Kopnin in Physics

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Landau Days, 24 June, 2014

* ***start of Majorana physics in cond-mat***
* ***first topological flat band***
* ***flat band superconductivity***
* ***chiral anomaly in cond-matter*** **(Kopnin force)**
* **Kopnin mass**
* ***skyrmion lattice***
* ***semilocal topological objects*** **(Kopnin-Burlachkov skyrmions)**
* ***instantons in cond-matter*** **(Ivlev-Kopnin phase slip)**
* ***Witten cosmic string in cond-mat (rotating core of vortex)***
* ***cosmic string formation by propagating front*** **(Kopnin-Thuneberg mechanism)**
* ***force on moving mirror (AB interface)***
* ***event horizon***
* ***non-equilibrium superconductivity***
* ***superfluid turbulence*** **(Kopnin number)**

Iordanskii force

Gravitational Aharonov-Bohm effect

heat bath velocity $\mathbf{v}_n$

Kopnin force

Axial anomaly

$$\mathbf{F}_{\mathrm{Iordanskii}} = \kappa \times \rho_n(\mathbf{v}_s - \mathbf{v}_n)$$

Aharonov-Bohm scattering of quasiparticles on a vortex

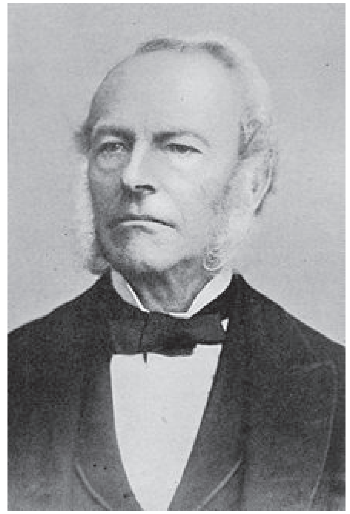
$$\mathbf{F}_{\mathrm{Kopnin}} = \kappa \times \mathbf{C}(T)(\mathbf{v}_n - \mathbf{v}_L)$$

momentum transfer from negative energy states in the core to heat bath

analog of baryogenesis

vacuum velocity $\mathbf{v}_s$

vortex velocity $\mathbf{v}_L$



$$\mathbf{F}_{\mathrm{Magnus}} = \kappa \times \rho(\mathbf{v}_L - \mathbf{v}_s)$$

momentum transfer between vortex and superfluid vacuum

Magnus–Joukowski lifting force in classical hydrodynamics

$$\mathbf{F}_{\mathrm{Magnus}} + \mathbf{F}_{\mathrm{Iordanskii}} + \mathbf{F}_{\mathrm{Kopnin}} + \mathbf{F}_{\mathrm{Stokes}} = 0$$

Kopnin functions $\mathbf{C}(T), \gamma(T)$

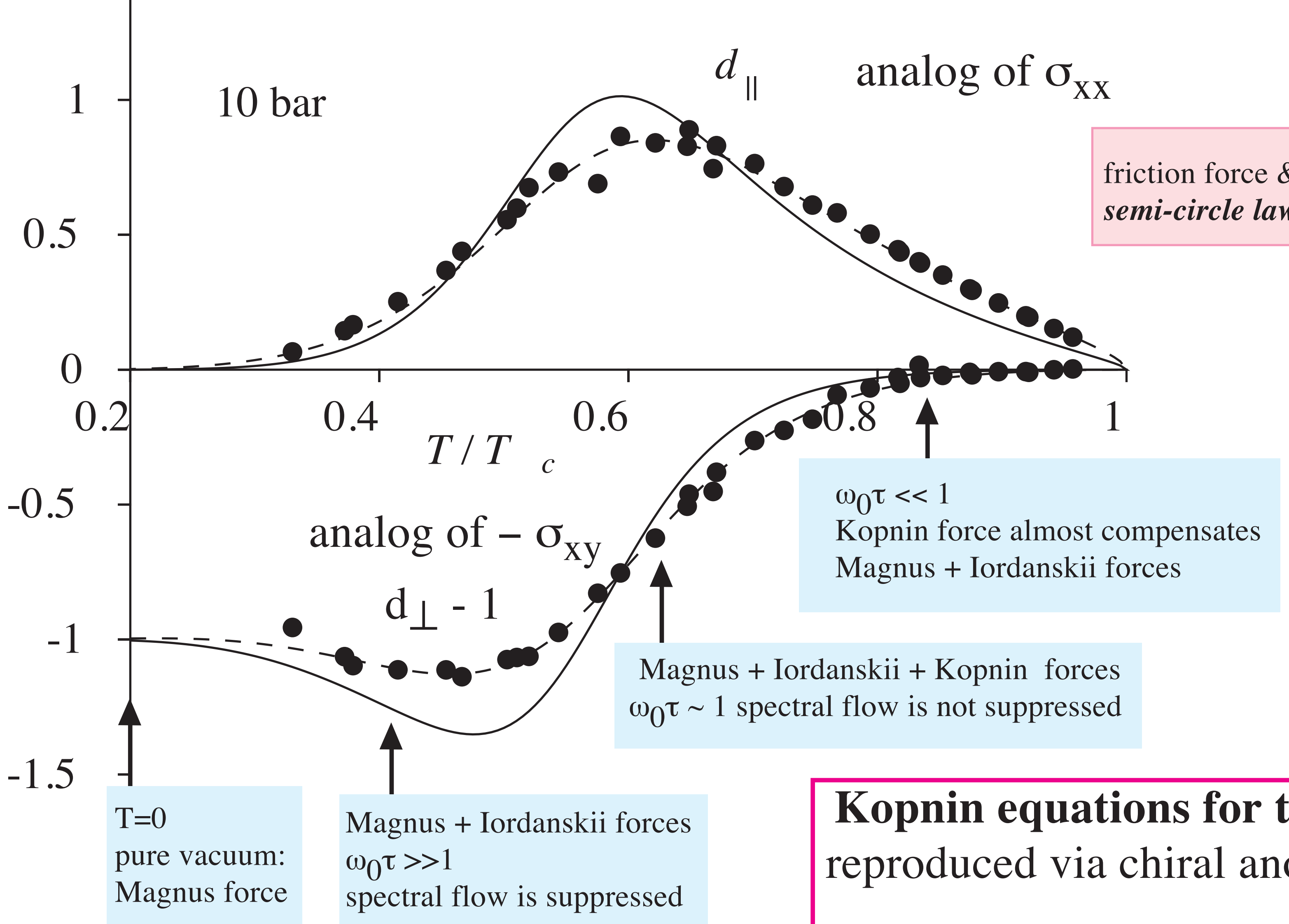
Stokes friction force

$$\mathbf{F}_{\mathrm{Stokes}} = -\gamma(T)\ (\mathbf{v}_L - \mathbf{v}_n)$$

Kopnin, Kravtsov, JETP **44**, 861 (1976)
Forces acting on vortices moving in a pure type II superconductor
Kopnin, GV, Parts, EPL **32**, 651 (1995)
On the spectral flow in vortex dynamics of superfluid 3He-B and superconductors

Observation of Kopnin force in Manchester experiments on ^{3}He-B vortices

$$\hat{z} \times (\mathbf{v}_L - \mathbf{v}_s) + d_\perp \hat{z} \times (\mathbf{v}_n - \mathbf{v}_L) + d_\parallel (\mathbf{v}_n - \mathbf{v}_L) = 0$$



friction force & nondissipative forces obey ***semi-circle law*** as σ_{xx} and σ_{xy} in QHE

$$\alpha + \boldsymbol{i}(1 - \alpha') = 1/(d_\parallel - \boldsymbol{i}(1 - d_\perp))$$

Kopnin equations for transport parameters

reproduced via chiral anomaly by Stone (1996)

$$1 - d_\perp = (\rho / \rho_s) \tan(\Delta/2T)\, (\omega_0\tau)^2 / [1+(\omega_0\tau)^2]$$

$$d_\parallel = (\rho / \rho_s) \tan(\Delta/2T)\, \omega_0\tau / [1+(\omega_0\tau)^2]$$

Kopnin number for superfluid hydrodynamics

equations of superfluid hydrodynamics with vorticity

$$\frac{\partial \mathbf{v}_{\mathrm{s}}}{\partial t} + \nabla\mu - \mathbf{v}_{\mathrm{s}} \times (\nabla \times \mathbf{v}_{\mathrm{s}}) = \mathbf{F}$$

$$\mathbf{F} = -\alpha'(\mathbf{v}_{\mathrm{s}} - \mathbf{v}_{\mathrm{n}}) \times (\nabla \times \mathbf{v}_{\mathrm{s}}) - \alpha\, \hat{\mathbf{n}} \times ((\mathbf{v}_{\mathrm{s}} - \mathbf{v}_{\mathrm{n}}) \times (\nabla \times \mathbf{v}_{\mathrm{s}}))$$

two dimensionless parameters in superfluid hydrodynamics:
non-dissipative $1-\alpha'$ & frictional α
their ratio is analog of Reynolds number for superfluid

$$\boxed{\mathrm{Ko} = (1-\alpha')/\alpha}$$

$$\mathrm{Ko} \sim \omega_0\tau$$

Reynolds & Kopnin numbers in 2-fluid hydrodynamics

phase diagram of turbulent flow in Fermi superfluids

$$\boxed{Ko = (1 - \alpha')/\alpha}$$

Kopnin number $Ko \sim \omega_0 \tau$

$$\boxed{Re_\nu = UR/\nu_n} \ll 1$$

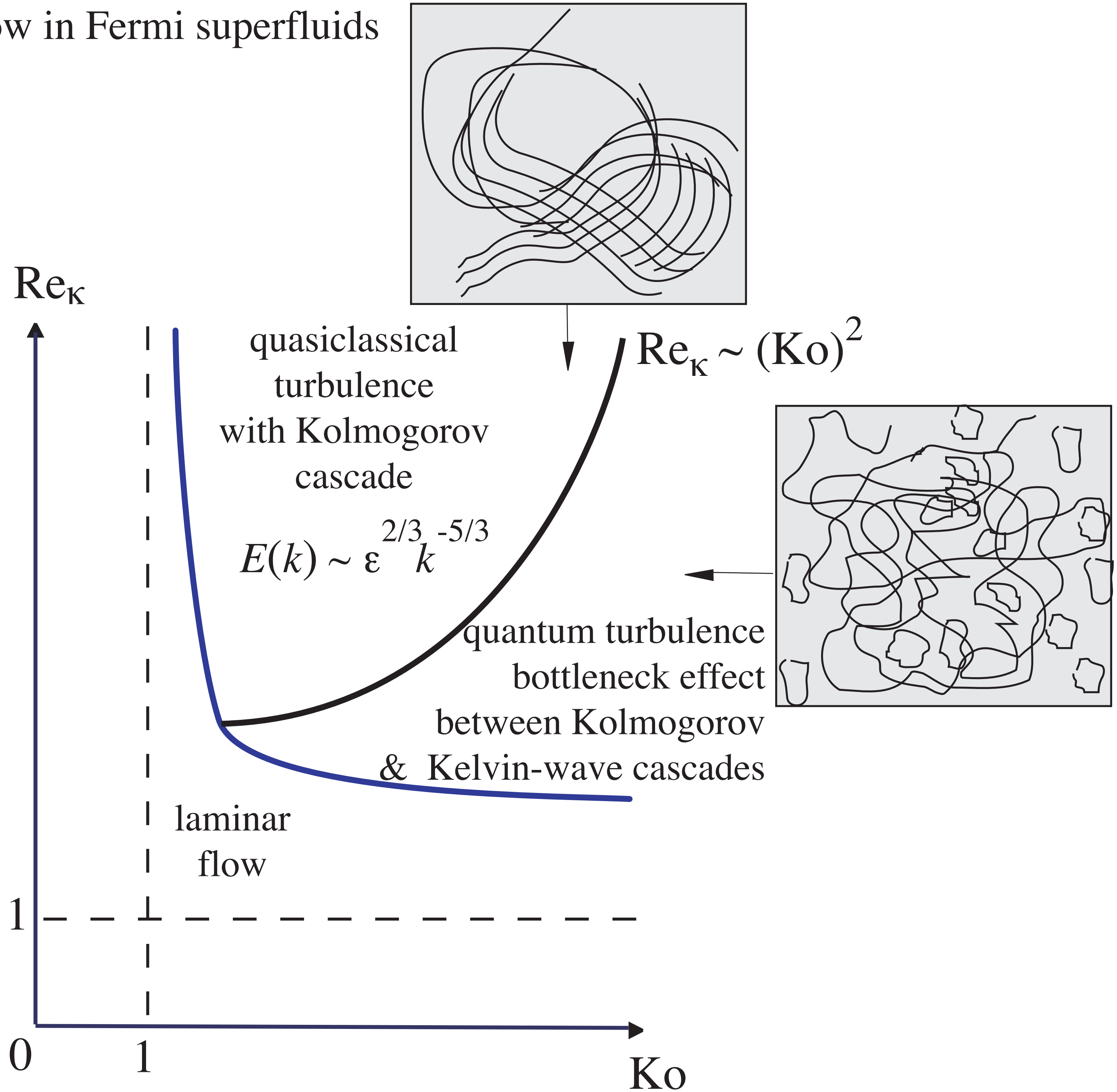
conventional
Reynolds number

ν_n – viscosity
of normal component

$$\boxed{Re_\kappa = UR/\kappa}$$

vorticity
Reynolds number

κ – circulation quantum

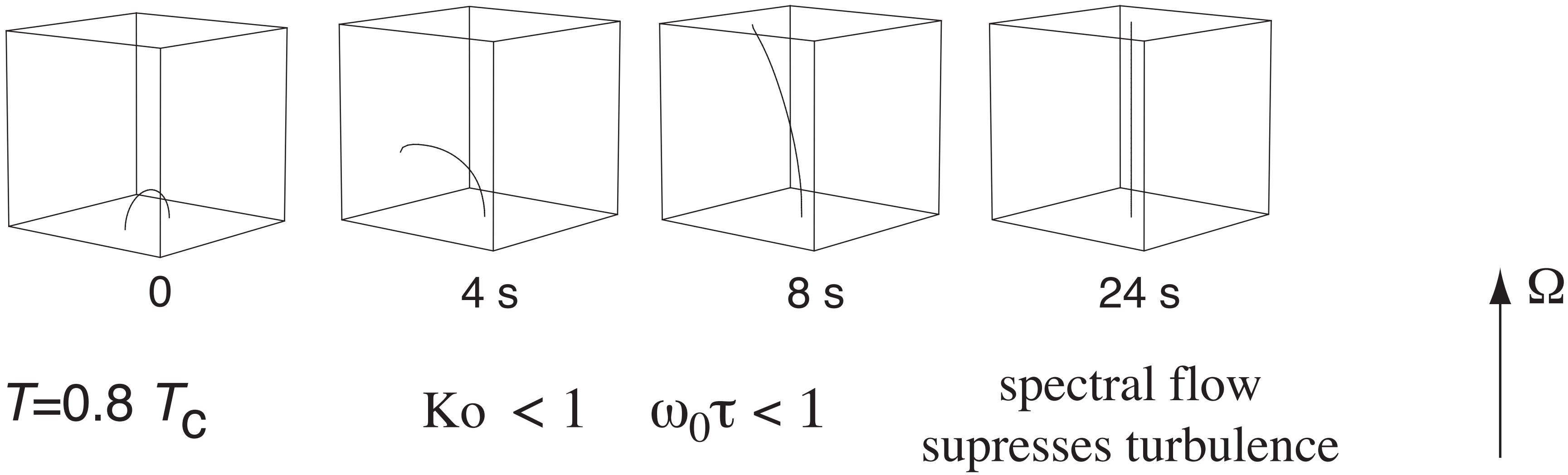


Kopnin number for superfluid turbulence

two dimensionless parameters in superfluid hydrodynamics:
non-dissipative $1-\alpha'$ & frictional α
their ratio is analog of Reynolds number for superfluid

$$\mathrm{Ko} = (1-\alpha')/\alpha$$

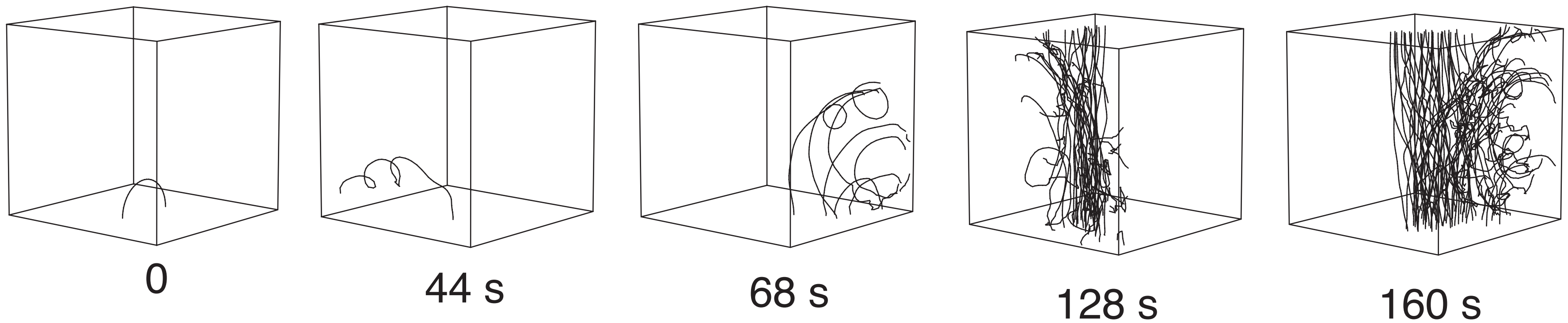
$$\mathrm{Ko} \sim \omega_0\tau$$



T=0.8 T_c Ko < 1 $\omega_0\tau < 1$

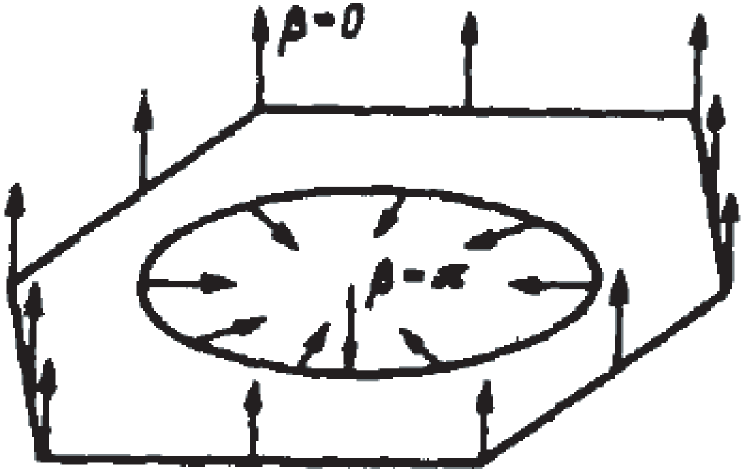
spectral flow
supresses turbulence

transition to superfluid turbulence occurs at Ko ~ 1



T=0.4 T_c Ko > 1 $\omega_0\tau > 1$

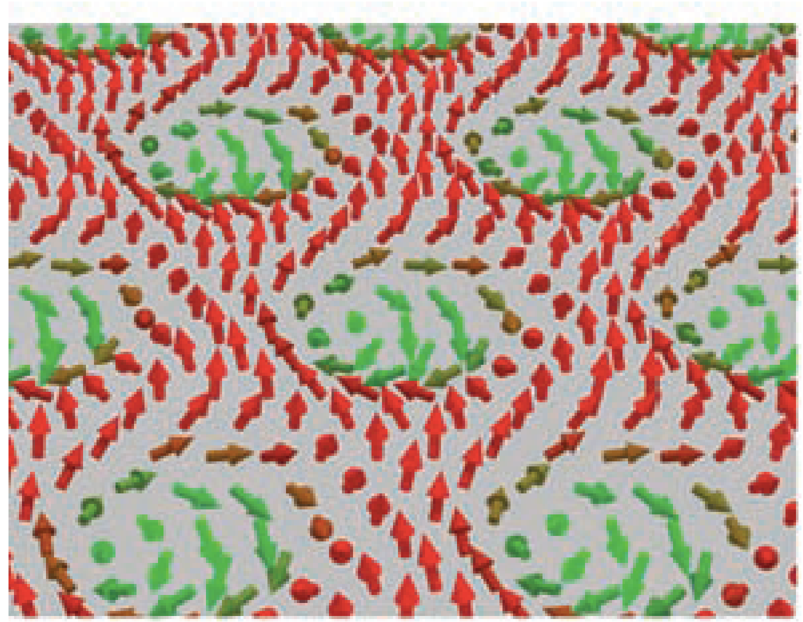
skyrmion lattices in 3He-A & chiral superconductors



GV & Kopnin, On the rotating 3He-A
JETP Lett. 25, 22 (1977)

lattice of semilocal skyrmion

Burlachkov & Kopnin
ZhETF **92**, 1110 (1987)



Science 13 February 2009:
Vol. 323 no. 5916 pp. 915-919
Skyrmion Lattice in a Chiral Magnet
S. Mühlbauer et al.

