Superconductor-metal/insulator transition in 2D electron systems with strong spin-orbit coupling

Igor Burmistrov

in collaboration with

Igor Gornyi (Karlsruhe Inst. of Tech. & Ioffe Phys. Tech. Inst.)

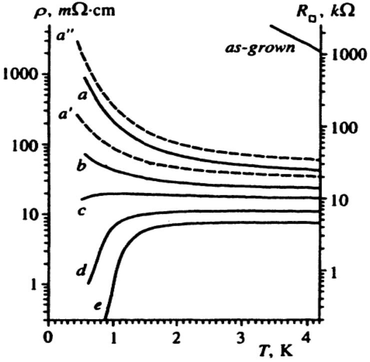
Alexander Mirlin (Karlsruhe Inst. of Tech. & Peterburg Nucl. Phys. Inst.)

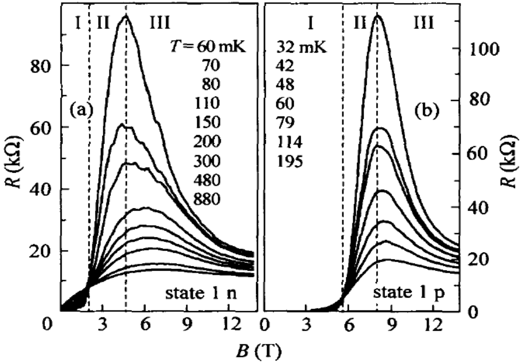
- amorphous Mo-Ge films (thickness $b = 15 \div 1000$ Å)
  [Graybeal, Beasley (1984)]
- Bi and Pb layers on amorphous Ge ($b = 4 - 75$ Å)
  [Strongin et al. (1971); Haviland et al. (1989)]
- ultrathin Be films ($b = 4 - 15$ Å) [Bielejec et al. (2001)]
- amorphous thick In-O films ($b = 100 - 2000$ Å) [Shahar, Ovadyahu (1992);]
  [Gantmakher et al. (1996, 1998, 2000); Sambandamurthy et al.(2004); Sacépé et al. (2011)]
- thin TiN films [Baturina et al. (2007)]
- $Li_x ZrNCl$ powders [Kasahara et al. (2009)]

√ $LaAlO_3/SrTiO_3$ interface [Caviglia et al. (2008), Gariglio et al. (2009)]

√ δ-doped Nb:$SrTiO_3$ films [Kim et al. (2012)]

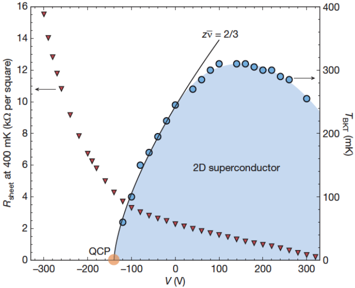
√ monolayer MoS_2 [Ye et al. (2012, 2014); Taniguchi et al. (2012)]



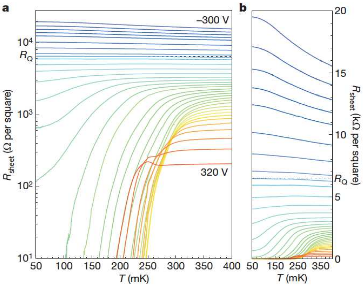


[Gantmakher et al. (1996, 1998)]

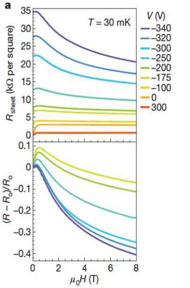
amorphous In-O film ($b \approx 200$ Å): resistance vs temperature (left), perpendicular (middle) and parallel (right) magnetic field



