHz
injected vortex
multipli-cation
turbulent burst
expansion

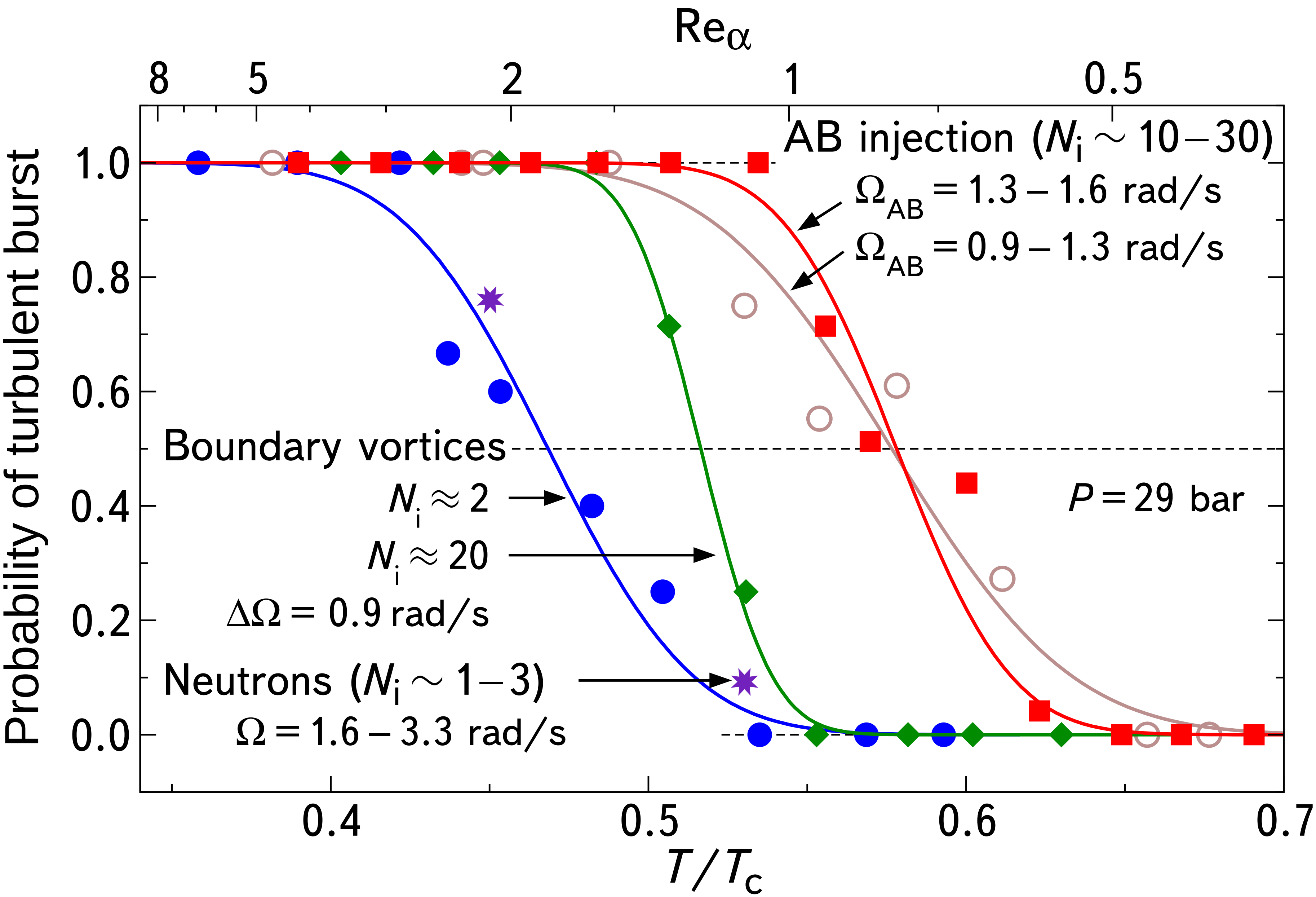
TRANSITION TO TURBULENCE



Finne et al, Nature **424**, 1022 (2003)

TRANSITION TO TURBULENCE

Injection with different methods:

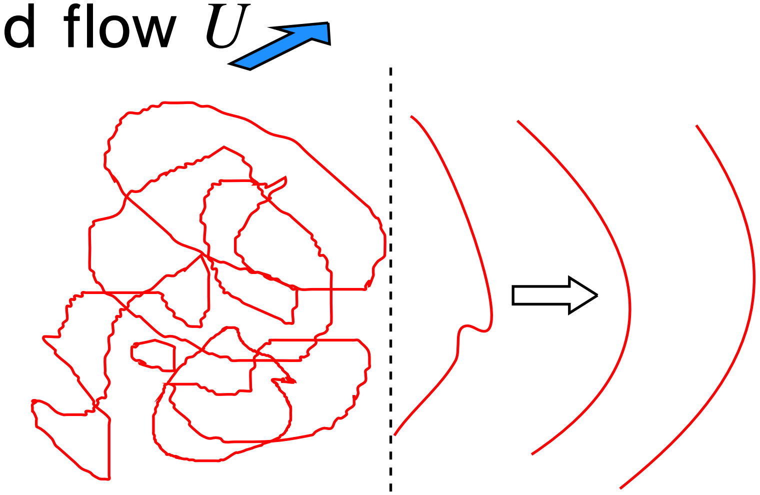


(N_i — number of injected vortices)

KOPNIN MODEL OF TRANSITION TO TURBULENCE

More microscopic approach and account for experimental details.

PRL **92**, 135301 (2004)



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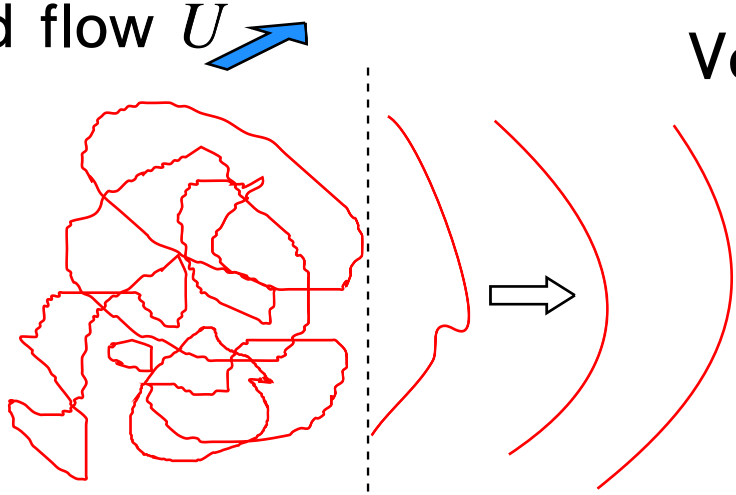
applied flow U



Multiplication region of seed vortices

Extraction region

Vortex density L:

$$\dot{L} = \beta[UL^{3/2} - \kappa L^2], \quad \beta = \underbrace{A(1-\alpha')}_{\text{multiplication}} - \underbrace{B\alpha}_{\text{extraction}}.$$

Density increases if $\beta > 0$ or

$\mathrm{Re}_\alpha \gtrsim 1$.

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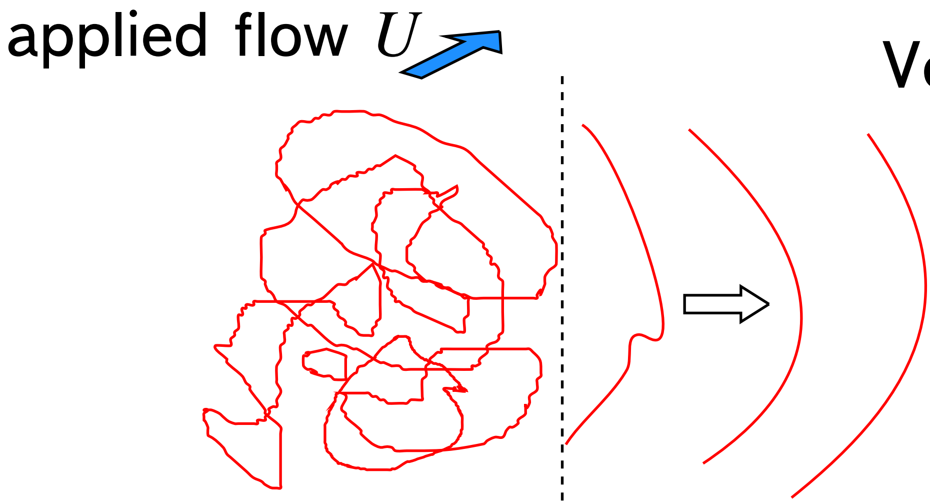

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Turbulence with stationary (wall-clamped) normal component:

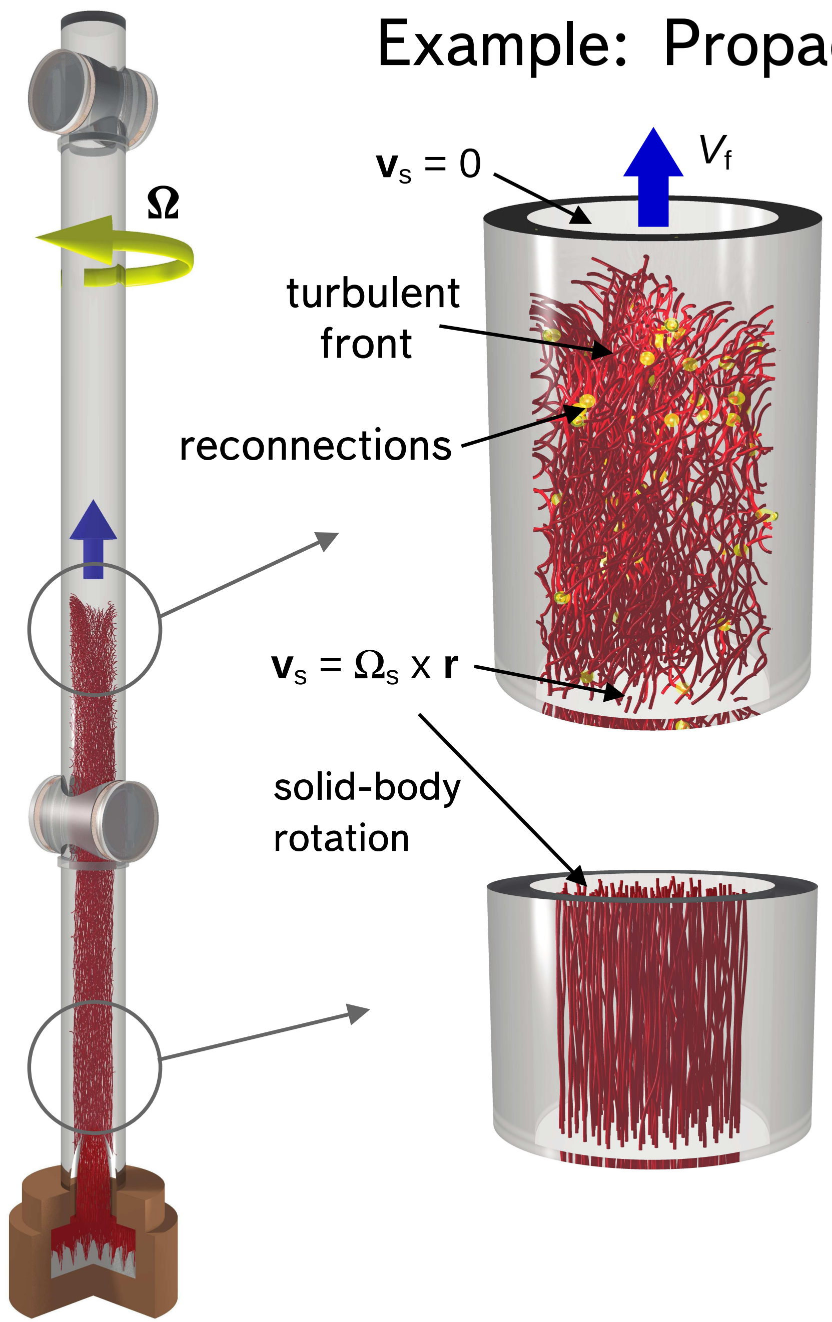
Viscosity of normal component $\nu \gg \alpha(\rho_s/\rho_n)\kappa$.

In ^{3}He $\nu \sim 10^3\kappa$, in ^{4}He $\nu \sim \kappa$.

VORTEX DYNAMICS AT VERY LOW TEMPERATURES

Turbulence in superfluid enhances energy dissipation, which remains finite in $T \to 0$ limit: Dissipation anomaly via energy cascade.

Example: Propagating turbulent vortex front.



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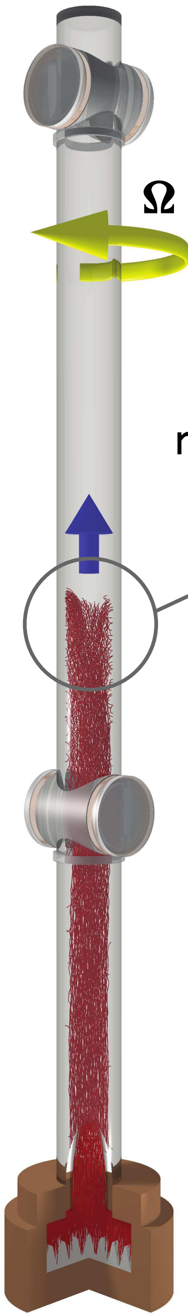

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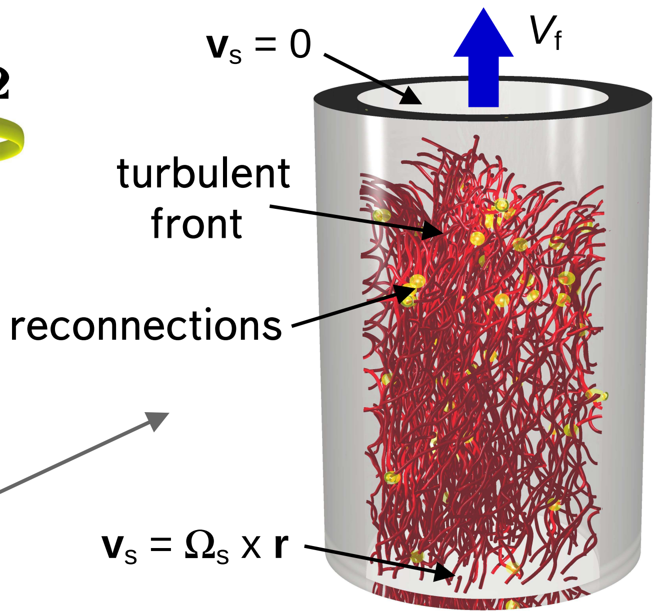

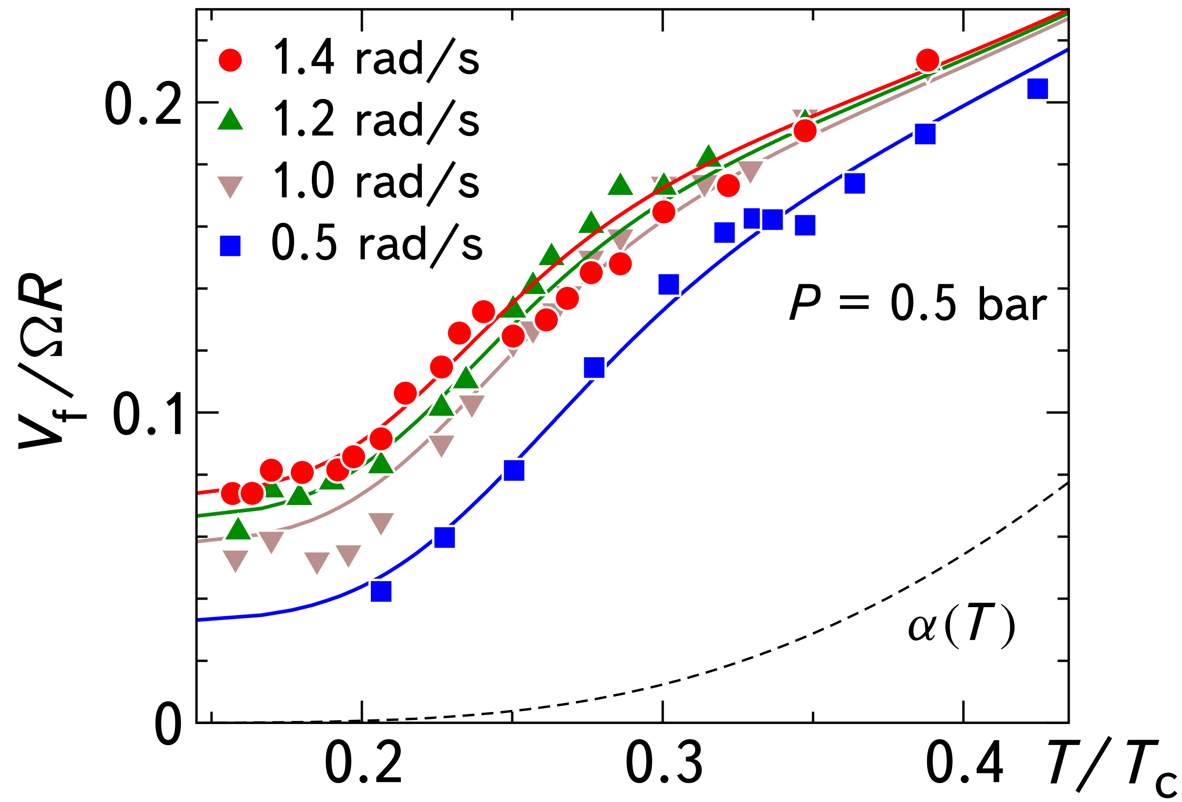

Energy dissipation:

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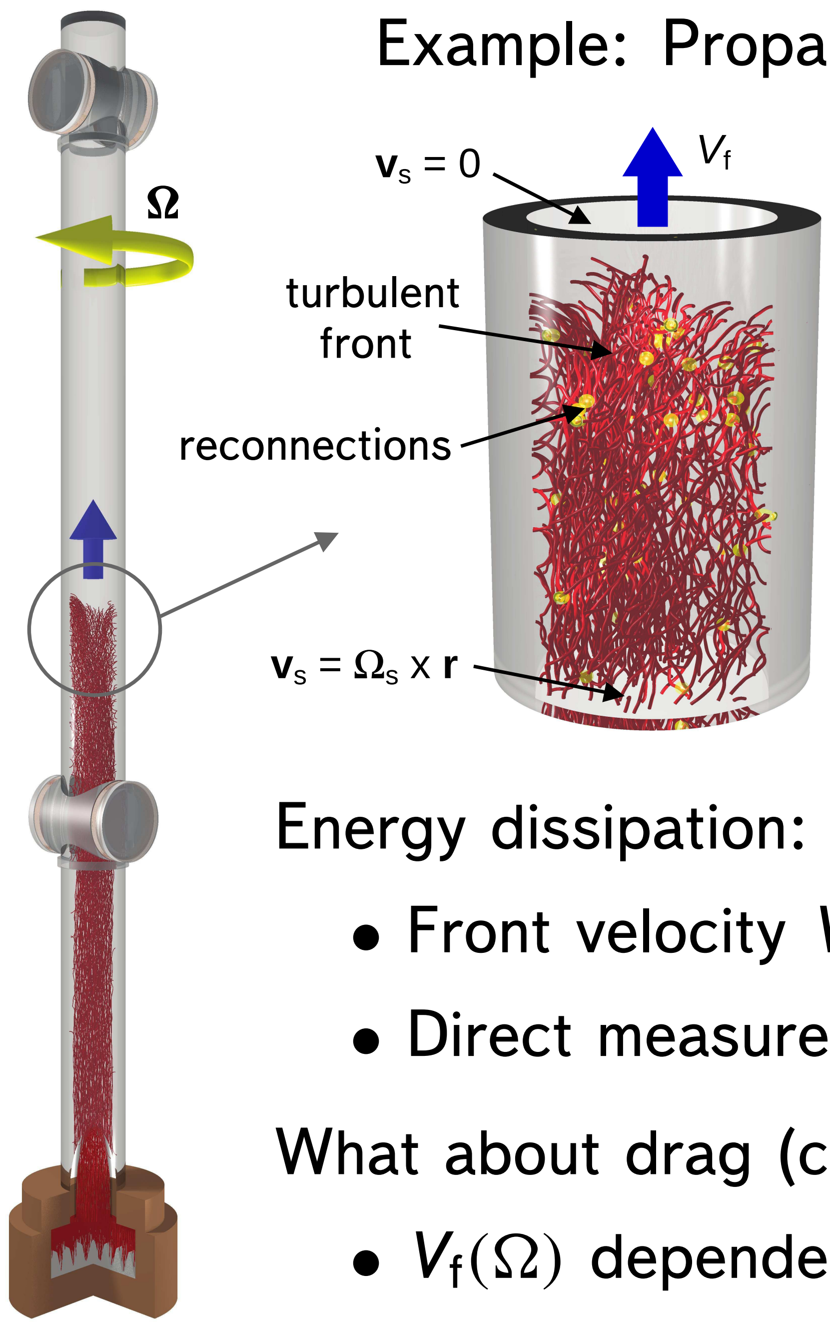

