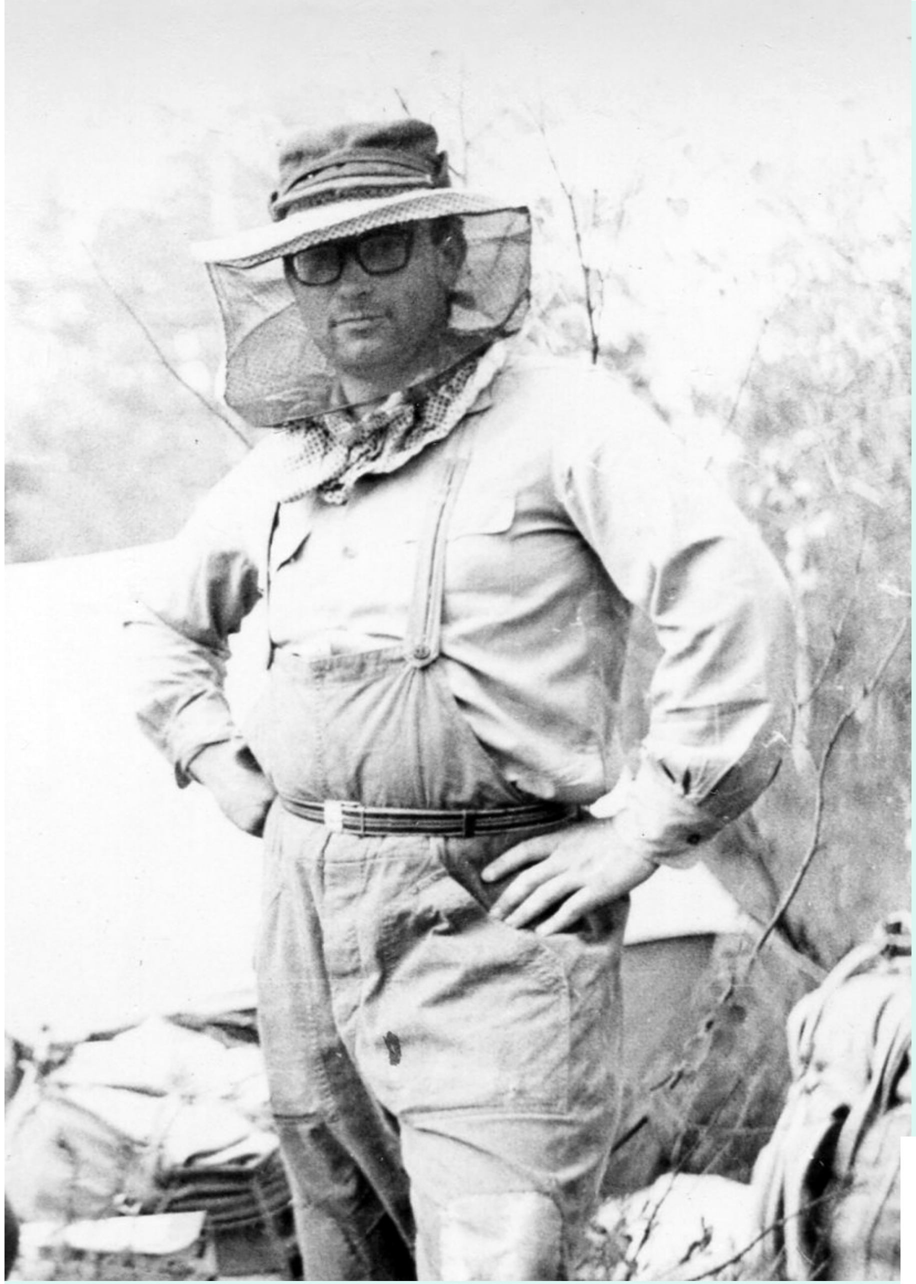
Commemoration symposium of Nikolai Kopnin

Landau Institute for Theoretical Physics
Chernogolovka, Russia

24 June 2014



Memory to Kolya Kopnin

Где-то там, средь чухонских озер и болот
Наш походный начальник груз по жизни несет.
Груз сей будет доставлен, знаю я, потому,
Что подвластен Колуну старик одному.

Half quantum vortices

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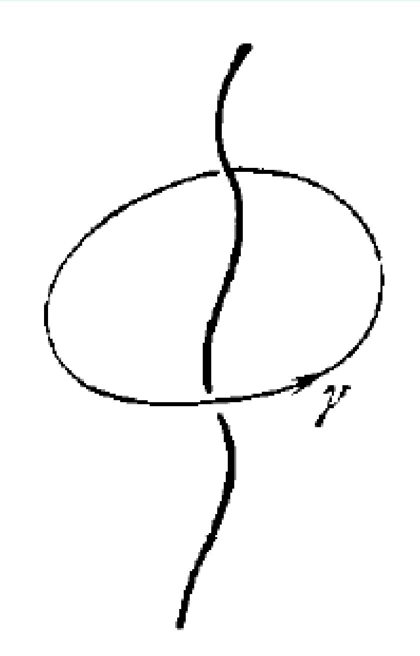
Outline

Vortices in equal spin pairing superfluids
Media with half quantum vortices
One quantum vortices in rotating vessel with He-3A
Half-quantum vortices in rotating vessel with He-3A
Half-quantum vortices in Sr2RuO4
Half-quantum vortices in rotating vessel with polar phase of He-3
Superfluid He-3 in aerogel
Superfluid He-3 in anisotropic aerogel
Phase diagram of superfluid He-3 in anisotropic aerogel
Superfluid He-3 in nematically ordered aerogel
Nuclear Magnetic Resonance in polar state
Conclusion

Vortices in separable equal spin pairing superfluids



$$\Psi = \Delta(e^{i\varphi_1}|\uparrow\uparrow\rangle + e^{i\varphi_2}|\downarrow\downarrow\rangle) = \Psi_{orb}\Psi_{spin}$$

$$\Psi_{orb} = \Delta e^{i\frac{\varphi_1+\varphi_2}{2}}, \quad \Psi_{spin} = e^{i\frac{\varphi_1-\varphi_2}{2}}|\uparrow\uparrow\rangle + e^{i\frac{\varphi_2-\varphi_1}{2}}|\downarrow\downarrow\rangle$$

One quantum vortex

$$\varphi_1 \rightarrow \varphi_1 + 2\pi, \qquad \varphi_2 \rightarrow \varphi_2 + 2\pi$$

$$\Psi_{orb} \rightarrow \Psi_{orb}, \qquad \Psi_{spin} = const, \qquad \Psi \rightarrow \Psi$$

Half quantum vortex

$$\varphi_1 = const, \qquad \varphi_2 \rightarrow \varphi_2 + 2\pi$$

$$\Psi_{orb} \rightarrow -\Psi_{orb}, \qquad \Psi_{spin} \rightarrow -\Psi_{spin}, \qquad \Psi \rightarrow \Psi$$