Semilocal cosmic object in cond-mat physics

A. Achucarro & T. Vachaspati
Phys. Rep. **327** (2000) 347
SEMILOCAL & ELECTROWEAK STRINGS

$$G = SU(2)_{\text{global}} \times U(1)_{\text{local}}$$

$$H = U(1)_{\text{semilocal}}$$

We should point out that systems related to the semilocal model have been studied in condensed matter. In **[28]**, the system was an unconventional superconductor where the role of the global SU(2) group was played by the spin rotation group.

[28] Burlachkov & Kopnin ZhETF 92, 1110 (1987)

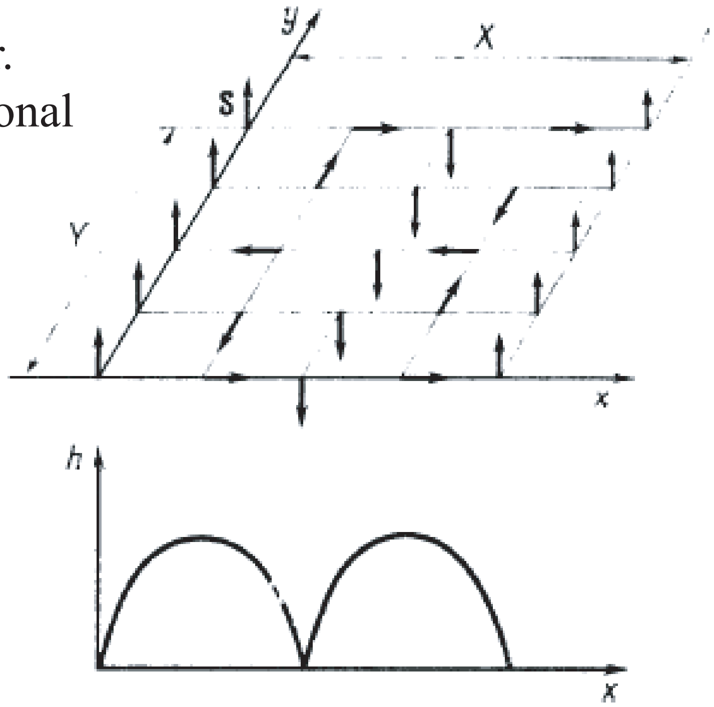
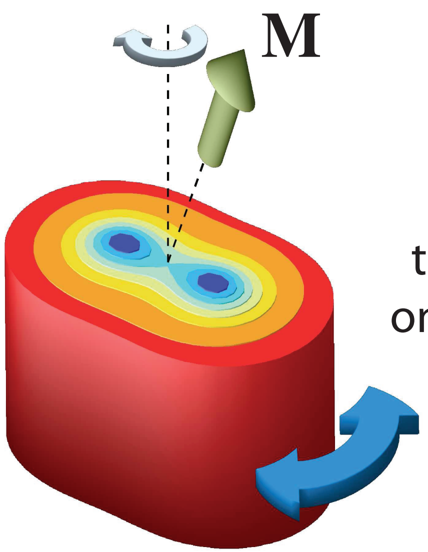


FIG. 1. Unit cell of the spin texture. The cell (nonsingular vortex) is rectangular and carries four quanta of magnetic flux. The coordinate axes are chosen so that $m_y < m_x$. The polar angle θ of the spin vector **S** depends only on the coordinate x and the azimuthal angle $\varphi = ky$. On the dashed line the direction of rotation of the spin is reversed (i.e., $k \to (-k)$). The magnetic field is $h = h(x)$.

Rotating vortex core: An instrument for detecting the core excitations (1998)



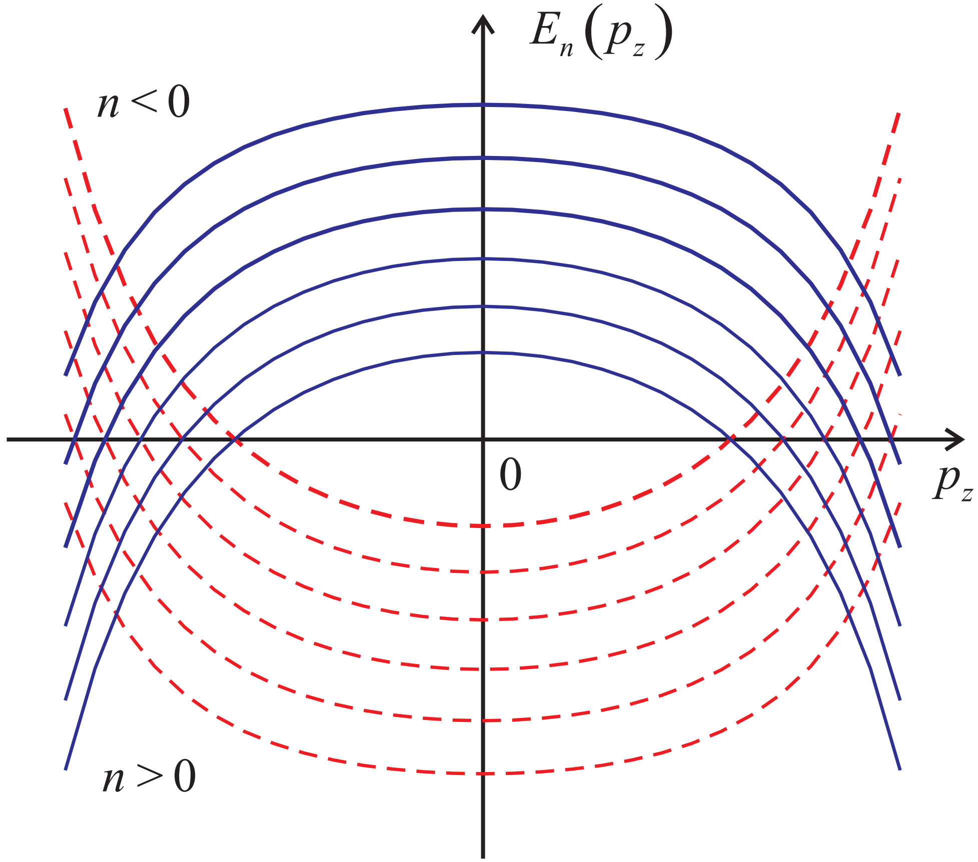
twistino: Goldsone
on broken symmetry
vortex core

excited by NMR

relaxation due to
fermion zero
modes in the core

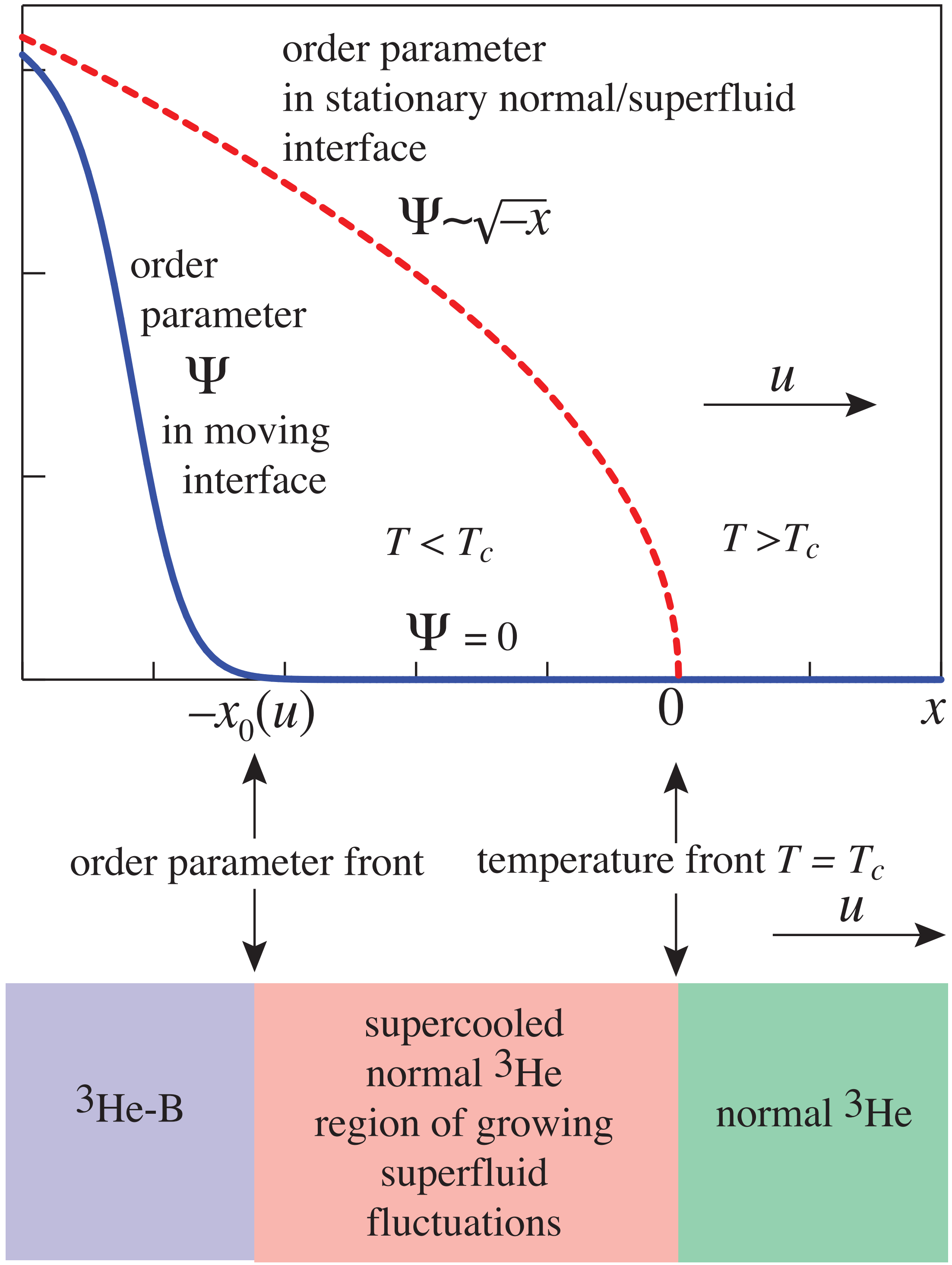
Witten string

core fermions



Kopnin-Thuneberg mechanism of string formation

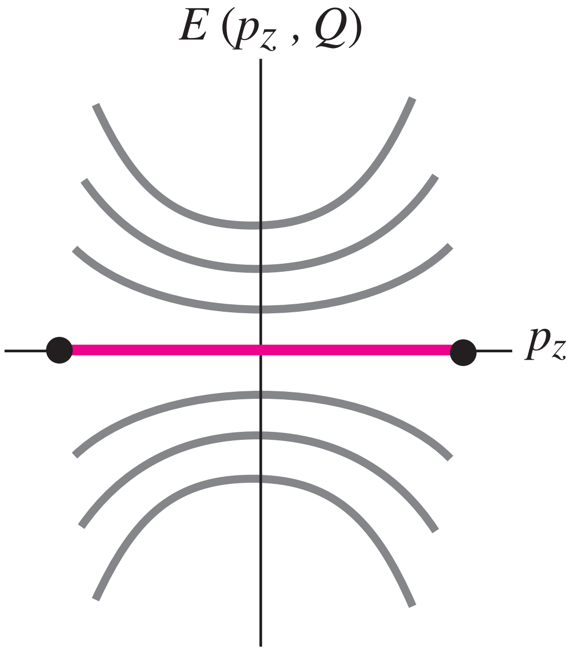
Kopnin & Thuneberg
PRL 83, 116 (1999)



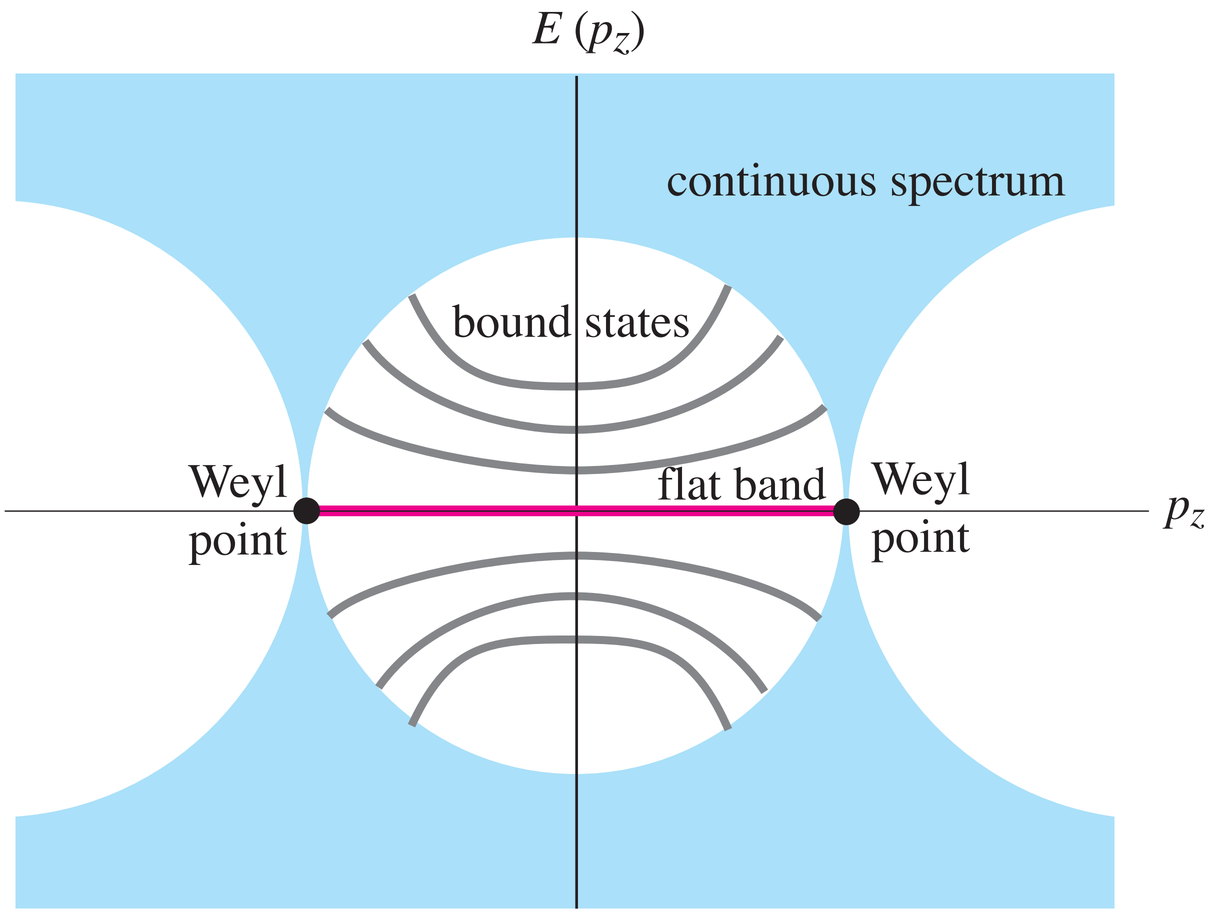
Start of Majorana cond-mat physics
Start of topologically protected flat bands

topologically protected Majorana flat band in vortex core of superfluids with Weyl points

Kopnin-Salomaa, PRB **44**, 9667 (1991)



flat band of bound states terminates on zeroes of continuous spectrum (i.e on Weyl points)



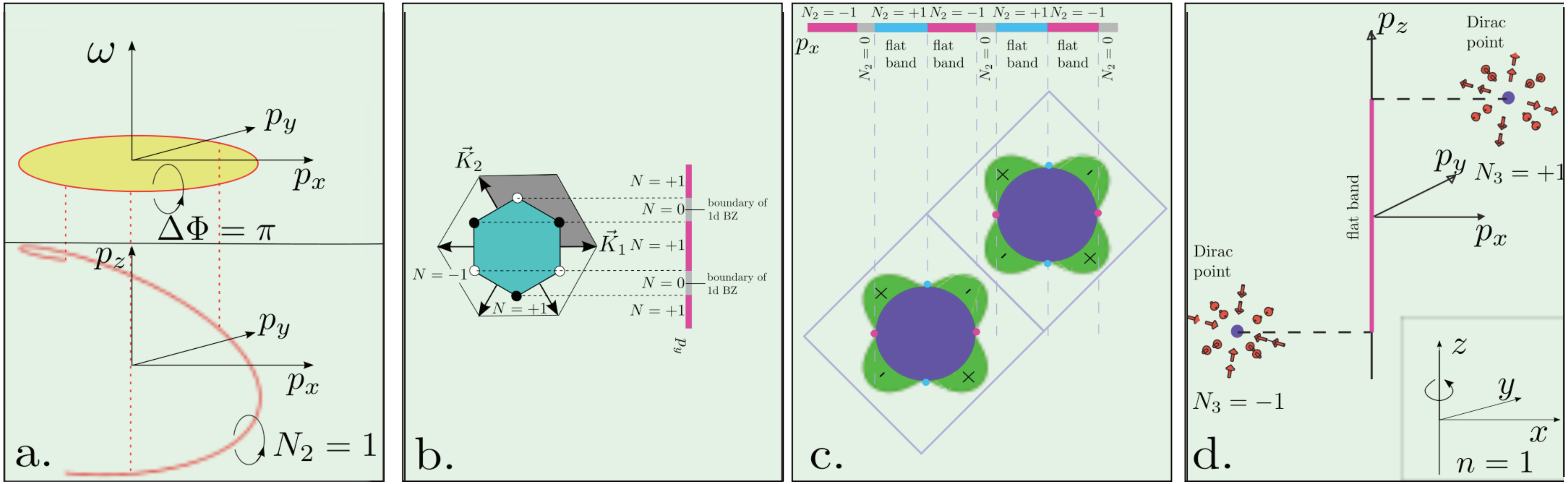
Topological flat bands

Flat band in multilayered graphene (Heikkilä-Kopnin-GV)

Flat band on graphene edge (Ryu-Hatsugai)

Flat band in cuprates (Kashivaya-Tanaka)

Flat band in vortex core (Kopnin-Salomaa)



Applications

1. Kopnin force
chiral anomaly
application to Standard Model, QCD, superconductivity

2. Burlachkov-Kopnin skyrmion
semilocal skyrmions in chiral superconductor
application to cosmic strings

3. Kopnin-Salomaa Majorana modes
flat band of Majorana modes in vortex core of chiral superfluid
application to Majorana physics & room-T superconductivity

4. Kopnin number
transition to superfluid turbulence
application to physics of turbulence

5. Kopnin mass
chiral Weyl particles in magnetic field at finite chemical potential
application to QCD & topological matter

6. Kopnin-Thuneberg mechanism
application to cosmic string creation

My work with Nikolai Kopnin:

-Sharing office for several years (2007-2011)

-Project on nonequilibrium superconductivity in 2006

-Flat band superconductivity 2011-2013

-Many times asking for his advice in physics, technical matters, different aspects of superconductivity, career matters…

Nikolai is well-known as a kind and friendly person and a thorough physicist. I wish to share one additional thing I would like to learn from him: Nikolai really knew the value of time. First, he never seemed to be too busy to discuss physics or other things. Second, when needed, he was able to use whatever time was required to finish a detailed calculation. Nikolai’s attitude seems to have been ”Choose what you want to do, and then do it.” He did not spend his time with (for him) useless things. I am honored and pleased to have had the possibility to work with such a great scientist and great man.