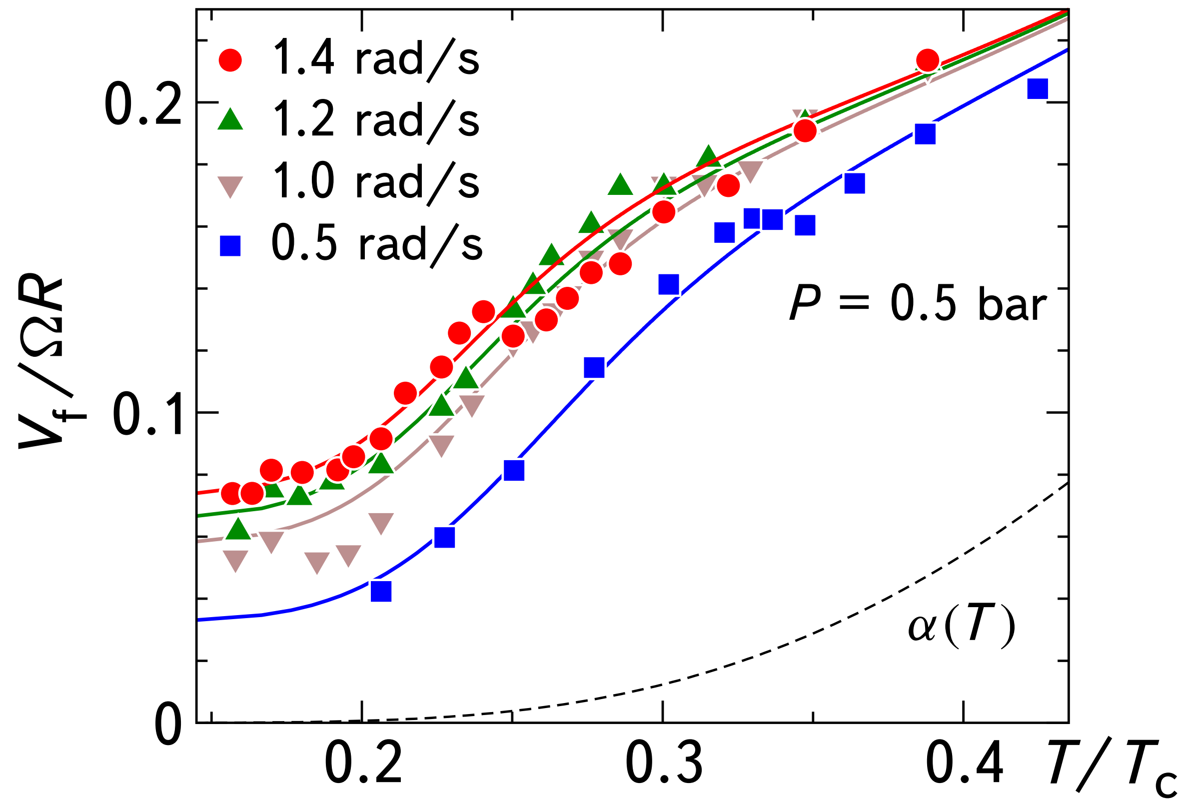

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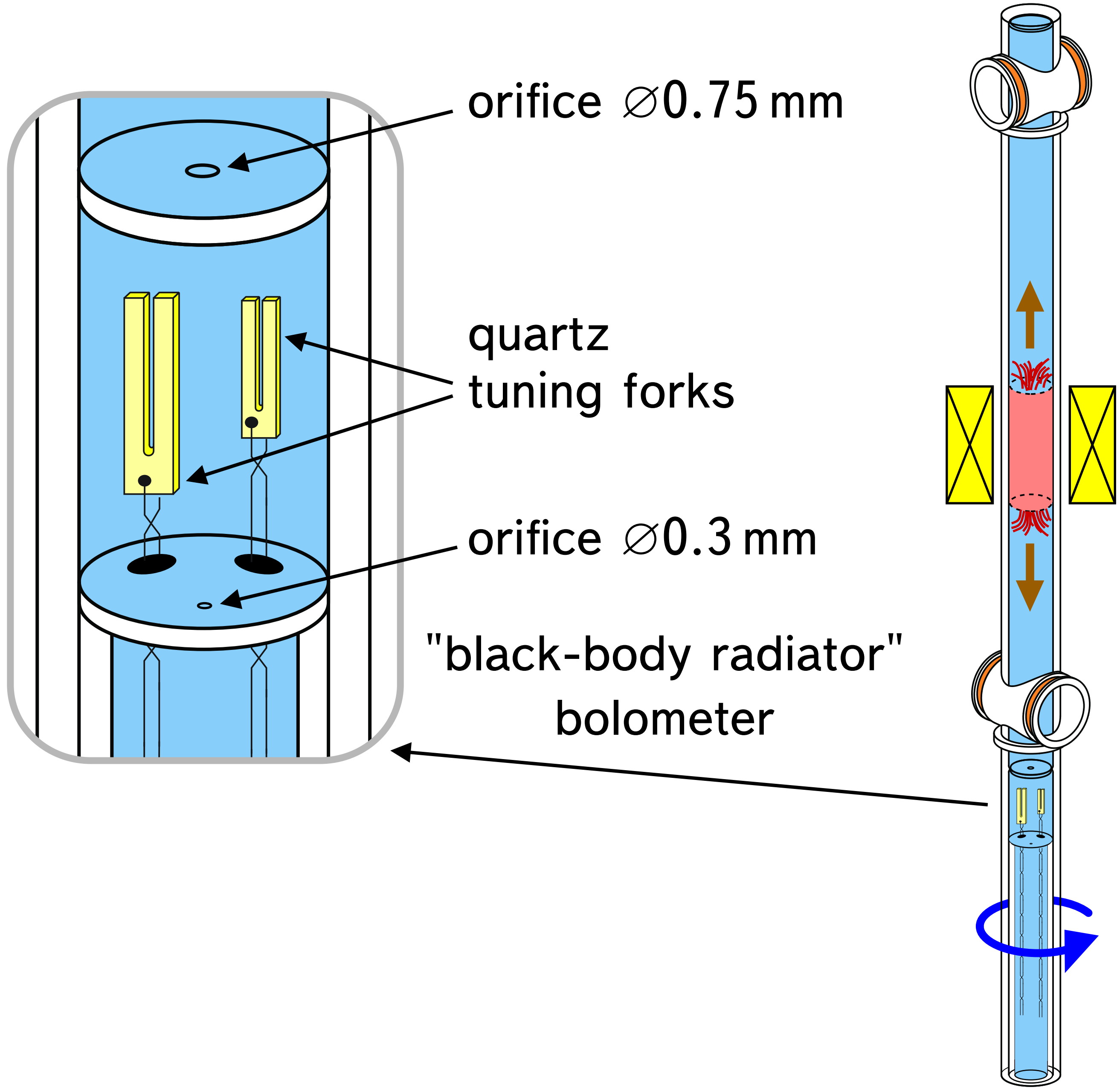
What about drag (coupling to the walls)?

- $V_f(\Omega)$ dependence
- Measurement of the front rotation

THERMAL SIGNAL FROM THE FRONT PROPAGATION

Thermal measurements:

- Sensitivity 0.1 pW
- Thermal time constant 25 s



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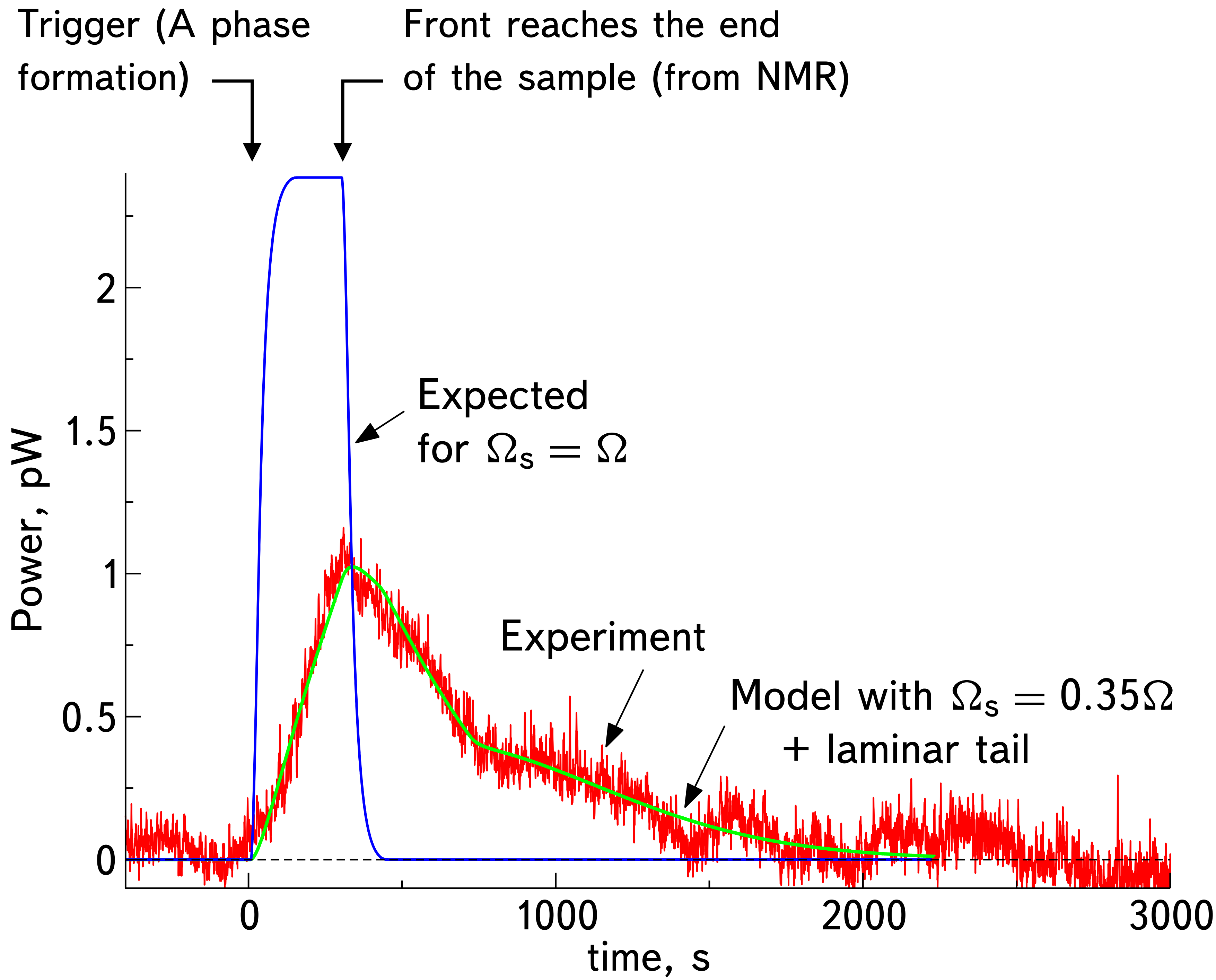

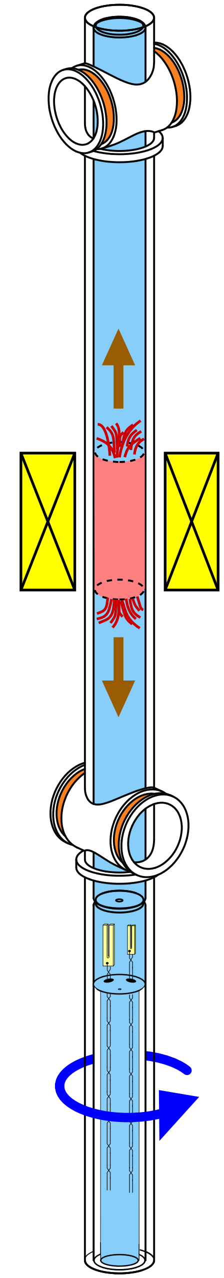

$P = 29\,\mathrm{bar}$, $T = 0.20T_c$, $\Omega = 1.2\,\mathrm{rad/s}$

PRL **107**, 135302 (2011)

PHENOMENOLOGICAL MODEL OF THE FRONT PROPAGATION

In turbulent motion averaged hydrodynamics should include *effective* friction:

For energy: $$\alpha_{\mathsf{en}} = C_{\mathsf{en}}\,\alpha(T) + \tilde{\alpha}_{\mathsf{en}}.$$

Laminar dissipation at the outer scale

Turbulent cascade

For angular momentum: $$\alpha_{\mathsf{am}} = C_{\mathsf{am}}\,\alpha(T) + \tilde{\alpha}_{\mathsf{am}}.$$

Mutual friction from the normal component

Walls, turbulence, etc

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Front velocity (energy dissipation): $V_{\text{f}} = \alpha_{\text{en}}\,\Omega_{\text{s}} R.$

Rotation behind the front Ω_{s} depends on the angular momentum transfer:

- Spin-up from the rotating container. Parameter $\text{Re}_\alpha \approx 1/\alpha_{\text{am}}$.
- Momentum transfer by line tension. Parameter $\text{Re}_\lambda = UR/\lambda$.

$$U = \Omega R$$

$$\lambda = \frac{\kappa}{4\pi} \ln \frac{\text{intervortex spacing}}{\text{core size}}$$

Simple model: $$\frac{\Omega_{\text{s}}}{\Omega} = \frac{1}{1 + \text{Re}_\alpha/\text{Re}_\lambda}$$

Nature Commun. **4**, 1614 (2013)

PHENOMENOLOGICAL MODEL OF THE FRONT PROPAGATION

In turbulent motion averaged hydrodynamics should include *effective* friction:

For energy: $\alpha_{\mathrm{en}} = C_{\mathrm{en}}\,\alpha(T) + \tilde{\alpha}_{\mathrm{en}}.$

Laminar dissipation at the outer scale — Turbulent cascade

For angular momentum: $\alpha_{\mathrm{am}} = C_{\mathrm{am}}\,\alpha(T) + \tilde{\alpha}_{\mathrm{am}}.$

Mutual friction from the normal component — Walls, turbulence, etc

Front velocity (energy dissipation): $V_{\mathrm{f}} = \alpha_{\mathrm{en}}\,\Omega_{\mathrm{s}} R.$

Rotation behind the front Ω_{s} depends on the angular momentum transfer:

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Nature Commun. **4**, 1614 (2013)

Decoupled regime (small α):

$$\Omega_{\mathrm{s}} \approx (\mathrm{Re}_\lambda/\mathrm{Re}_\alpha)\,\Omega, \quad V_{\mathrm{f}} \propto \Omega^2.$$

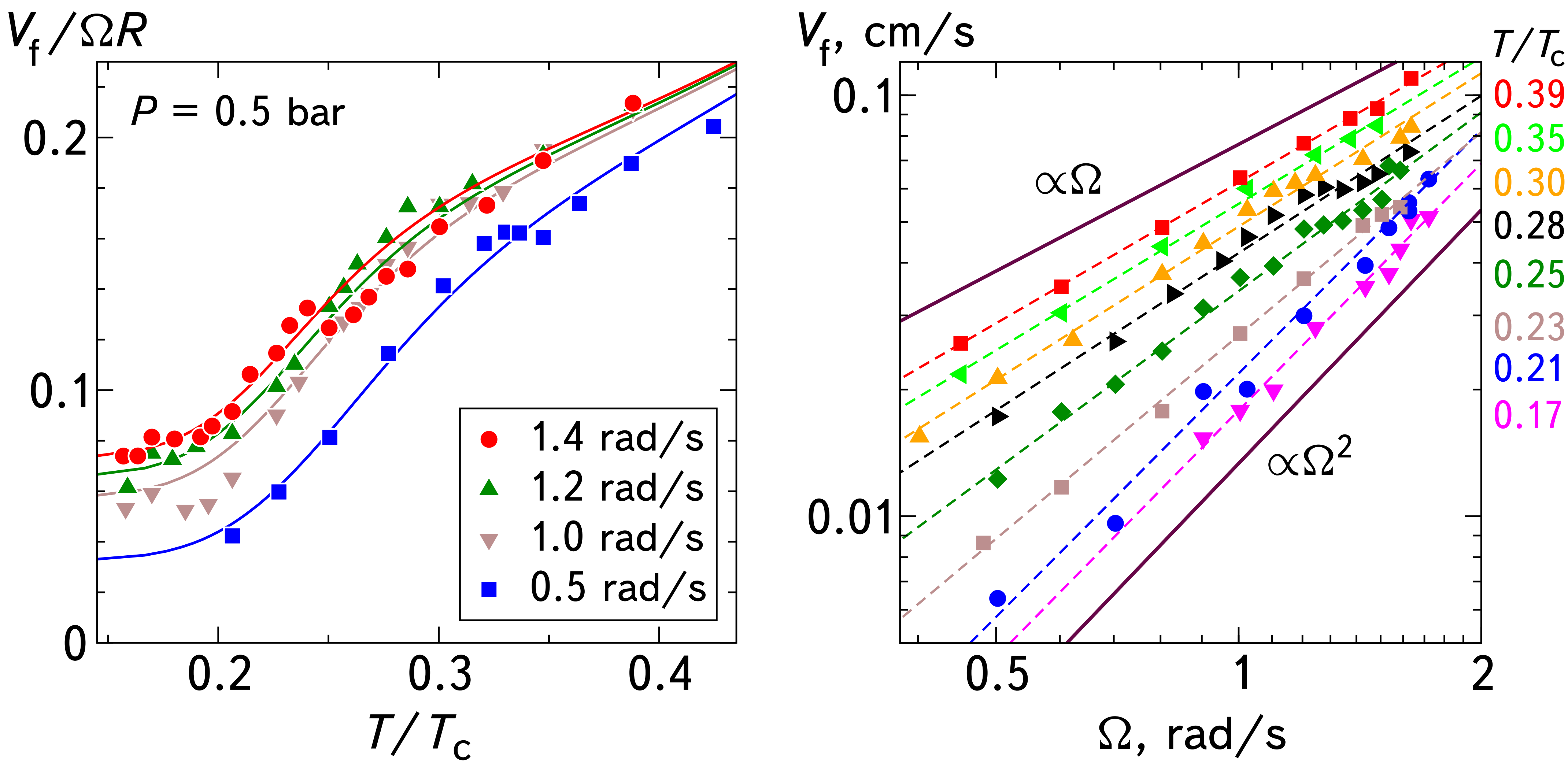
Coupled regime (large α):

$$\Omega_{\mathrm{s}} \approx \Omega, \quad V_{\mathrm{f}} \propto \Omega.$$

$\mathrm{Re}_\alpha \sim \mathrm{Re}_\lambda$

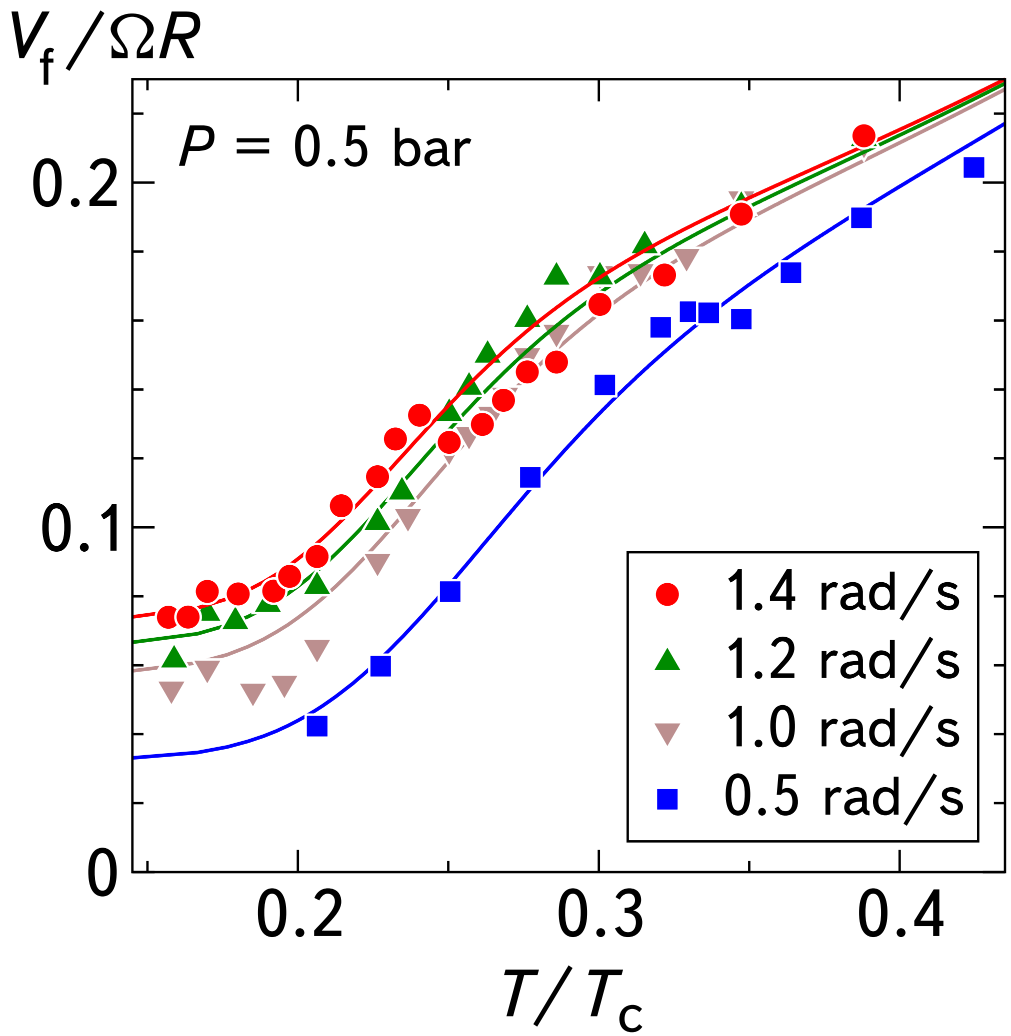
T/T_{c}

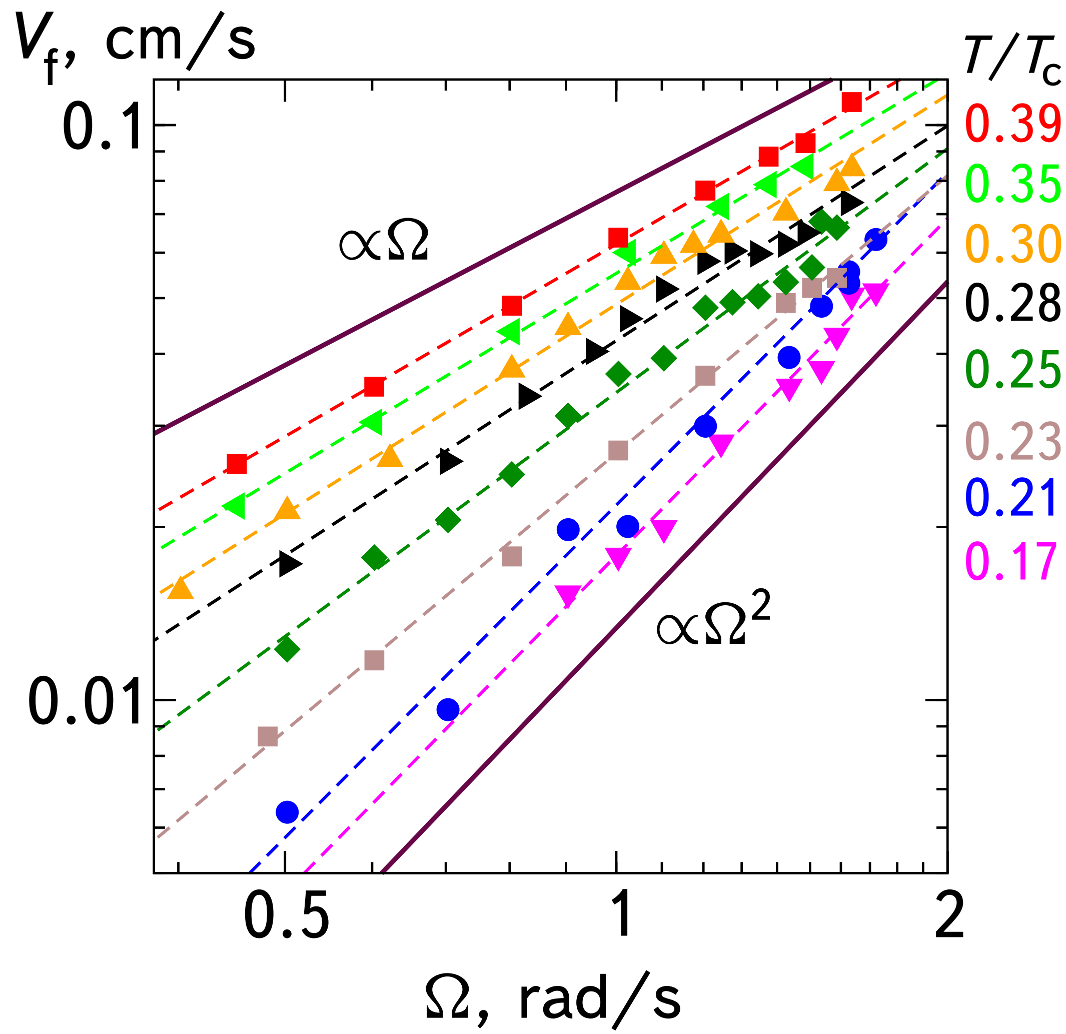
FRONT VELOCITY VS TEMPERATURE AND ROTATION



Nature Commun. **4**, 1614 (2013)

FRONT VELOCITY VS TEMPERATURE AND ROTATION





Energy: $\alpha_{en} = C_{en}\,\alpha(T) + \tilde{\alpha}_{en}$

$C_{en} \approx 0.52$

$\tilde{\alpha}_{en} \approx 0.20$

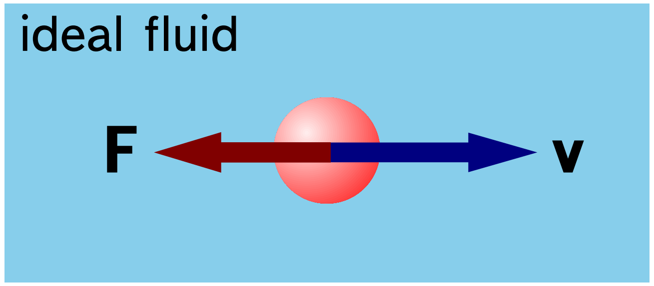
Angular momentum: $\alpha_{am} = C_{am}\,\alpha(T) + \tilde{\alpha}_{am}$

$C_{am} \approx 1.33$

$\tilde{\alpha}_{am} \approx 0.002$

Nature Commun. **4**, 1614 (2013)

NEW REGIME OF SUPERFLUID HYDRODYNAMICS



D'Alembert's paradox (1752):

drag $\mathbf{F} = 0$, dissipation $\mathbf{F} \cdot \mathbf{v} = 0$.