G.Volovik & V.M. 1976

Media with HQV

1. Supefluid ^{3}He-A

2. Superfluid polar phase in nematically ordered aerogel,

3. Superconducting Sr_2RuO_4, J.Jang et al, Science 2011

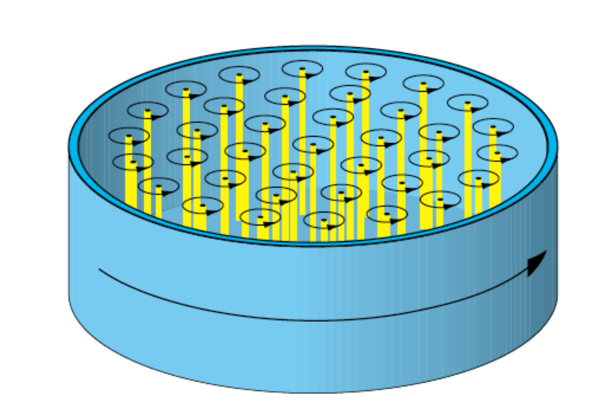
4. Exciton-polariton condensate, K.Lagudakis et al, Science 2009

$$\sim \mathbf{e}_\lambda \exp(i\varphi)$$

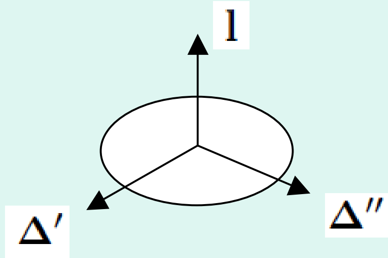
5. Charge Density Waves - CDW
   Spin Density Waves - SDW
   Super Solids
   Fulde-Ferrel-Larkin-Ovchinnikov - FFLO superconducting states

$$\Psi(x,y) = A\cos(\mathbf{k}\boldsymbol{\rho} + \phi(x,y))e^{i\varphi(x,y)}$$

One quantum vortices in rotating vessel with He-3A



$$\Psi^A_\alpha = \Psi^{spin}_\alpha \Psi^{orb}(\mathbf{k}) = A^A_{\alpha i}\hat{k}_i = \Delta(T)V_\alpha(\Delta'_i + i\Delta''_i)\hat{k}_i/\sqrt{2}$$



$$\mathbf{\Delta}' + i\mathbf{\Delta}'' = e^{i\varphi}(\hat{x} + i\hat{y}) \qquad \mathbf{l} = \mathbf{\Delta}' \times \mathbf{\Delta}'' = const \qquad \mathbf{V} = const$$

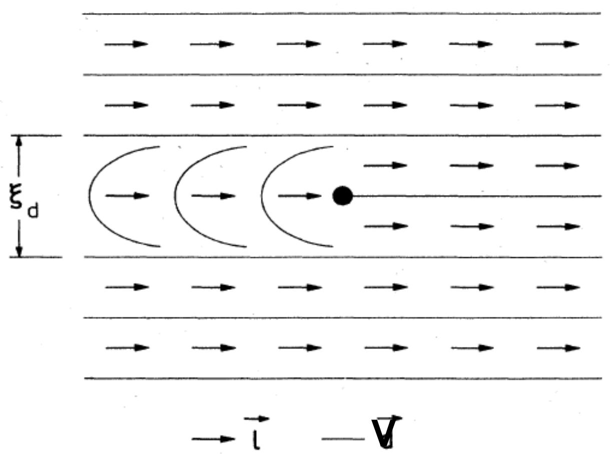
$$\mathcal{F}_\nabla = \int d^3\mathbf{r}\left(K_1 \frac{\partial A_{\alpha i}}{\partial x_j}\frac{\partial A^\star_{\alpha i}}{\partial x_j} + K_2 \frac{\partial A_{\alpha i}}{\partial x_j}\frac{\partial A^\star_{\alpha j}}{\partial x_i} + K_3 \frac{\partial A_{\alpha i}}{\partial x_i}\frac{\partial A^\star_{\alpha j}}{\partial x_j}\right)$$

$$f_{n=1} = \pi|\Delta|^2(2K_1 + K_2 + K_3)\ln\frac{r_v}{\xi}$$

$$F_{n=1} = n_v f_{n=1}$$

$$n_v = \frac{2\Omega}{\Gamma} \qquad \Gamma = \frac{h}{2m_3} = 0,66\cdot 10^{-3}\frac{cm^2}{sec} \qquad r_v = n_v^{-1/2} \approx 10^{-2}cm \quad at \quad \Omega \approx 3\ rad/sec$$

Half-quantum vortices in rotating vessel with He-3A



$$\Psi^A_\alpha = \Psi^{spin}_\alpha \Psi^{orb}(\mathbf{k}) = A^A_{\alpha i}\hat{k}_i = \Delta(T)V_\alpha(\Delta'_i + i\Delta''_i)\hat{k}_i/\sqrt{2}$$

$$\mathbf{\Delta}' + i\mathbf{\Delta}'' = e^{i\varphi/2}(\hat{y} + i\hat{z}) \qquad \mathbf{l} = \mathbf{\Delta}' \times \mathbf{\Delta}'' = \hat{x} \qquad \mathbf{V} = \hat{x}\cos\frac{\varphi}{2} - \hat{y}\sin\frac{\varphi}{2}$$

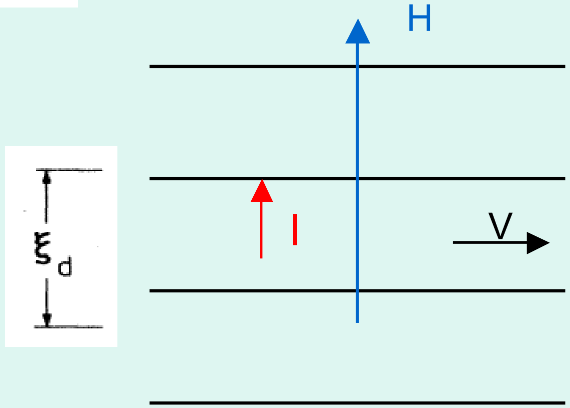
$$F_{so} = \frac{g_D}{5|\Delta|^2}\left(A_{\alpha\alpha}A^\star_{\beta\beta} + A_{\alpha i}A^\star_{i\alpha} - \frac{2}{3}A_{\alpha i}A^\star_{\alpha i}\right) = \frac{g^A_D}{5}\left(\frac{1}{3} - (\mathbf{V}\mathbf{l})^2\right)$$

$$\xi_d \approx 10^{-3} cm$$

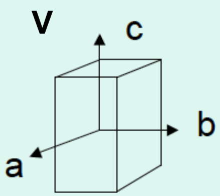
$$f_{n=1/2} = \frac{\pi}{2}|\Delta|^2(2K_1 + K_2 + K_3)\ln\frac{\xi_d}{\xi} + \mathcal{O}\left(|\Delta|^2 K_{123}\frac{r_v}{\xi_d}\right)$$

$$F_{n=1/2} = 2n_v f_{n=1/2} = n_v\left[\pi|\Delta|^2(2K_1 + K_2 + K_3)\ln\frac{\xi_d}{\xi} + \mathcal{O}\left(|\Delta|^2 K_{123}\frac{r_v}{\xi_d}\right)\right] >$$

$$> F_{n=1} = n_v f_{n=1} = n_v \pi|\Delta|^2(2K_1 + K_2 + K_3)\ln\frac{r_v}{\xi}$$