Tero Heikkilä

Towards room-temperature flat-band superconductivity

T. Heikkilä & G. Volovik

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Landau Days, 24 June, 2014

1. metal as dominating topological material

* Fermi surface as vortex in p-space
* finite DOS at zero energy as source of conventional superconductivity

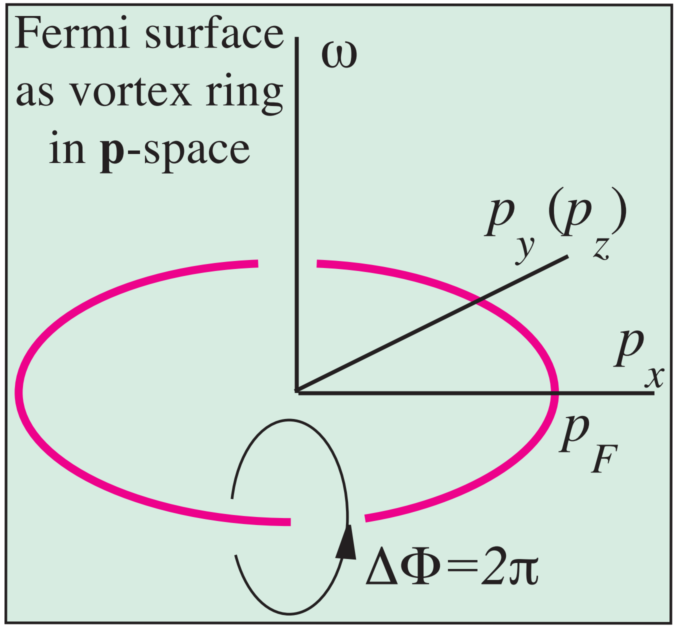
2. flat band as novel form of topological matter

* from Fermi surface to Khodel-Shaginyan flat band
* flat band near van Hove singularity
* surface flat bands in materials with nodal lines: cuprate superconductors, graphene
* flat bands in the vortex core (Kopnin-Salomaa 1991)
* flat bands in the core of other topological defects (dislocation, domain wall, soliton, ...)

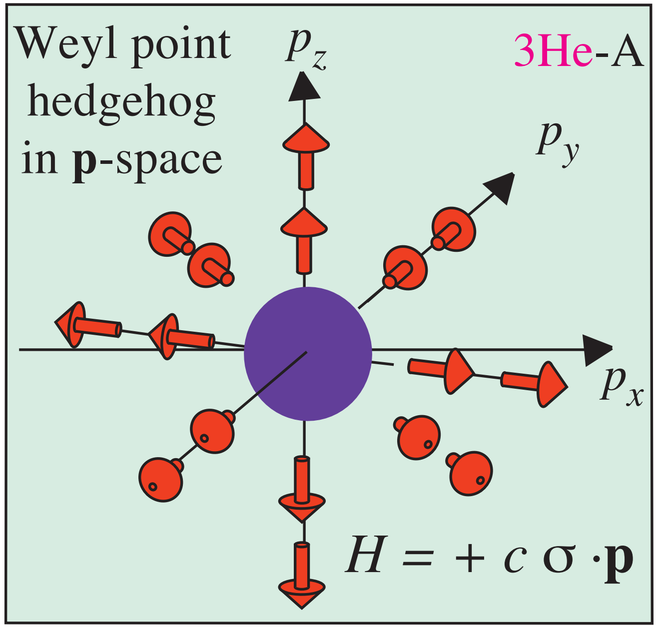
3. flat band as route to room T_c superconductivity

* flat band & superconductivity in rhombohedral graphite (Heikkila-Kopnin)
* graphite & its twist interfaces (screw dislocation network & possible flat band)
* room-T superconductivity in graphite?

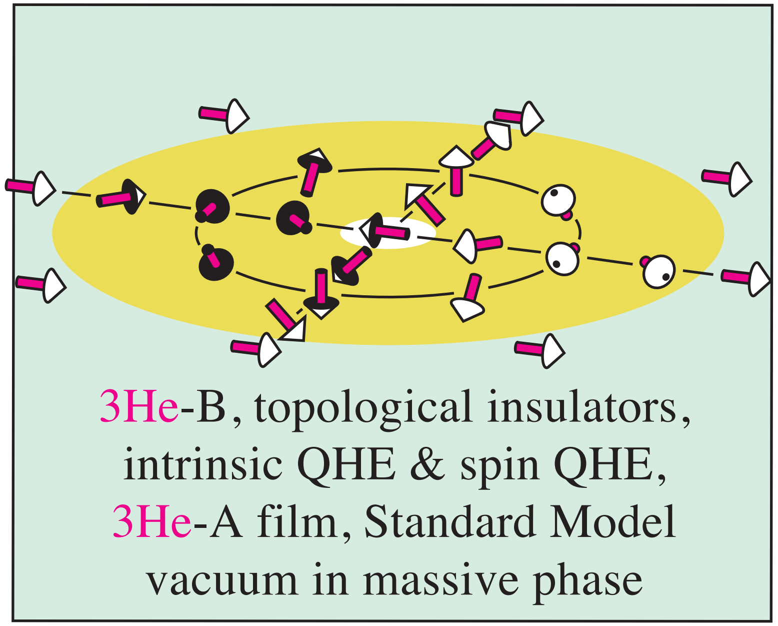
gapless topological vacua as defects in p-space



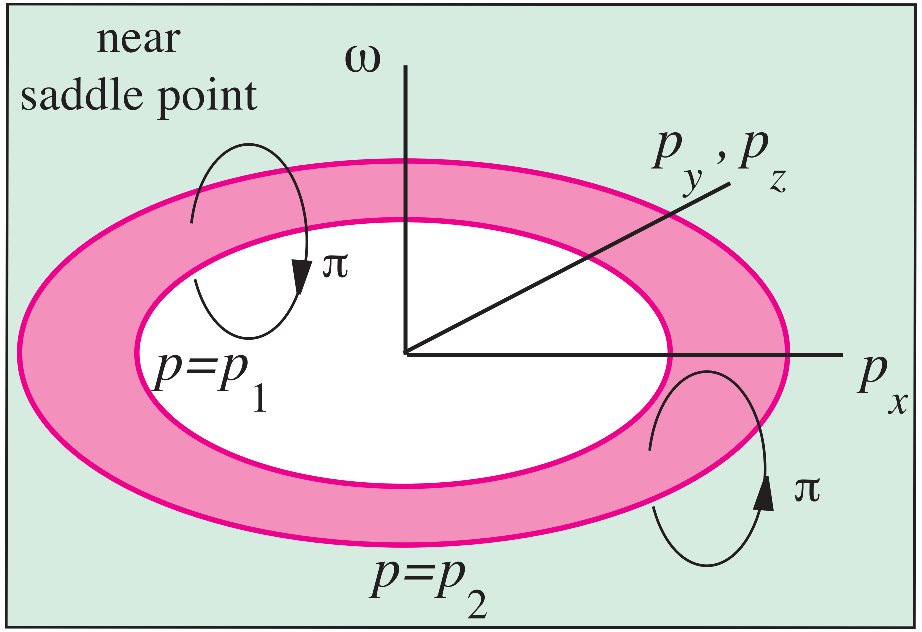
metals, normal 3He



3He-A, vacuum of Standard Model,
topological semimetals (Abrikosov)

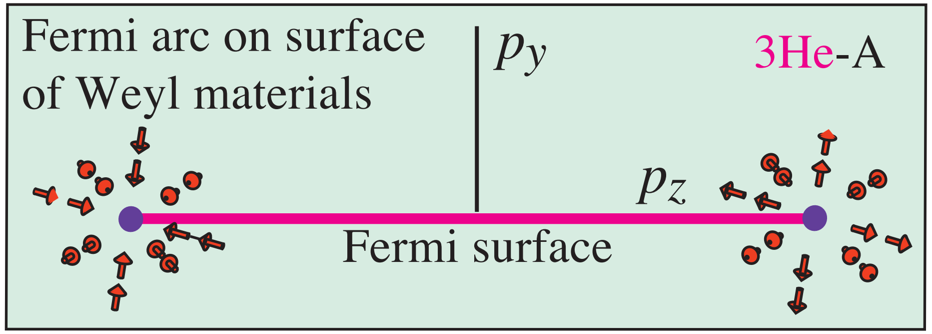


fully gapped topological matter -
skyrmion in **p**-space
dimensional reduction of Horava-2005
K-theory classification

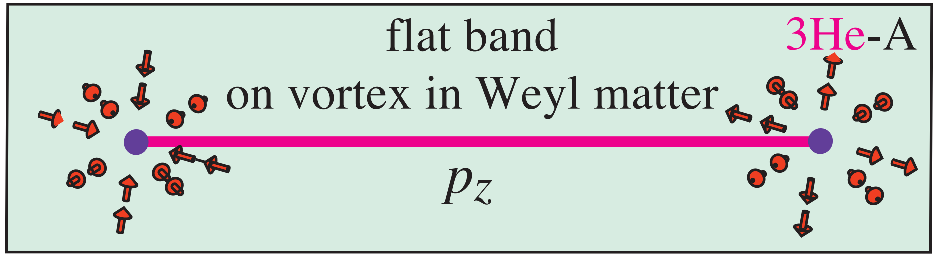


Khodel-Shaginyan flat band: π-vortex in **p**-space

bulk - surface
correspondence



bulk - vortex
correspondence



Kopnin-Salomaa flat band on vortex:
Dirac string in vortex terminating on monopole in bulk

Topological matter

bulk - edge correspondence:

topology in bulk protects:
gapless fermions on surface of fully gapped systems;
higher order nodes in nodal topological materials

2D Quantum Hall insulator & 3He-A film	gapless chiral edge states (GV 1992)
3D topological insulator	gapless Dirac fermions (Volkov-Pankratov 1985)
superfluid 3He-B	gapless Majorana fermions (Salomaa-GV 1988)
3He-A, Weyl semimetal with point nodes	Fermi arc (nodal line) on surface (Tutsumi et al 2011)
graphene with Dirac nodes	dispersionless 1D flat band (Ryu-Hatsugai 2002)
semimetal with Fermi lines	2D flat band on the surface (Heikkila-Kopnin-GV 2010)

bulk - defect correspondence:

topology in bulk protects gapless fermions inside topological defect (vortex / cosmic string)

relativistic string	fermion zero modes in core (Jackiw-Rossi 1981)
3He-A with Weyl points	1D flat band in the core (Kopnin-Salomaa 1991)
2D 3He-A & 2D p+ip superconductor	Majorana fermions in the core (GV 1999)

two major universality classes of gapless topological matter

Landau theory of Fermi liquid

vacua with Fermi surface:
metals, normal ^{3}He

universal properties of metals emerge from topological stability of **Fermi surface**

Standard Model + gravity

vacua with Weyl, Dirac, Majorana points:
^{3}He-A, planar phase, Weyl semimetal, vacuum of SM

gravity & SM emerge from topological stability of **Fermi (Weyl) point**

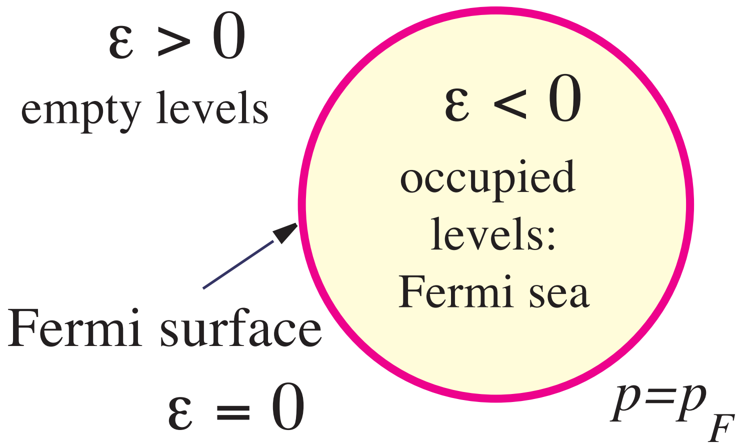
$$g^{\mu\nu}(p_\mu - eA_\mu - e\tau\cdot\mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau\cdot\mathbf{W}_\nu) = 0$$

Theory of topological matter:
Nielsen, TKNN, Volkov, So, Ishikawa, Matsuyama, Haldane, Yakovenko, Kaplan, Read, Horava, Kitaev, Ludwig, Schnyder, Ryu, Furusaki, S-C Zhang, Kane, Liang Fu, ...

metal is dominating topological material

Energy spectrum of
non-interacting gas of fermionic atoms

$$\varepsilon(p) = \frac{p^2}{2m} - \mu = \frac{p^2}{2m} - \frac{p_F^{\,2}}{2m}$$



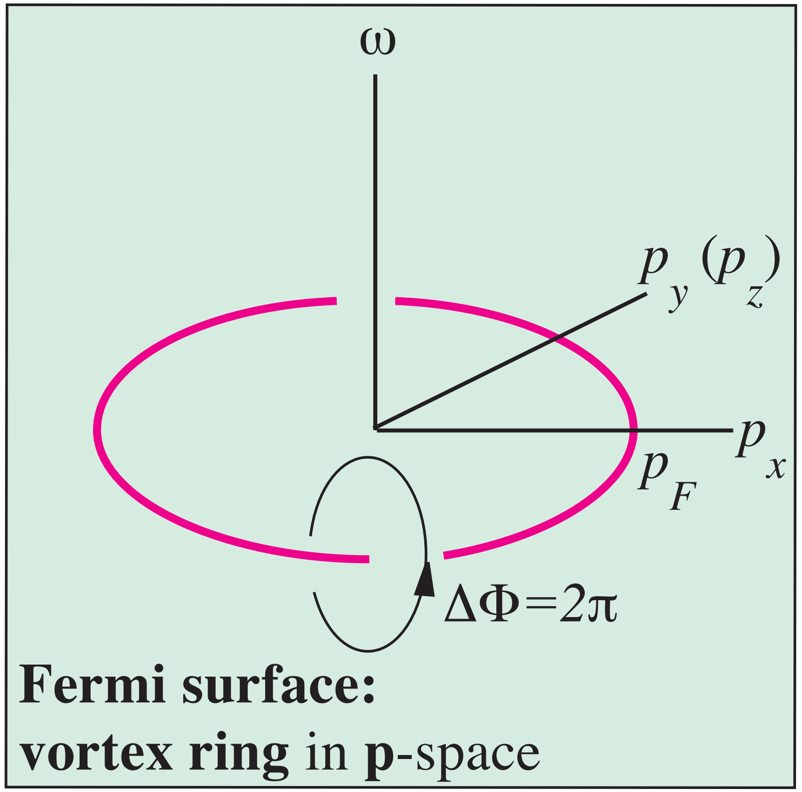
why metals are ubiquitous ?

because Fermi surface is topologically protected

Fermi surface = vortex in p-space

Green's function

$$G^{-1} = i\omega - \varepsilon(p)$$



phase of Green's function

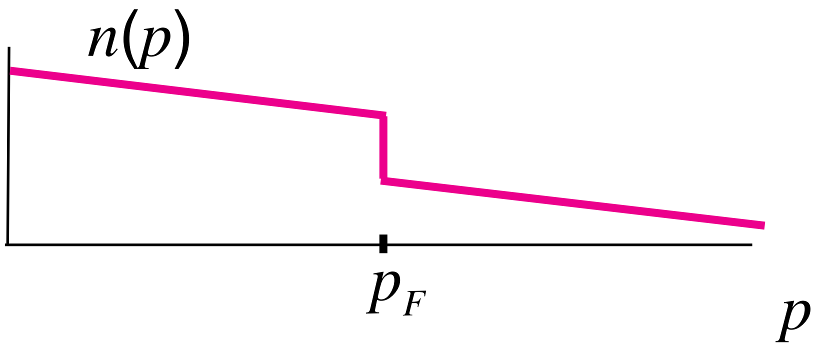
$$G(\omega,\mathbf{p}) = |G|e^{\,i\Phi}$$

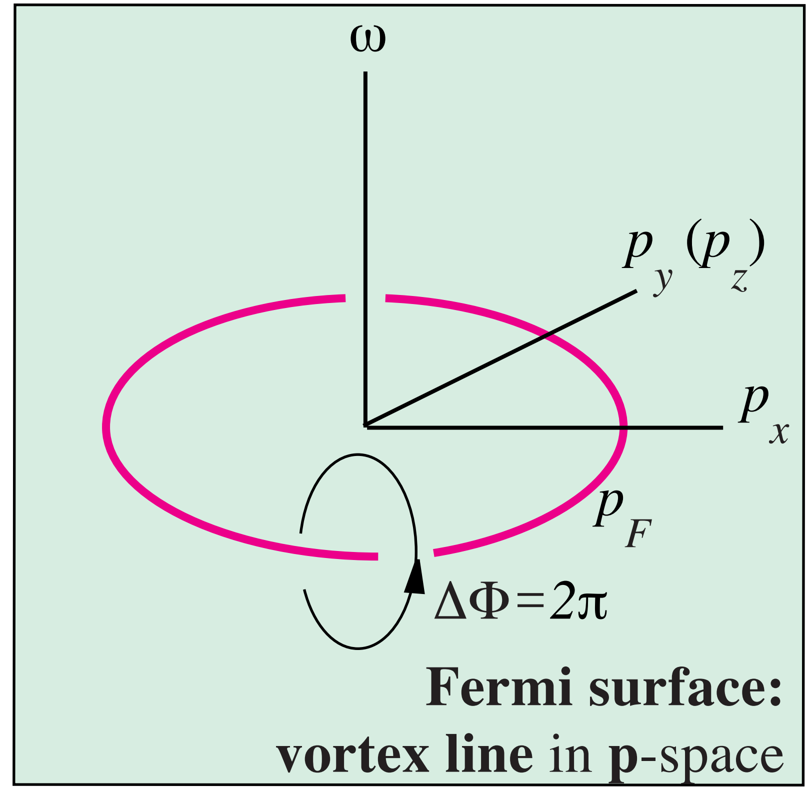
has winding number $N = 1$

Migdal jump, non-Fermi liquids & p-space topology

* Singularity at Fermi surface is robust to perturbations:
winding number $N=1$ cannot change continuously,
interaction (perturbative) cannot destroy singularity

* Typical singularity: Migdal jump





$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$

* Other types of singularity with the same winding number:
Luttinger Fermi liquid,
marginal Fermi liquid, pseudo-gap ...