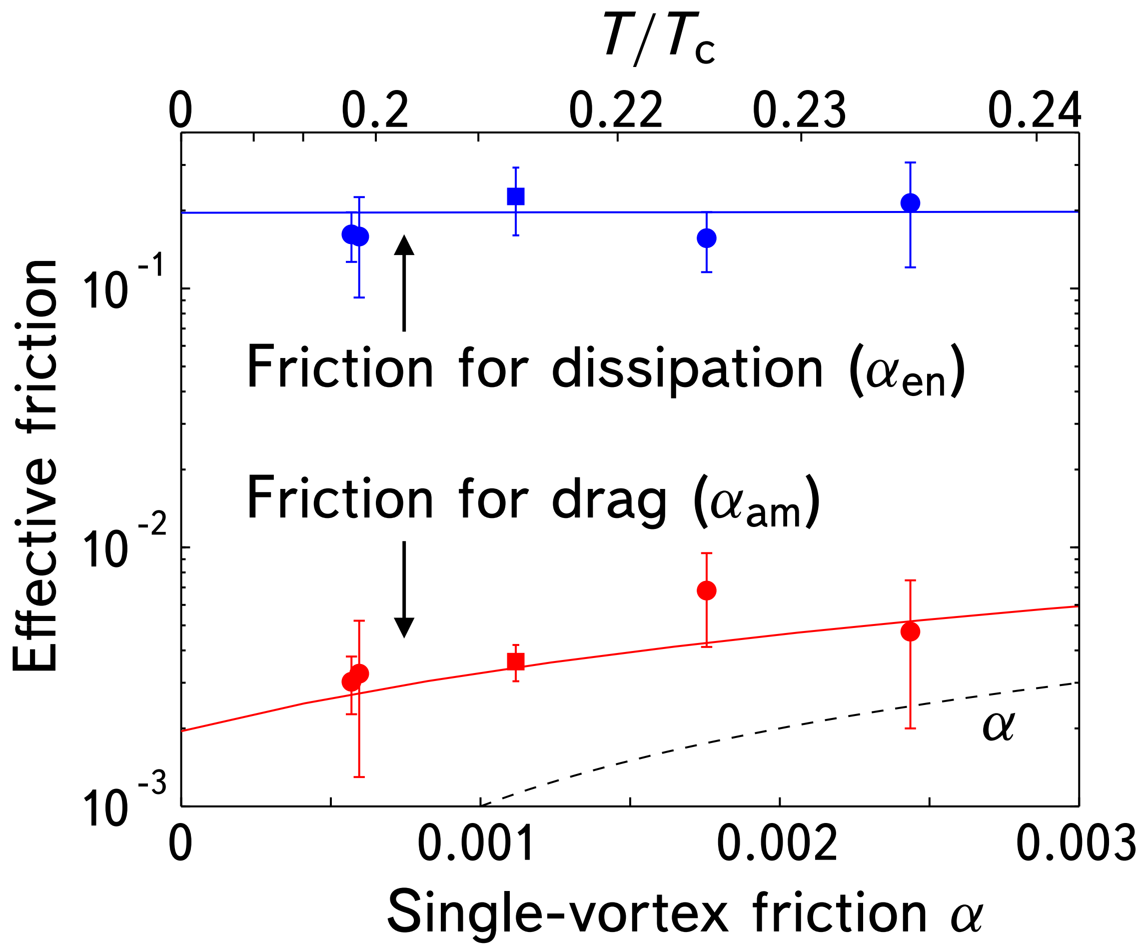
(classical solution: Prandtl 1904 – boundary layers)

Owing to quantum turbulence (QT) in superfluids even when $T \to 0$:

dissipation > 0,

drag $\to 0$.

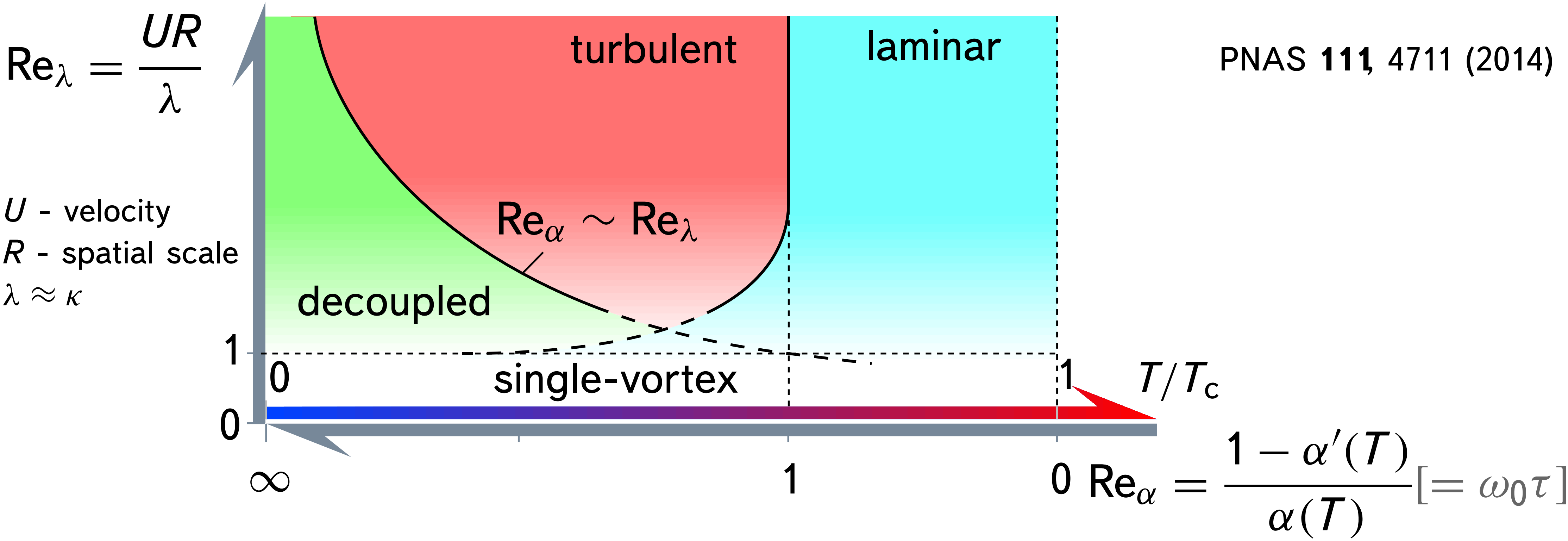
No effective boundary layers, decoupling of superfluid from the reference frame.



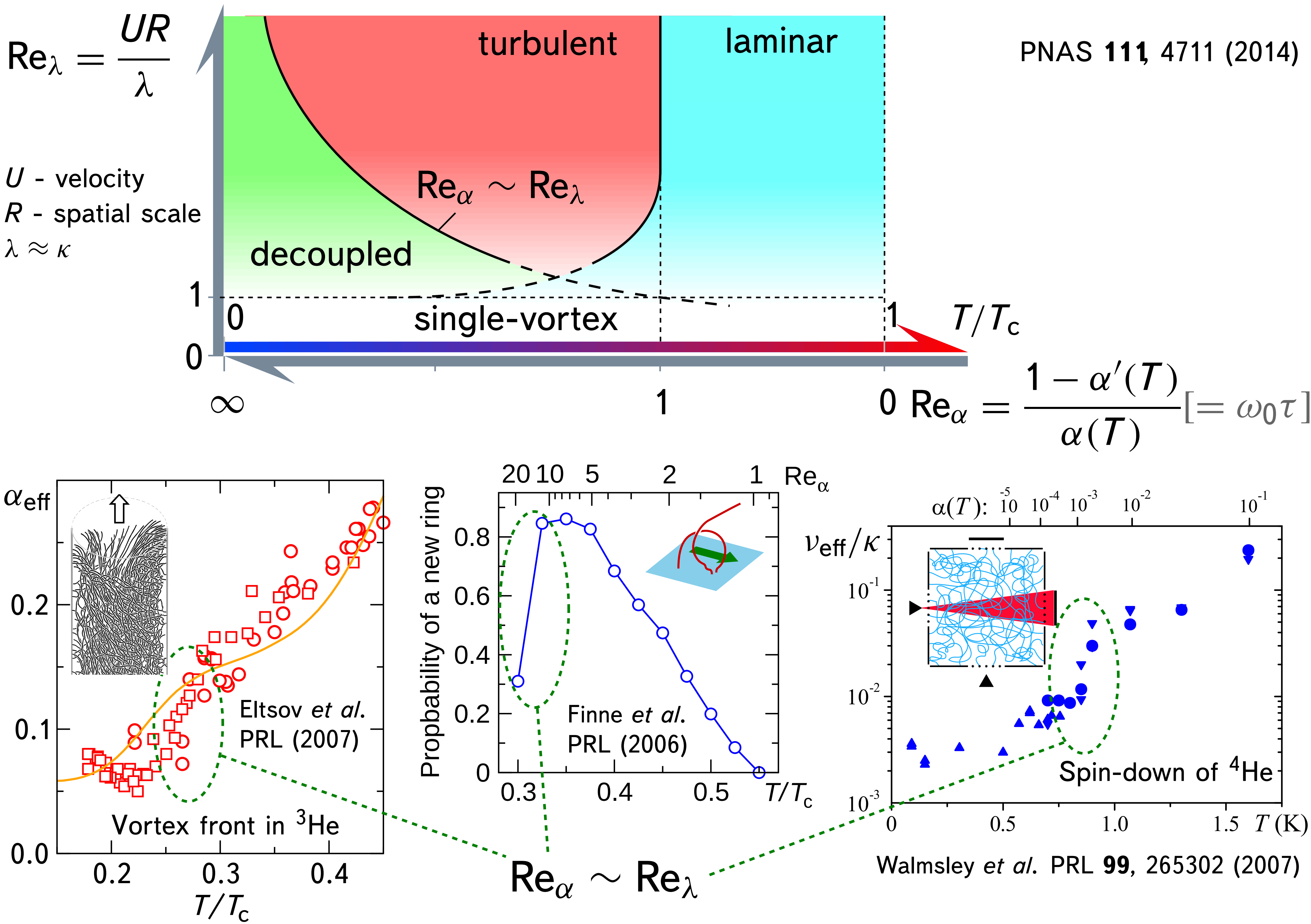
Nature Commun. **4**, 1614 (2013)

REGIMES OF SUPERFLUID HYDRODYNAMICS

PNAS **111**, 4711 (2014)



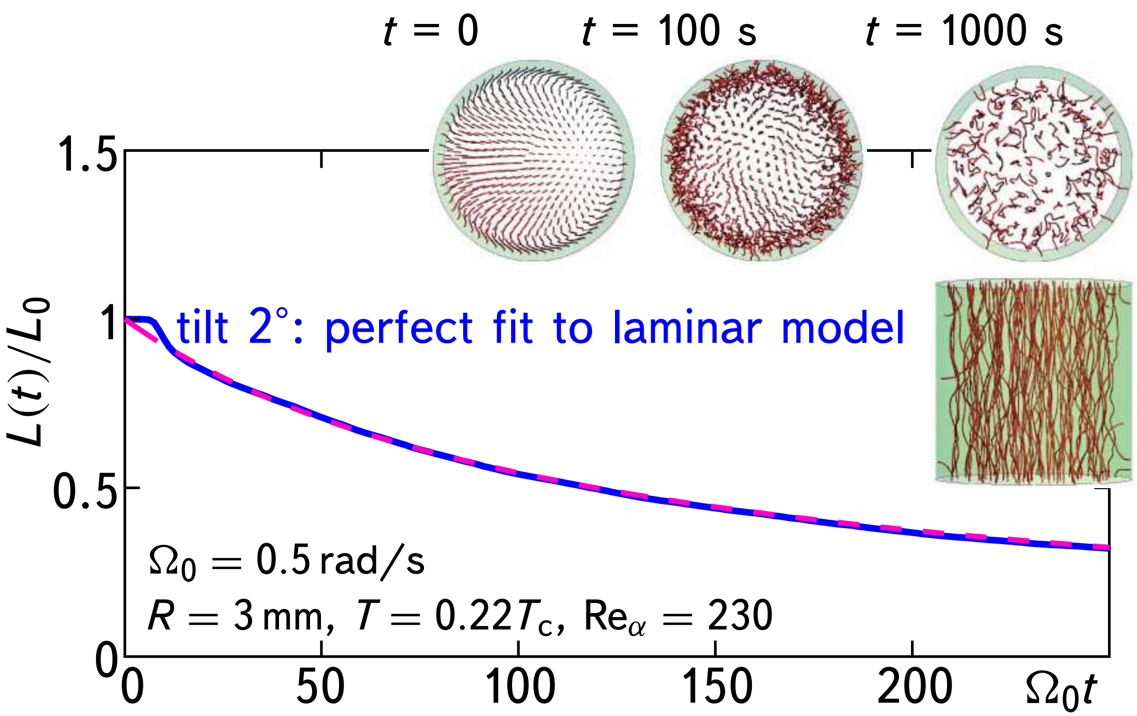
REGIMES OF SUPERFLUID HYDRODYNAMICS

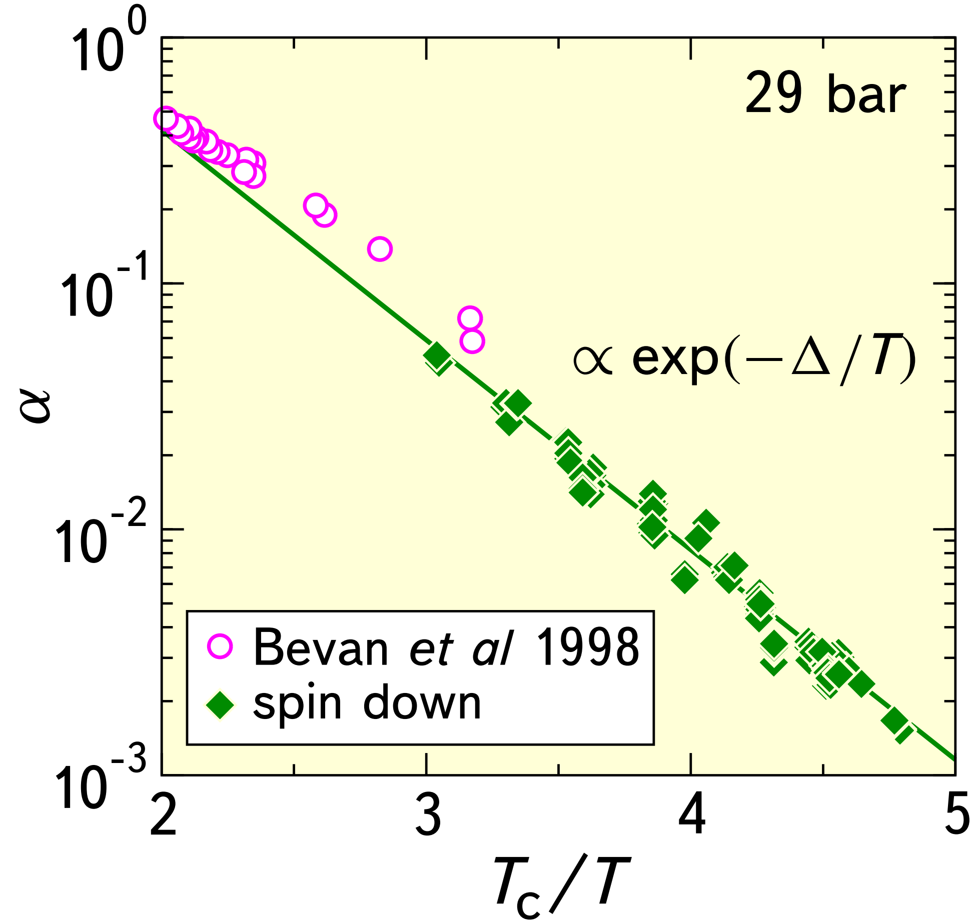


MUTUAL FRICTION AT LOW TEMPERATURES

Kopnin theory: $\alpha \sim \dfrac{\hbar E_F}{\Delta^2 \tau_n} \exp(-\Delta/T)$.

Measurement: laminar spin-down.





PRL **105**, 125301 (2010)

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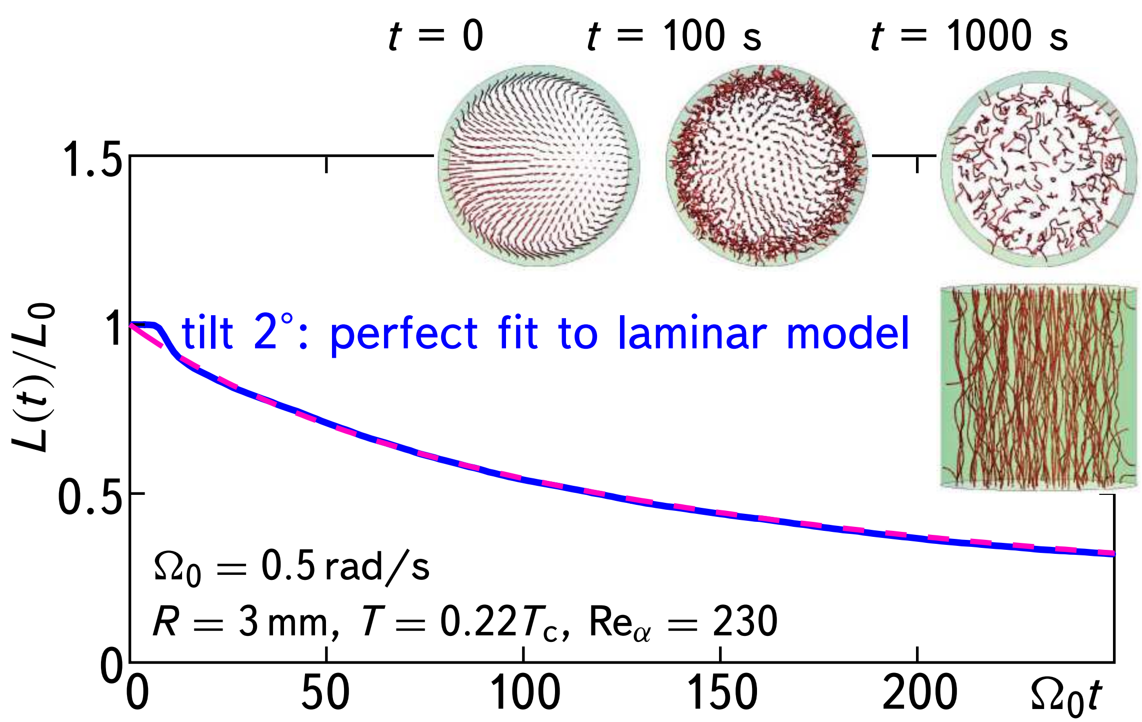
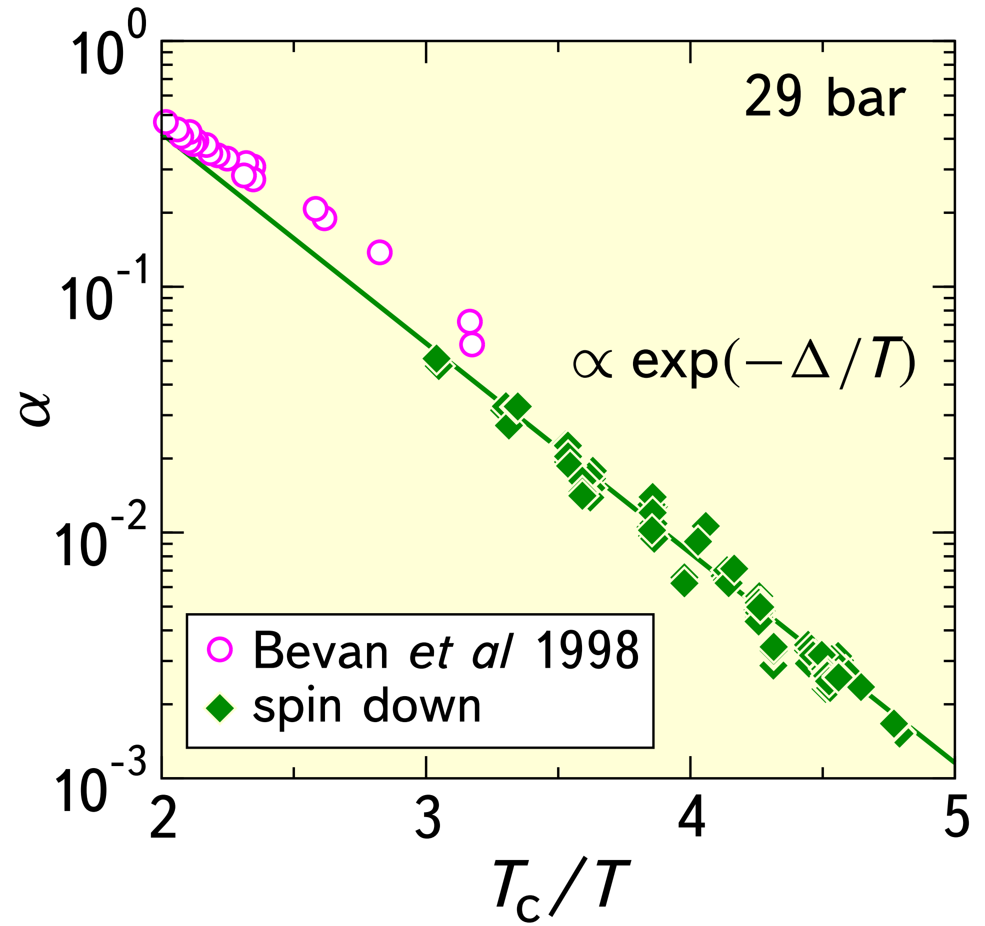


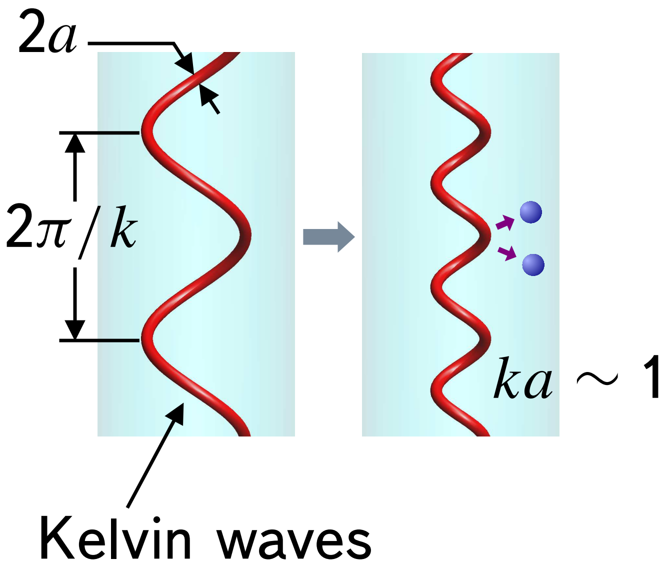


PRL **105**, 125301 (2010)

Open questions in $T \to 0$ limit:

- Finite τ (walls, kinks) and thus finite α?
- Core states "overflow" and $\mathbf{v}_L$-dependent α?
- Interaction with Kelvin waves?