phase diagram



resistance vs T

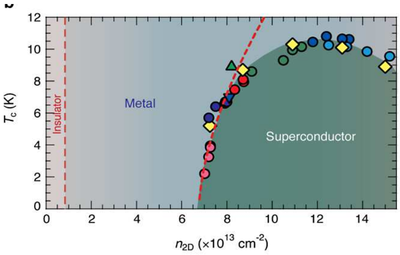


resistance vs $H_\perp$

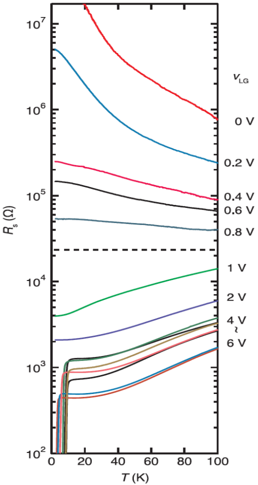
[from Caviglia et al. (2008)]

spin-orbit splitting $\Delta_{so} \sim 0.2 \div 1 \cdot 10^2$ K (gate tunable)

$m_*/m_e = 3$	$n \sim 10^{12} \div 10^{14}$ cm^{-2}	$\mu \sim 10^3 \div 6 \cdot 10^3$ cm^2/V·s	$\epsilon \sim 10^4$
$l \sim (0.7 \div 4.6) \cdot 10^{-6}$ cm	$\varkappa^{-1} \sim 10^{-8}$ cm	$E_F \sim 7.6 \div 760$ K	$1/\tau \sim 8.4 \div 12.6$ K



phase diagram



resistance vs T

[from Ye et al. (2012,2014)]

Edel'stein (1989)

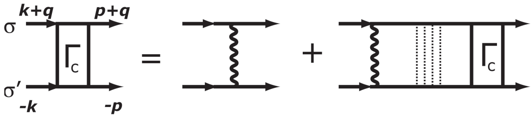
Gor'kov, Rashba (2001)

Dimitrova, Feigelman (2003,2007)

see recent review: "Non-centrosymmetric superconductors: Introduction and overview", Eds. E. Bauer and M. Sigrist, Springer 2012

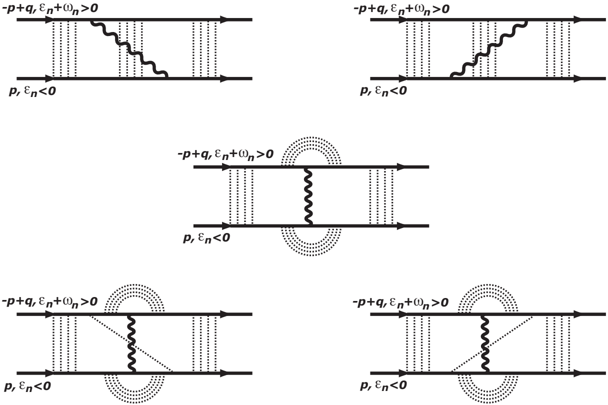
- nonmagnetic impurities do not affect s-wave superconductors

  Cooper-instability is the same for clean and diffusive electrons:



- mean free path l does not enter expression for T_c

in the presence of spin-orbit coupling nonmagnetic impurities can affect T_c [see Samokhin (2012)]



diagrams for renormalization of attraction in the Cooper channel

- perturbation theory

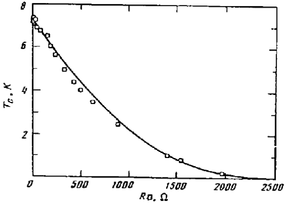
$$\frac{\delta T_c}{T_c^{BCS}} = -\frac{e^2}{6\pi^2\hbar} R_\Box \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^3 < 0$$

[Ovchinnikov (1973); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982)]

- RG theory (in the lowest order in disorder and Cooper-channel attraction)

T_c vanishes at the sheet resistance

$$R_\Box \sim \left(\ln \frac{1}{T_c^{BCS}\tau}\right)^{-2}$$



[Finkelstein (1987)]

the problem: RG theory predicts infinite resistance at T_c!?