* Example: zeroes in $G(\omega,\mathbf{p})$ have the same $N=1$ as poles

$$G(\omega,\mathbf{p}) = \frac{Z(p,\omega)}{i\omega - \varepsilon(p)} \qquad Z(p,\omega) = (\omega^2 + \varepsilon^2(p))^\gamma$$

zeroes in $G(\omega,\mathbf{p})$
for $\gamma > 1/2$

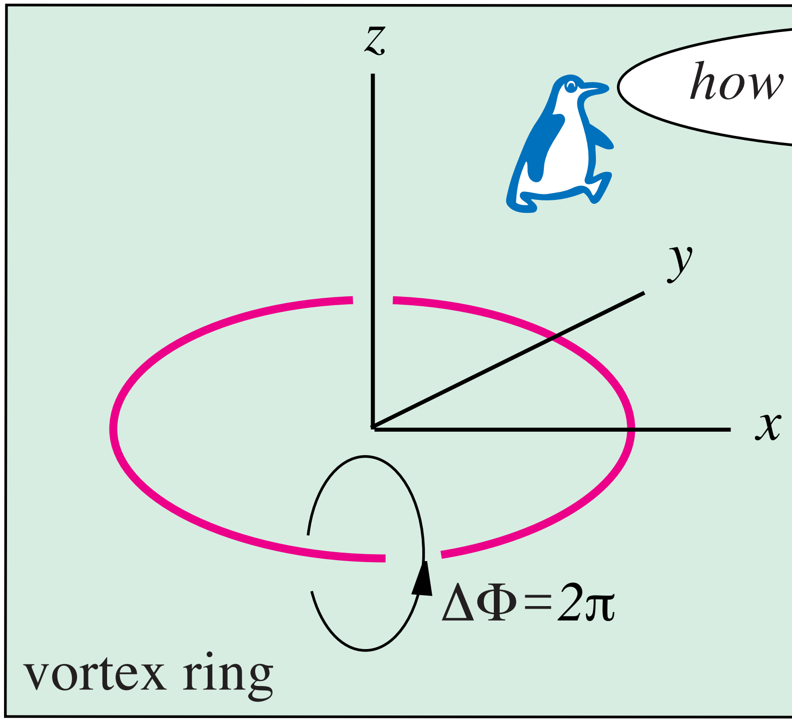
* Important for interacting systems, where quasiparticles are ill defined

* Fermi surface exists in superfluids/superconductors, examples: 3He-A in flow &
Gubankova-Schmitt-Wilczek, PRB74 (2006) 064505,
but Luttinger theorem is not applied

quantized vortex in **r**-space $\equiv$ **Fermi surface** in **p**-space

homotopy group π_1

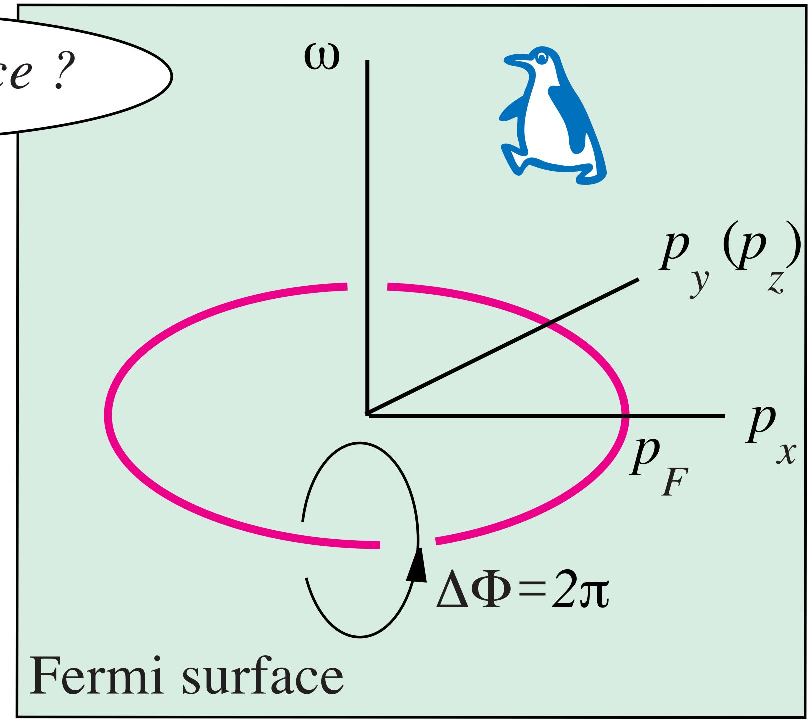
Topology in **r**-space



how is it in **p**-*space ?*

winding number
$N_1 = 1$

Topology in **p**-space



$\Psi(\mathbf{r}) = |\Psi| \, e^{\,i\Phi}$

scalar order parameter
of superfluid & superconductor

$G(\omega,\mathbf{p}) = |G| e^{\,i\Phi}$

Green's function (propagator)

classes of mapping $S^1 \rightarrow U(1)$
manifold of
broken symmetry vacuum states

classes of mapping $S^1 \rightarrow GL(\mathrm{n,C})$
space of
non-degenerate complex matrices

non-topological flat bands due to interaction

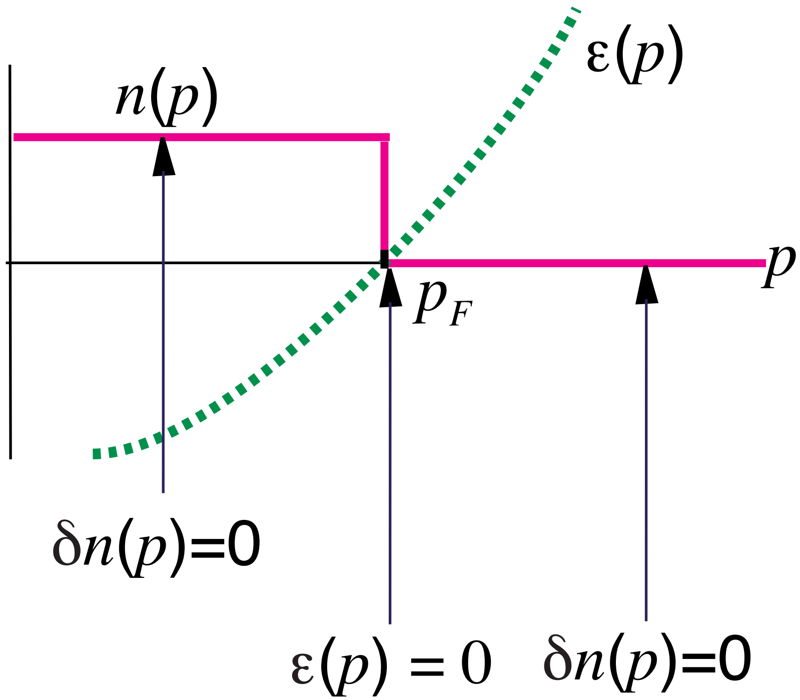
Khodel-Shaginyan fermion condensate

JETP Lett. **51**, 553 (1990)
GV, JETP Lett. **53**, 222 (1991)
Nozieres, J. Phys. (Fr.) **2**, 443 (1992)

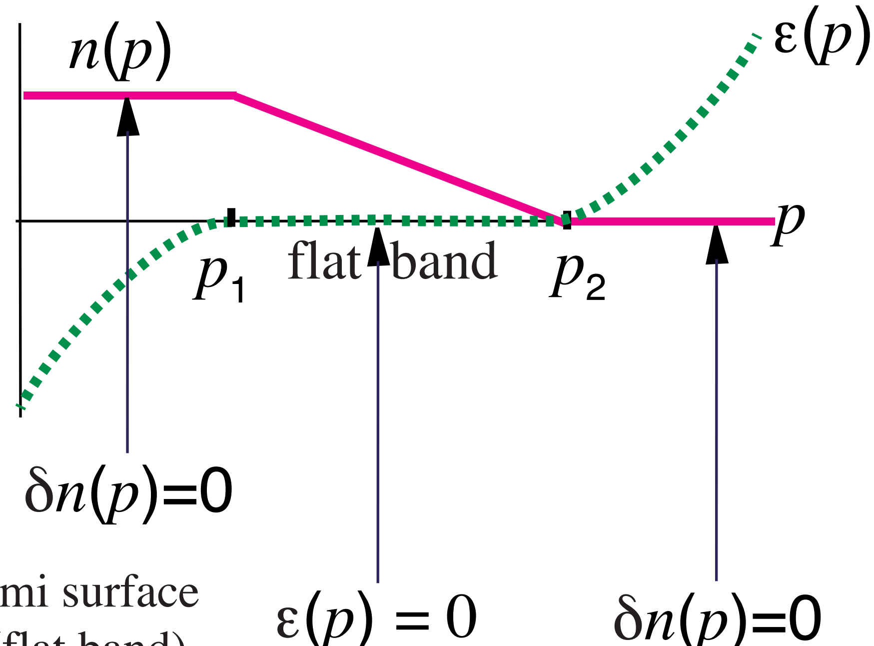
$$E\{n(p)\}$$

$$\delta E\{n(p)\} = \int \varepsilon(p)\,\delta n(p)\,d^{\mathrm{d}}p = 0$$

solutions: $\varepsilon(p) = 0$ or $\delta n(p)=0$



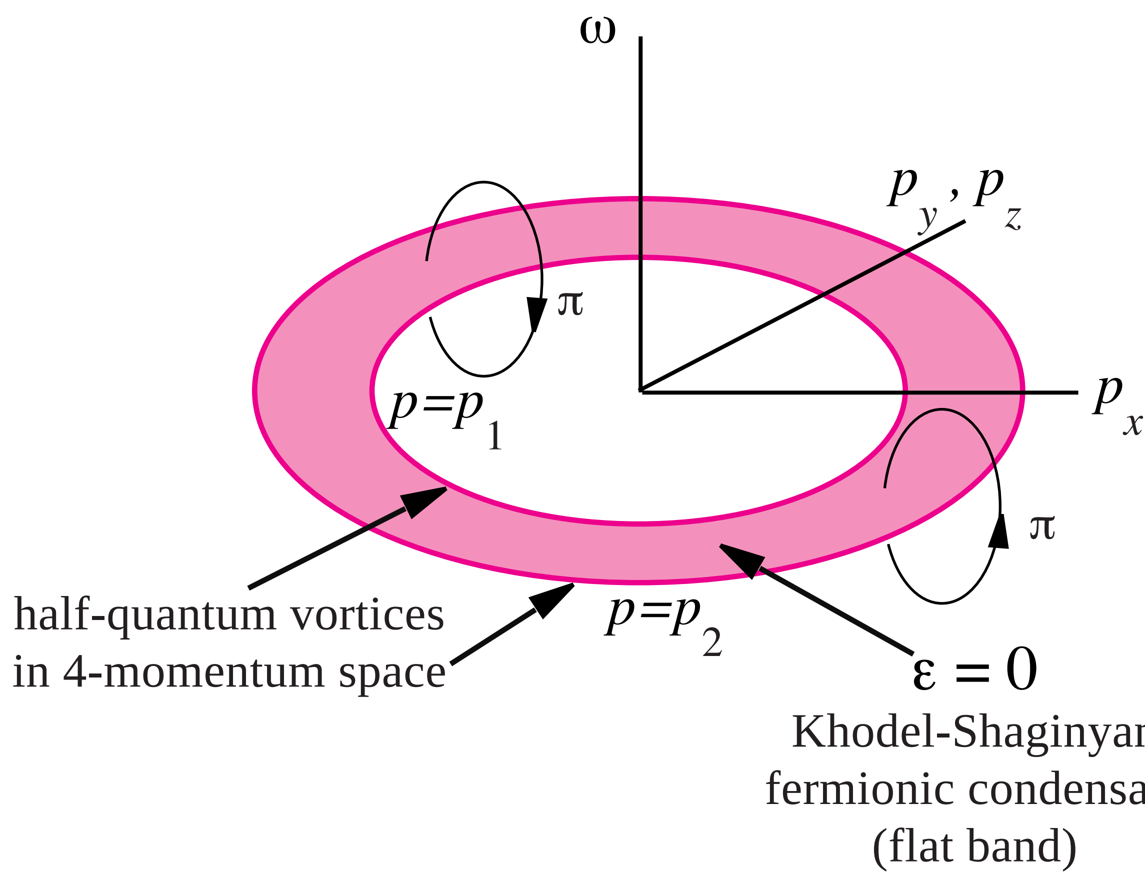
splitting of Fermi surface
to Fermi ball (flat band)



S.-S. Lee
Non-Fermi liquid from a charged black hole: A critical Fermi ball
PRD 79, 086006 (2009)

anti-de Sitter/conformal field theory correspondence

Flat band as momentum-space dark soliton terminated by half-quantum vortices



phase of Green's function changes by π across the "dark soliton"

Fermi Condensation Near van Hove Singularities Within the Hubbard Model on the Triangular Lattice

Dmitry Yudin,[1] Daniel Hirschmeier,[2] Hartmut Hafermann,[3] Olle Eriksson,[1]
Alexander I. Lichtenstein,[2] and Mikhail I. Katsnelson[4,5]

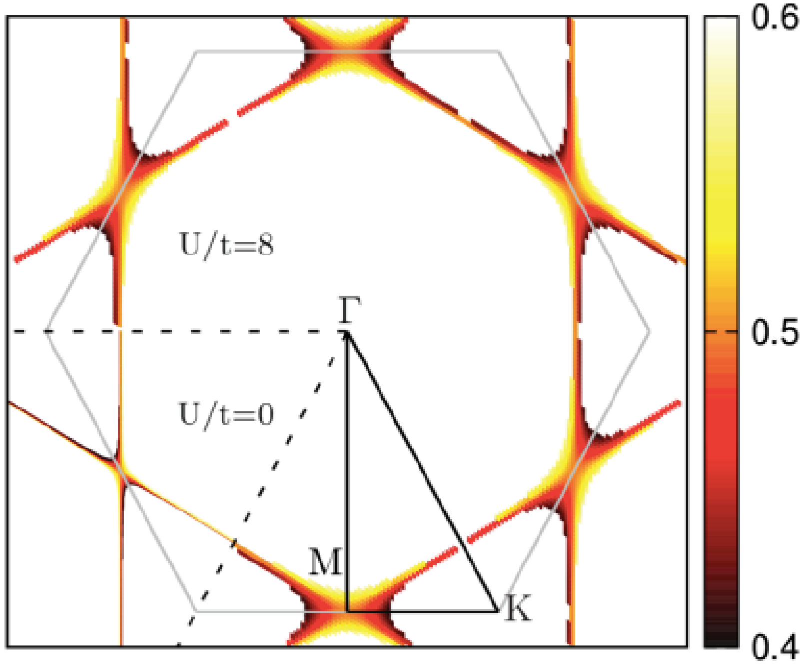
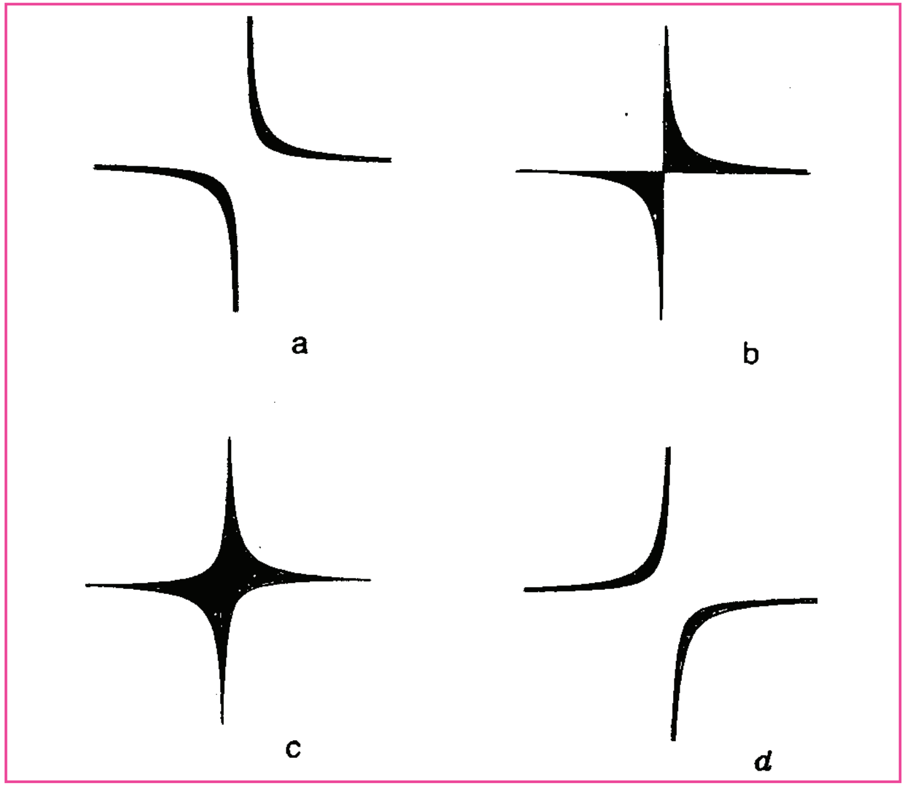


FIG. 4 (color online). Broadened Fermi surface within ±0.1 electrons for $U/t = 8$ and $T/t = 0.1$. The lower left sextant shows the noninteracting result.