MUTUAL FRICTION AT ULTRA-LOW TEMPERATURES



Theoretical prediction for finite dissipation at $T \to 0$:

Cascade of Kelvin waves towards larger k

$\Rightarrow$ interaction with bound fermions

$\Rightarrow$ emission of bulk quasiparticles.

Silaev, PRL **108**, 045303 (2012)

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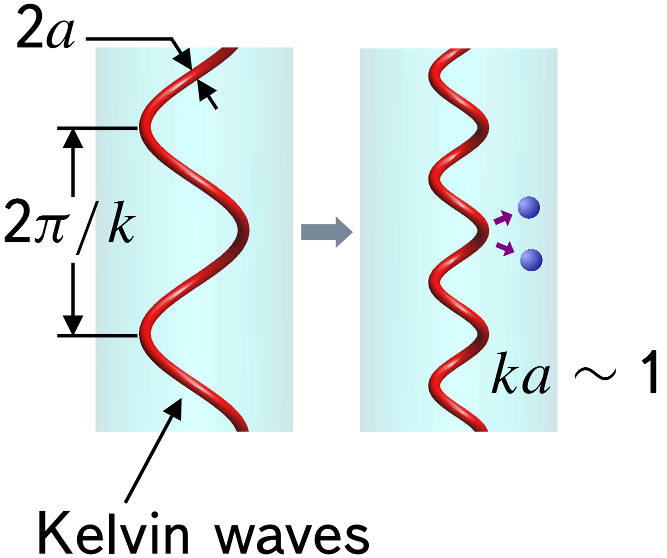

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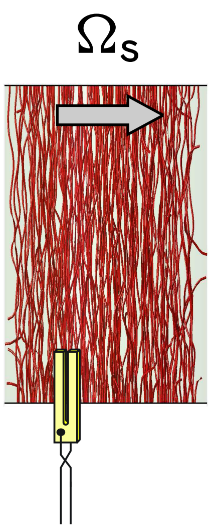
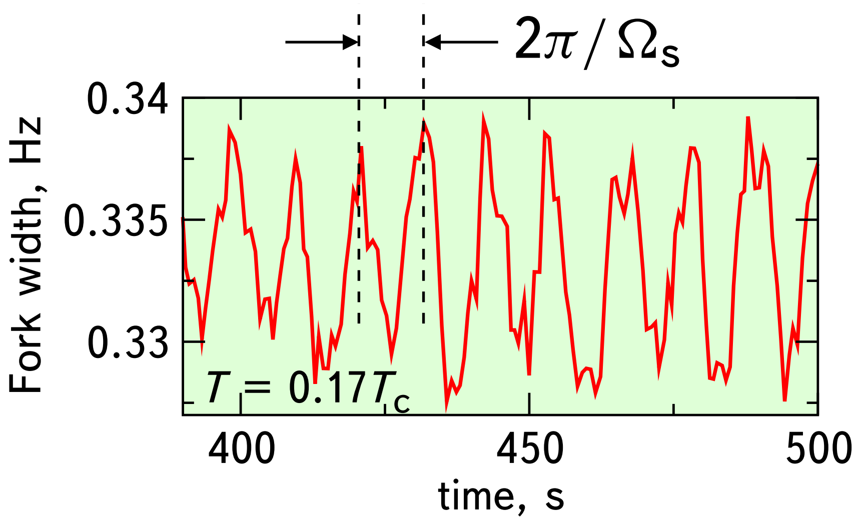
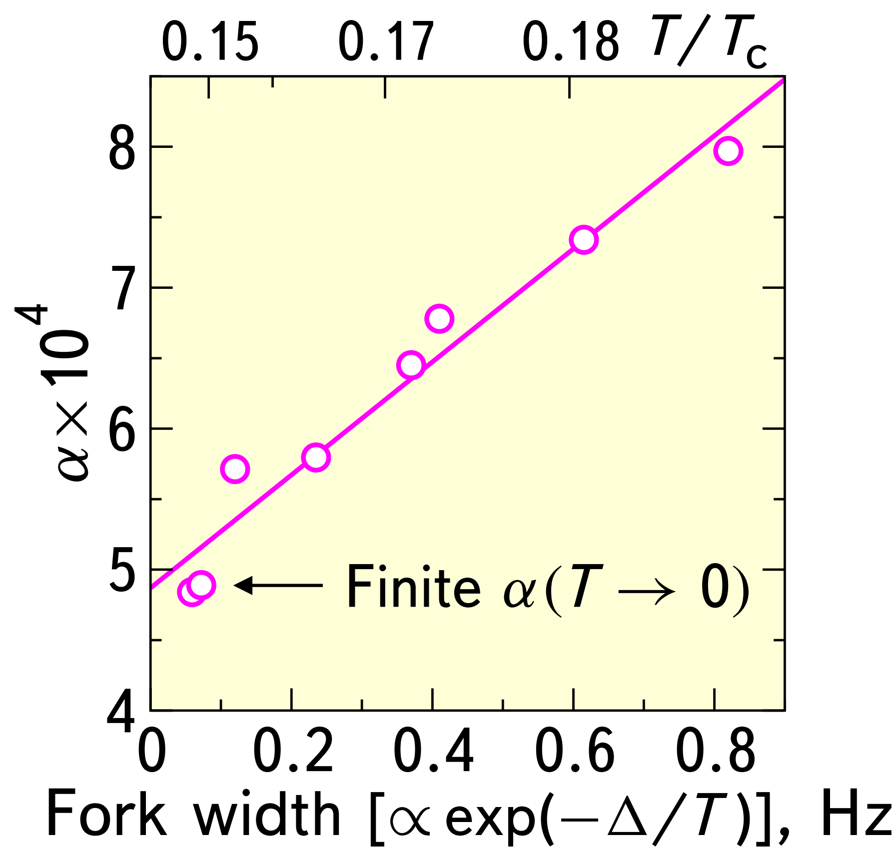
Silaev, PRL **108**, 045303 (2012)

Observation of spin-down by Andreev reflection:



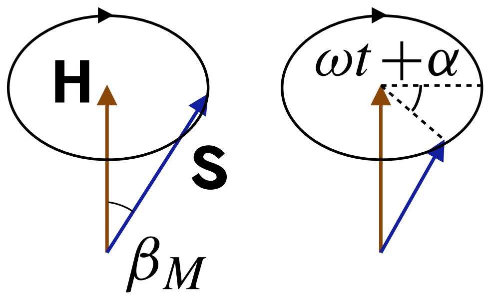


Hosio et al, PRB **85**, 224526 (2012)



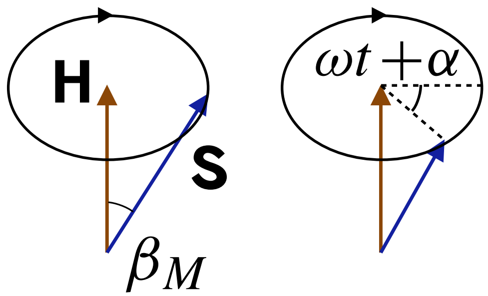
PROBING VORTEX-CORE- AND SURFACE-BOUND FERMIONS WITH MAGNON BEC

TRAPPED MAGNON CONDENSATES IN ^{3}He-B



Spin waves $\rightarrow$ magnons with spin $-\hbar$: $\hat{N}_{\mathrm{m}} = \dfrac{S - \hat{S}_z}{\hbar}$

TRAPPED MAGNON CONDENSATES IN ^{3}He-B



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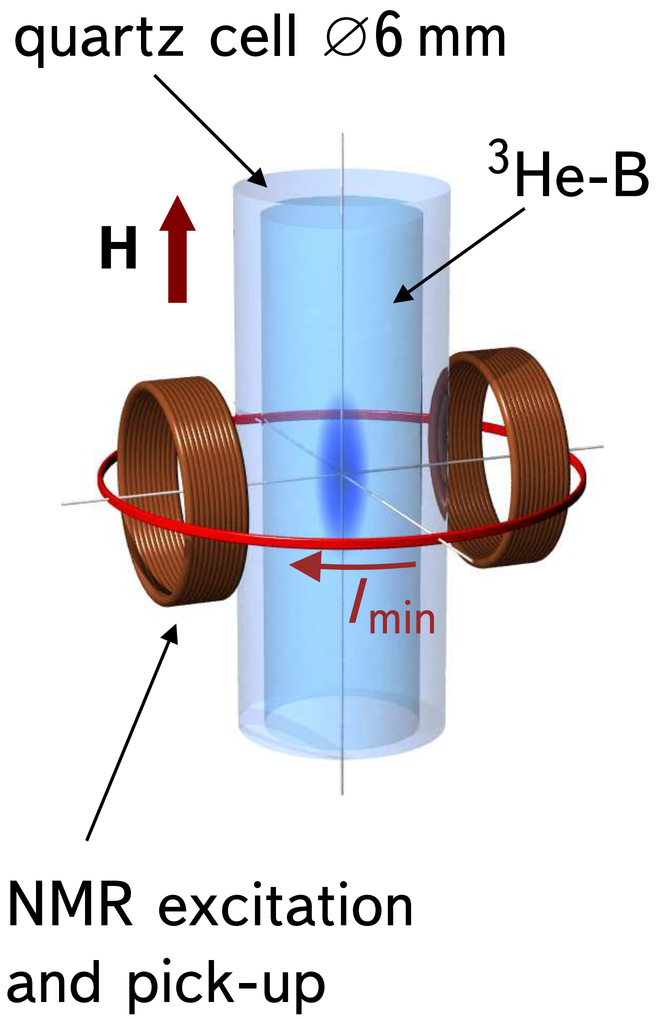
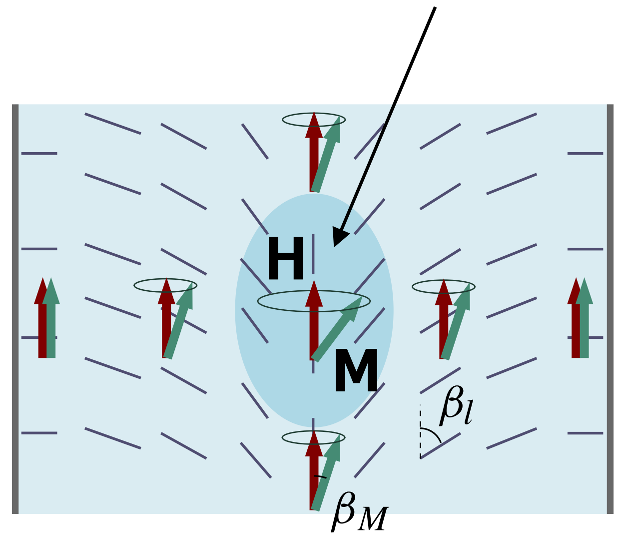
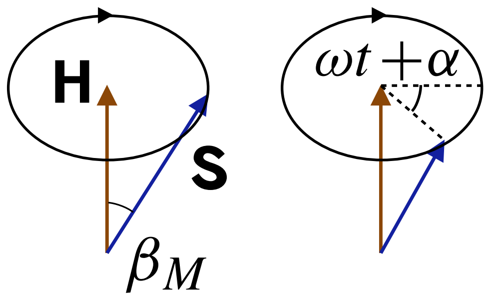
Magnon condensate in ^{3}He-B: $\Psi(\mathbf{r}) \propto \sin\dfrac{\beta_M(\mathbf{r})}{2} e^{i\omega t + i\alpha(\mathbf{r})}$



Coherently precessing magnetization



TRAPPED MAGNON CONDENSATES IN ^{3}He-B



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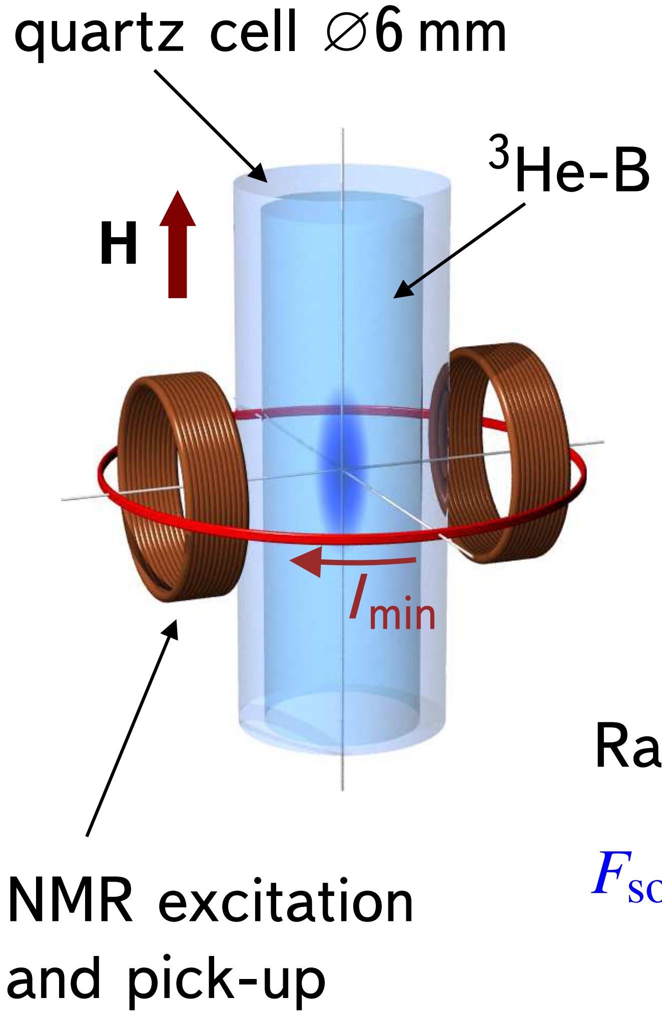
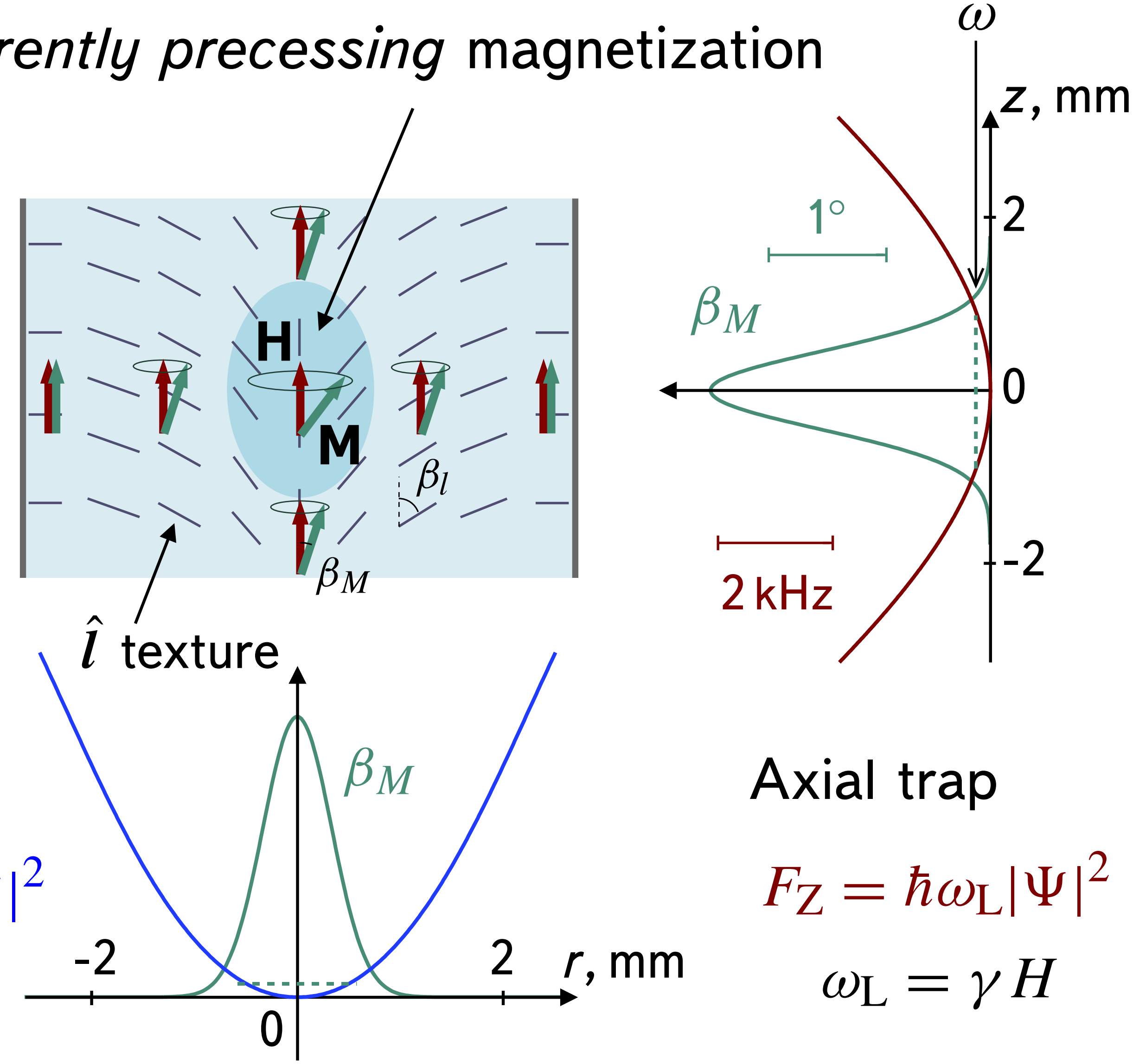




Radial trap

$F_{\rm so} \propto \sin^2 \dfrac{\beta_l}{2} |\Psi|^2$

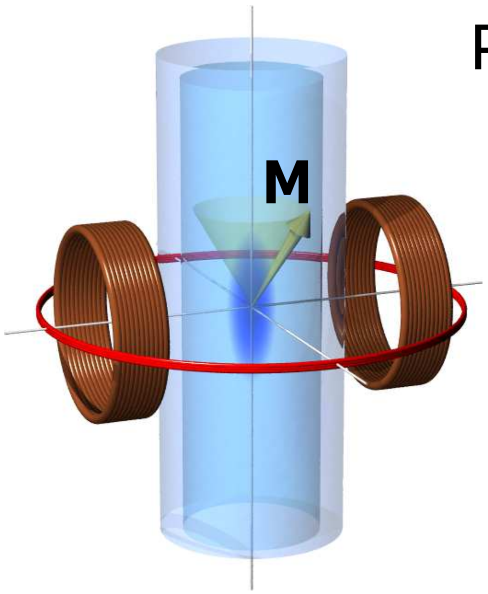
Axial trap

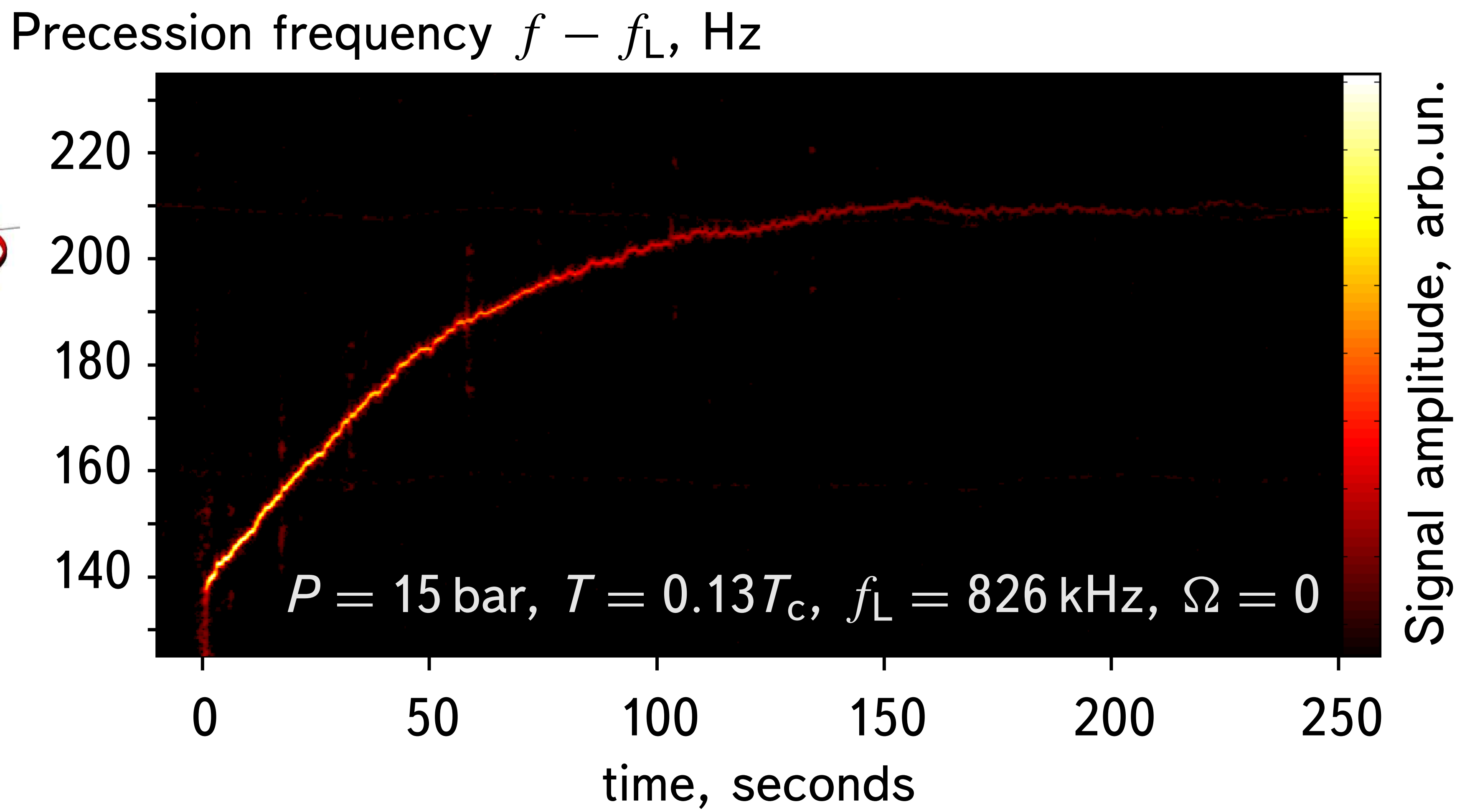
$F_{\rm Z} = \hbar\omega_{\rm L} |\Psi|^2$

$\omega_{\rm L} = \gamma H$

COHERENT PRECESSION OF MAGNON CONDENSATE

Spontaneous coherence: Excitation with higher frequency or noise.