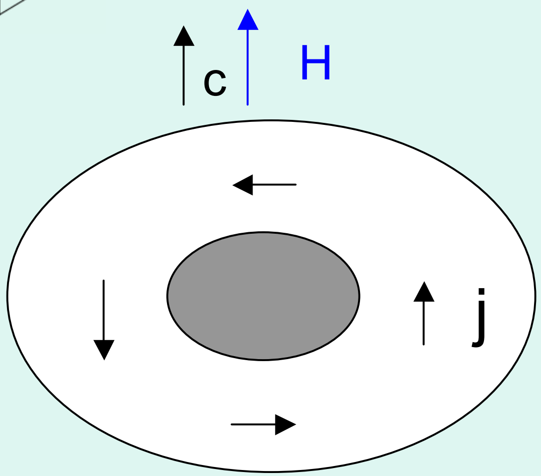
G.Volovik & M.Salomaa, 1985

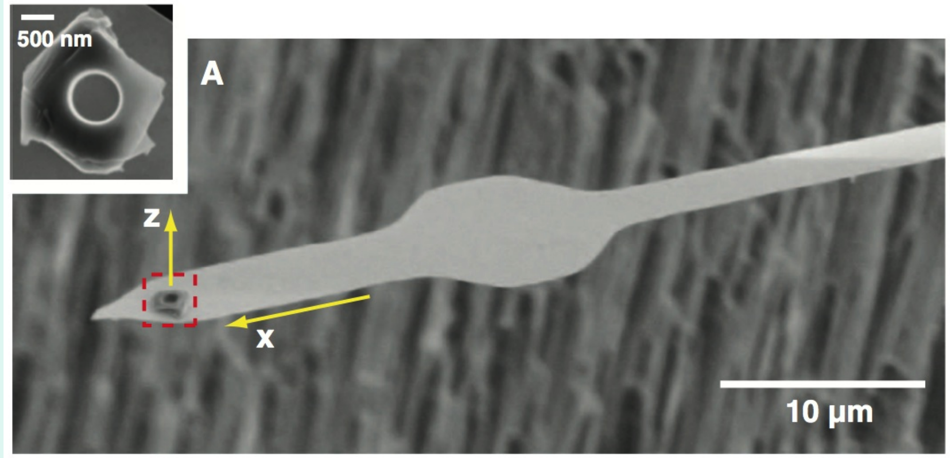


$$\mathbf{d}(\mathbf{k}) = \mathbf{V}\Delta(\mathbf{k})e^{i\varphi}$$

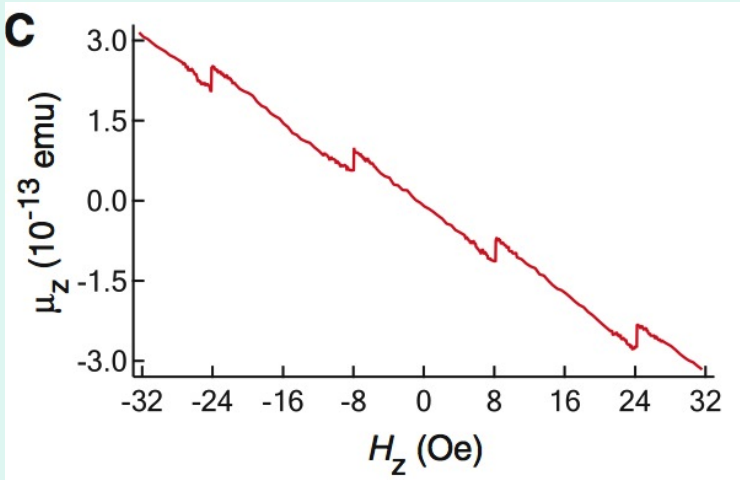
$$\xi_d \geq 50\mu m$$

Sr_2RuO_4 - A-phase type superconductor





Superconducting Sr_2RuO_4 ring magnetic moment



$$\mathbf{j} = \frac{c}{4\pi\lambda^2}\left(\nabla\varphi - \frac{2\pi}{\Phi_0}\mathbf{A}\right)$$

$$\oint \mathbf{j} d\gamma = \frac{c}{4\pi\lambda^2}\left(2\pi N - 2\pi\frac{\Phi}{\Phi_0}\right)$$

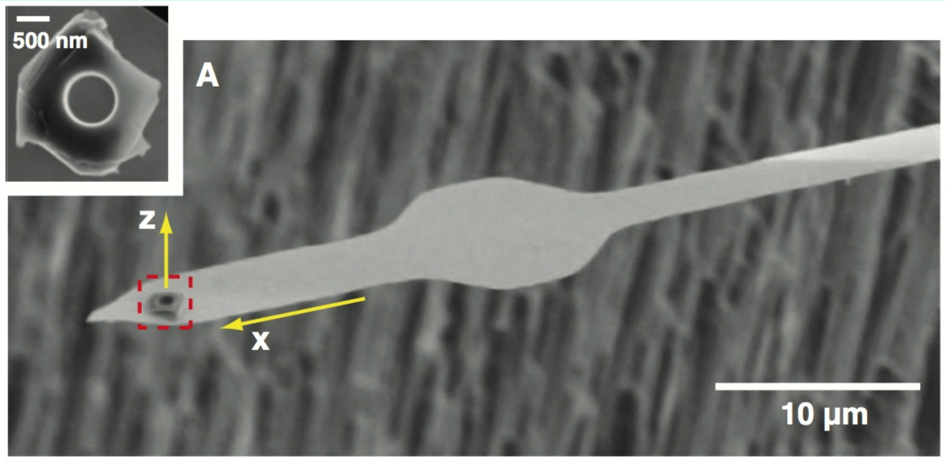
$$\mu_z = \frac{1}{2c}\int (\mathbf{j}\times\mathbf{r})_z dV = \Delta\mu_z N - \chi_M H$$