Письма в ЖЭТФ, том 59, вып.11, стр.798 - 802, ©1994 г. 10 июня

ON FERMI CONDENSATE: NEAR THE SADDLE POINT AND WITHIN THE VORTEX CORE

G.E. Volovik

superconducting transition temperature

$$\boxed{1 = g \int \frac{d^3p}{2h^3} \frac{1}{E(p)}}$$

$$E^2(p) = \Delta^2 + \varepsilon^2(p)$$

finite DoS of Fermi surface gives superconducting transition

$$E^2(p) = \Delta^2 + v_F^{\,2}(p - p_F)^2$$

normal superconductors:
exponentially suppressed
transition temperature

$$\boxed{T_c = T_F \exp(-1/g\nu)}$$

interaction *DOS*

extremely high DoS of flat band gives high transition temperature

$$E(p) = \Delta \quad \text{in flat band}$$

flat band superconductivity:
linear dependence
of T_c on coupling g

$$\boxed{T_c \sim g V_{FB}}$$

coupling

flat band volume

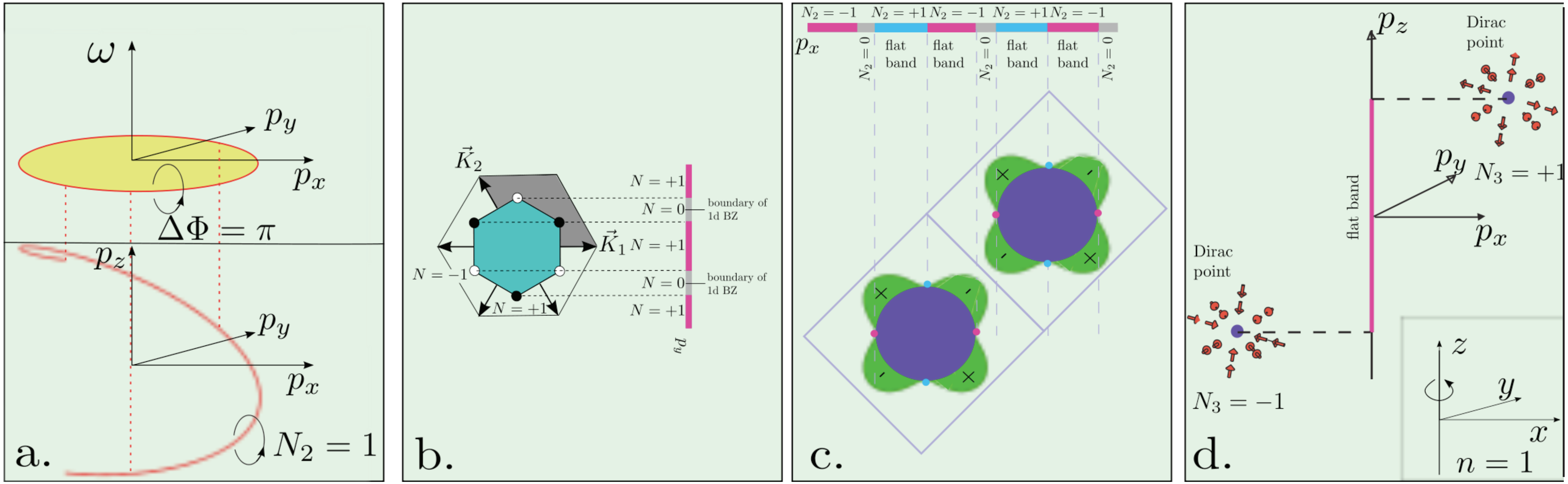
Topological flat bands

Flat band in multilayered graphene

Flat band on graphene edge (Ryu-Hatsugai)

Flat band in cuprates (Kashivaya-Tanaka)

Flat band in vortex core (Kopnin-Salomaa)



graphene

topology of graphene nodes

$$N = \frac{1}{4\pi i} \mathbf{tr}\left[\mathbf{K} \oint dl\, \mathbf{H}^{-1} \partial_l \mathbf{H}\right]$$

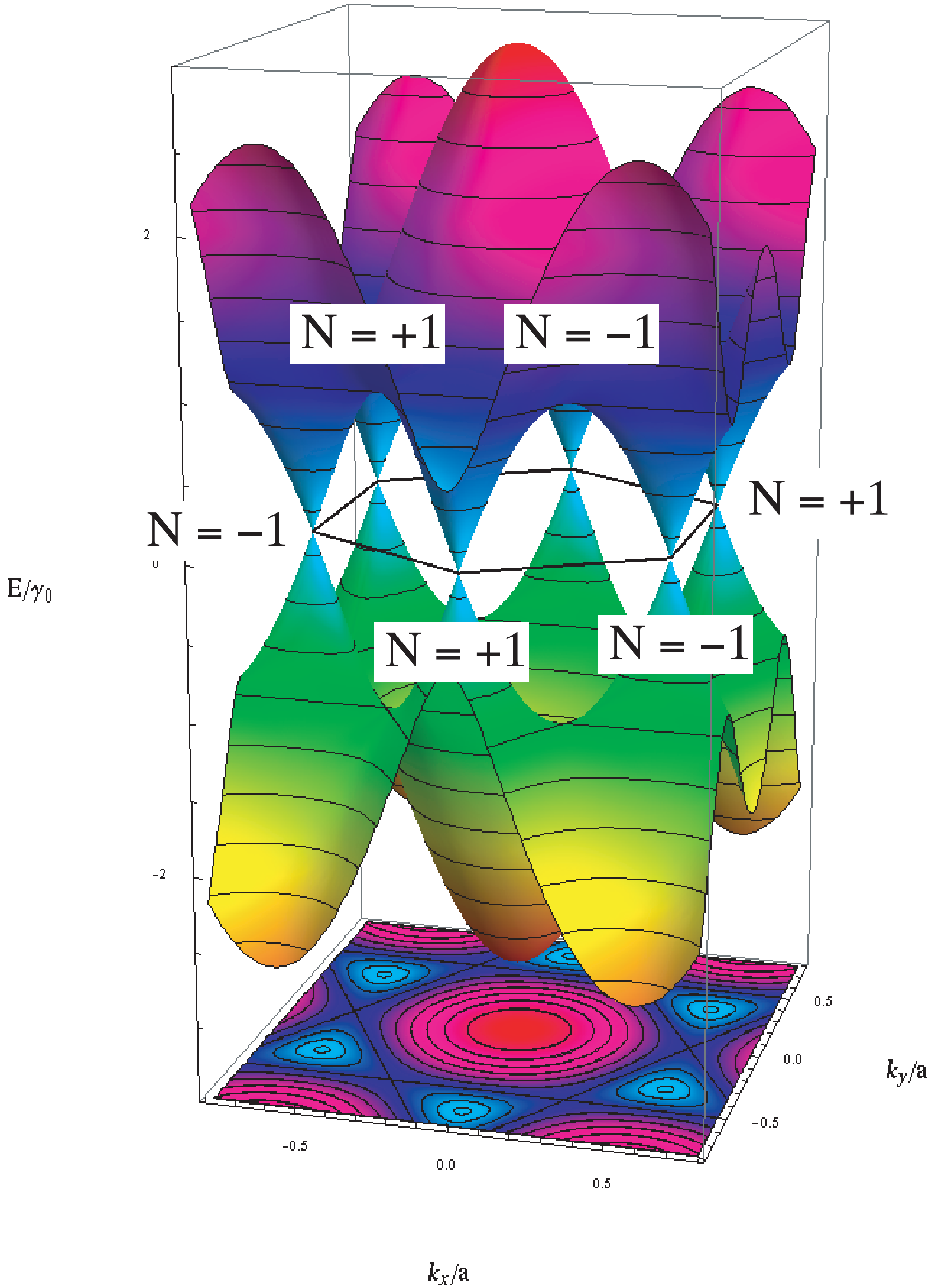
K - symmetry operator,
commuting or anti-commuting with **H**

close to nodes:

$$\mathbf{H}_{N=+1} = \tau_x p_x + \tau_y p_y$$

$$\mathbf{H}_{N=-1} = \tau_x p_x - \tau_y p_y$$

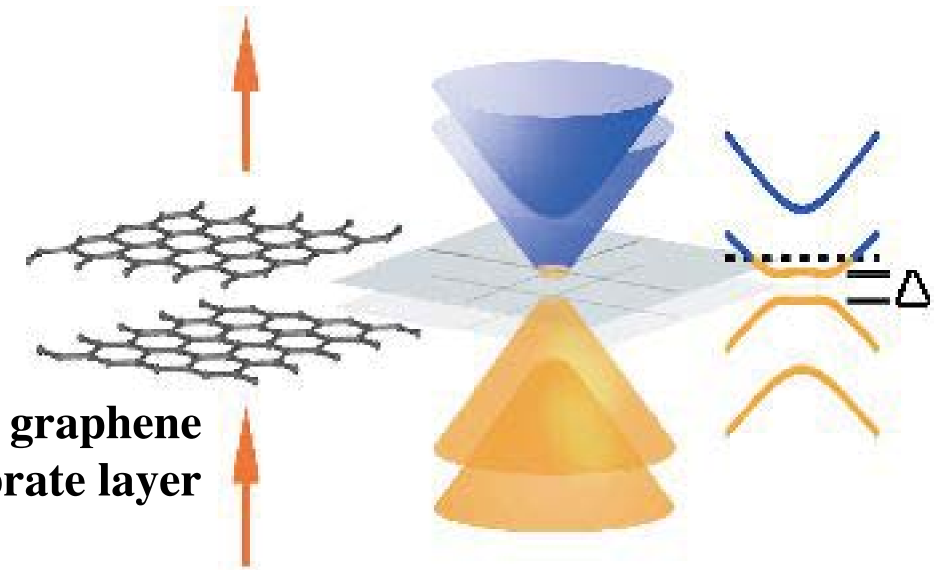
$$\mathbf{K} = \tau_z$$



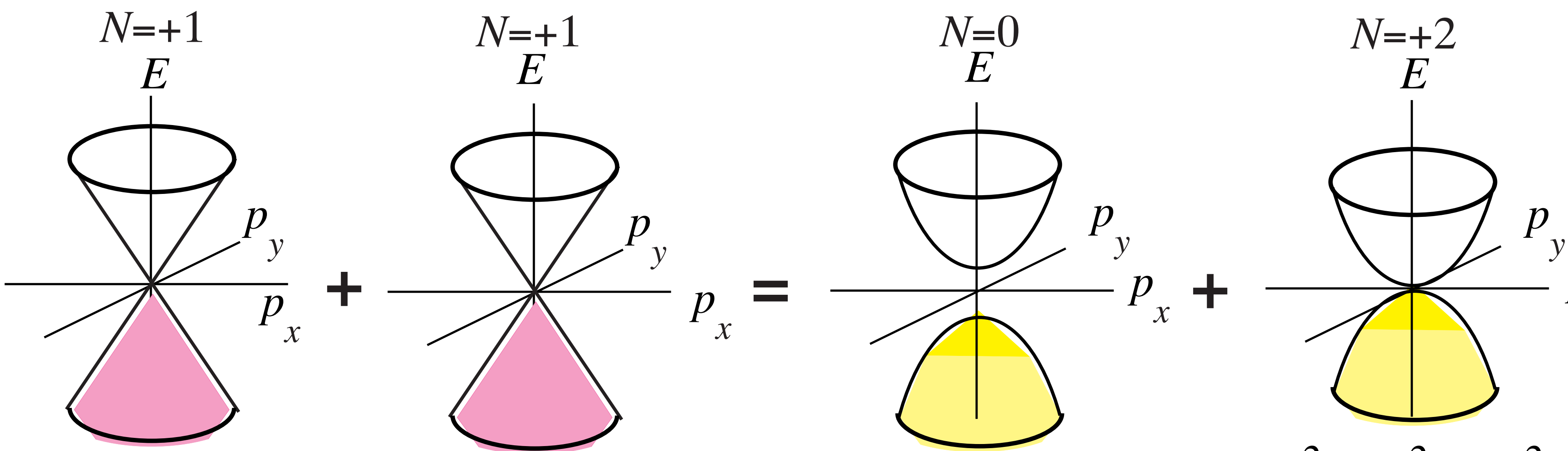
Summation of topological charges in action

exotic fermions:
massless fermions with quardatic,
cubic & quartic dispersion
semi-Dirac fermions

$$N = \frac{1}{4\pi i}\,\mathbf{tr}\left[\mathbf{K}\oint dl\, \mathbf{H}^{-1}\nabla_l \mathbf{H}\right]$$



bilayer graphene
double cuprate layer



$E=cp$ $\quad$ $E=cp$

relativistic massless fermions

$E^2 = 2c^2p^2 + 4m^2$

Dirac fermion

$E^2 = (p^2/\,2m)^2$

nonrelativistic Dirac fermion with quadratic dispersion

re-entrant violation of Lorentz invariance & neutrino physics Klinkhamer & GV arXiv:1109.6624

nonlinear Dirac fermions

N=1: Dirac fermions with linear dispersion

$$H = \begin{pmatrix} 0 & p_x + ip_y \\ p_x - ip_y & 0 \end{pmatrix} = \sigma_x p_x + \sigma_y p_y$$

N=2: Dirac fermions with quadratic dispersion

$$H = \begin{pmatrix} 0 & (p_x + ip_y)^2 \\ (p_x - ip_y)^2 & 0 \end{pmatrix}$$

Dirac fermions with nonlinear dispersion

$$H = \begin{pmatrix} 0 & (p_x + ip_y)^N \\ (p_x - ip_y)^N & 0 \end{pmatrix}$$

N=3: Dirac fermions with cubic dispersion

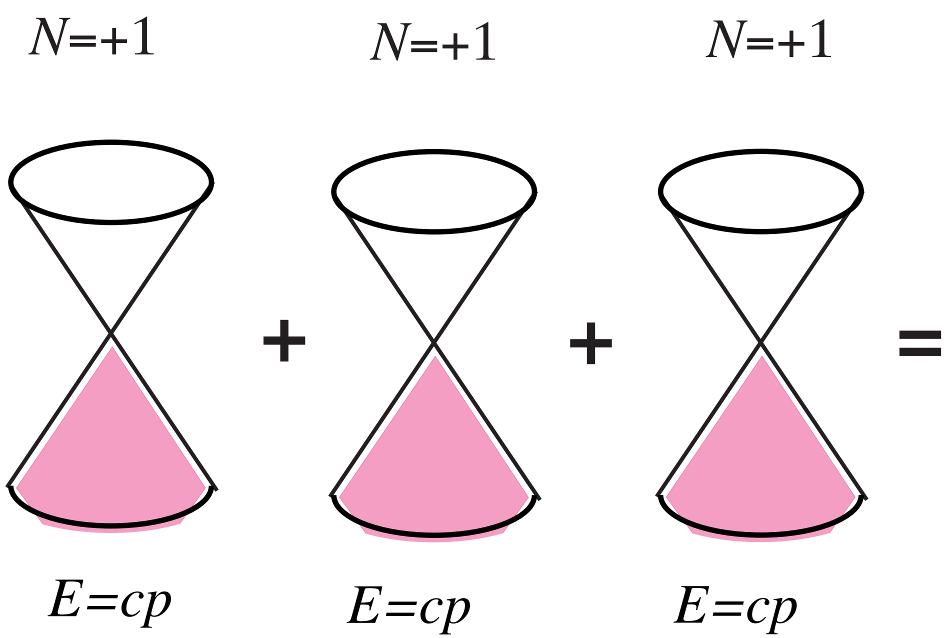
$$H = \begin{pmatrix} 0 & (p_x + ip_y)^3 \\ (p_x - ip_y)^3 & 0 \end{pmatrix}$$

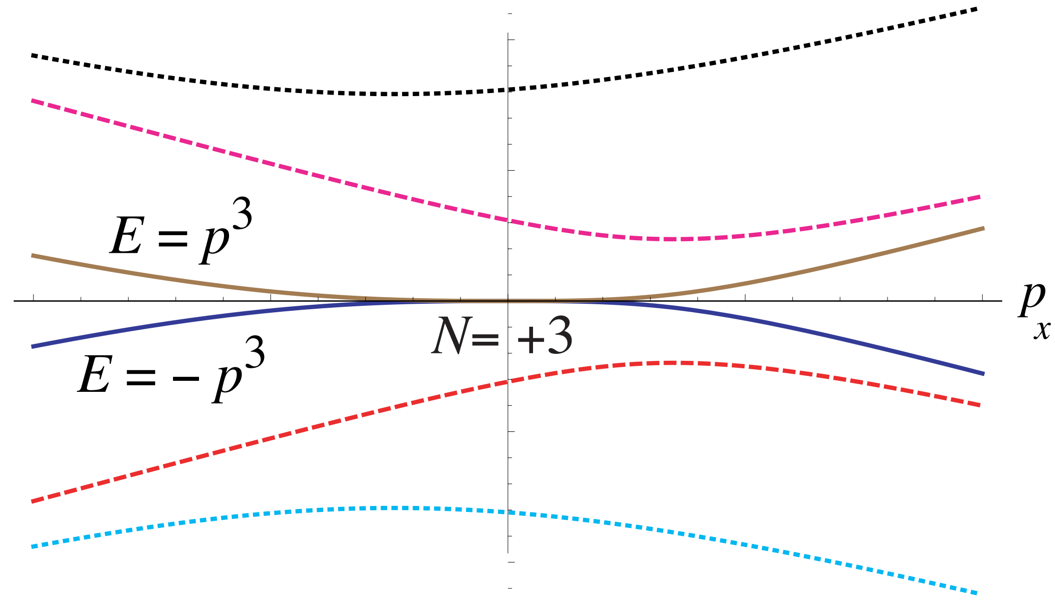
multiple Fermi point

T. Heikkila & GV arXiv:1010.0393

cubic spectrum in trilayer graphene

$$N = 1 + 1 + 1 = 3$$





multilayered graphene

$$N = 1 + 1 + 1 + \ldots$$

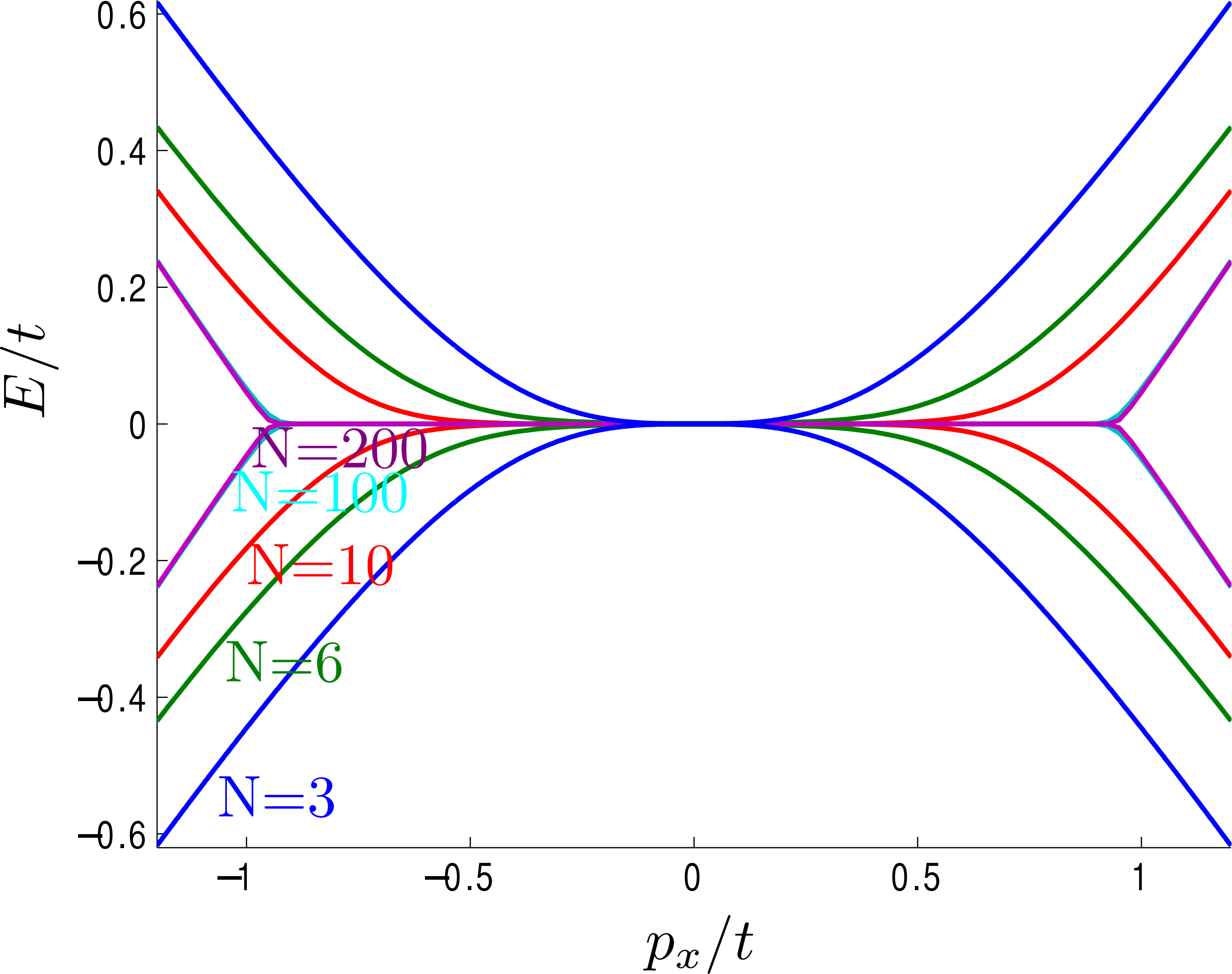
spectrum in the outer layers

$$E = p^N$$

$$E = - p^N$$

what kind of induced gravity emerges near degenerate Fermi point?