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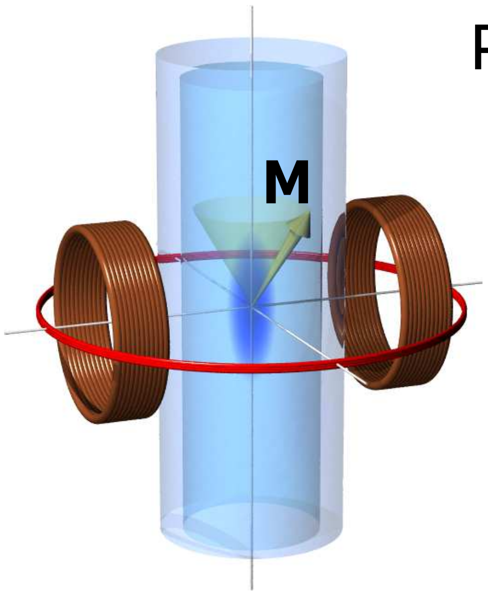
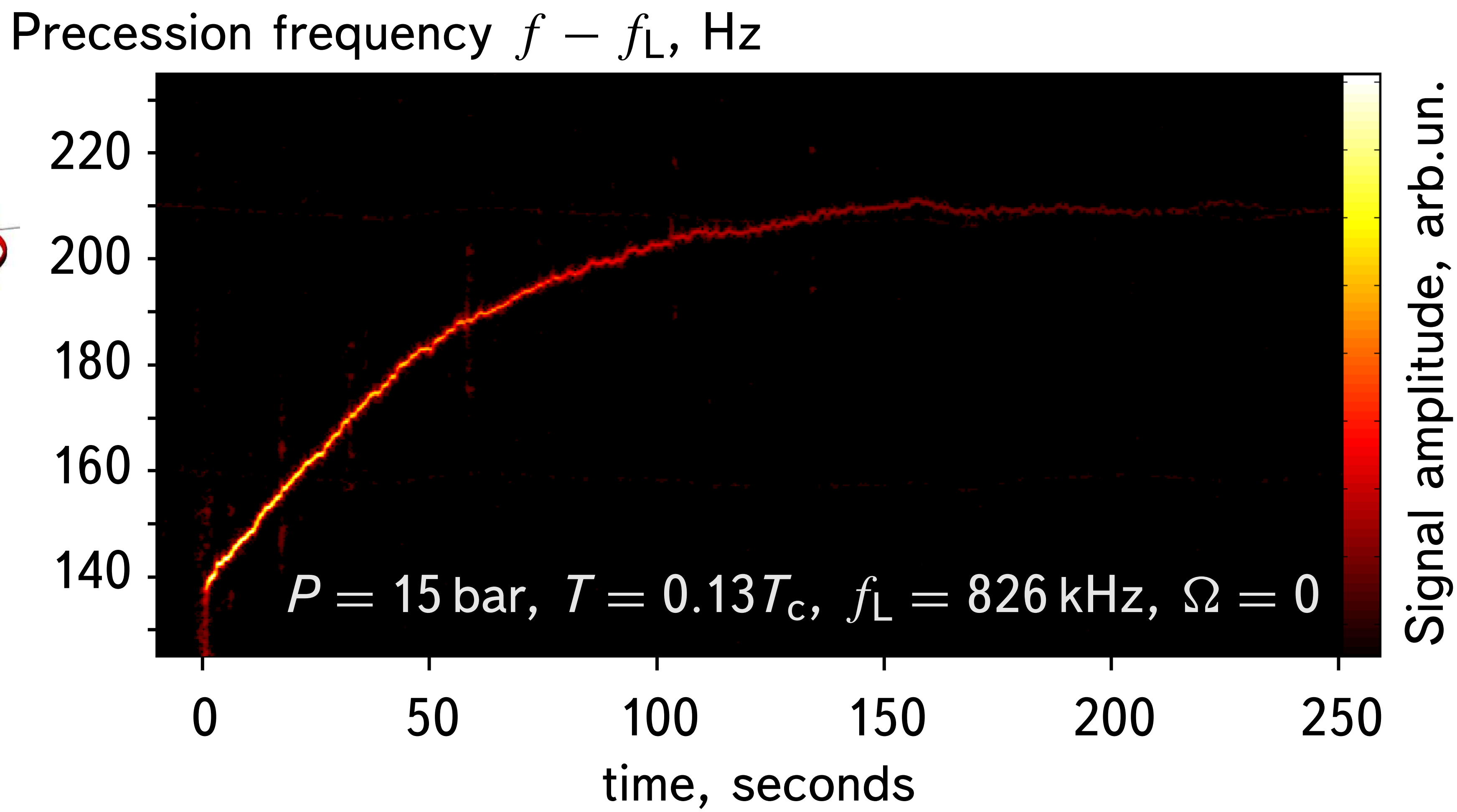
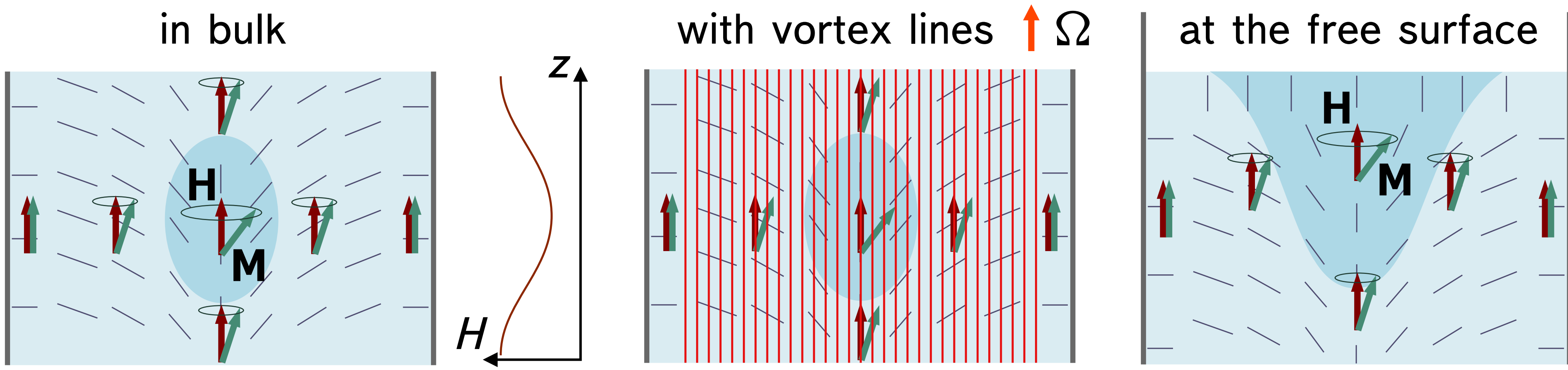




- Self-modification of the trap.
- Sensitive probe of relaxation sources.

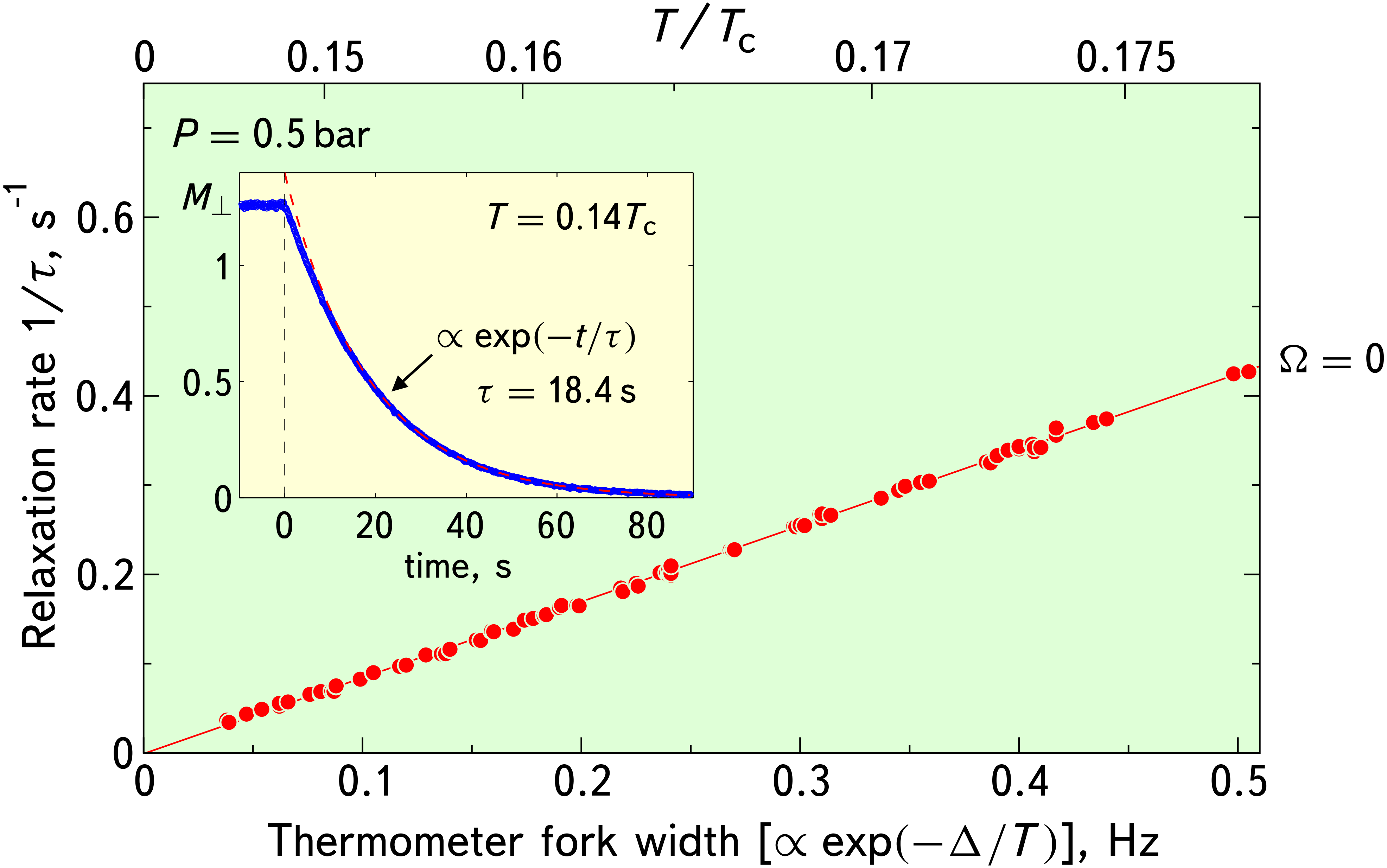
MOTIVATION FOR RELAXATION STUDIES

Long life time of the magnon BEC in the $T \to 0$ limit makes them a sensitive probe for extra relaxation sources.



We are looking for the contribution from fermions bound to the surface or the cores of quatized vortices to the relaxation of magnon condensates.

RELAXATION IN THE VORTEX STATE



Relaxation rate: $1/\tau = 1/\tau_0 + C\,\exp(-\Delta/T)$

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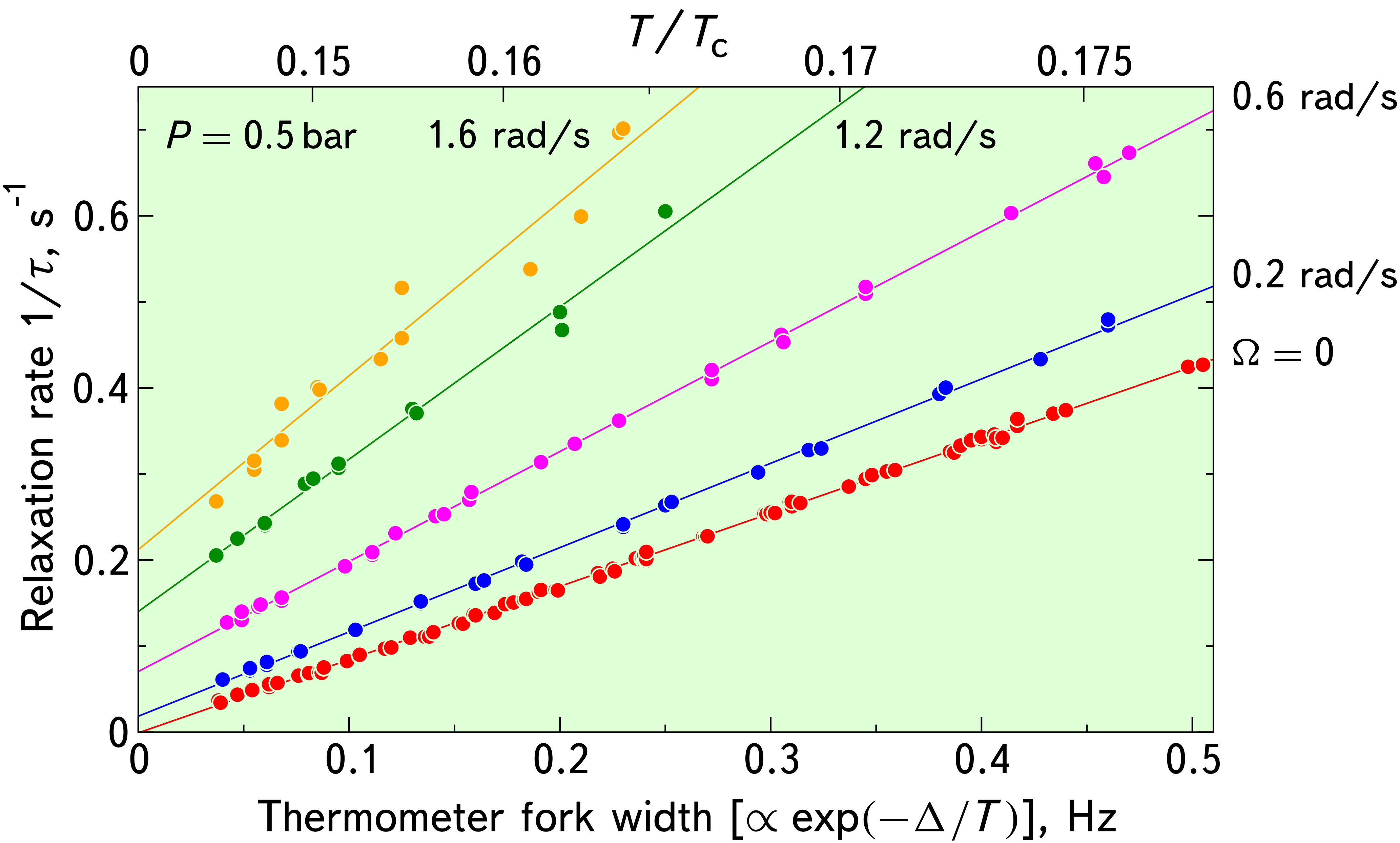
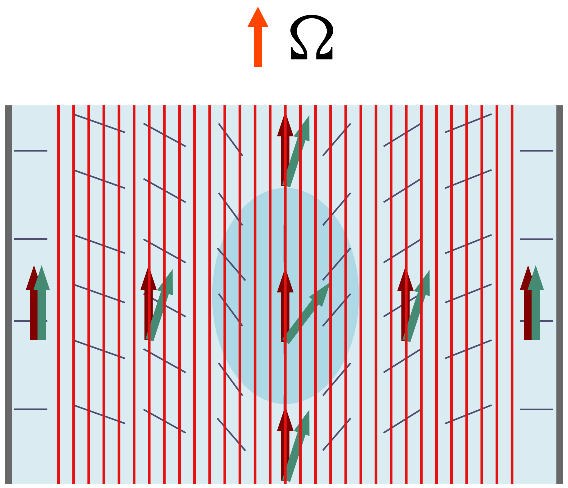
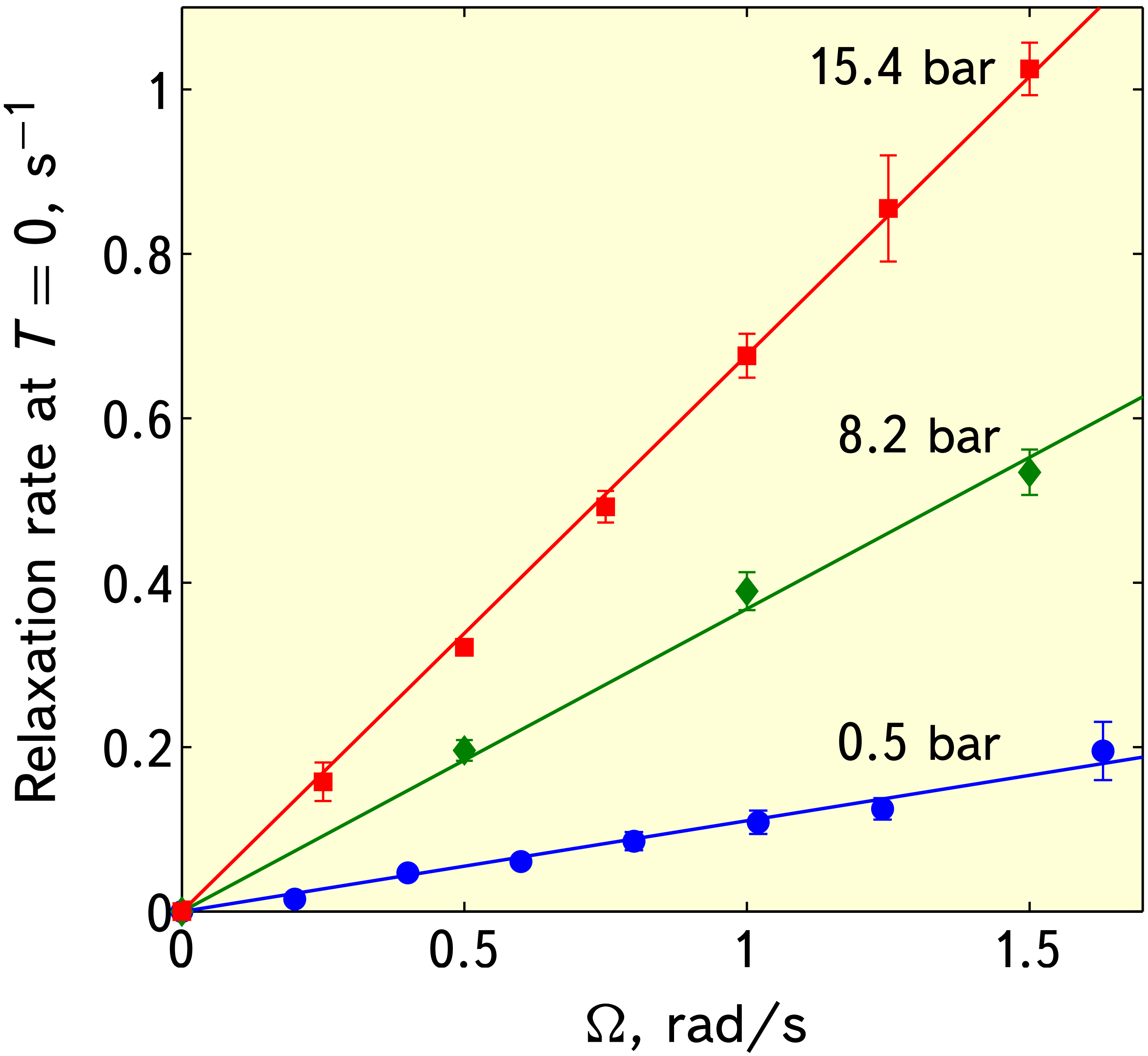


Relaxation rate: $1/\tau = 1/\tau_0(\Omega) + C(\Omega)\exp(-\Delta/T)$

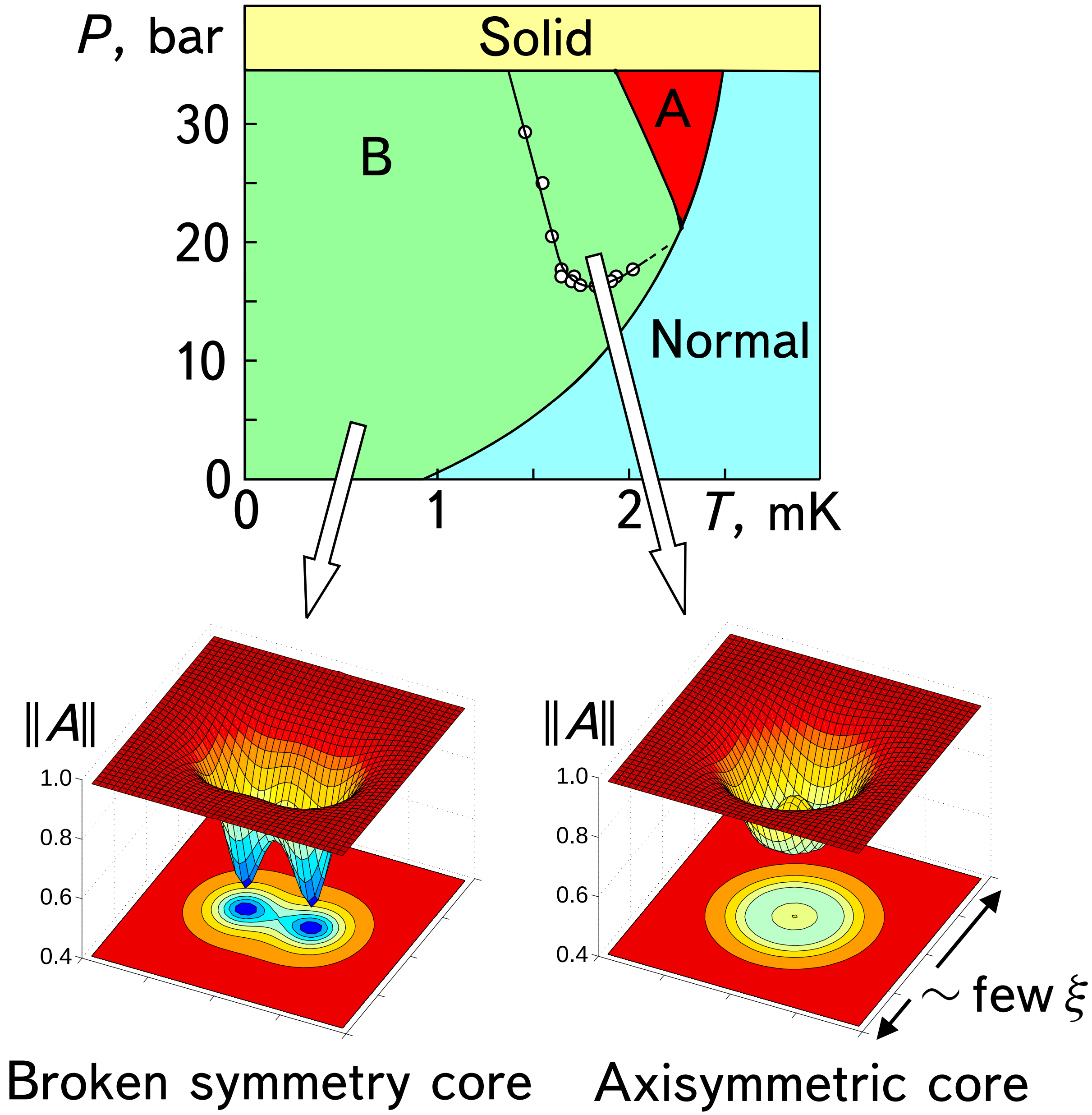
DEPENDENCE OF RELAXATION ON VORTEX DENSITY





Relaxation $\propto$ vortex density and increases with decreasing core size.