$$H_{c1}^z = \frac{\Phi_0}{2\pi R^2} \approx 8\ G$$

$$\Delta\mu_z = 4.4\times 10^{-14} emu$$

J.Jang et al Science 2011

Half quantum vortices in Sr2RuO4



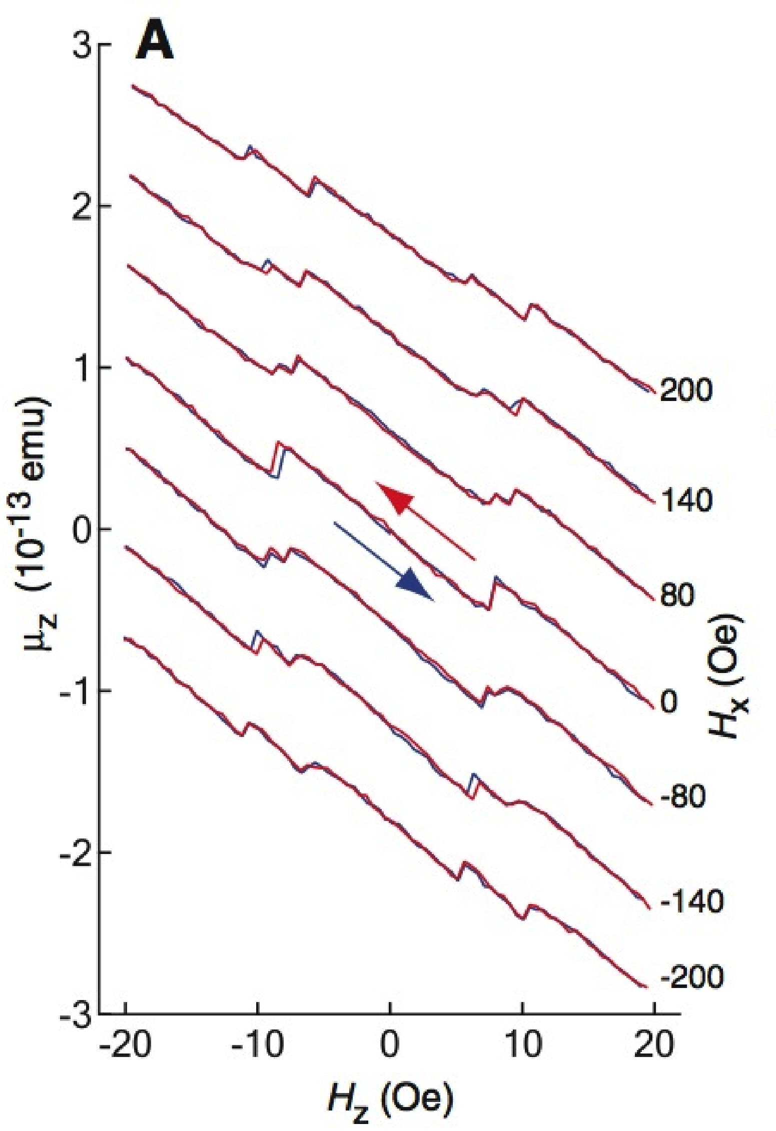
$$H_{c1}^x = \frac{2\Phi_0}{\pi d^2} \approx 250\ G$$

$$\Delta\mu_z = \frac{1}{2} \times 4.4 \times 10^{-14} emu$$

$$\mathbf{V} = \hat{z}\cos\varphi/2 + \hat{x}\sin\varphi/2$$

$$\mathbf{j}_{spin} = \rho_s \frac{\hbar}{2m} \nabla \frac{\varphi}{2}$$

$$\mu_x = \frac{e}{2mc} \int (\mathbf{j} \times \mathbf{r})_x dV \approx 10^{-16} emu$$

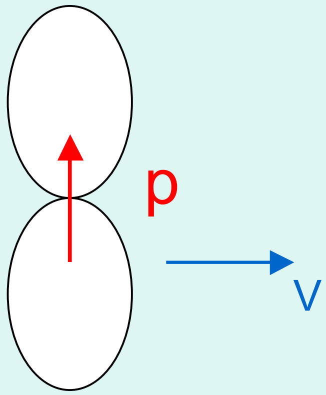


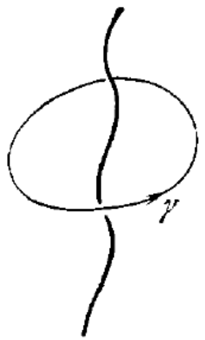
J.Jang et al Science 2011

Half-quantum vortices in rotating vessel with superfluid polar phase of He-3

$$\Psi_\alpha^{pol} = \Psi_\alpha^{spin}\Psi^{orb}(\mathbf{k}) = A_{\alpha i}^{pol}\hat{k}_i = \Delta(T)V_\alpha p_i \hat{k}_i e^{i\phi}$$

Cooper pair in polar state





Half-quantum vortex

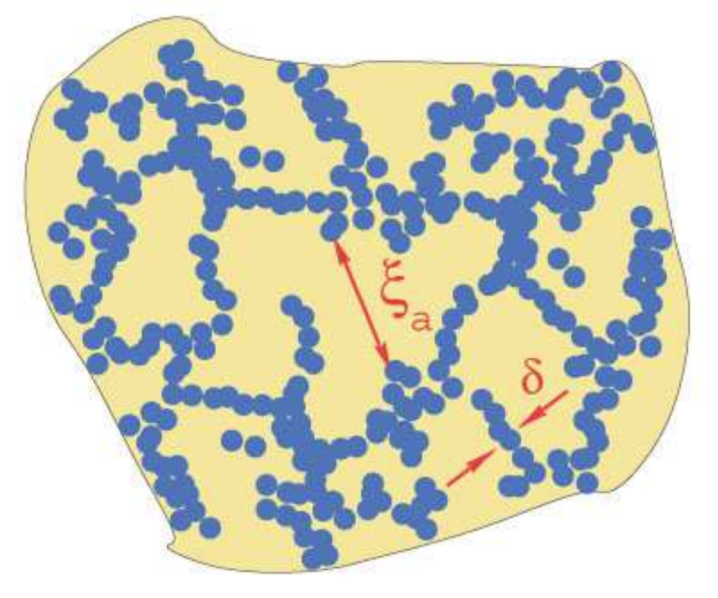
$$\phi = \varphi/2 \qquad \mathbf{p} = const \qquad \mathbf{V} = \hat{x}\cos\frac{\varphi}{2} - \hat{y}\sin\frac{\varphi}{2}$$

$$F_{so} = \frac{g_D}{5|\Delta|^2}\left(A_{\alpha\alpha}A^\star_{\beta\beta} + A_{\alpha i}A^\star_{i\alpha} - \frac{2}{3}A_{\alpha i}A^\star_{\alpha i}\right) = \frac{2}{5}g_D^{pol}\left((\mathbf{Vp})^2 - \frac{1}{3}\right)$$

$$F_{n=1} = n_v f_{n=1} = F_{n=1/2} = 2n_v f_{n=1/2} = n_v \pi|\Delta|^2(2K_1 + K_2 + K_3)\ln\frac{r_v}{\xi}$$

Now, there is no problem with spin-orbit coupling but where to get a polar phase ?

Superfluid He-3 in aerogel



A sketch of silica aerogel showing regions containing He-3 (yellow)
threated by strands and aggregates of silica (blue)

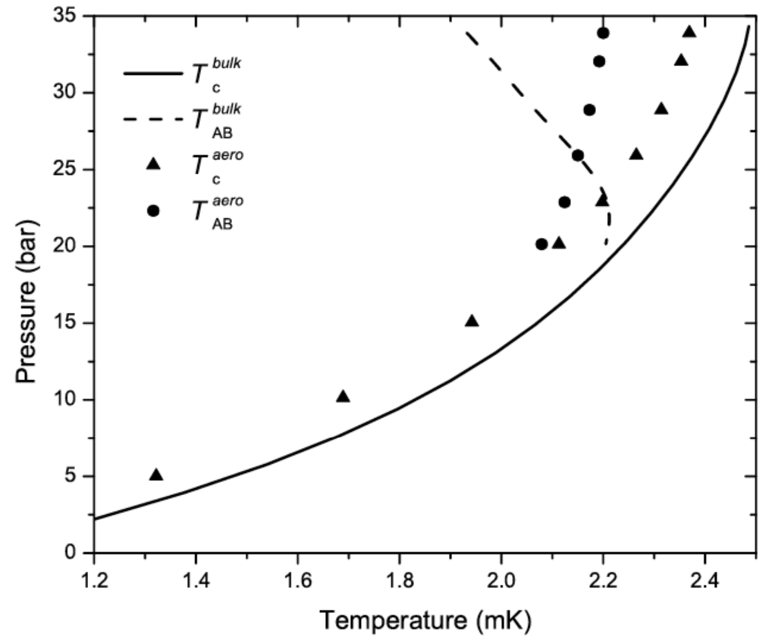


FIG. 4. Phase diagram of superfluid ^{3}He confined to 99.3% porosity aerogel in a magnetic field of 28.4 mT. Triangles represent $T_{c,\text{aero}}$, while circles represent the equilibrium value of $T_{AB,\text{aero}}$. For comparison, the bulk phase diagram is also given.