- BCS model in the basis of electron states ϕ_ε for a given disorder

  [Bulaevskii, Sadovskii (1984); Ma, Lee (1985); Kapitulnik, Kotliar (1985)]

  superconductivity survives as long as

$$T_c^{BCS} \gtrsim \delta_\xi \propto \xi^{-d}$$

  where ξ – localization length, d – dimensionality

- enhancement of T_c near Anderson transition $\left(T_c^{BCS} \propto \exp(-2/\lambda)\right)$

$$T_c \propto \lambda^{d/|\Delta_2|}$$

  [Feigelman, Ioffe, Kravtsov, Yuzbashyan (2007); Feigelman, Ioffe, Kravtsov, Cuevas (2010)]

  [I.S.B., Gornyi, Mirlin (2012)]

  where $\Delta_2 < 0$ – multifractal exponent for inverse participation ratio

Question to answer

How resistance and magnetoresistance are described near the superconductor-metal/insulator transition within RG approach?

- free electrons in d-dimensions

$$H_0 = \int d^d\boldsymbol{r}\,\overline{\psi}_\sigma(\boldsymbol{r})\left[-\frac{\nabla^2}{2m}\right]\psi_\sigma(\boldsymbol{r})$$

where $\sigma = \pm 1$ is spin projection

- scattering off white-noise random potential

$$H_{\text{dis}} = \int d^d\boldsymbol{r}\,\overline{\psi}_\sigma(\boldsymbol{r})\,V(\boldsymbol{r})\psi_\sigma(\boldsymbol{r}), \qquad \langle V(\boldsymbol{r})\,V(0)\rangle = \frac{1}{2\pi\nu\tau}\delta(\boldsymbol{r})$$

where ν denotes the thermodynamics density of states

- electron-electron interaction

$$H_{\text{int}} = \frac{1}{2}\int d^d\boldsymbol{r_1}d^d\boldsymbol{r_2}\;U(|\boldsymbol{r_1}-\boldsymbol{r_2}|)\,\overline{\psi}_\sigma(\boldsymbol{r_1})\psi_\sigma(\boldsymbol{r_1})\overline{\psi}_{\sigma'}(\boldsymbol{r_2})\psi_{\sigma'}(\boldsymbol{r_2})$$

- Coulomb repulsion with BCS-type attraction ($\lambda > 0$):

$$U(\boldsymbol{R}) = \frac{e^2}{\epsilon R} - \frac{\lambda}{\nu_0}\delta(\boldsymbol{R})$$

- short-ranged repulsion with BCS-type attraction ($\lambda > 0$):

$$U(\boldsymbol{R}) = u_0 \left[1 + \frac{R^2}{a^2}\right]^{\alpha/2} - \frac{\lambda}{\nu}\delta(\boldsymbol{R}), \qquad \alpha > d, \qquad u_0 > 0$$

- assumptions

$$\mu \gg \tau^{-1} \gg T$$

where

μ – chemical potential
τ – transport mean-free time
T – temperature

- particle-hole channel:

$$H_{\text{int}}^{\text{p-h}} = \frac{1}{2\nu} \int_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \sum_{j=0}^{3} F_j(q) m_j(\boldsymbol{q}) m_j(-\boldsymbol{q})$$

where $l = v_F \tau$ denotes mean-free path, $m_j(\boldsymbol{q}) = \int_{\boldsymbol{k}} \bar{\psi}_\sigma(\boldsymbol{k}+\boldsymbol{q}) s_j^{\sigma\sigma'} \psi_{\sigma'}(\boldsymbol{k})$ and

$$F_0(q) = F_s, \qquad F_{1,2,3}(q) = F_t$$

- particle-particle channel:

$$H_{\text{int}}^{\text{p-p}} = -\frac{F_c}{\nu} \int_{ql \lesssim 1} \frac{d^d \boldsymbol{q}}{(2\pi)^d} \int \frac{d^d \boldsymbol{k}_1 \, d^d \boldsymbol{k}_2}{(2\pi)^{2d}} \bar{\psi}_\sigma(\boldsymbol{k}_1) \bar{\psi}_{-\sigma}(-\boldsymbol{k}_1 + \boldsymbol{q}) \psi_{-\sigma}(\boldsymbol{k}_2 + \boldsymbol{q}) \psi_\sigma(-\boldsymbol{k}_2)$$