BROKEN SYMMETRY OF VORTEX CORES IN ^{3}He-B



Ikkala, Hakonen, Bunkov, Krusius et al 1982-
Salomaa, Volovik, Thuneberg et al

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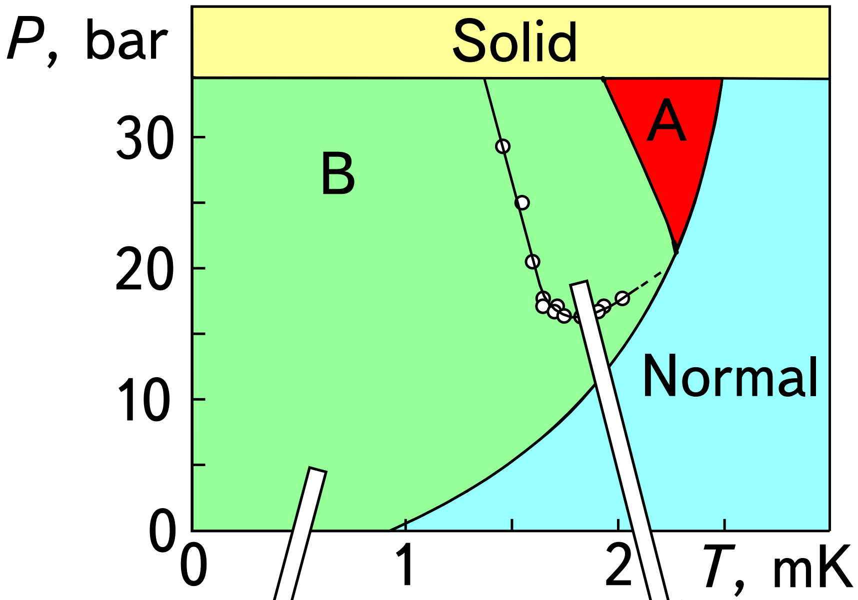
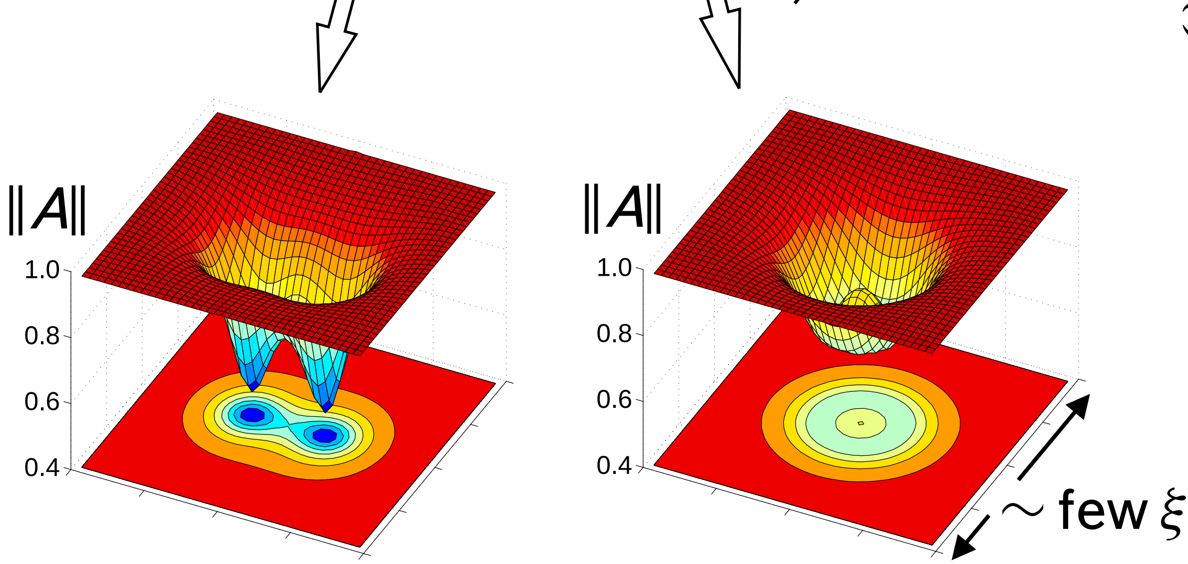
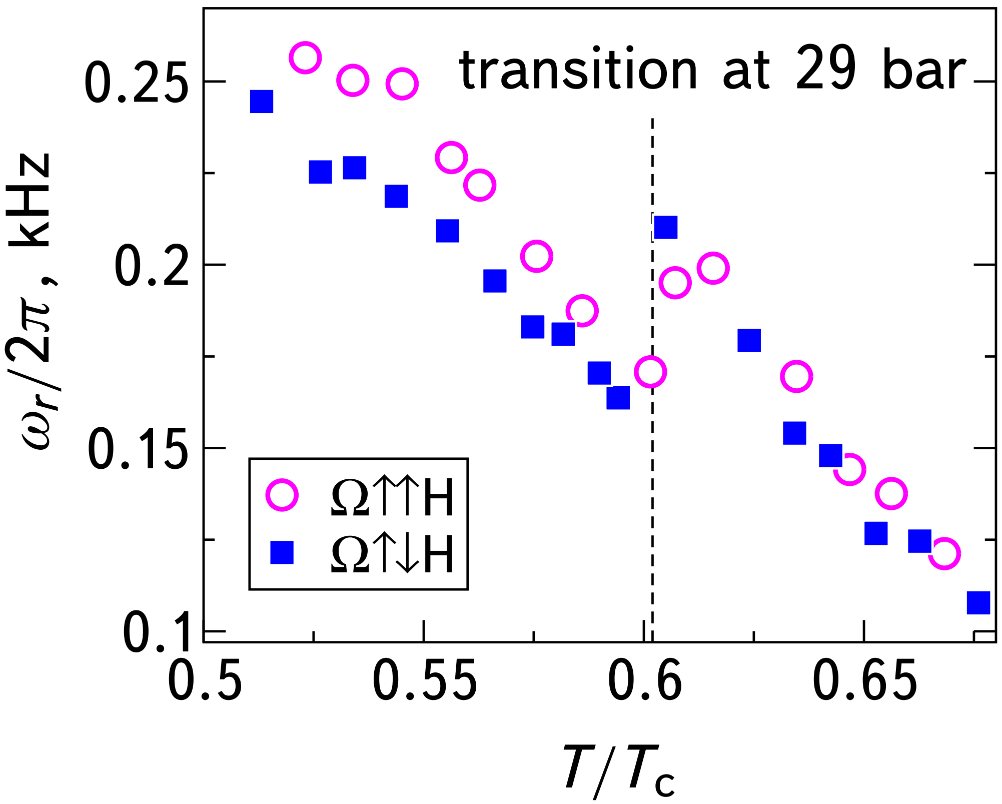




Broken symmetry core   Axisymmetric core

Effect on the textural potential well for magnons:



Ikkala, Hakonen, Bunkov, Krusius et al 1982-
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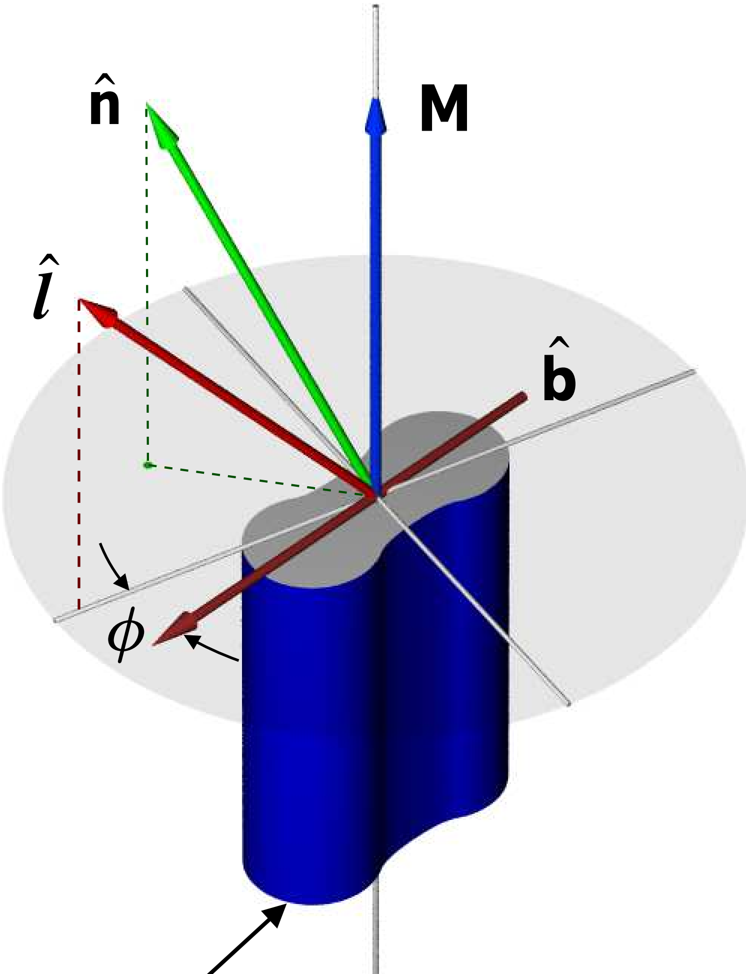
RELAXATION OF SPIN PRECESSION VIA VORTEX CORES

Energy of the non-axisymmetric core: $F = T_D(\hat{\mathbf{b}}\cdot\hat{\mathbf{n}})^2 - T_H(\hat{\mathbf{b}}\cdot\hat{\boldsymbol{l}})^2$

spin-orbit (dipolar) energy

magnetic anisotropy energy



Core of the non-axisymmetric vortex

Core motion: $f\dot{\phi} = -\dfrac{\delta F}{\delta \phi} + K\partial_z^2\phi$

(Kondo et al 1991)

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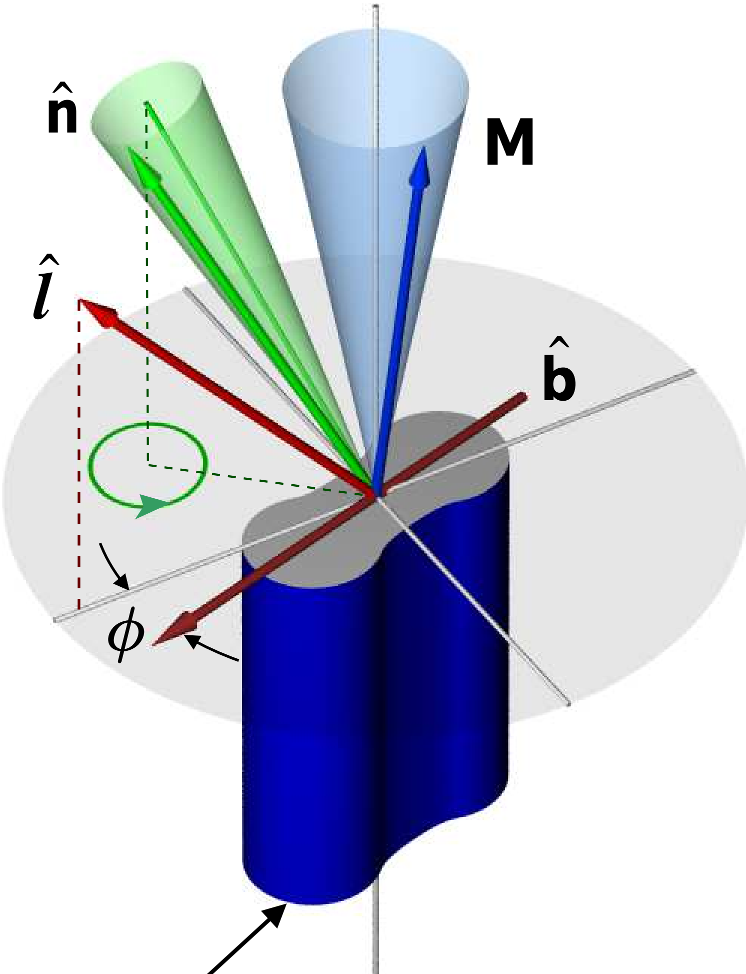

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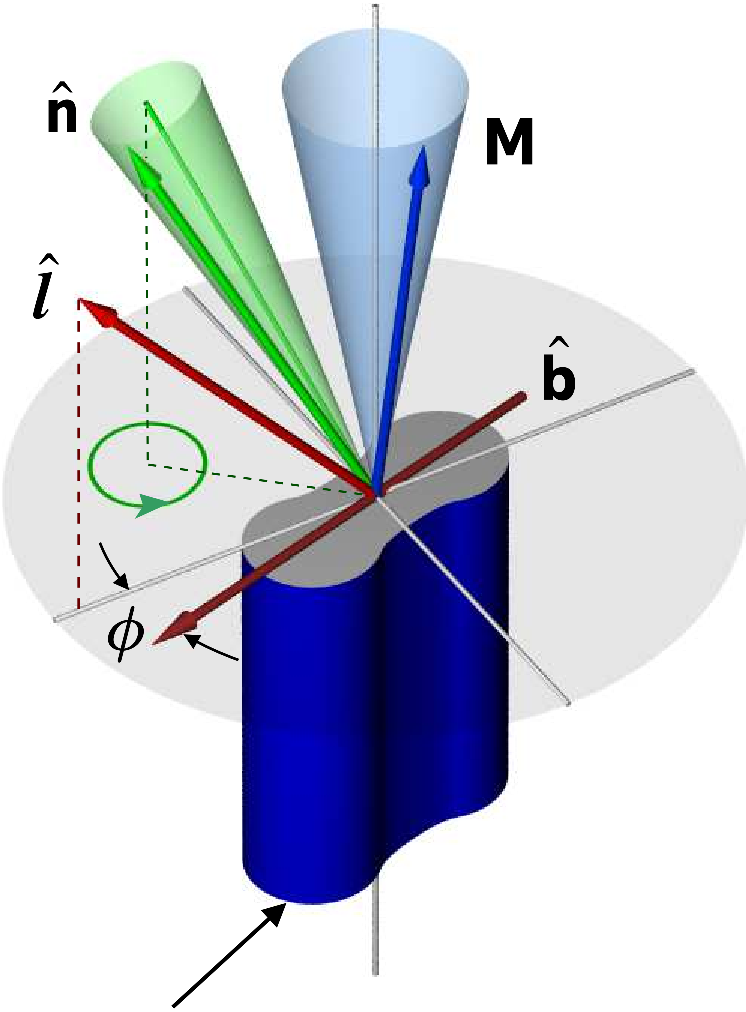

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Power $\langle \dot{\phi}^2 \rangle / f$ dissipates the Zeeman energy $\Rightarrow$

$$\frac{1}{\tau_{\mathrm{v}}} = N_{\mathrm{v}} \, \langle n_{0\perp}^{-2} \rangle \, \frac{\gamma^2}{\chi_{\mathrm{B}}} \, f \, \left(\frac{T_D}{T_H} \right)^2$$

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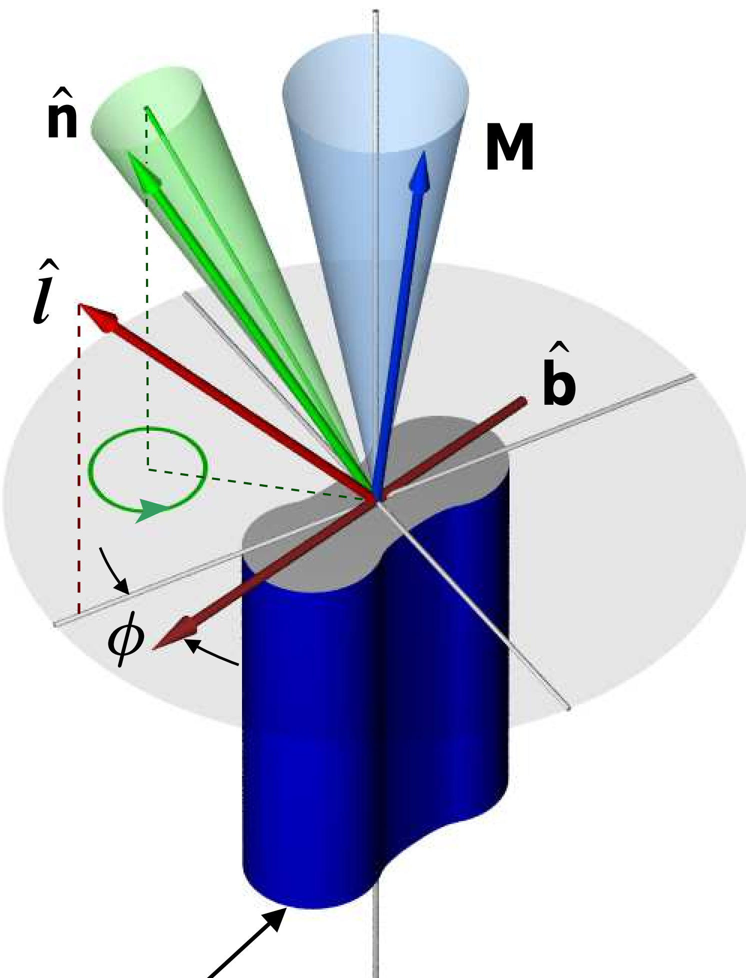
spin-orbit (dipolar) energy — magnetic anisotropy energy



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Information on the **core-bound fermions**

Information on the **order parameter** in the core

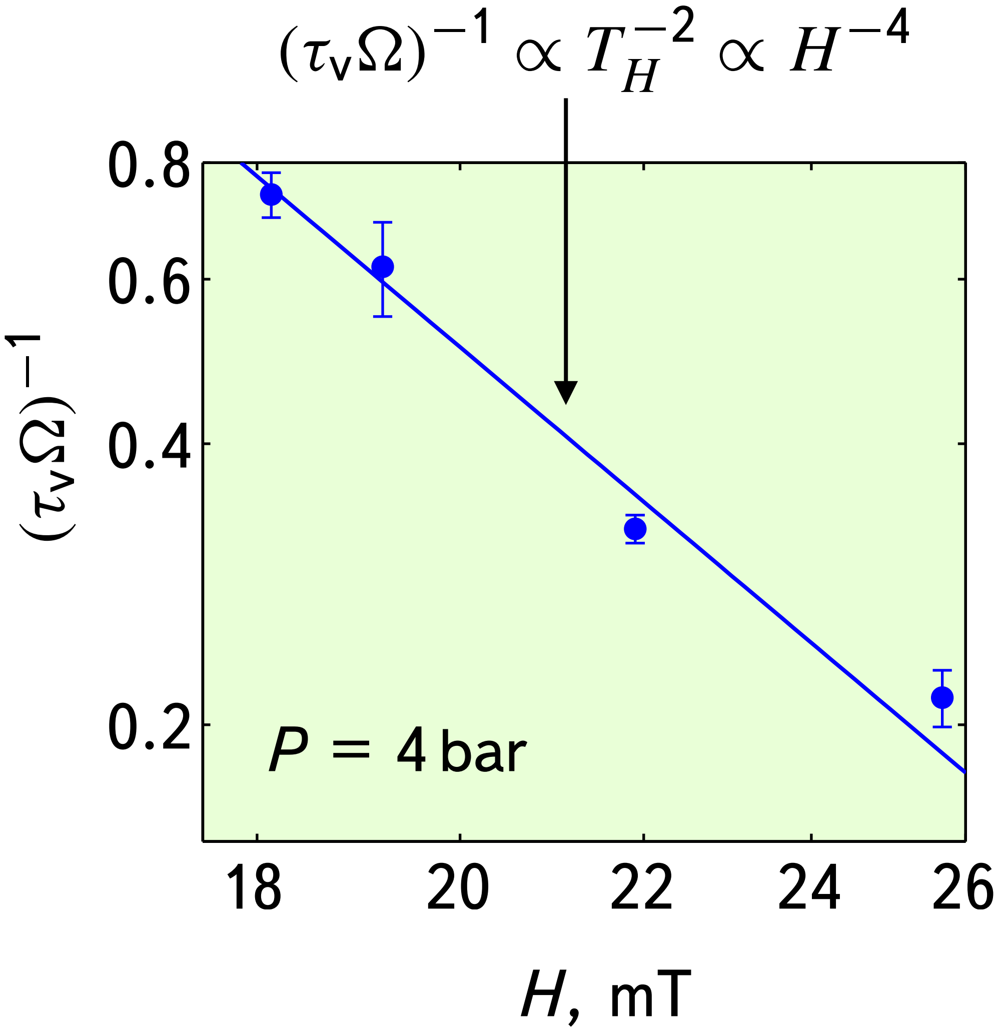
Kopnin & Volovik, PRB **57**, 8526 (1998)

DEPENDENCE OF RELAXATION ON PRESSURE AND MAGNETIC FIELD

$$\frac{1}{\tau_\mathsf{v}} = N_\mathsf{v} \langle n_{0\perp}^{-2} \rangle \frac{\gamma^2}{\chi_\mathsf{B}} f \left(\frac{T_D}{T_H} \right)^2$$

$N_\mathsf{v} \propto \Omega,\ f \sim k_\mathsf{F}(k_\mathsf{F} R_\mathsf{c})^2,\ R_\mathsf{c} \sim (m_\mathsf{eff}/m_3)\xi_0,$

$T_D \sim g_D \Delta_0^2 R_\mathsf{c} \xi_0,\ T_H \sim (\chi_\mathsf{N} - \chi_\mathsf{B}) H^2 R_\mathsf{c} \xi_0$