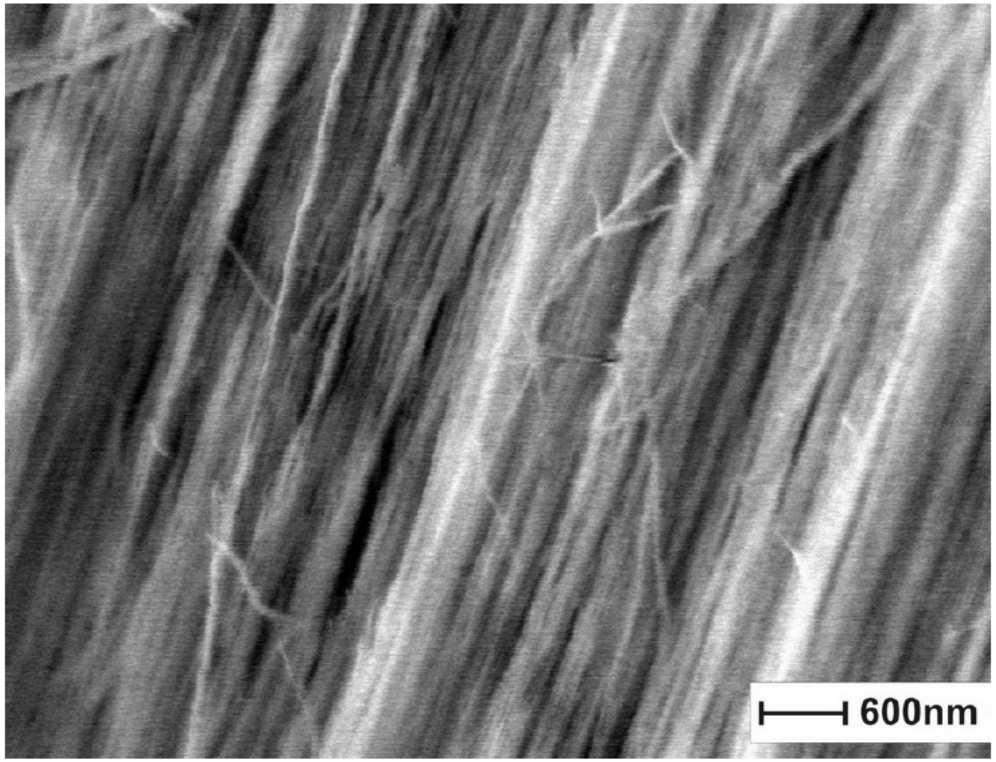
Baumgardner and Osheroff, PRL 2004

In absence of aerogel anisotropy all superfluid phases with p-pairing have the same critical temperature

$$F = \alpha_0(T - T_c)A_{\alpha i}A^{\star}_{\alpha i} + fourth\ \ order\ \ terms = \alpha_0(T - T_c)|\Delta|^2 + fourth\ \ order\ \ terms$$

$$(T_c - T_{c0})/T_{c0} \approx -\xi\delta/\xi_a^2$$

Superfluid He-3 in nematically ordered aerogel



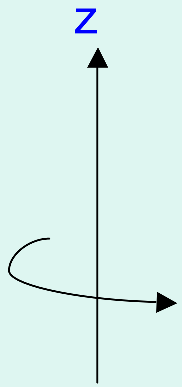
$Al_2O_3 \cdot H_2O$ strands
with a characteristic diameter of $\sim 5 - 10$ nm
and a characteristic separation of $\sim 70 - 80$ nm.
Strands are oriented along nearly the same direction
at a macroscopic distance $\sim 3 - 5$ mm.

Dmitriev et al, JETP Lett 2012

Superfluid He-3 in anisotropic aerogel

Anisotropy lifts the degeneracy between superfluid phases

$$F^{(2)} = F_i^{(2)} + F_a^{(2)} = \alpha_0(T - T_c)|\Delta|^2 + \eta_{ij} A_{\alpha i} A^{\star}_{\alpha j}$$



Media uniaxial anisotropy with anisotropy axis along $\hat{z}$ is given by the traceless tensor

$$\eta_{ij} = \eta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

(i) B phase $A^B_{\alpha i} = \Delta R_{\alpha i} e^{i\phi}$ $\quad F_a^{(2)}(A^B_{\alpha i}) = 0$

$\eta < 0$

(ii) A-phase $A_{\alpha i} = \frac{\Delta}{\sqrt{2}} V_\alpha (\hat{x}_i + i\hat{y}_i)$ $\quad \mathbf{l} = \hat{z}$ $\quad F_a^{(2)} = \eta|\Delta|^2$ $\quad T_c^A = T_c - \eta/\alpha_0$

$\eta > 0$

(iii) A phase $A_{\alpha i} = \frac{\Delta}{\sqrt{2}} V_\alpha \left(\hat{z}_i + i(\hat{x}_i \cos\varphi(\mathbf{r}) + \hat{y}_i \sin\varphi(\mathbf{r}))\right)$ $\quad \mathbf{l} = -\hat{x}\sin\varphi(\mathbf{r}) + \hat{y}\cos\varphi(\mathbf{r})$

$$F_a^{(2)} = -\eta|\Delta|^2/2 \qquad T_c^A = T_c + \eta/2\alpha_0$$

(iv) Polar phase $A_{\alpha i} = \Delta V_\alpha \hat{z}_i e^{i\phi}$ $\quad F_a^{(2)} = -2\eta|\Delta|^2$ $\quad T_c^{polar} = T_c + 2\eta/\alpha_0$

Axipolar phase in uniaxially anisotropic aerogel

$$F_{cond} = \alpha A^\star_{\alpha i} A_{\alpha i} + \eta_{ij} A_{\alpha i} A^\star_{\alpha j}$$
$$+\beta_1 |A_{\alpha i} A_{\alpha i}|^2 + \beta_2 A^\star_{\alpha i} A_{\alpha j} A^\star_{\beta i} A_{\beta j} + \beta_3 A^\star_{\alpha i} A_{\beta i} A^\star_{\alpha j} A_{\beta j} + \beta_4 (A^\star_{\alpha i} A_{\alpha i})^2 + \beta_5 A^\star_{\alpha i} A_{\beta i} A^\star_{\beta j} A_{\alpha j}$$