$$F_s = \nu U(q) + F_t \qquad \text{singlet (p-h) channel}$$

$$F_t = -\frac{\nu}{2}\int_0^{2\pi}\frac{d\theta}{2\pi}\,U_{scr}\Big(2k_F \sin\frac{\theta}{2}\Big) \qquad \text{triplet (p-h) channel}$$

$$F_c = \frac{F_t}{2} - \frac{\nu}{4}\int_0^{2\pi}\frac{d\theta}{2\pi}\,U_{scr}\Big(2k_F\Big|\cos\frac{\theta}{2}\Big|\Big) = F_t \qquad \text{singlet (p-p) channel}$$

where $U_{scr}(q) = U(q)/[1+\nu U(q)]$ stands for the statically screened interaction

BCS attraction only ($\lambda \ll 1$): $\qquad -F_s = F_t = F_c = \lambda/2$

Coulomb interaction only ($\varkappa/k_F \ll 1$):

$$F_s \to \infty, \quad F_t = F_c \approx -\frac{\varkappa}{4\pi k_F}\ln\frac{4k_F}{\varkappa}$$

where inverse static screening length $\varkappa = 2\pi e^2 \nu$

nonlinear sigma-model action: [Finkelstein(1983)]

$$\mathcal{S} = -\frac{g}{32}\,\mathrm{Tr}(\nabla Q)^2 + 4\pi T z\,\mathrm{Tr}\,\eta Q - \frac{\pi T}{4}\sum_{\alpha,n,r,j}\int_{\boldsymbol{r}} \Gamma_{rj}\,\mathrm{tr}\left[t_{rj} J^{\alpha}_{n,r} Q\right]\mathrm{tr}\left[t_{rj}\left(J^{\alpha}_{n,r}\right)^{T} Q\right]$$

where the matrix field Q (Matsubara, replica, spin and particle-hole spaces) obeys

$$Q^2(\boldsymbol{r}) = 1, \qquad \mathrm{tr}\, Q(\boldsymbol{r}) = 0, \qquad Q(\boldsymbol{r}) = C^T Q^T(\boldsymbol{r}) C,$$

g – conductivity in units e^2/h,
z – Finkelstein's parameter
Γ_{rj} – interaction parameters:

$$\Gamma = \begin{pmatrix} \Gamma_s & \Gamma_t & \Gamma_t & \Gamma_t \\ \Gamma_c & 0 & 0 & 0 \\ \Gamma_c & 0 & 0 & 0 \\ \Gamma_s & \Gamma_t & \Gamma_t & \Gamma_t \end{pmatrix}$$

$SU(4)$ generators in spin and particle-hole spaces (τ_r and s_j are Pauli matrices)

$$t_{rj} = \tau_r \otimes s_j, \quad r, j = 0, 1, 2, 3$$

matrices involved:

$$\Lambda^{\alpha\beta}_{nm} = \mathrm{sgn}\, n\, \delta_{nm}\delta^{\alpha\beta} t_{00}, \quad \eta^{\alpha\beta}_{nm} = n\,\delta_{nm}\delta^{\alpha\beta} t_{00}$$
$$J^{\alpha}_{n,0} = J^{\alpha}_{n,3} = I^{\alpha}_{n,-}, \quad J^{\alpha}_{n,1} = J^{\alpha}_{n,2} = I^{\alpha}_{n,+}$$
$$(I^{\gamma}_{k,\pm})^{\alpha\beta}_{nm} = \delta_{n\pm m,k}\delta^{\alpha\beta}\delta^{\alpha\gamma} t_{00}$$

- convenient dimensionless interaction parameters: $\gamma_{s,t,c} = \Gamma_{s,t,c}/z$