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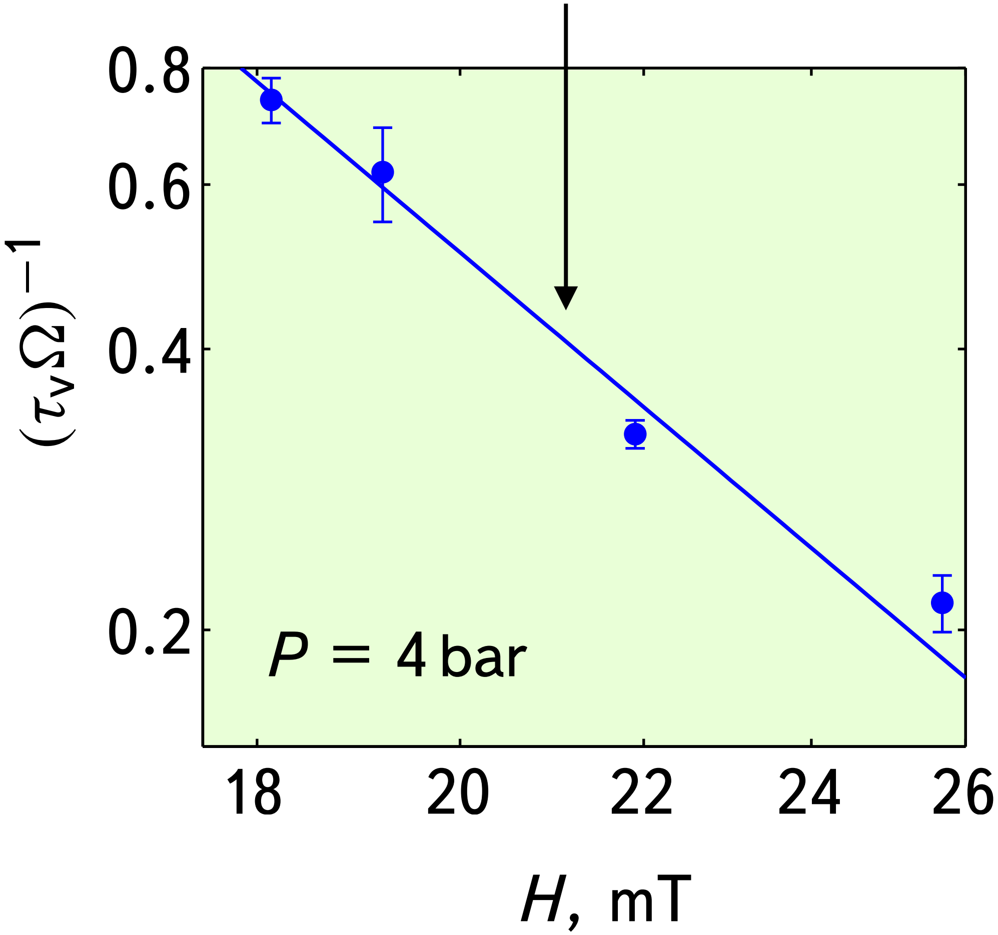
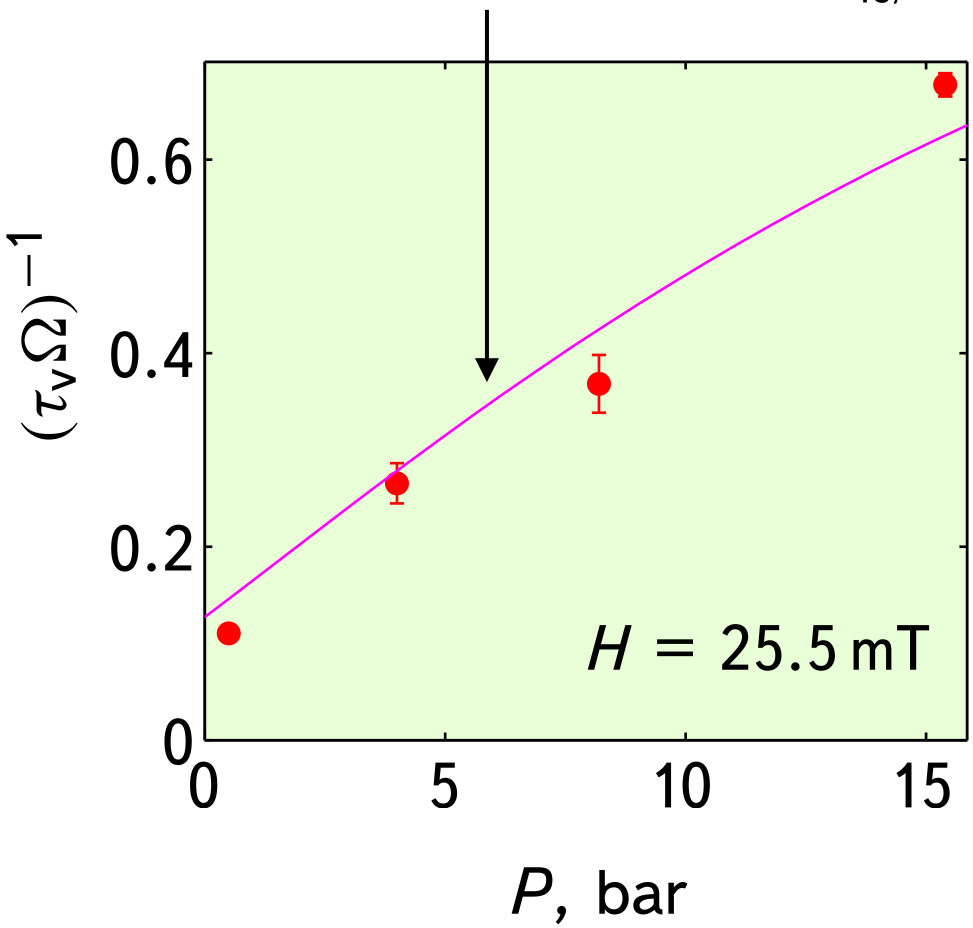
$$T_D \sim g_D\Delta_0^2 R_\mathrm{c}\xi_0,\ \ T_H \sim (\chi_\mathrm{N}-\chi_\mathrm{B})H^2 R_\mathrm{c}\xi_0$$

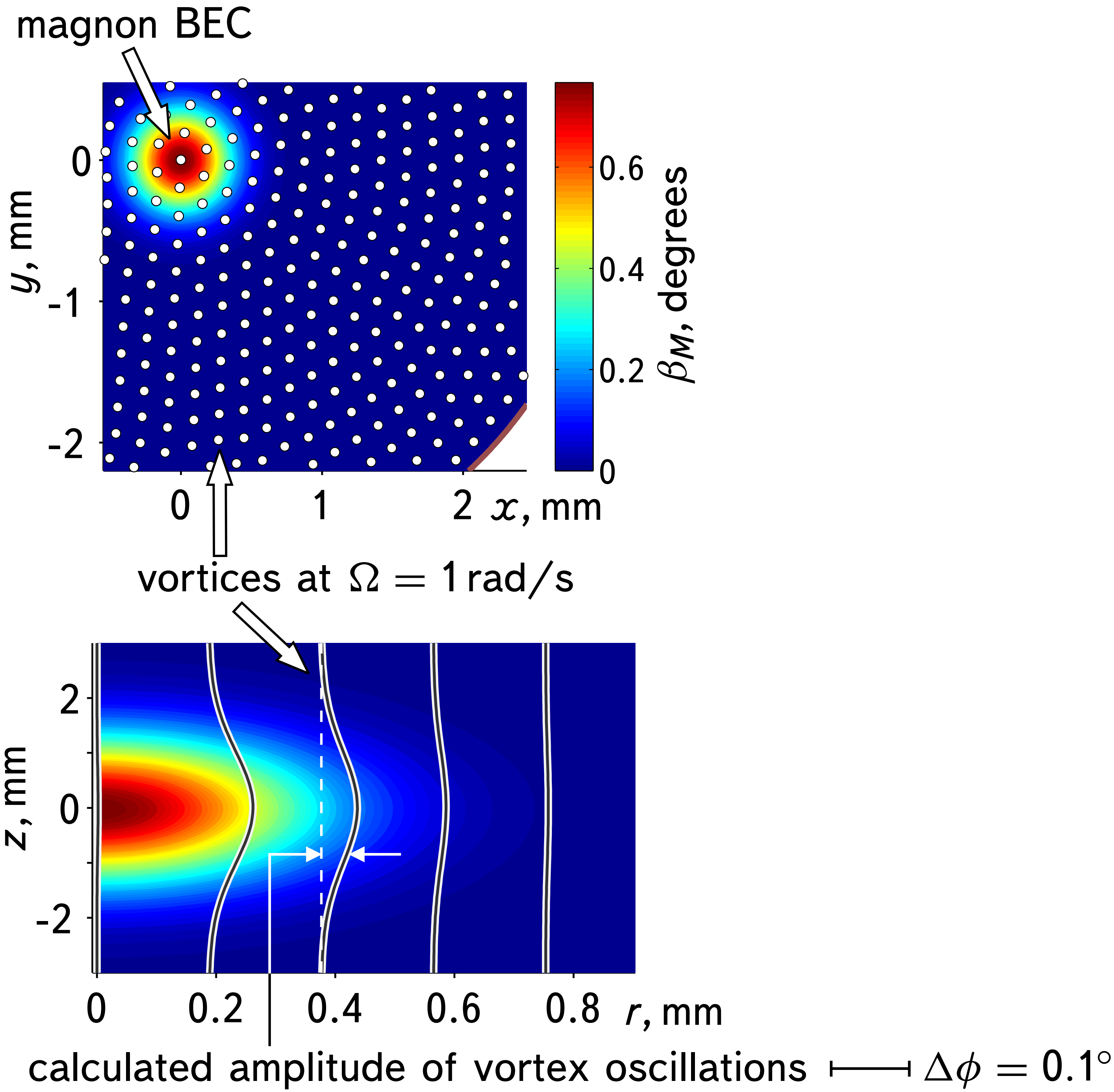
$$(\tau_\mathrm{v}\Omega)^{-1} \propto T_H^{-2} \propto H^{-4}$$



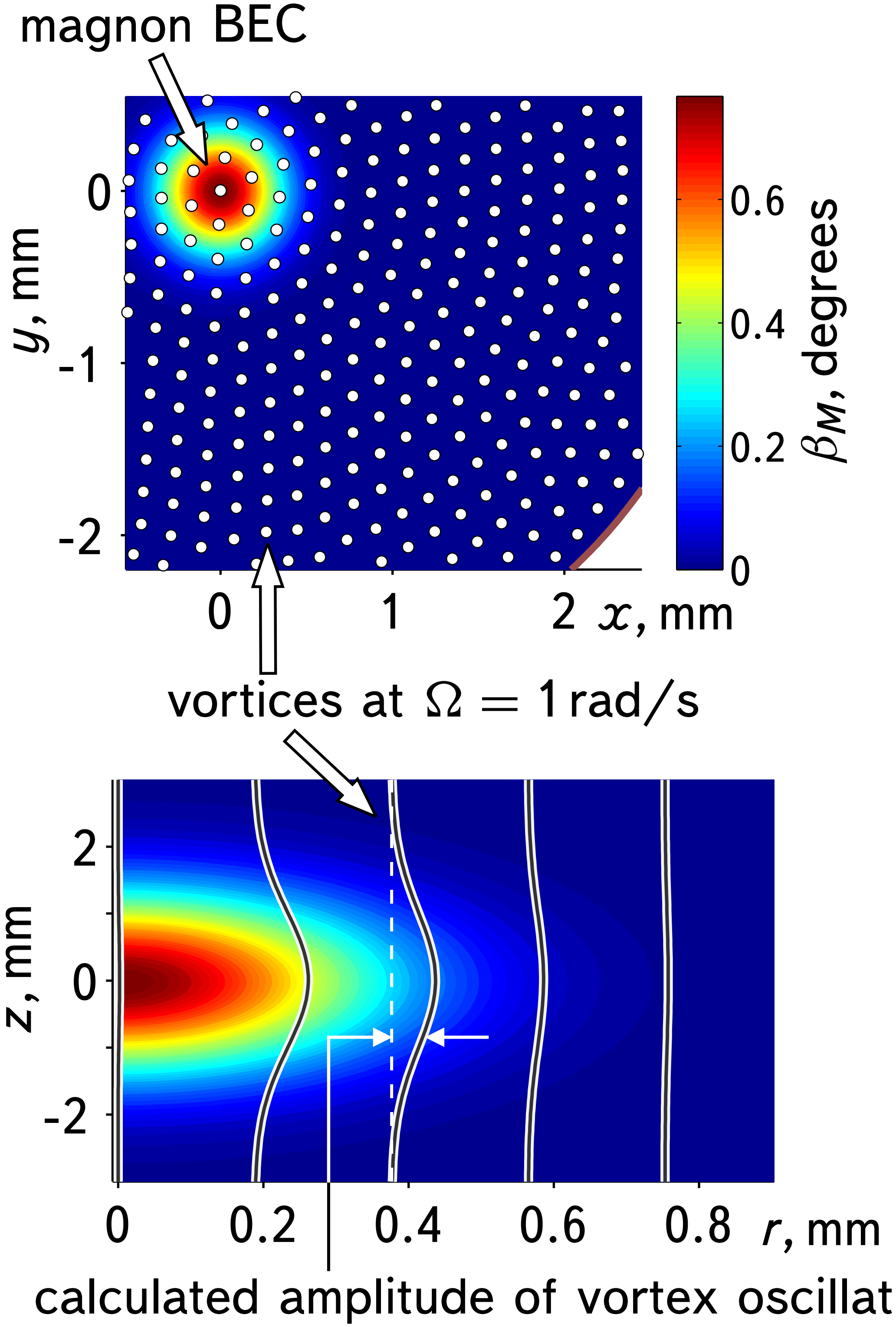
$$(\tau_\mathrm{v}\Omega)^{-1} \propto g_D^2\,\Delta_0^4\,\xi_0^2\,\frac{(1.5+F_{0a})^3(1+F_{0a})^2}{1+F_{1s}/3}$$



CALCULATION OF VORTEX-INDUCED RELAXATION



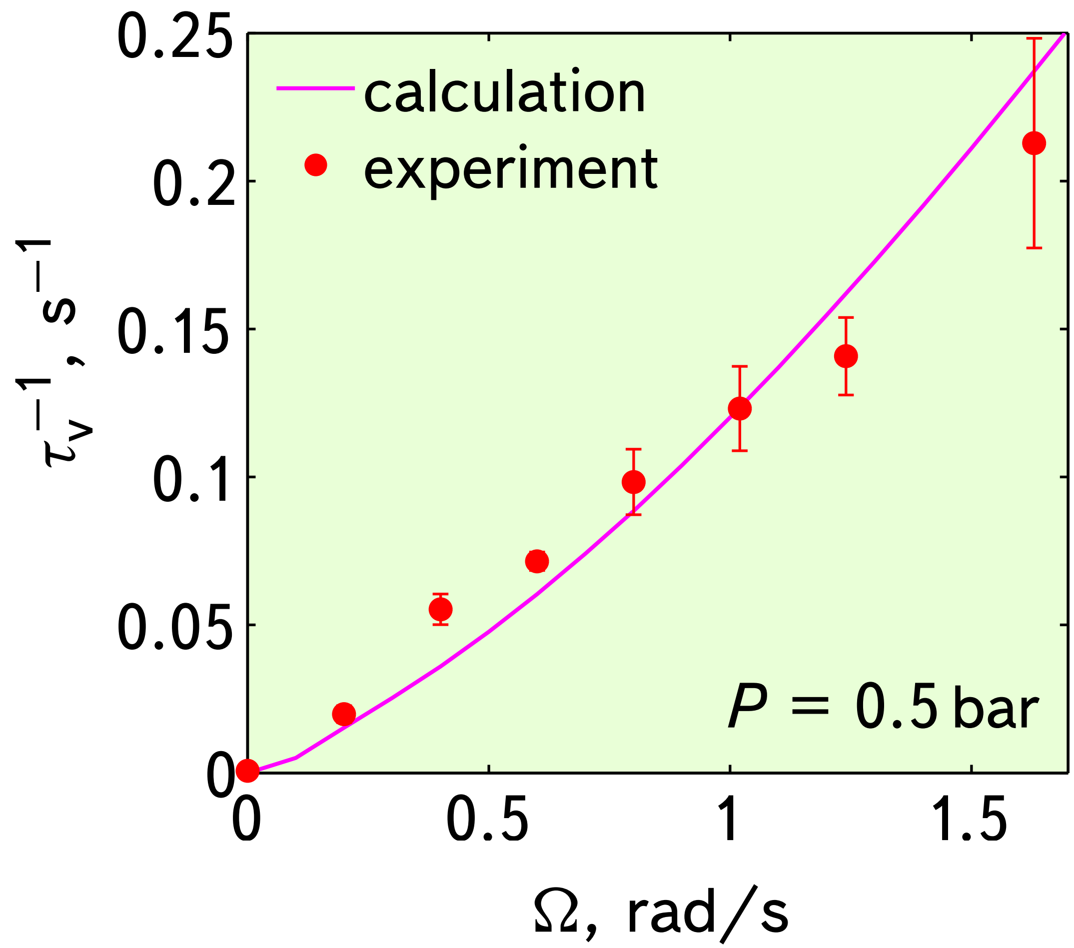
CALCULATION OF VORTEX-INDUCED RELAXATION



Core friction f:

estimated $\sim 10^{-18}$ erg s/cm,

fit $2 \cdot 10^{-19}$ erg s/cm

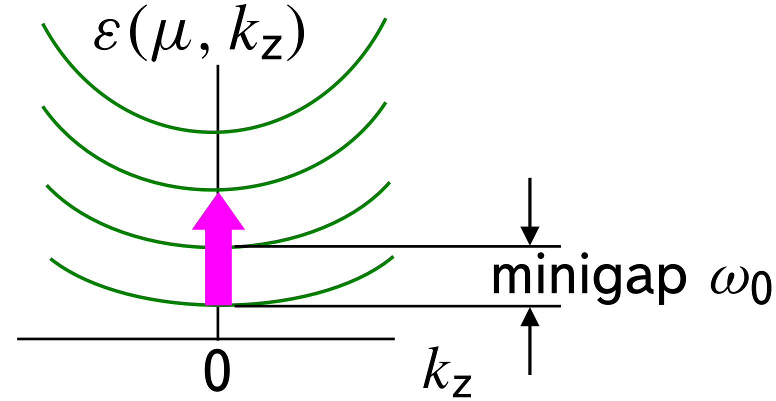


RESONANT ABSORPTION FROM VORTEX CORES

Resonant absorption at $\omega = n\omega_0$ owing to van Hove singularities at $k_z = 0$.

Possible to observe in ^{3}He-B at low T since

$$\omega_0 \tau = \mathrm{Ko} = 1/\alpha \gg 1.$$



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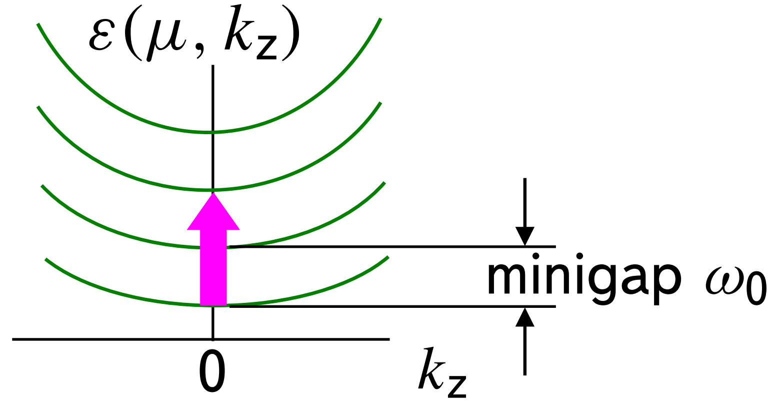
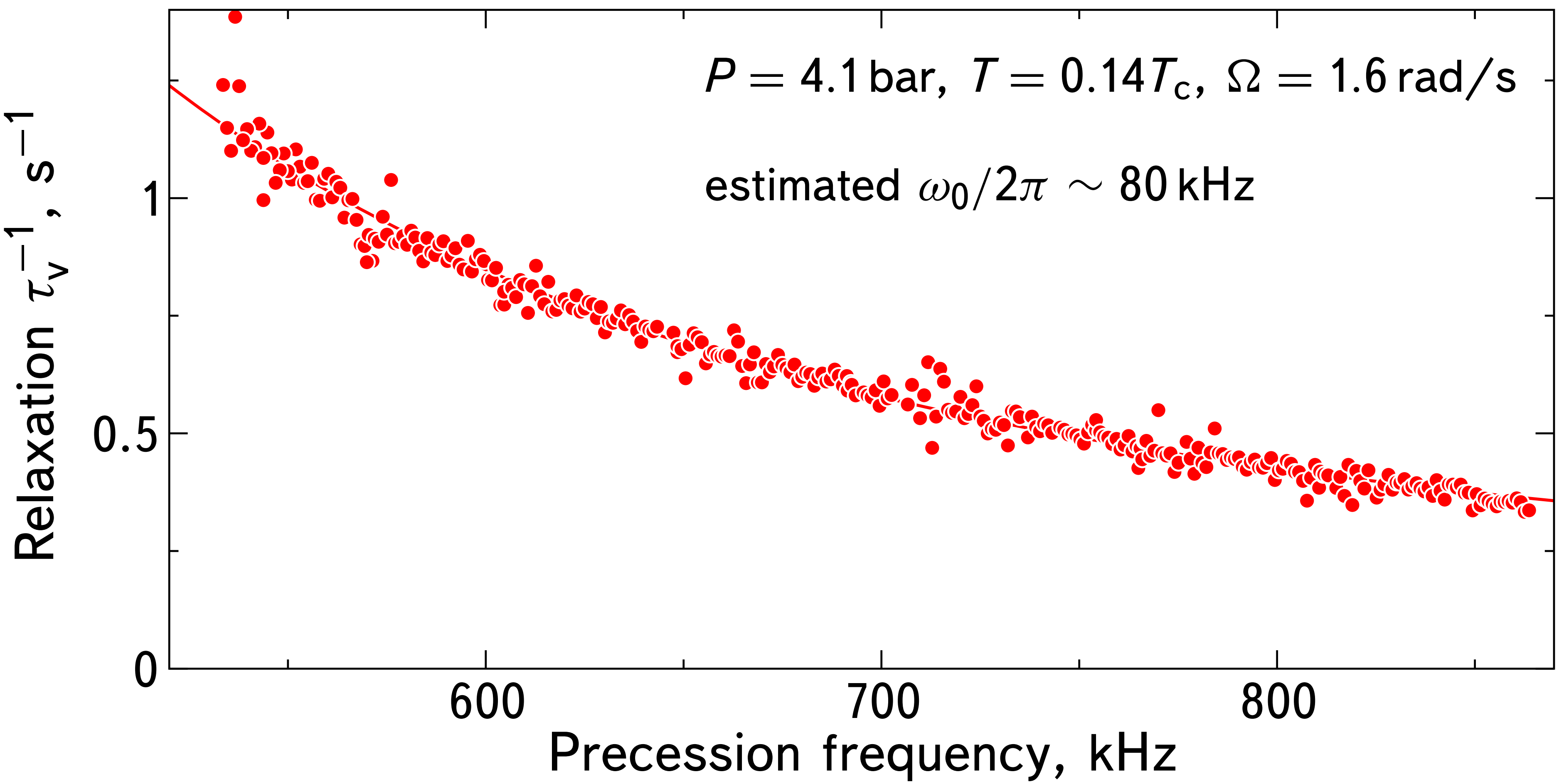
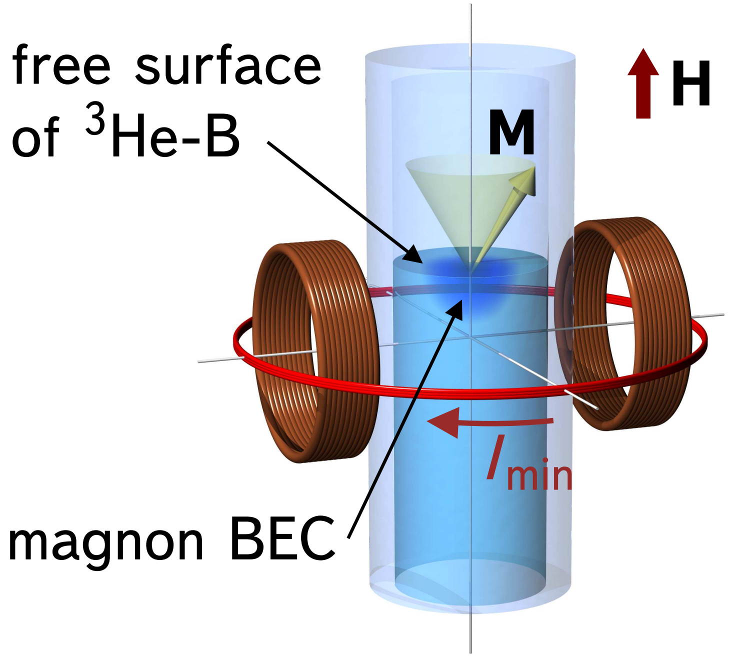
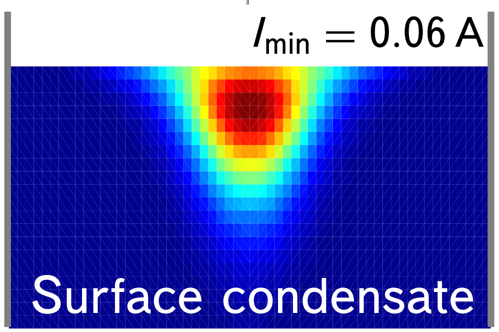


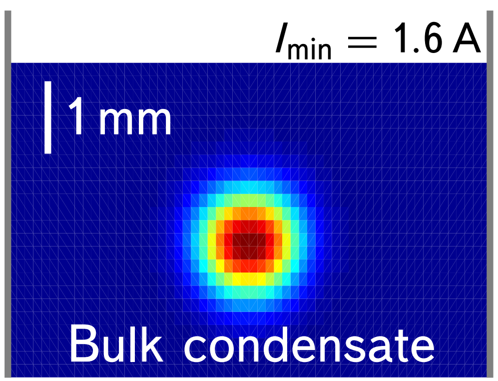


RELAXATION OF THE MAGNON BEC AT THE FREE SURFACE

Majorana bound states are predicted to exist at the surfaces of ^{3}He-B.