Axipolar phase $A_{\alpha i} = V_\alpha \left[a \hat{z}_i + ib(\hat{x}_i \cos\varphi(\mathbf{r}) + \hat{y}_i \sin\varphi(\mathbf{r}))\right]$

$$F_{cond} = (\alpha - 2\eta) a^2 + (\alpha + \eta) b^2 + \beta_{12} (a^2 - b^2)^2 + \beta_{345} (a^2 + b^2)^2$$

$$\beta_{12} = \beta_1 + \beta_2, \qquad \beta_{345} = \beta_3 + \beta_4 + \beta_5$$
$$\beta_{12345} = \beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5$$

polar

$$T_{c1} = T_c + 2\frac{\eta}{\alpha_0}$$

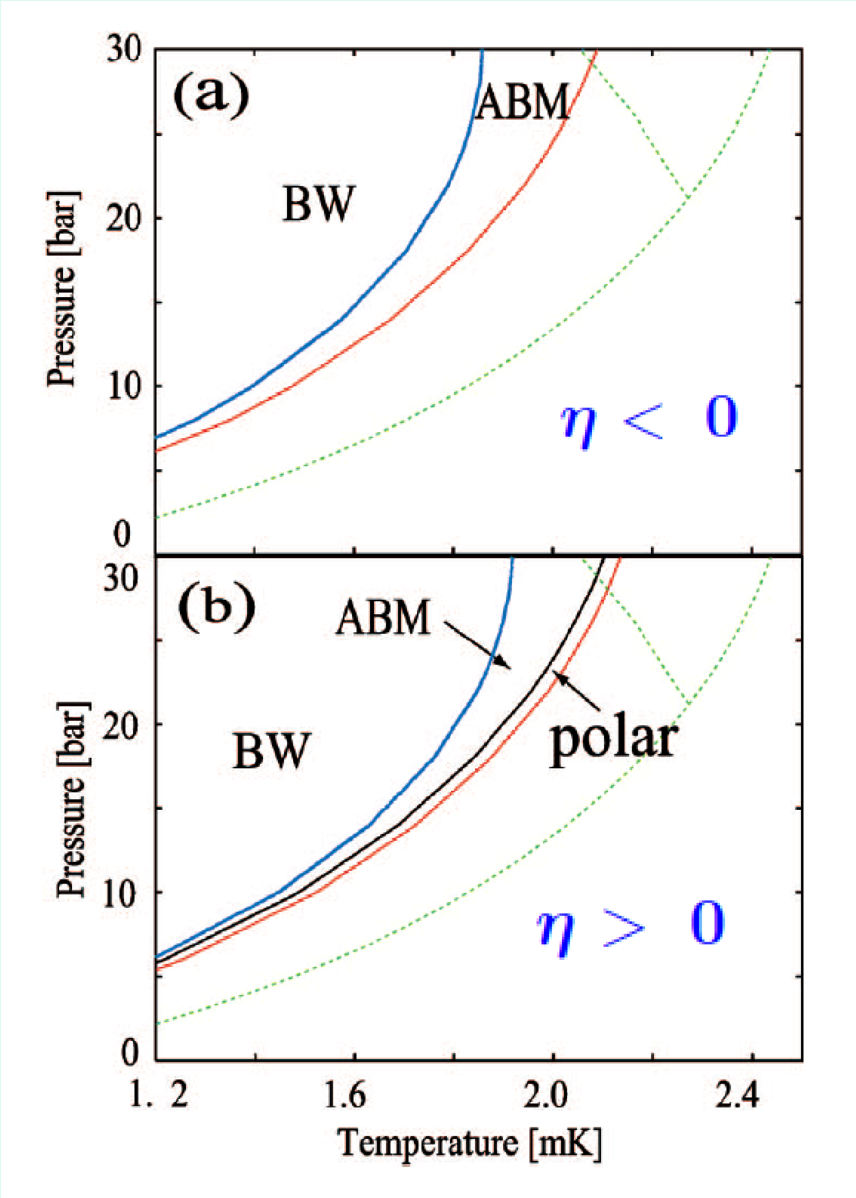
$$a^2 = a_0^2 = -\frac{\alpha_0 (T - T_{c1})}{2\beta_{12345}}, \qquad b = 0$$

axipolar

$$T_{c2} = T_c - \frac{\eta}{\alpha_0} \frac{3\beta_{345} - \beta_{12}}{2\beta_{12}} = T_c - \frac{5 - 1.2\delta}{2 + 0.15\delta} \frac{\eta}{\alpha_0}$$

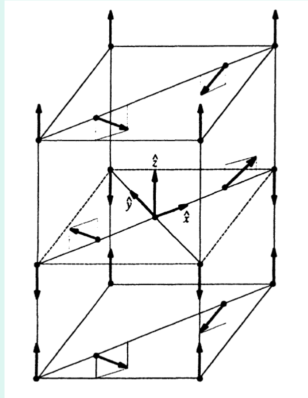
$$a = a_0 + \delta a, \qquad \delta a = -\frac{\beta_{345} - \beta_{12}}{2\beta_{12345}} \frac{b^2}{a_0}, \qquad b^2 = -\frac{\alpha_0 (T - T_{c2})}{4\beta_{345}}$$

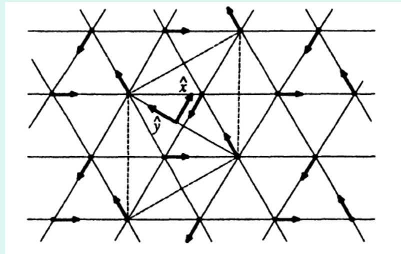
Phase diagram of superfluid He-3 in anisotropic aerogel

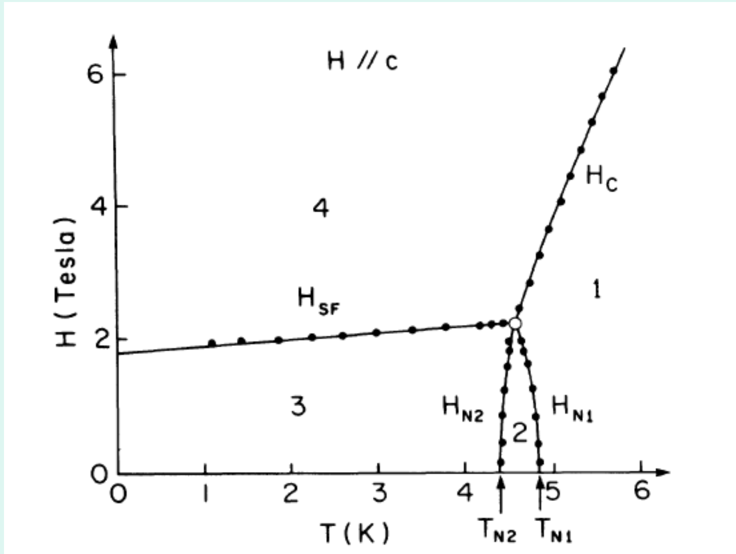


Aoyama and Ikeda, PRB 2006

$CsNiCl_3$







Zhu and Walker, PRB1987; Plumer et al, PRL 1988

Small anisotropy

At small anisotropy when $T_{c1} \approx T_{c2}$ we deal in fact with one phase transition from normal to superfluid state with order parameter amplitudes

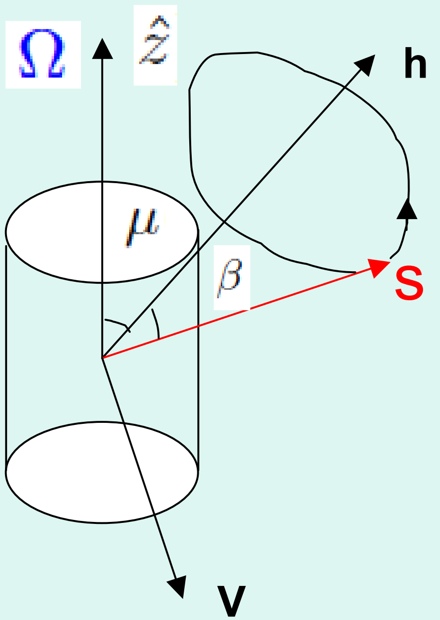
$$a = b = \Delta_A/\sqrt{2} \qquad \Delta_A^2 = -\frac{\alpha_0(T - T_{cA})}{2\beta_{345}} \qquad T_{cA} = T_c + \frac{\eta}{2\alpha_0}$$

and randomly distributed $\varphi(\mathbf{r})$ angle due to local anisotropy in Imry-Ma clusters.