- initial values (at the energy scale $\min\{\omega_D, \tau^{-1}\}$):

$$\gamma_{s0} = -\frac{F_s}{1+F_s}, \qquad \gamma_{t0} = -\frac{F_t}{1+F_t},$$

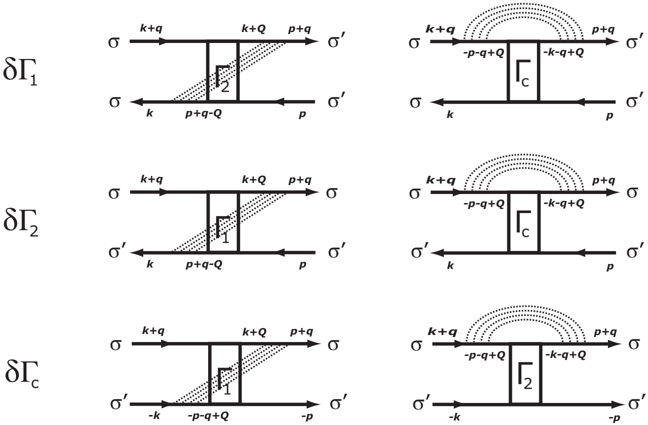
$$\gamma_{c0} = -\frac{F_c}{1 - F_c \ln \max\{1, \omega_D \tau\}} = -\frac{1}{\ln \frac{\min\{\omega_D, \tau^{-1}\}}{T_c^{BCS}}}$$

where $T_c^{BCS} = \omega_D \exp(-1/F_c)$

- BCS attraction only ($\lambda \ll 1$, $\omega_D \tau \ll 1$): $\gamma_{c0} = \gamma_{t0} = -\gamma_{s0} = -\lambda/2$

- Coulomb interaction only ($\varkappa/k_F \ll 1$):

$$\gamma_{s0} = -1, \quad \gamma_{t0} = \gamma_{c0} \approx \frac{\varkappa}{4\pi k_F} \ln \frac{4k_F}{\varkappa}$$



where $\Gamma_1 = (\Gamma_t - \Gamma_s)/2$ and $\Gamma_2 = \Gamma_t$

[Ovchinnikov (1973); Maekawa, Fukuyama (1982); Takagi, Kuroda (1982)]

$$\frac{dt}{dy} = t^2 \Big[\overbrace{\frac{n-1}{2}}^{WL/WAL} + \overbrace{f(\gamma_s) + n f(\gamma_t)}^{A-A} \overbrace{-\gamma_c}^{DOS} \overbrace{-2\gamma_c^2}^{A-L} \Big]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + n\gamma_t + 2\gamma_c\big)$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - [n(n-3)+1]\gamma_t - 2\gamma_c(1+2\gamma_t)\Big)$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\Big[(1+\gamma_c)(\gamma_s - n\gamma_t) - 2\gamma_c^2 + 2n\gamma_c\Big(\gamma_t - \ln(1+\gamma_t)\Big)\Big] - 2\gamma_c^2$$

where running RG scale $y = \ln L/l$ and $f(x) = 1 - (1 + 1/x)\ln(1+x)$

$n = 3$ – $SU(2)$ spin-rotational symmetry preserved
$n = 1$ – spin-rotational symmetry is broken down to $U(1)$
$n = 0$ – spin-rotation symmetry is fully broken

lowest order in γ_c: Finkelstein (1984, 1985); Castellani, Di Castro, Forgacs, Sorella (1984); Ma, Fradkin (1986)

all orders in γ_c but seems to be wrong: Belitz, Kirkpatrick (1994); Dell'Anna (2013)

- surface-asymmetry induced spin-orbit coupling

$$H_{so} = \alpha[\boldsymbol{\sigma} \times \boldsymbol{k}]_z \quad \Longrightarrow \quad \Delta_{so} = \alpha p_F$$

[Bychkov, Rashba (1984)]