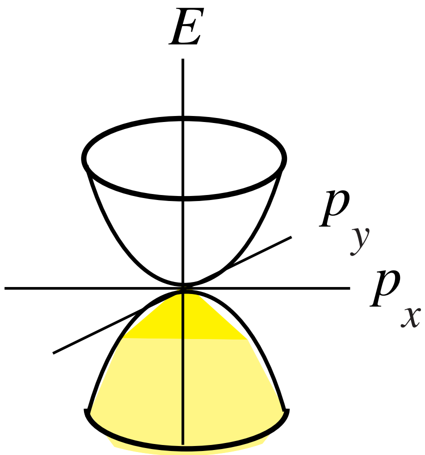
route to topological flat band on the surface of 3D material



0.6
0.4
0.2
0
−0.2
−0.4
−0.6
E/t
−1
−0.5
0
0.5
1
p_x/t
N=200
N=100
N=10
N=6
N=3

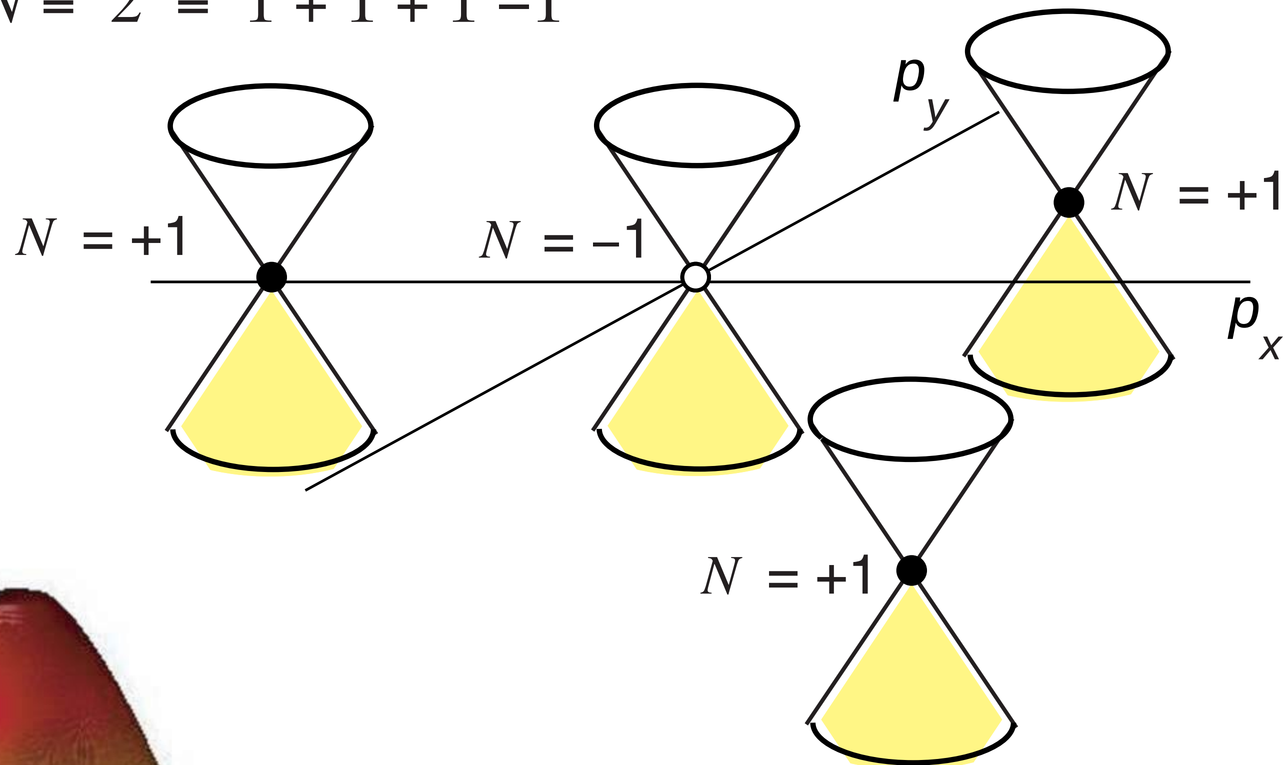
Splitting of Dirac and Weyl points

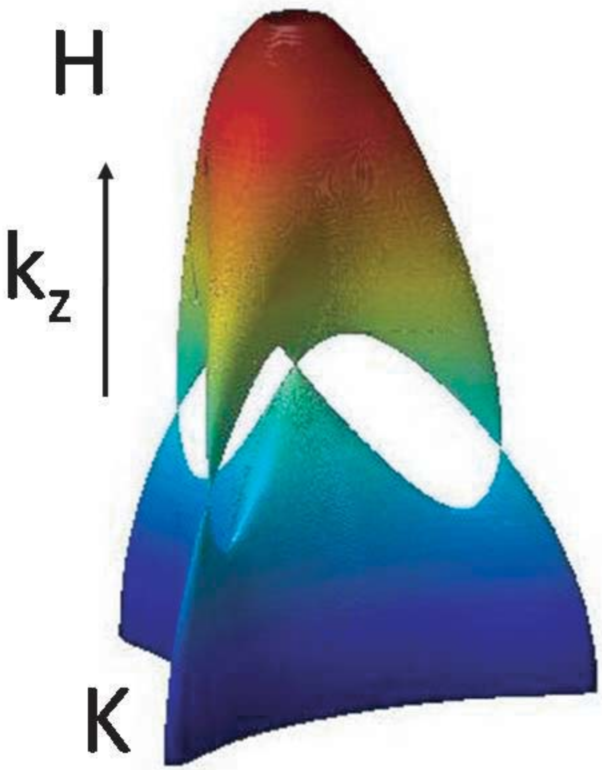
Splitting of quadratic point or trigonal warping



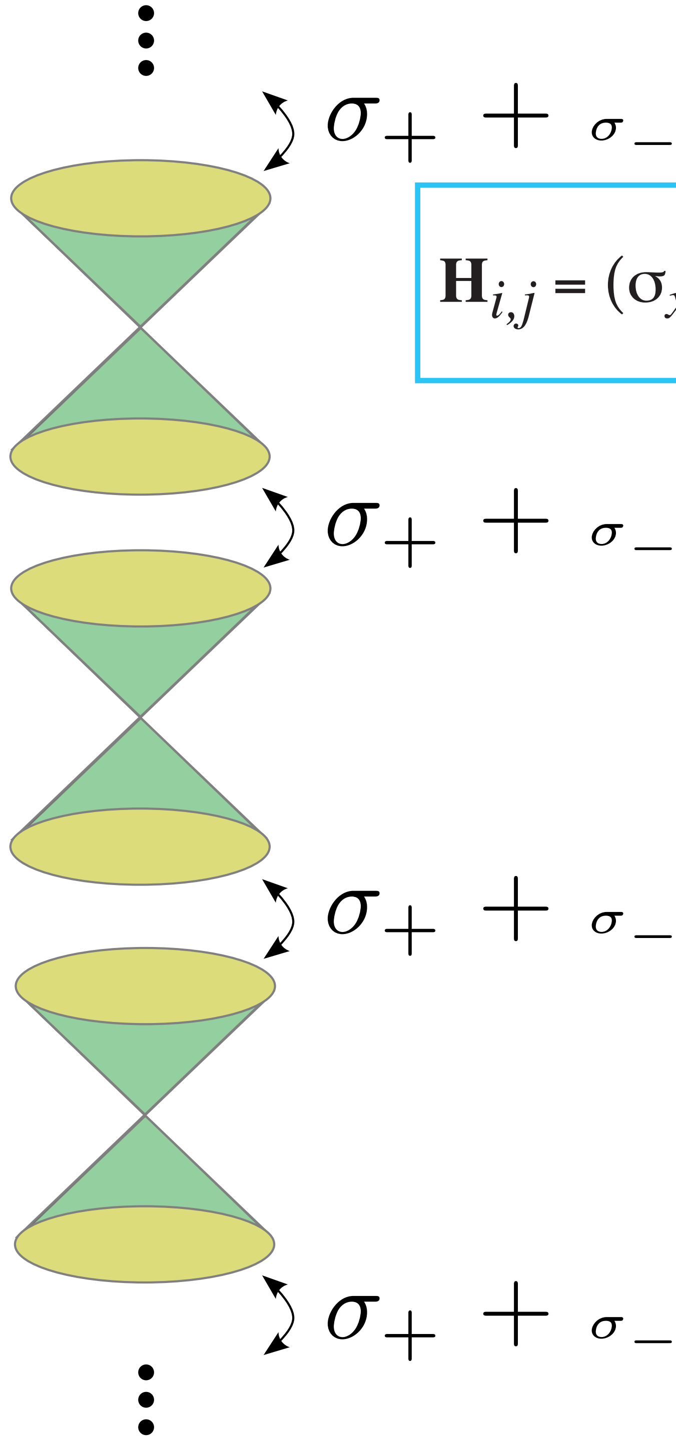
$N = +2$

$$N = 2 = 1 + 1 + 1 - 1$$

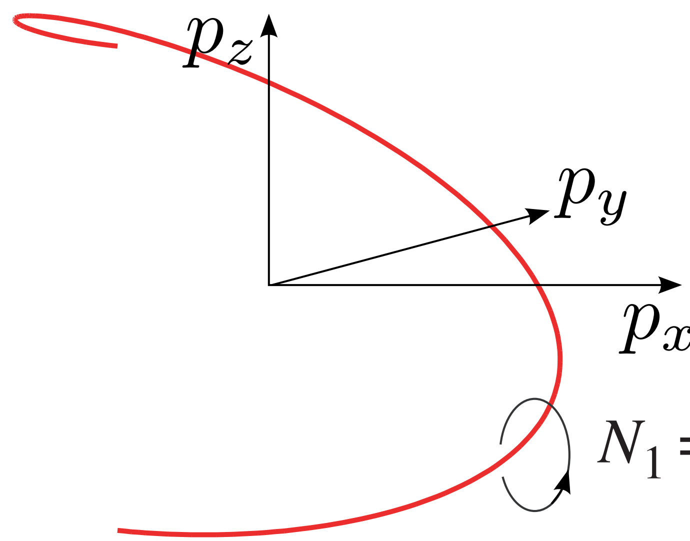




formation of nodal spiral in bulk (together with flat band on the surface) by stacking of graphene layers



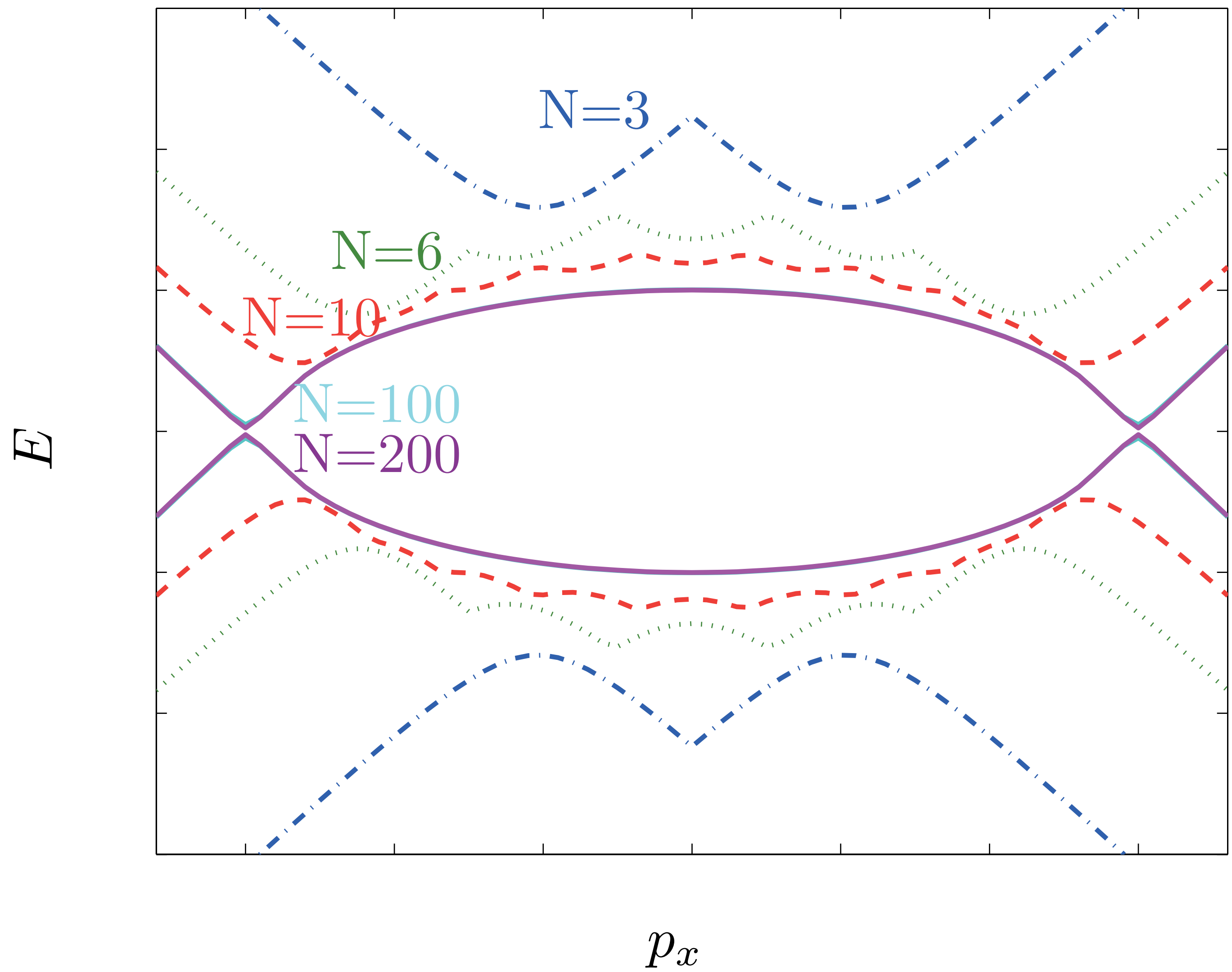
$$\mathbf{H}_{i,j} = (\sigma_x p_x + \sigma_y p_y)\delta_{i,j} + (\sigma_+ t_+ + \sigma_- t_-)\delta_{i,j+1}$$



$$t_+ > t_-$$

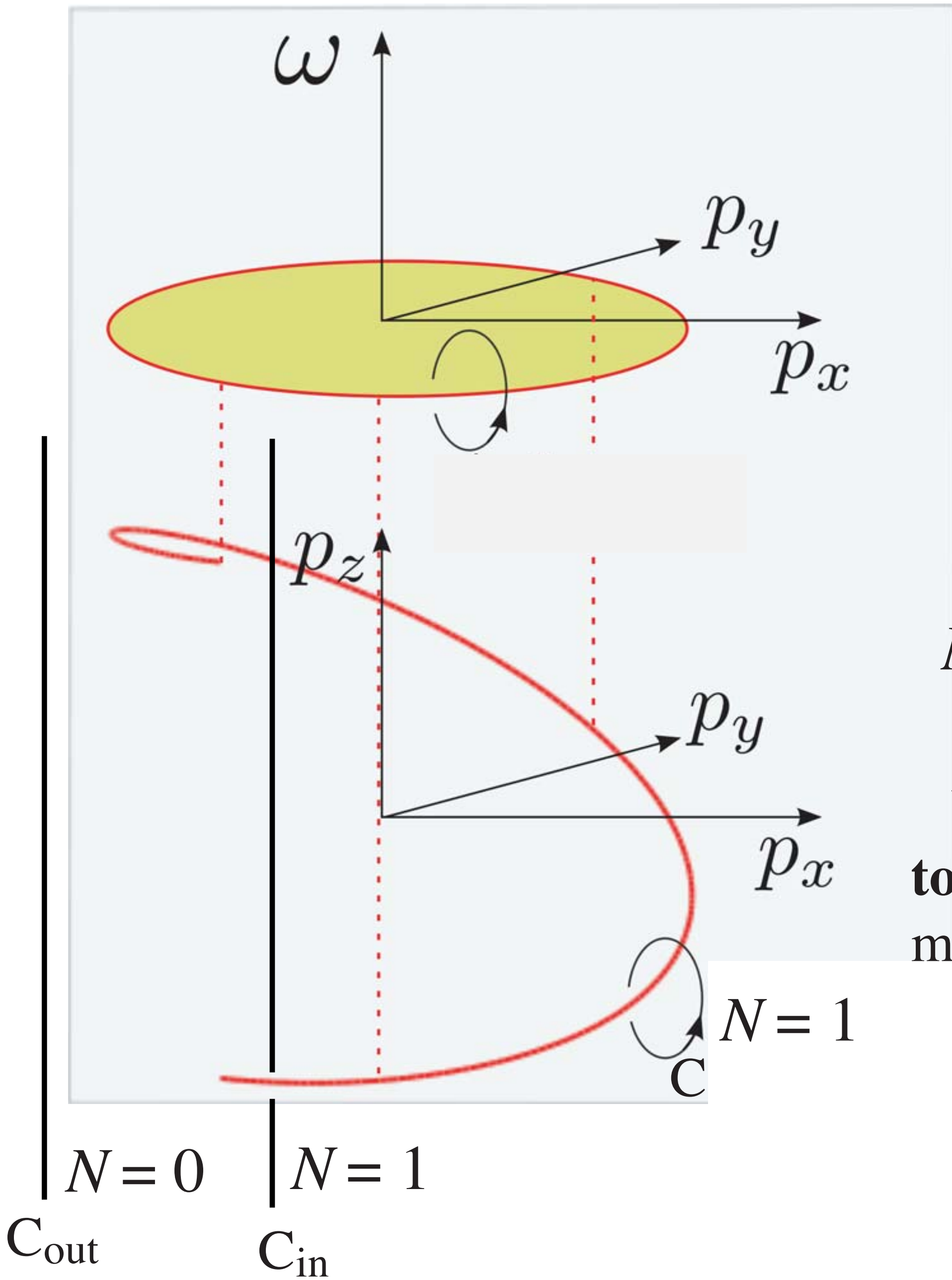
$$N_1 = \frac{1}{4\pi i}\, \mathbf{tr}\left[\mathbf{K} \oint dl\, \mathbf{H}^{-1} \nabla_l\, \mathbf{H}\right]$$

Emergence of nodal line from gapped branches by stacking graphene layers



example of topological bulk-surface correspondense:
Nodal spiral generates flat band on the surface

projection of spiral on the surface determines boundary of flat band



$$N = \frac{1}{4\pi i}\, \mathbf{tr}\, [\mathbf{K} \oint_C dl\, \mathbf{H}^{-1} \nabla_l\, \mathbf{H}]$$

at each (p_x,p_y) except the boundary of circle one has 1D fully gapped state (insulator)

$N_{out}(p_x,p_y) = 0$ trivial 1D insulator

$N_{in}(p_x,p_y) = 1$ topological 1D insulator

topological insulator has 1D gapless **edge state**
manifold (p_x,p_y) of edge states forms **flat band**