Nuclear Magnetic Resonance in polar state

$$2\omega\Delta\omega = 2\omega(\omega-\gamma H) = -\frac{2\gamma^2}{\chi}\frac{\partial\langle F_{so}\rangle|_{min}}{\partial\cos\beta}$$

I.Fomin,1976



$$F_{so} = C_1\left(A_{\alpha\alpha}A^\star_{\beta\beta} + A_{\alpha i}A^\star_{i\alpha}\right) + C_2\left(2A_{zz}A^\star_{\beta\beta} + c.c.\right) + const$$

$$A_{\alpha i} = V_\alpha\left[a\hat{z}_i + ib(\hat{x}_i\cos\varphi(\mathbf{r}) + \hat{y}_i\sin\varphi(\mathbf{r}))\right]$$

$$F_{so} = [2(C_1+C_2)a^2 - C_1 b^2]\langle(\mathbf{V}(t)\hat{z})^2\rangle$$

$$F_{so} = 2(C_1+C_2)\frac{\alpha_0(T_{c1}-T)}{2\beta_{12345}}\langle(\mathbf{V}(t)\hat{z})^2\rangle$$

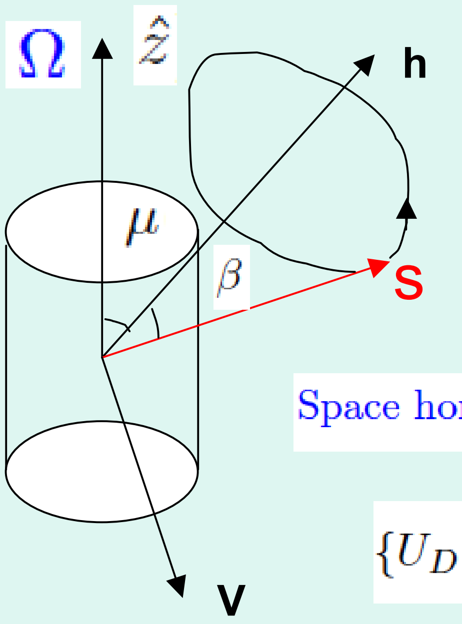
$$\mathbf{V}(t) = R_x(\mu)R_z(-\omega_L t)R_y(\beta)R_z(\omega_L t+\varphi)\hat{x}$$

$$U_D = \langle(\mathbf{V}(t)\hat{z})^2\rangle = \frac{1}{4}\sin^2\mu\left[(\cos\beta+1)^2\sin^2\varphi + (\cos\beta-1)^2/2\right] + \frac{1}{2}\cos^2\mu(1-\cos^2\beta)$$

$$2\omega\Delta\omega = -\Omega^2_{pol}\frac{\partial\, U_D|_{min}}{\partial\cos\beta}, \qquad \Omega^2_{pol} = \frac{2\gamma^2}{\chi}2(C_1+C_2)\frac{\alpha_0(T_{c1}-T)}{2\beta_{12345}}$$

NMR in homogeneous and nonhomoheneos polar state

$$A_{\alpha i} = \Delta V_\alpha \hat{z}_i e^{i\phi}$$



$$U_D = \langle (\mathbf{V}(t)\hat{z})^2 \rangle$$

$$U_D = \frac{1}{4}\sin^2\mu\left[(\cos\beta+1)^2\sin^2\varphi + (\cos\beta-1)^2/2\right] + \frac{1}{2}\cos^2\mu(1-\cos^2\beta)$$

Space homogeneous $\mathbf{V} = const$ - (i) $\Omega = 0$, (ii) $\Omega > \Omega_{c1}$ - single quantum vortices

$$\{U_D\}_{min}(\varphi = 0) = \frac{1}{8}\sin^2\mu(\cos\beta-1)^2 + \frac{1}{2}\cos^2\mu(1-\cos^2\beta)$$

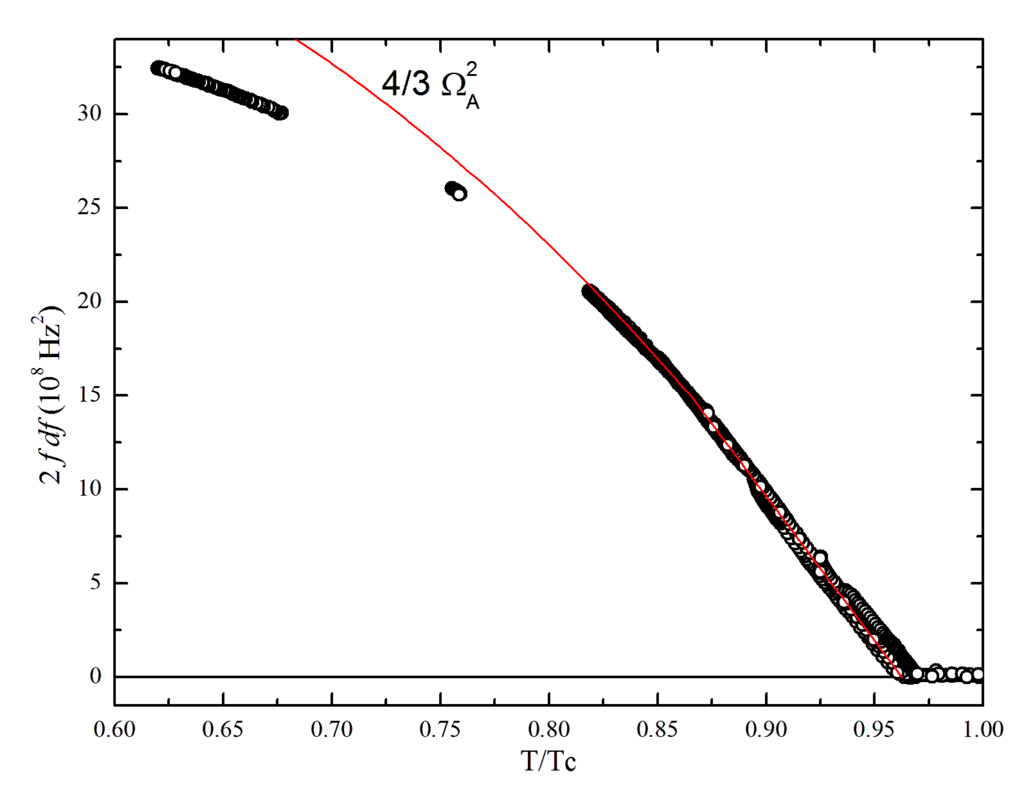
$$2\omega\Delta\omega = \Omega_{pol}^2\left(\cos\beta + \frac{1}{4}\sin^2\mu(1-5\cos\beta)\right)$$