- D'yakonov-Perel' spin relaxation (under assumption $\Delta_{so} \ll \tau^{-1}$):

$$\frac{1}{\tau_s^x} = \frac{1}{\tau_s^y} = \frac{1}{2\tau_s^z} \sim \frac{1}{\tau_{so}} \sim \Delta_{so}^2 \tau \ll \frac{1}{\tau}$$

spin-relaxation length scale $l_{so} \sim v_F/\Delta_{so} \gg l$

- spin-orbit skew scattering (from $\boldsymbol{k}$ to $\boldsymbol{k'}$) $H_{sos} = v_s[\boldsymbol{k} \times \boldsymbol{k'}]_z \sigma_z$
- spin-relaxation rates

$$\frac{1}{\tau_s^x} = \frac{1}{\tau_s^y} = 0, \qquad \frac{1}{\tau_s^z} \sim \nu \left\langle \left| v_s[\boldsymbol{k} \times \boldsymbol{k'}]_z \right|^2 \right\rangle$$

[Hikami, Larkin, Nagaoka (1980)]

spin-relaxation length scale $l_s^z \sim l\sqrt{\tau_s^z/\tau} \gg l$ (provided $\tau_s^z \gg \tau$)

RG equations / lowest order in $t = 2/(\pi g)$, but exact in γ's

$$\frac{dt}{dy} = t^2\left[\frac{n-1}{2} + f(\gamma_s) + nf(\gamma_t) - \gamma_c - 2\gamma_c^2\right]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + n\gamma_t + 2\gamma_c\big)$$

$$\frac{d\gamma_t}{dy} = -\frac{t}{2}(1+\gamma_t)\Big(\gamma_s - [n(n-3)+1]\gamma_t - 2\gamma_c(1+2\gamma_t)\Big)$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[(1+\gamma_c)(\gamma_s - n\gamma_t) - 2\gamma_c^2 + 2n\gamma_c\Big(\gamma_t - \ln(1+\gamma_t)\Big)\right] - 2\gamma_c^2$$

assumption: $1/\tau_{so} \ll 1/\tau_s^z \ll 1/\tau$

$n = 3 \quad \tau_{so}^{-1} \ll 1/\tau_s^z \ll T \quad L \ll l_s^z \ll l_{so}$

$n = 1 \quad \tau_{so}^{-1} \ll T \ll 1/\tau_s^z \quad l_s^z \ll L \ll l_{so}$

$n = 0 \quad T \ll \tau_{so}^{-1} \ll 1/\tau_s^z \quad l_s^z \ll l_{so} \ll L$

$$\frac{dt}{dy} = t^2\left[-\frac{1}{2} + f(\gamma_s) - \gamma_c - 2\gamma_c^2\right]$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + 2\gamma_c\big)$$
$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[(1+\gamma_c)\gamma_s - 2\gamma_c^2\right] - 2\gamma_c^2$$

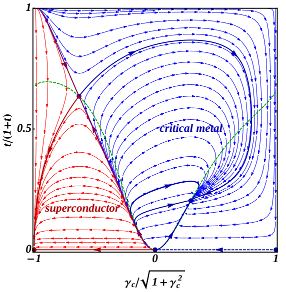
where $f(x) = 1 - (1 + 1/x)\ln(1+x)$

superconducting phase: $\gamma_c \to -\infty$, $t \sim -1/\gamma_c \to 0$

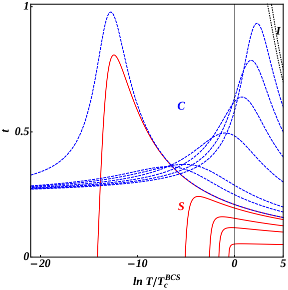
(noninteracting) supermetallic phase: $t = 0$, $\gamma_s = \gamma_c = 0$

insulating phase: $t = \infty$ (within one-loop RG it is not possible)

$$\frac{dt}{dy} = t^2\left[-\frac{1}{2} + 1 - \gamma_c - 2\gamma_c^2\right]$$
$$\frac{d\gamma_c}{dy} = \frac{t}{2}\left[1 + \gamma_c + 2\gamma_c^2\right] - 2\gamma_c^2$$