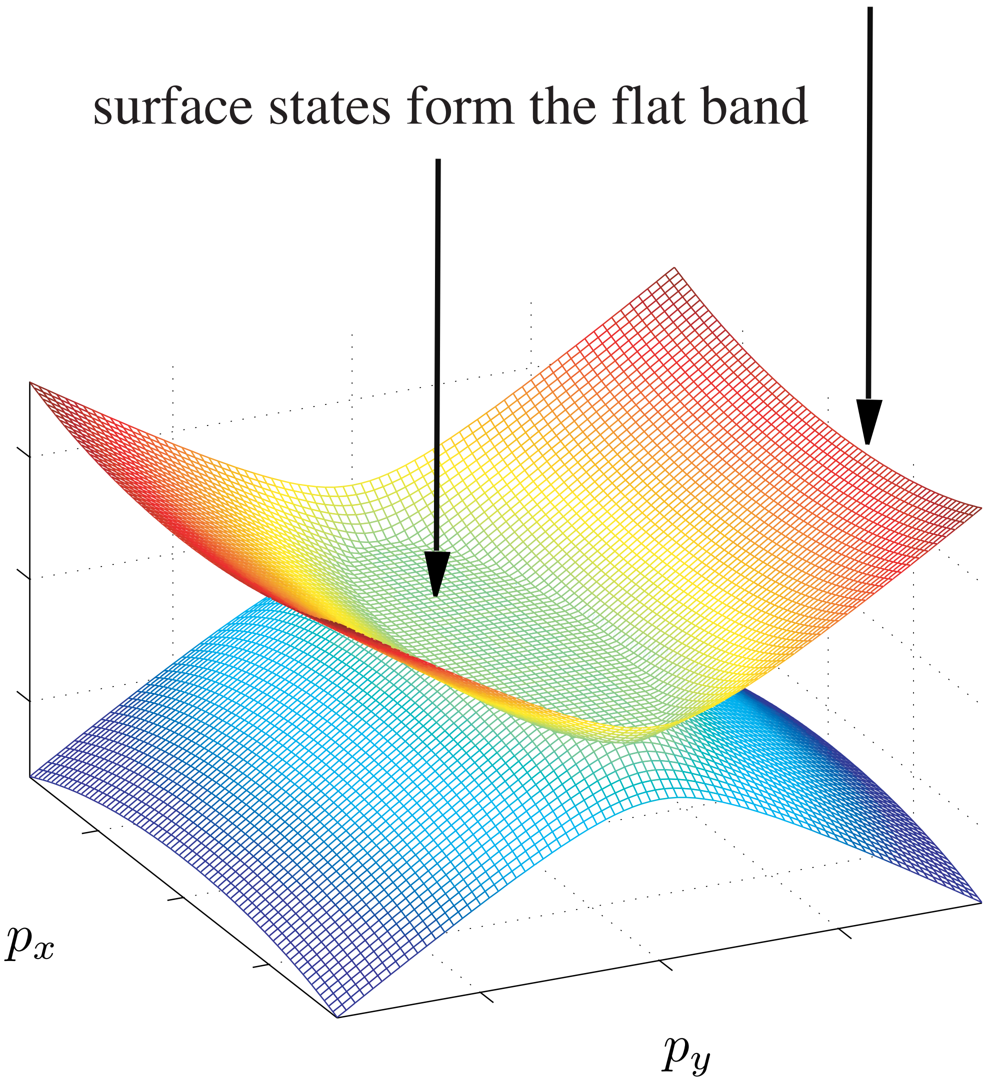
Nodal spiral generates flat band on the surface

projection of nodal spiral on the surface determines boundary of flat band

lowest energy states:

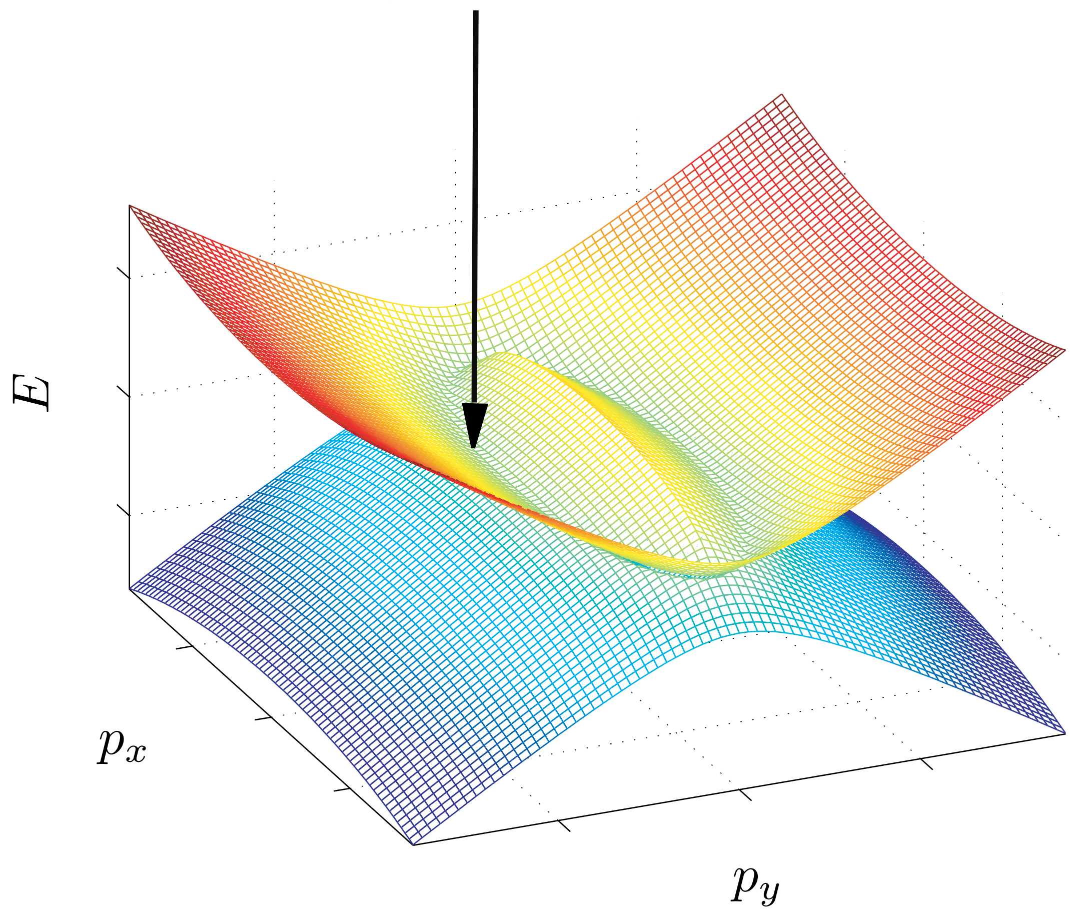
bulk states

surface states form the flat band



energy spectrum in bulk
(projection to p_x , p_y plane)

nodal line



Modified nodal spiral in rhombohedral graphite: spiral of Fermi surfaces (McClure 1969)

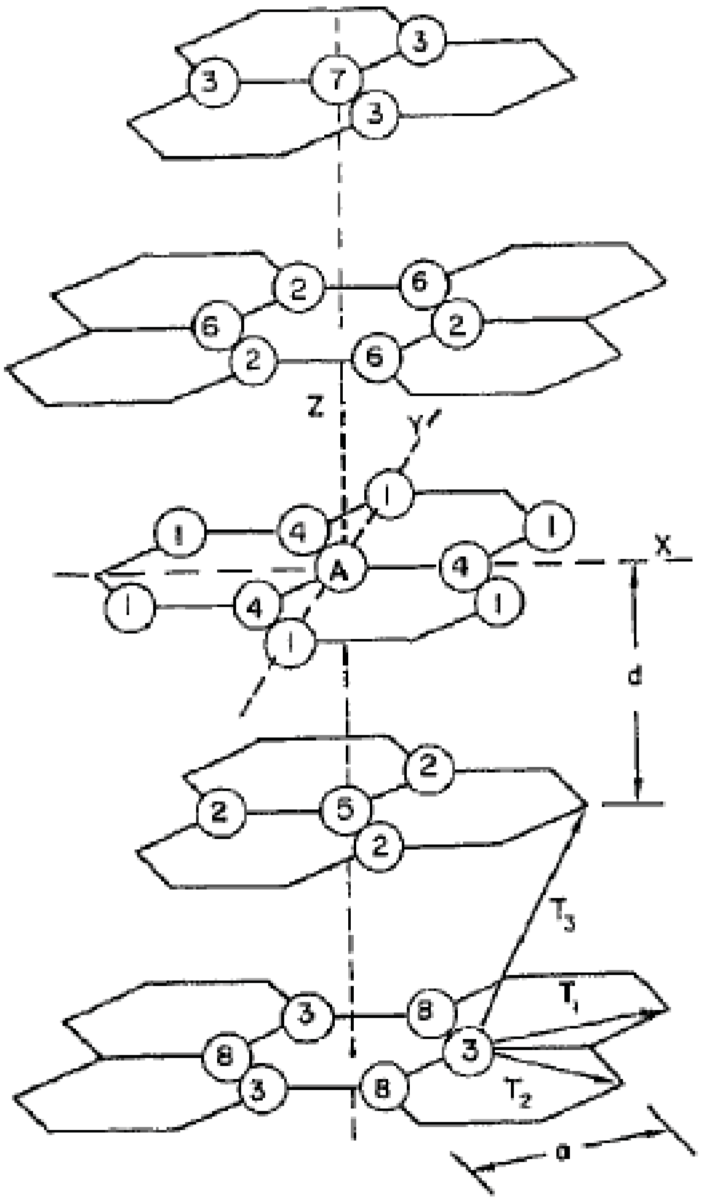
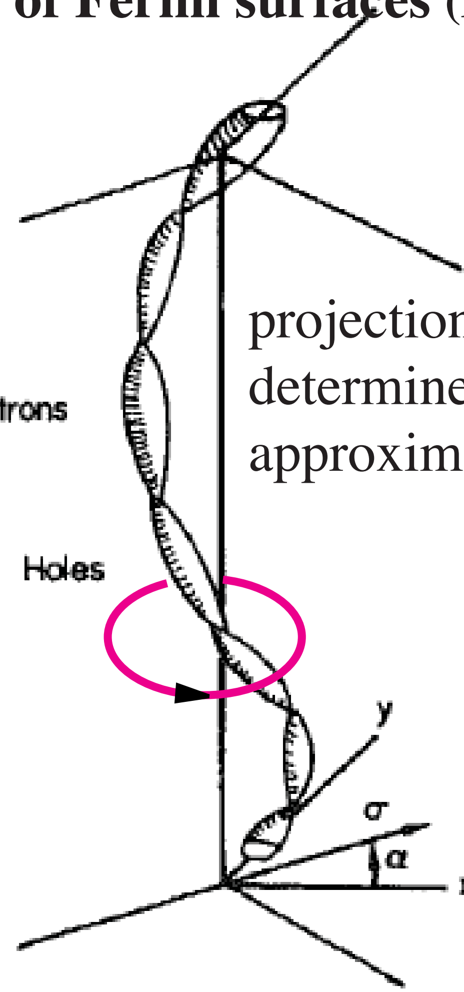


Fig. 1. The crystal lattice of rhombohedral graphite. The numbering of the groups of neighbors of the central A atom is explained in the text.

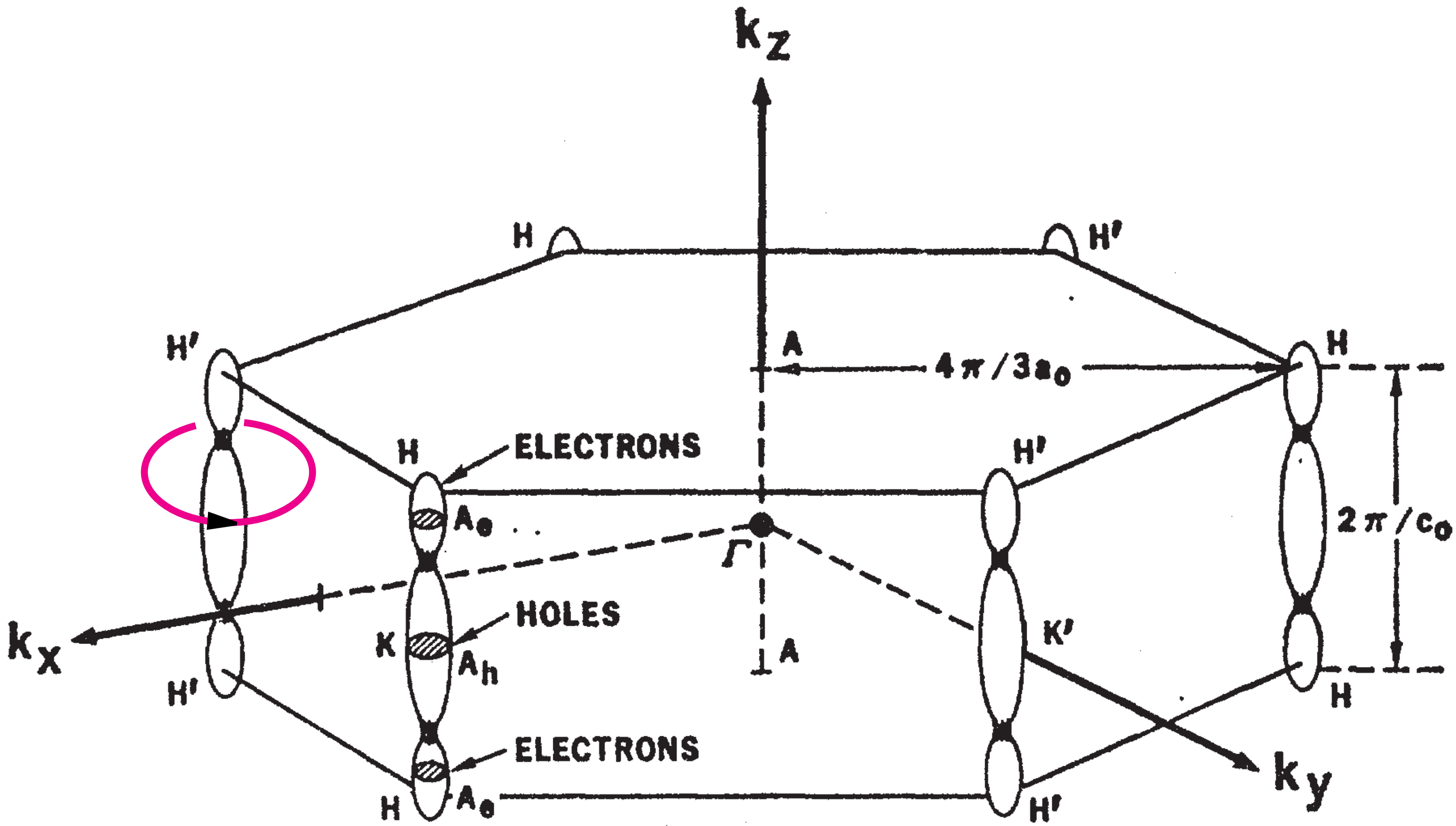


projections on surfaces determine approximate flat band

Fig. 2. The Fermi surface of rhombohedral graphite. The surface is centered on one of the six vertical zone edges. The widths of the surfaces have been magnified by more than an order of magnitude.

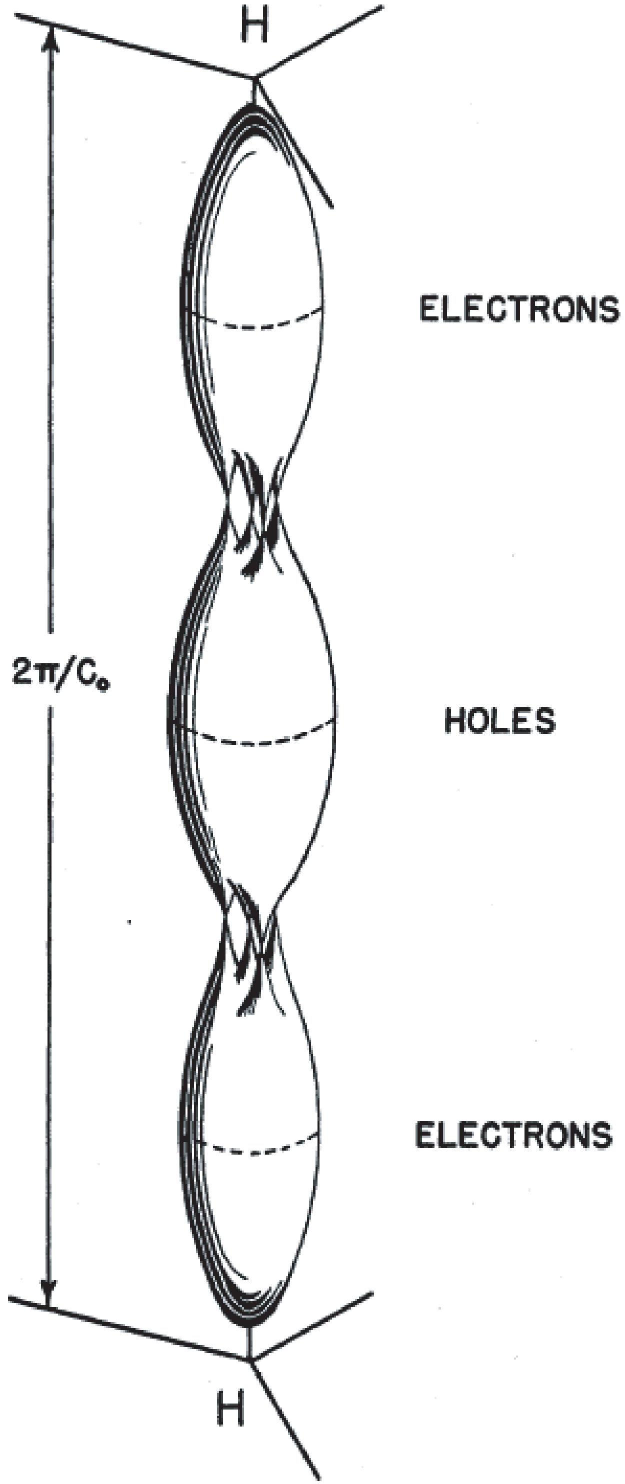
for conventional graphite: approximate flat band on the lateral surface

Nodal lines in graphite tranformed to chain of electron and hole FS

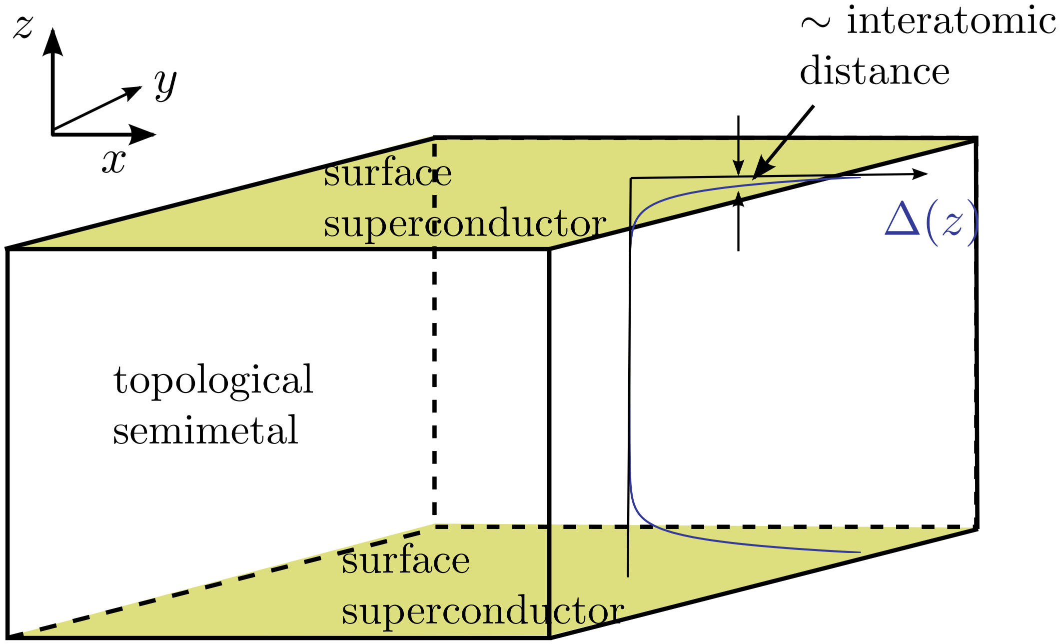


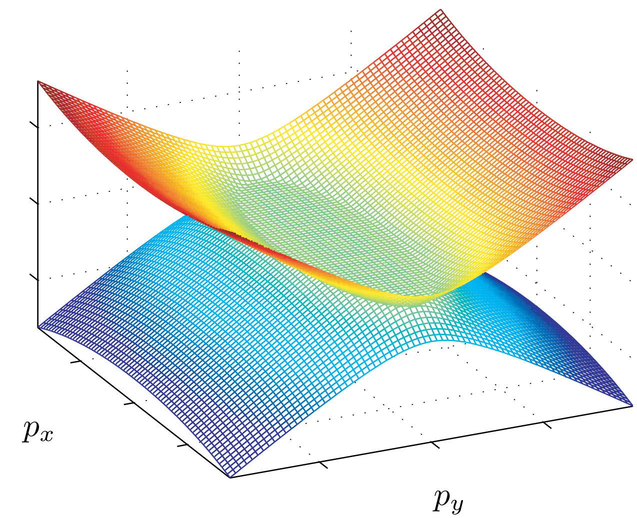
for conventional graphite:
approximate flat band
on the lateral surface

FIG. 4. The Fermi surface for pure graphite. The central surface contains holes and the outer surfaces contain electrons. The length-to-width ratio of each surface is about 13. The trigonal anisotropy is exaggerated for clarity.