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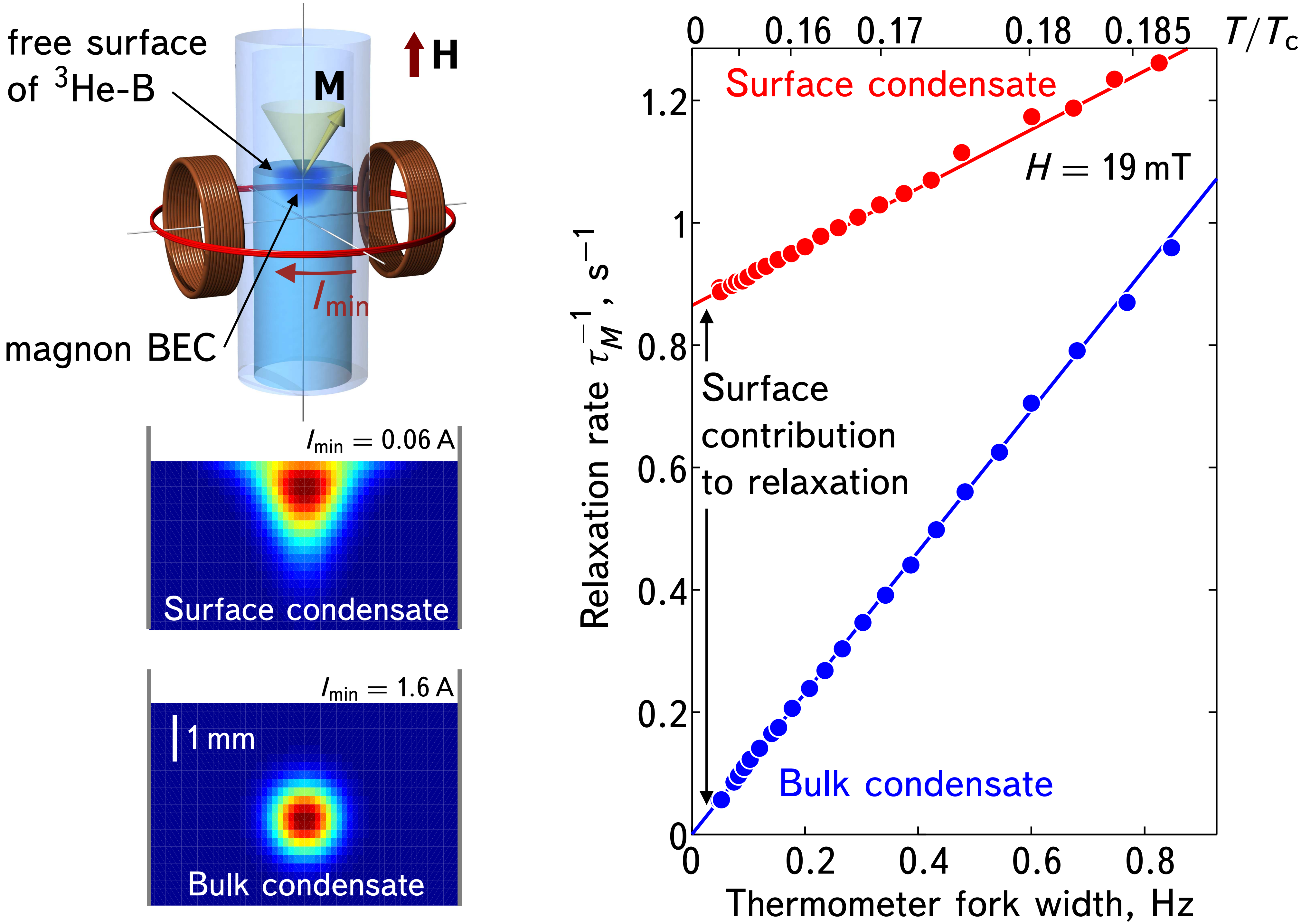
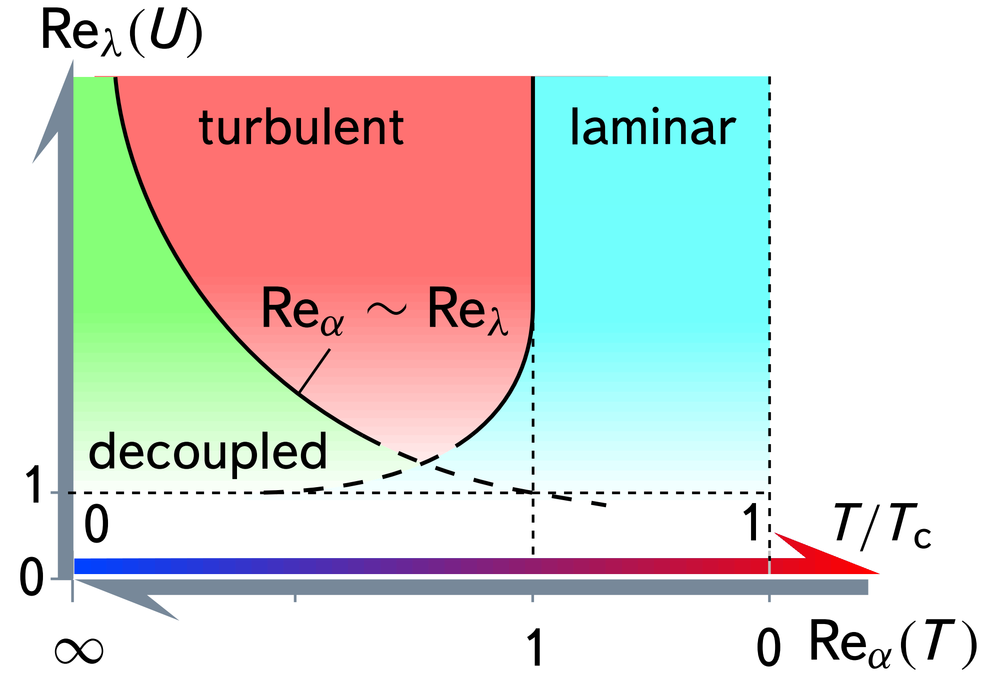


SUMMARY

- Kopnin force dominates mutual friction in Fermi superfluids. Decreasing of the friction with temperature causes profound transitions in the superfluid hydrodynamics, which are observed in the experiments in the rotating cryostat.



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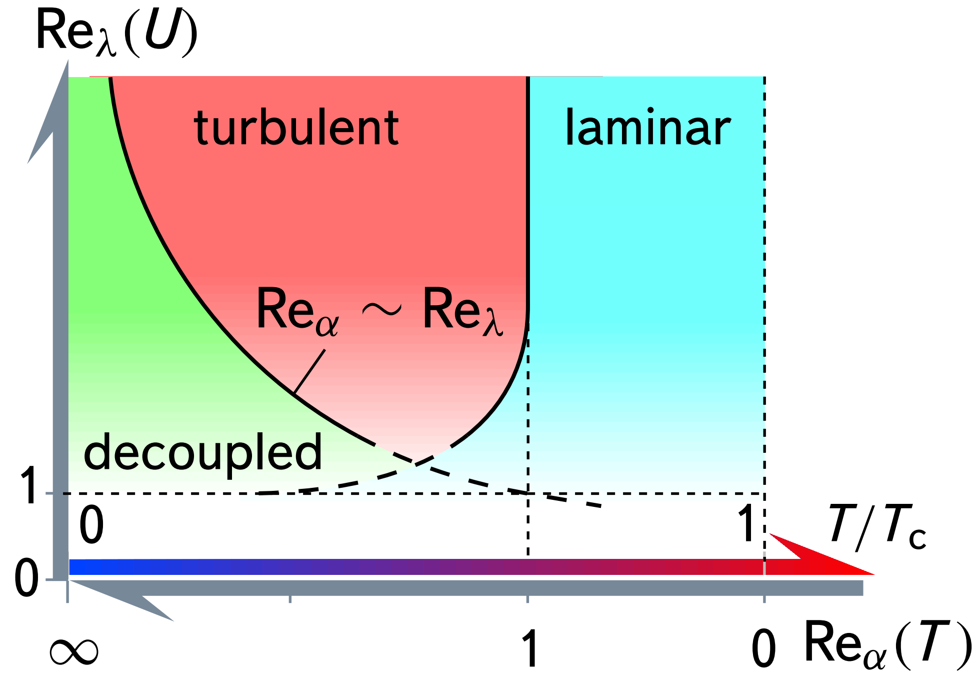
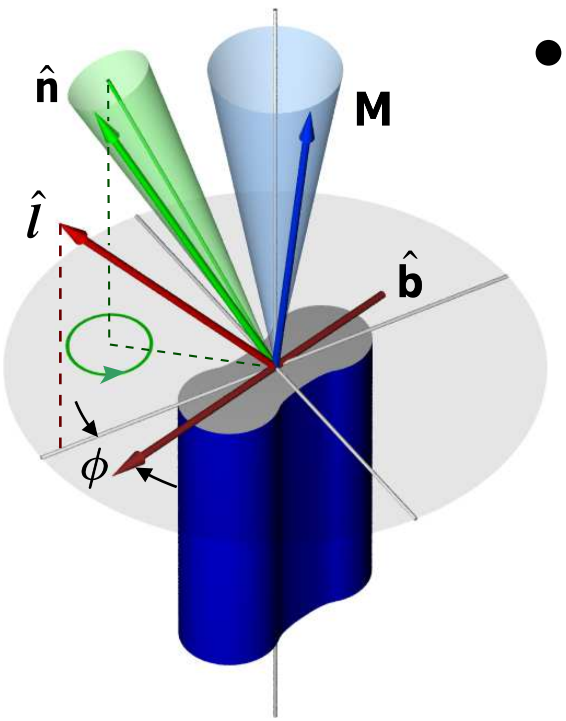
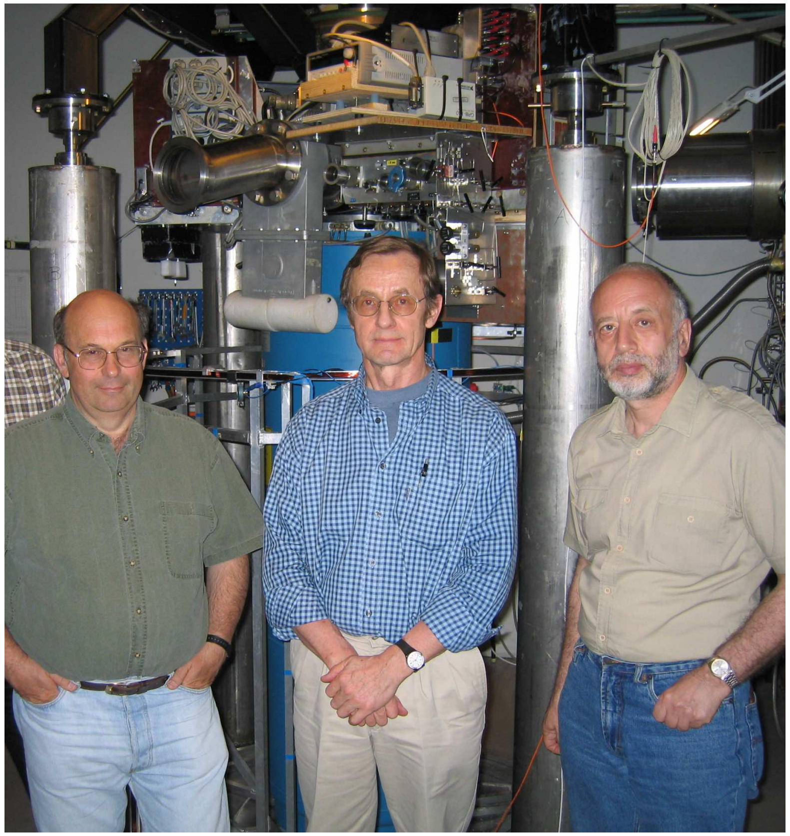




- Magnon BEC is a sensitive tool to measure magnetic relaxation from vortex-core- and surface-bound fermions in ^{3}He-B in $T \to 0$ limit.

— Potential to probe individual transitions between quasiparticle levels in the core.

— Majorana nature of surface-bound fermions via anisotropy of relaxation for **H** direction when $H \lesssim 3\,\mathrm{mT}$.



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Vladimir Dmitriev, *Kapitza Institute, Russia*

Rich Haley, *Lancaster University, UK*

Lev Levitin, *Royal Holloway, UK*

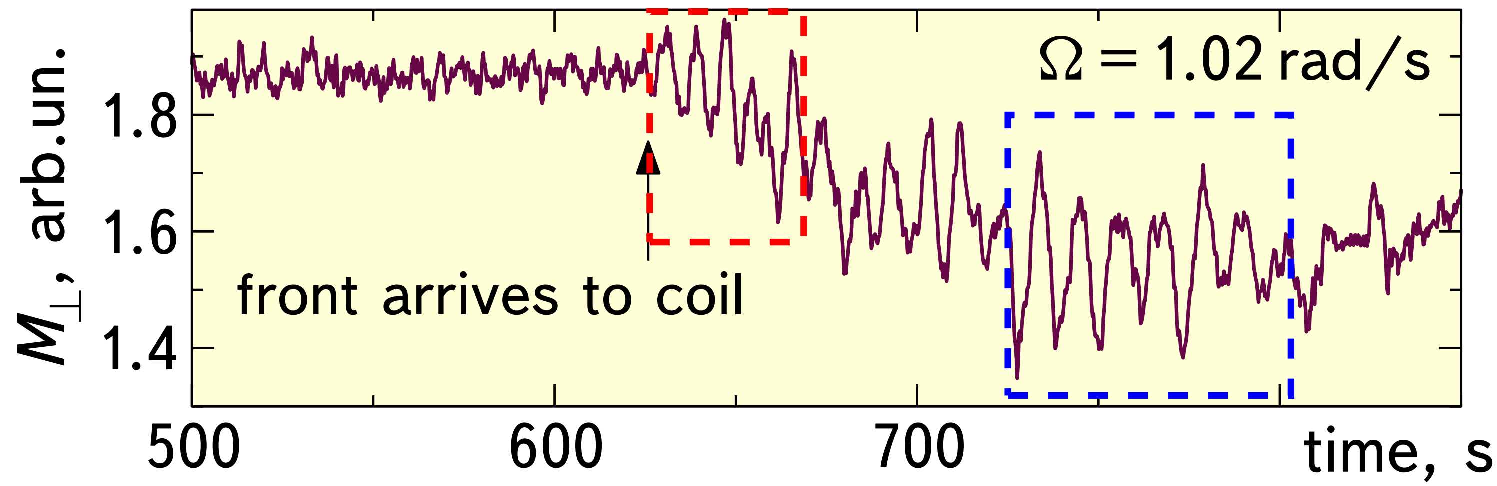
Victor L'vov, *Weizmann Institute, Israel*

Paul Walmsley, *University of Manchester, UK*

FRONT ROTATION IN THE EXPERIMENT

Vortex arrangement is not axially symmetric and rotates

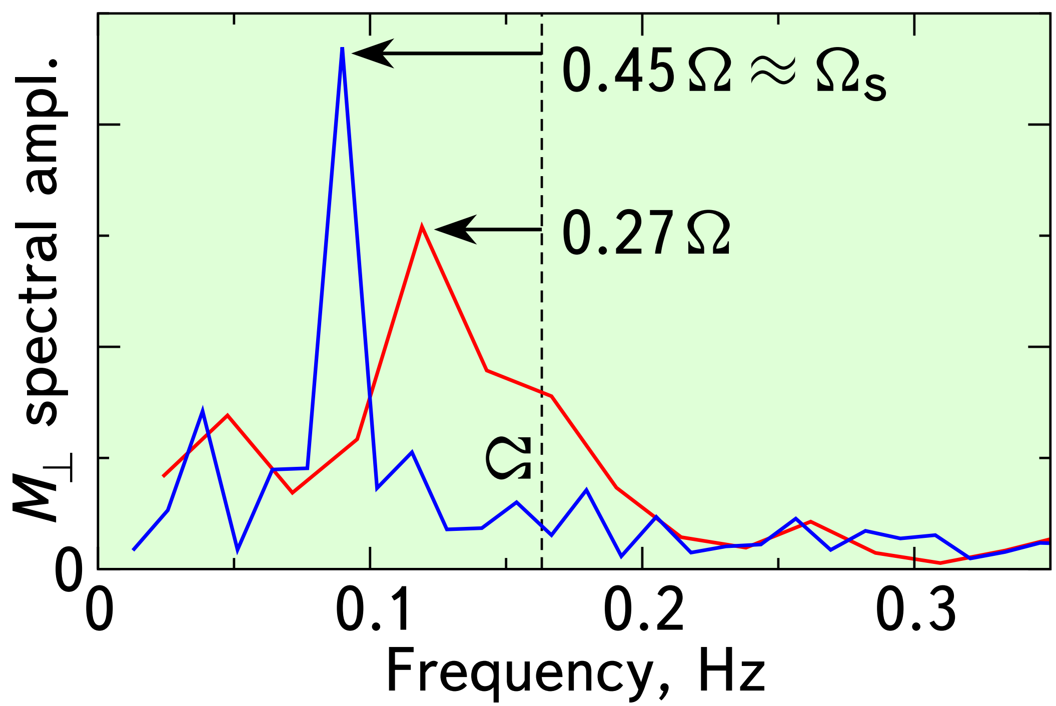
$\Rightarrow$ NMR signal is periodically modified.



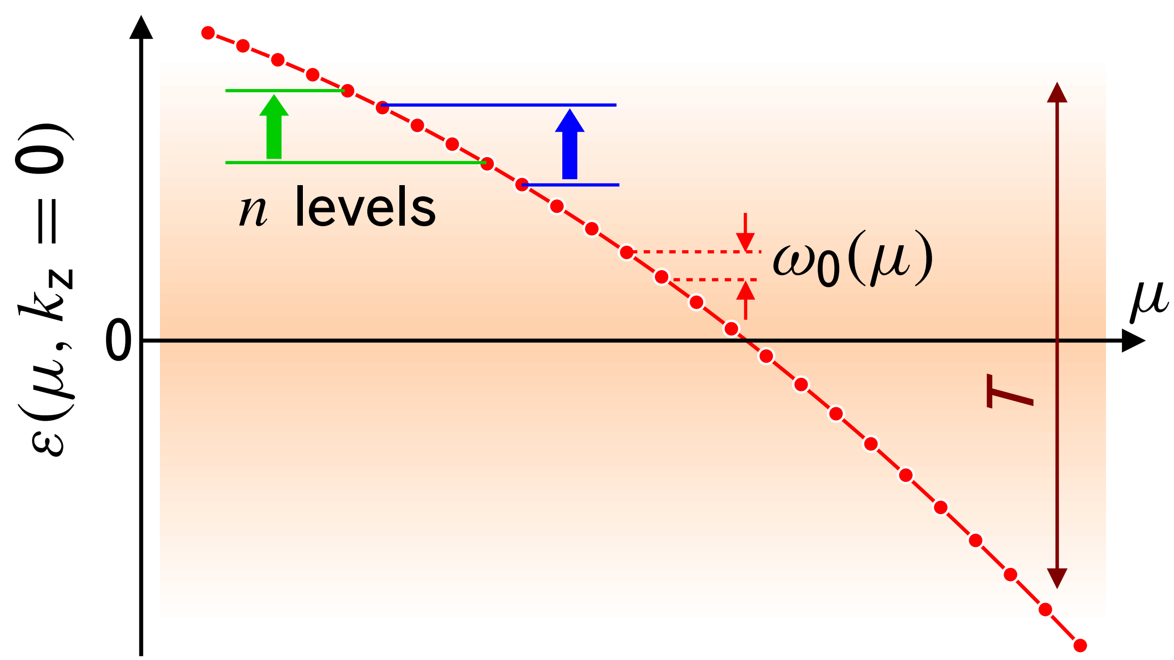
Precession in simulations:

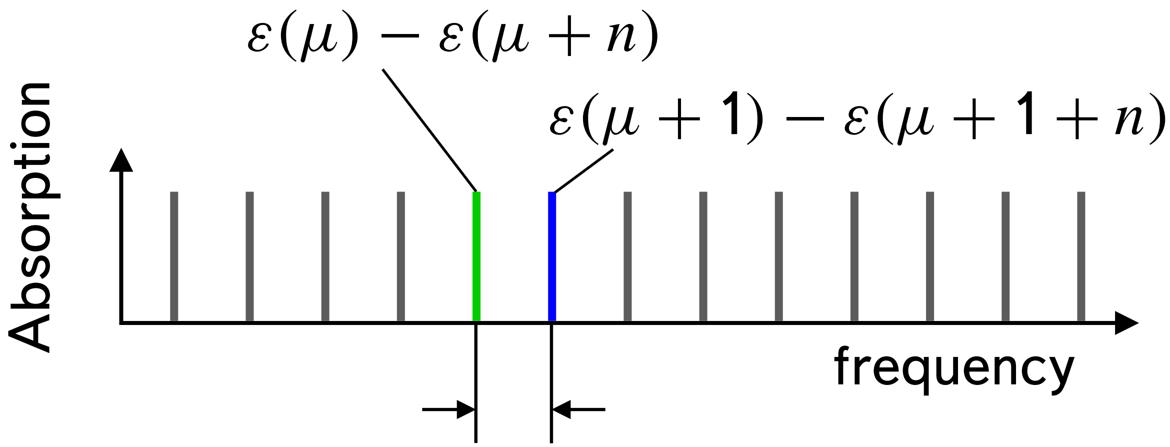
cluster: $\approx \Omega_s$

front: $\approx 0.65\Omega_s$



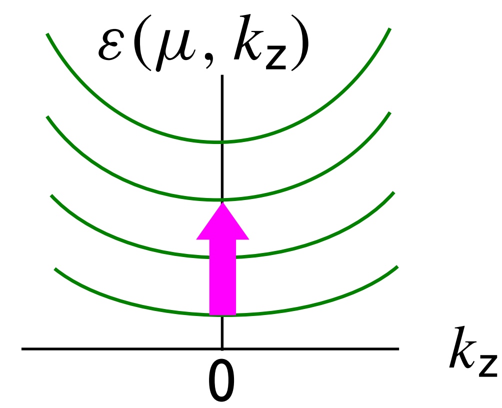
FREQUENCY COMB FROM THE CORE-BOUND FERMIONS





$$\omega_{\text{comb}} = n \left| \frac{d\omega_0}{d\mu} \right| = \frac{\omega}{\omega_0} \left| \frac{d\omega_0}{d\mu} \right| \ll \omega_0$$

Absorption peaks at $k_z = 0$



+ non-linearity in $\varepsilon(\mu) \Rightarrow$ frequency comb

Possibility for observation:

$$\omega_0 \sim 100\,\text{kHz},\ \omega \sim 1\,\text{MHz}$$

$$\omega_{\text{comb}} \sim \frac{\omega}{\xi_0 k_F} \sim 1\,\text{kHz}$$

$$\alpha < 10^{-3},\ 1/\tau < 100\,\text{Hz}$$

Peak width $1/\tau$: $(\omega_0 \tau)^{-1} \sim$ mutual friction α.

[PRB **85**, 224526 (2012)]