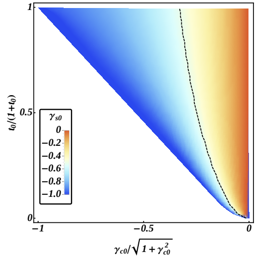
RG flow



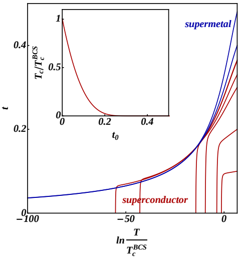
resistance vs temperature

for $\gamma_{c0} = -0.2$ and $t_0 = 0.05 \div 0.75$

$$\frac{dt}{dy} = t^2\left[\frac{1}{2} - \frac{1+\gamma_s}{\gamma_s}\ln(1+\gamma_s) - \gamma_c - 2\gamma_c^2\right]$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + 2\gamma_c\big)$$
$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[(1+\gamma_c)\gamma_s - 2\gamma_c^2\right] - 2\gamma_c^2$$



phase diagram



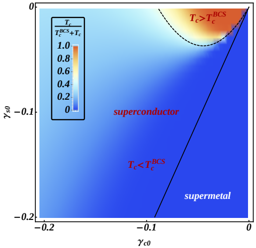
resistance vs temperature

for $\gamma_{c0} = -0.15$, $\gamma_{s0} = -0.4$ and $t_0 = 0.1 \div 0.5$

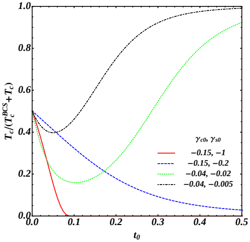
$$\frac{dt}{dy} = t^2\left[\frac{1}{2} - \frac{1+\gamma_s}{\gamma_s}\ln(1+\gamma_s) - \gamma_c - 2\gamma_c^2\right]$$

$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + 2\gamma_c\big)$$

$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[(1+\gamma_c)\gamma_s - 2\gamma_c^2\right] - 2\gamma_c^2$$



dependence of T_c on t_0



dependence of T_c on γ_{s0} and γ_{c0} for $t_0 = 0.5$

[I.S.B., Gornyi, Mirlin (2012)]

$|\gamma_{s0}| \lesssim |\gamma_{c0}| \ll t_0 \ll 1$

$$\frac{dt}{dy} \approx -\frac{t^2}{2} \qquad \frac{d\gamma_s}{dy} = -\frac{t}{2}\big(\gamma_s + 2\gamma_c\big) \quad \frac{d\gamma_c}{dy} = -\frac{t}{2}\gamma_s - 2\gamma_c^2$$

three-step RG:

(i) from $t = t_0$ to $t \approx t_0/2$: approaching BCS-like line $\gamma_c = -\gamma_s = \gamma$

(ii) increase of $|\gamma|$ from $|\gamma_{c0}|$ to $|\gamma_*| \sim \sqrt{|\gamma_{c0}|t_0}$:

$$\frac{d|\gamma|}{dy} = \frac{t}{2}|\gamma| \quad \Longrightarrow \quad |\gamma| \sim \frac{|\gamma_{c0}|t_0}{t} \quad \Longrightarrow \quad |\gamma_*| \sim t_* \sim \sqrt{|\gamma_{c0}|t_0}$$

(iii) superconducting instability with initial attraction γ_*

$$\frac{d\gamma}{dy} = -2\gamma^2 \quad \Longrightarrow \quad \ln\frac{1}{T_c\tau} \sim (|\gamma_{c0}|t_0)^{-1/2} \quad \Longrightarrow \quad T_c \gg T_c^{BCS}$$