Surface superconductivity in topological semimetals: route to room temperature superconductivity





Extremely high DoS of flat band gives high transition temperature:

$$1 = g \int \frac{d^2p}{2h^2} \frac{1}{E(p)}$$

normal superconductors:
exponentially suppressed
transition temperature

$$T_c = T_F \exp(-1/g\nu)$$

interaction ***DOS***

$$\mathrm{DoS} = \nu(\varepsilon) \sim \varepsilon^{2/N-1}$$

N is number of layers

flat band superconductivity:
linear dependence
of T_c on coupling g

$$T_c \sim g S_{\mathrm{FB}}$$

coupling ***flat band area***

$N = 4$: $\nu(\varepsilon) \sim \varepsilon^{-1/2}$ Kopaev (1970); Kopaev-Rusinov (1987)

evidence of room-temperature superconductivity?

2000: Kopelevich Y, Esquinazi P, Torres J H S and Moehlecke S
J. of Low Temp. Phys. **11**, 691–702

2002: Kempa H, Esquinazi P and Kopelevich Y
Phys. Rev. B **65** 241101

2007: Kopelevich Y and Esquinazi P
J. of Low Temp. Phys. **146** 629

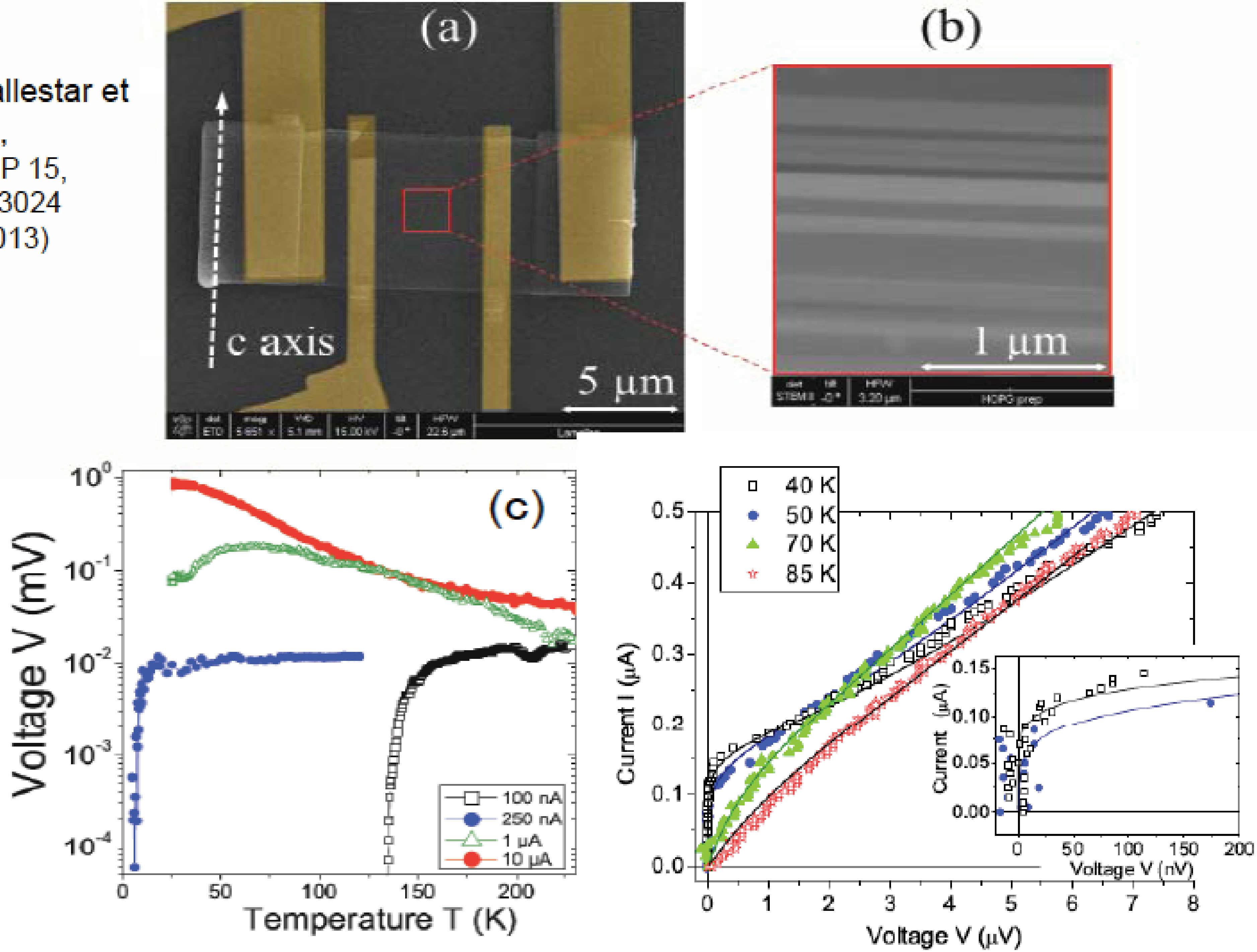
2012: Scheike T, Böhlmann W, Esquinazi P, Barzola--Quiquia J, Ballestar A and Setzer A.
Advanced Materials **24** 5826

2013: Scheike T, Esquinazi P, Setzer A and Böhlmann W , arXiv:1301.4395

Ballestar A, Barzola-Quiquia J, Scheike T and Esquinazi P
New J. Phys. **15** 023024

G. Larkins, Y. Vlasov, K. Holland, arXiv:1307.0581

Ballestar et al.,
NJP 15, 023024 (2013)

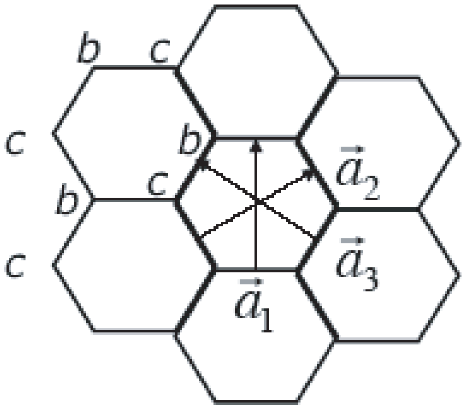


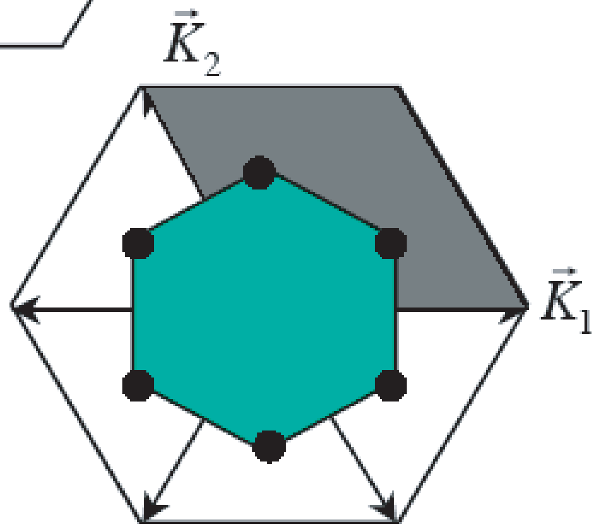
Flat band on graphene edge & on edge dislocation in graphite

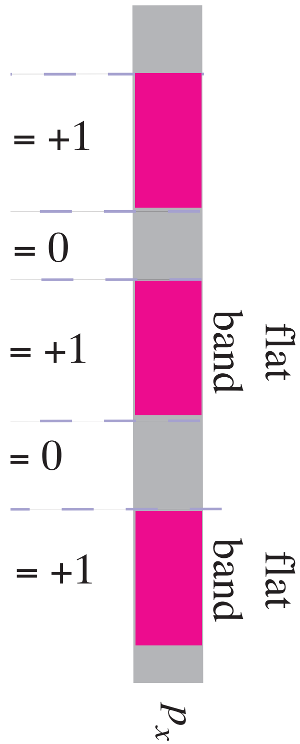
1D flat band on graphene edge (Ryu-Hatsugai)

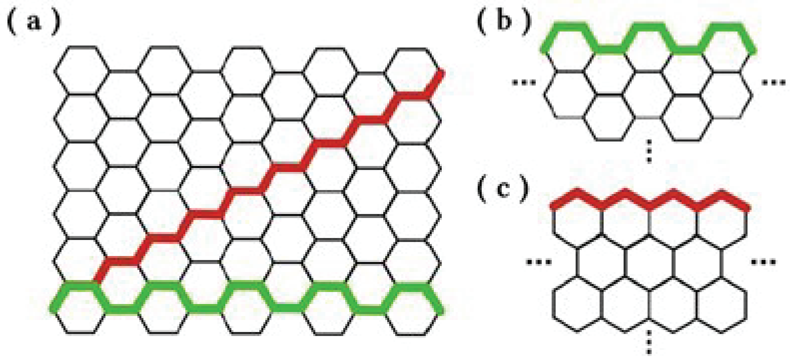
projections of Dirac points to the edge determine boundaries of the flat band

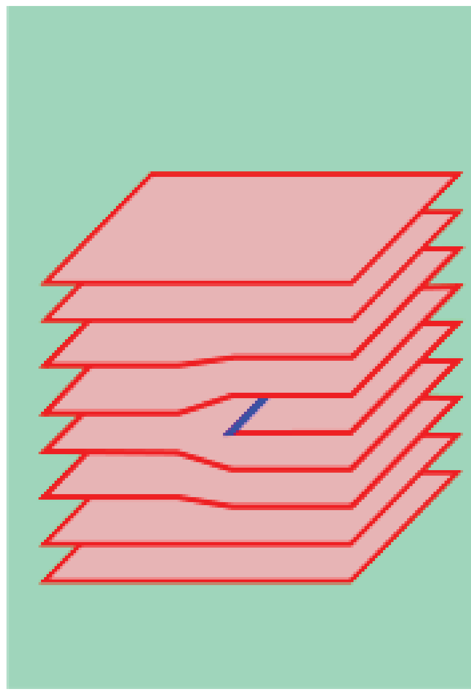
$$N = \frac{1}{4\pi i}\,\mathbf{tr}\,[\mathbf{K}\oint dl\,\mathbf{H}^{-1}\nabla_l\,\mathbf{H}]$$







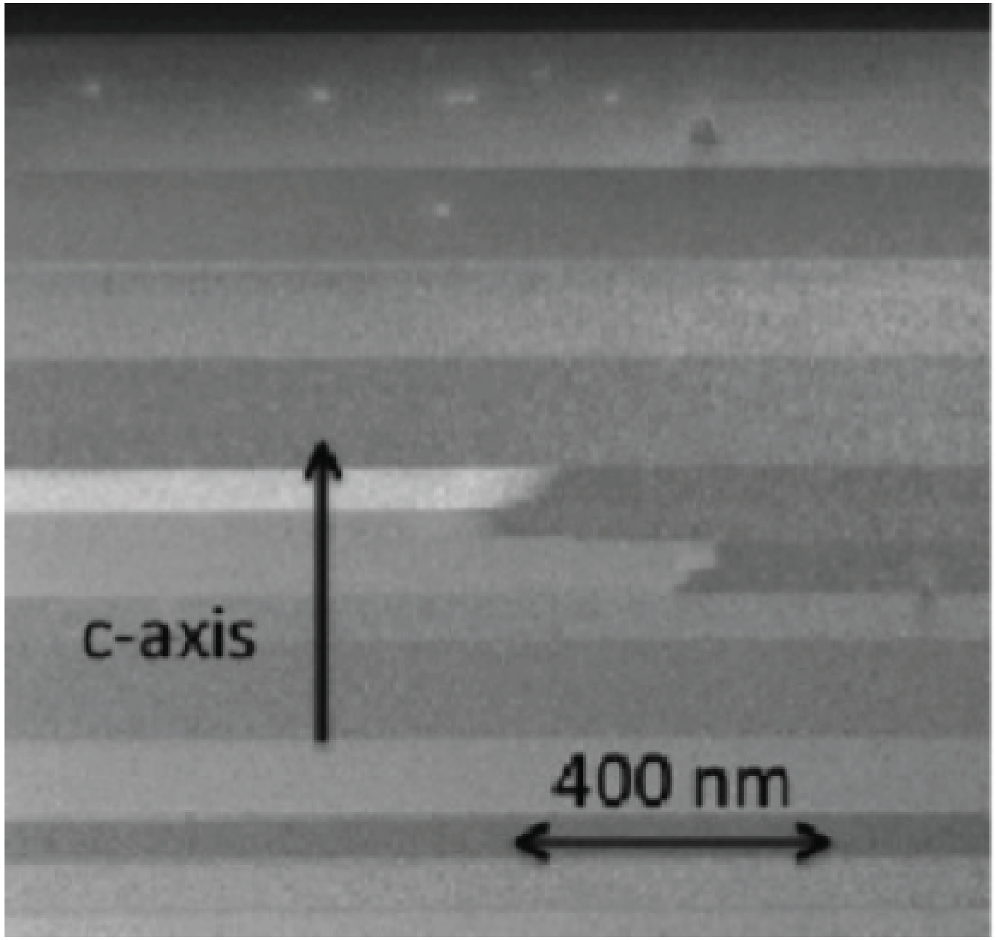




edge dislocation in graphite is the edge of graphene sheet dislocation contains 1D flat band

Twist interfaces in graphite

Screw dislocations network
at twist grain boundary



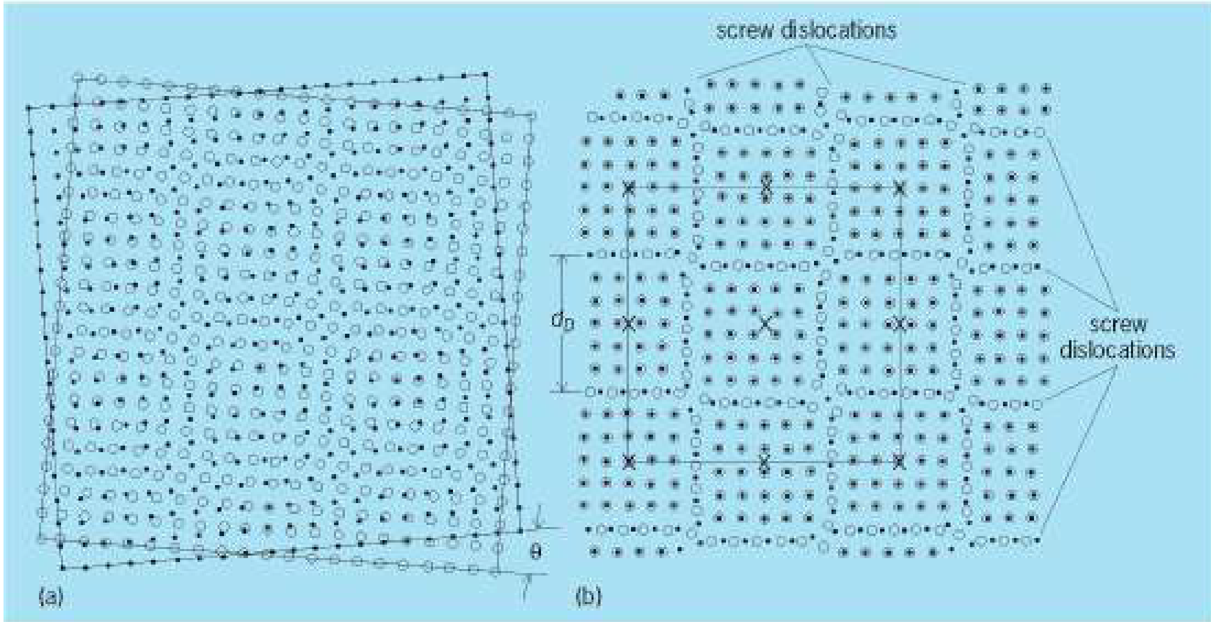


Fig. 2. Atomic configurations in the planes just above and below the boundary plane in a small-angle ⟨001⟩ twist grain boundary (*a*) Unrelaxed configuration. (*b*) Relaxed configuration, illustrating the formation of the screw dislocation network. The periodicity is that of the coincidence site lattice, the unit cell of which is indicated by the square. The crosses indicate the location of O-lattice points. (*After T. R. Schober and R. W. Balluffi, Quantitative observation of misfit dislocation arrays in low and high angle twist boundaries, Phil. Mag., 21:109–148, 1970*)

Esquinazi, et al

dislocation in graphite is the edge of graphene sheet
it has flat band

possible flat band superconductivity
within graphite interface

conclusions

flat band is generic phenomenon

1. Khodel-Shaginyan fermion condensate
(*in particular near van Hove singularity*)

2. Topological surface & edge flat bands
(*graphene, graphite, ...*)

3. Kopnin-Salomaa Majorana modes
flat band of Majorana modes in vortex core of chiral superfluid/superfconductor

4. Flat band on dislocation in graphite

5. Flat band in topological insulators

6. Flat band in artificial lattices (Kagome, spin 1, etc.)

7. Flat band in ultracold gases

flat band opens parametrically different scenario for onset of superconductivity