Space nonhomogeneous $\mathbf{V}(\mathbf{r})$ $\Omega > \Omega_{c1}$ - half-quantum vortices

$$\langle U_D\rangle_\varphi = \frac{1}{4}\sin^2\mu(\cos^2\beta+1) + \frac{1}{2}\cos^2\mu(1-\cos^2\beta)$$

$$2\omega\Delta\omega = \Omega_{pol}^2\left(\cos\beta - \frac{3}{2}\sin^2\mu\cos\beta\right)$$

Slope at small pressures



It is instructive to make formal comparison of the obtained frequency shift with the frequency shift that occurs in case of direct transition to the Imry-Ma A-phase. This case

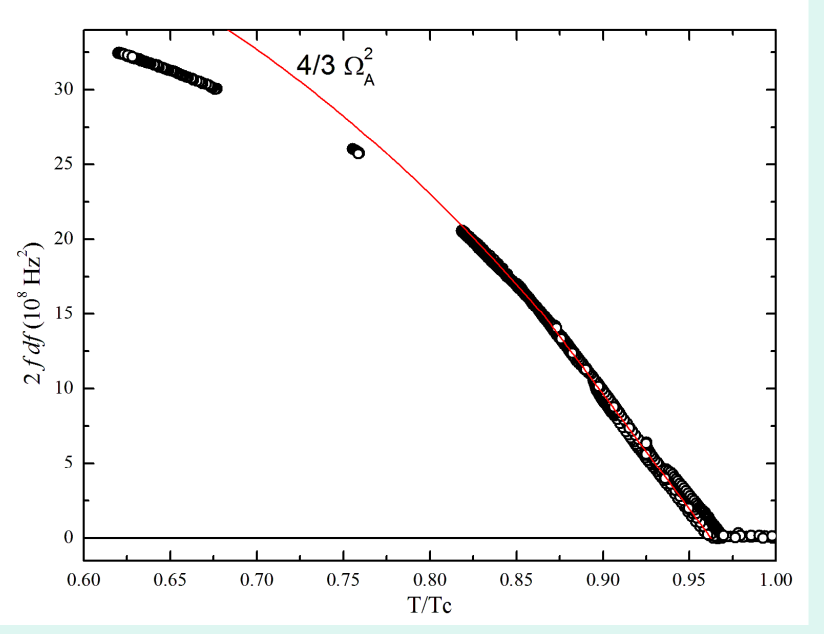
$$2\omega\Delta\omega = \frac{1}{2}\Omega_A^2\left(\cos\beta + \frac{1}{4}\sin^2\mu(1-5\cos\beta)\right), \qquad \Omega_A^2 = \frac{2\gamma^2}{\chi}(C_1+2C_2)\frac{\alpha_0(T_{cA}-T)}{2\beta_{345}}.$$

Then the slopes of linear temperature dependence of Ω_{pol}^2 and Ω_A^2 are related to each other as follows

$$\frac{d\Omega_{pol}^2}{dT} \Big/ \frac{d\Omega_A^2}{dT} = \frac{2(C_1+C_2)}{C_1+2C_2}\frac{\beta_{345}}{\beta_{12345}} = \frac{4(C_1+C_2)}{3(C_1+2C_2)}\frac{1-0.525\delta}{1-0.3\delta}.$$

Experimentally (Dmitriev, unpublished) this ratio at small pressures is close to 4/3.

Slope change below T_{c2}



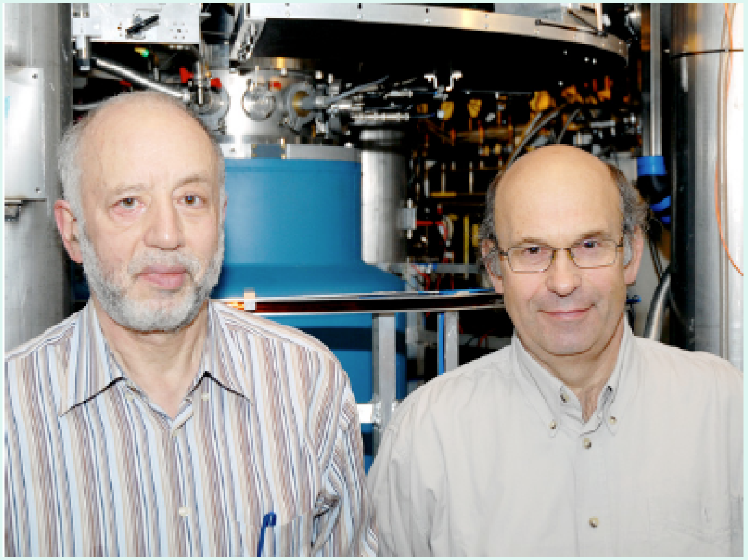
$$2\omega\Delta\omega = \Omega^2_{axipol}\left(\cos\beta + \frac{1}{4}\sin^2\mu(1-5\cos\beta)\right),$$

$$\Omega^2_{axipol} = \frac{2\gamma^2}{\chi}2(C_1+C_2)\frac{\alpha_0(T_{c1}-T)}{2\beta_{12345}}\left[T_{c1}-T+r(T-T_{c2})\right]$$

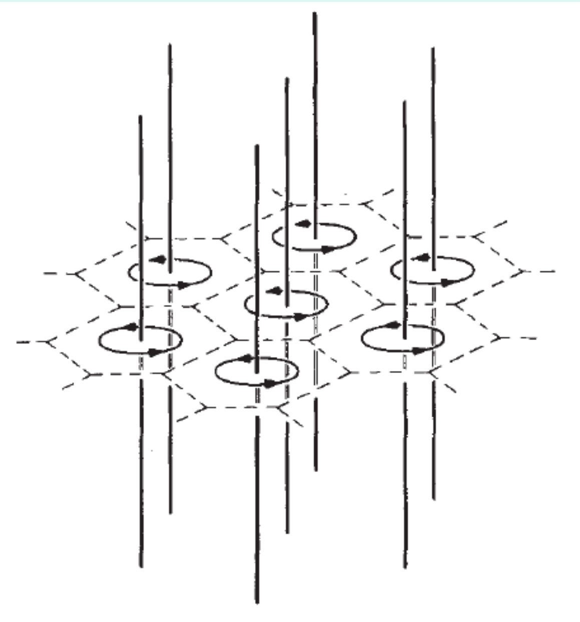
$$C_2 = 0$$

$$1-r = 1-\frac{1}{4\beta_{345}}\left[2(\beta_{345}-\beta_{12})+\frac{C_1}{C_1+C_2}\beta_{12345}\right]$$

$$r = \frac{3\beta_{345}-\beta_{12}}{4\beta_{345}} = \frac{5-3.22\delta}{8-4.2\delta}$$



Conclusion



The superfluid He-3 under rotation has been discussed first in 1977 by G.E.Volovik and N.B.Kopnin.
Since that time the singular single quanta vortices as well nonsingular vortices with 2 quanta of circulation have been revealed in rotating He-3A. He-3A supports also the existence of vortices with half-quantum of circulation. However, the half-quantum vortices in open geometry always possess an extra energy due to spin-orbital coupling leading to formation of domain wall at distances larger than dipole length ~10^{-3} cm from vortex axis. Fortunately the same magnetic dipole-dipole interaction does not prevent the existence of half-quantum vortices in the polar phase of superfluid He-3 recently discovered in peculiar porous media "nematically ordered" aerogel.