- perpendicular field $H_\perp$ suppresses cooperons at scales

$$L \gg l_H = \sqrt{\frac{\phi_0}{H_\perp}}$$

- parallel field $H_\parallel$ suppresses copperons at scales

$$L \gg l_Z = \frac{l_H}{\sqrt{2\pi g_L (1+\gamma_{t0}) t_0}}$$

- RG equations at $L \gg \min\{l_H, l_Z\}$

$$\frac{dt}{dy} = t^2\left[\frac{1}{2} - \frac{1+\gamma_s}{\gamma_s}\ln(1+\gamma_s)\right], \qquad \frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\gamma_s$$

the smooth dependence of T_c on H as well as dependence of initial parameters γ_{c0}, γ_{s0} and t_0 on H are not taken into account

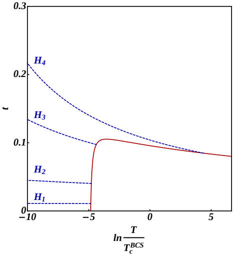
RG at lengthscales $L \ll l_{HZ}$

$$\frac{dt}{dy} = t^2 \left[\frac{1}{2} - \gamma_c - \frac{\gamma_c^2}{2} \right]$$

$$\frac{d\gamma_c}{dy} = \frac{t}{2} \left[1 + \gamma_c + 2\gamma_c^2 \right] - 2\gamma_c^2$$

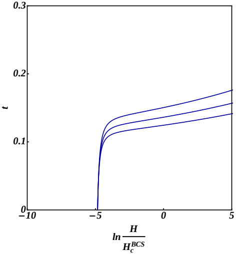
RG at lengthscales $L \gg l_{HZ}$

$$\frac{dt}{dy} = t^2$$



resistance vs T for fixed H

for $\gamma_{c0} = -0.15$ and $t_0 = 0.08$



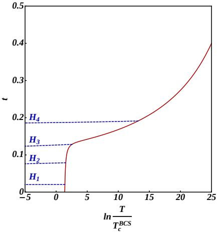
resistance vs H for fixed $T < T_c$

RG at lengthscales $L \ll l_{HZ}$

$$\frac{dt}{dy} = t^2\left[-\frac{1}{2} + f(\gamma_s) - \gamma_c - \frac{\gamma_c^2}{2}\right]$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\big(\gamma_s + 2\gamma_c\big)$$
$$\frac{d\gamma_c}{dy} = -\frac{t}{2}\left[(1+\gamma_c)\gamma_s - 2\gamma_c^2\right] - 2\gamma_c^2$$

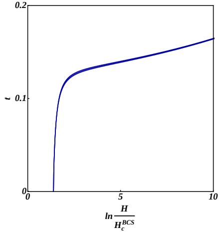
RG at lengthscales $L \gg l_{HZ}$

$$\frac{dt}{dy} = t^2 f(\gamma_s)$$
$$\frac{d\gamma_s}{dy} = -\frac{t}{2}(1+\gamma_s)\gamma_s$$



resistance vs T for fixed H

for $\gamma_{c0} = -0.04$, $\gamma_{s0} = -0.02$ and $t_0 = 0.4$



resistance vs H for fixed $T < T_c$

- superconducting transition in 2D is of Berezinsky-Kosterlitz-Thouless type
- our (mean-field) T_c is the upper estimate for T_c^{BKT}
- the description of BKT transition within nonlinear sigma-model approach is needed (in progress)

- $LaAlO_3/SrTiO_3$: the effective dielectric constant is about 10^4 (due to $SrTiO_3$),
- if static screening length $\varkappa^{-1} < l$, effective singlet-channel interaction is universal with $\gamma_s = -1$ (long-ranged)
- if static screening length $l < L_{T_c} < \varkappa^{-1}$, effective singlet-channel interaction is short-ranged

Conclusions

- one-loop RG equation but exact in the Cooper-channel interaction are derived
- the transition to the superconducting phase is analyzed
- in the case of broken spin-rotational symmetry monotonous magnetic-field dependence of resistance at $T < T_c$ is found