The Conductivity of the Half Filled Landau Level

S.V. Iordanski

Landau ITP

June 23, 2014

1) The theoretical description of 2DES with FQHE was developed in a short time after the experimental discovery (1982).Initially it was based on the assumption that the electron states must be constructed exclusively from the states of the first Ll like the famous Laughlin function but this approach does not give the general description of the possible states with FQHE. Later it was used the phenomenological Jains theory of "composite" fermions suggesting that electrons in strong m.field are "dressed" by some additional m.flux. That construction (and a more sophisticated Chern-Simons field) give some part of the observed fractions (1990) but not all.

2) In my work (2009) it was shown that the electron states with partially filled LI are thermodynamic unstable due to the formation of the quantized vortices lowering the electron free energy with the properly oriented m. moment in a fixed external m.field. This phenomenon has a close analogy to the vortex creation in a rotating vessel with liquid He_3. In a large enough sample the vortices define some vortex lattice. In a sense the vortex singularities play the function of the additional flux of "composite" fermions, but give the additional freedom in the physical description of 2DES in m.field like a phase transition.

3) The periodicity of the vortex lattice is not enough to have a simple band theory for 2DES in m.field due to the absence of the periodicity for the Hamiltonian (since the effective vector-potential is not periodic) giving rise to "m" translations. The simple band structure arise only for the rational flux of the "effective" field (m.+ vortex contribution) per the unit cell of the vortex lattice $Bs + K\Phi_0 = \frac{l}{n}\Phi_0$ where $\Phi_0 = \frac{2\pi e}{c\hbar}$ and l, n are co prime numbers, s is the area of unit cell, K is the number of vortices with unit circulation.

4)The lowest energy corresponds to the unit circulation of each vortex. One electron per each unit cell corresponds to the full filling of the band with a gap on its boundary and give the density $n_e = \frac{1}{s} = \frac{B}{\Phi_0}\frac{l-nK}{n}$ That formula gives all observed fractions and is more full then "composite" fermion classification

5)The specific properties of electrons in vortex lattices can be observed not only by FQHE which require the existence of the gap on the band boundary. There are specific cases when the vortex velocity contribution is fully compensating the external m.field flux e.g.

$$Bs + 2\Phi_0 = 0$$

corresponding to the density of the half filled Ll. In this case there are two vortices with the unit circulation in unit cell. There are also other cases with the larger number of vortices in unit cell.

6)I restrict my consideration by this case in connection with the experiments of the US group (R.L.Willet et all,1990-1993). On the surface of piezoelectric GaAs it was constructed a high quality geterostructure $Al_xGa_{1-x}As/GaAs$. In the volume of the piezoelectric was generated SAW which produced the electric field acting on 2DES at strong m.field. The ohmic conductivity of 2DES was measured at various electron densities.

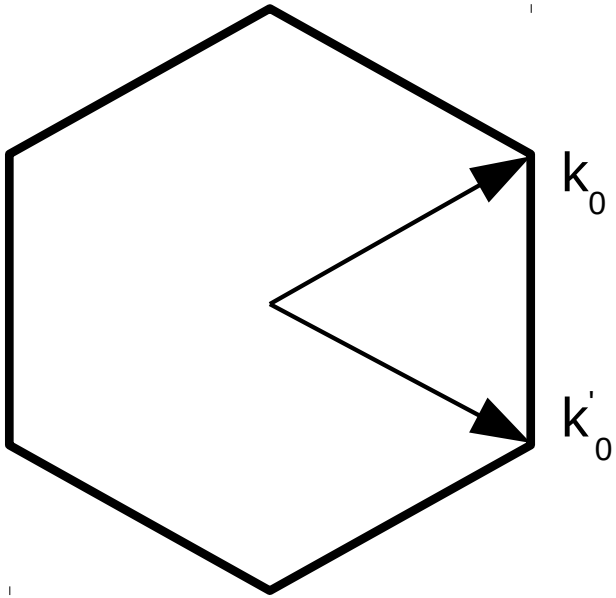
7)The largest conductivity was observed at the half filling of LI .The results were not explained in the full extent neither by "composite" fermion theory nor by Chern-Simons field both suggesting the existence of Fermi surface in 2DES. I shall try to construct the physical picture and calculate the proper conductivity in the model of vortex lattice.If the total flux of the effective field through the area of unit cell is zero the magnetic translations transforms into the ordinary abelian group of translations.

8)The main term in the hamiltonian at a strong m. field

$$H = \int \psi^{+}(\vec{r}) \frac{\hbar^2}{2m} \left[-i\vec{\nabla} - \frac{e}{c\hbar} \vec{A}_{eff} \right]^2 \psi(\vec{r}) d^2 r$$

We can choose the gauge with $\vec{A}_{eff} \equiv \vec{A}_{eff,y}$. The circulation of $\vec{A}_{eff}$ along the boundary of the unit cell is zero for any periodic vector function. The Furrier transform of $\vec{A}_{eff}(\vec{r}$ define the reciprocal lattice. Therefore we have a periodic hamiltonian with a periodic vector-potential instead of a standard periodic potential.

9)We assume that the Brilluin cell in the reciprocal lattice has the form of hexagon with two nonequivalent vectors on its boundary (special symmetry possible for two vortices in unit cell).
$\vec{k}_0 = (k_{0x}, k_{0y})$, $\vec{k'}_0 = (k_{0x}, -k_{0y})$ where $k_{0x} = \frac{2\pi}{3a}$, $k_{0y} = \frac{2\pi}{3a\sqrt{3}}$
here a is the size of the unit cell.


k0
k'0

10)It means that at the points $\vec{k}_0$ and $\vec{k'}_0$ we have two dimensional space group representation i.e. that we have two degenerate points where the energy of two different bands coincide or the energy gap is closed. There is no Fermi surfaces but two Fermi points for the electrons at the half filling of LI. That situation is well known for graphene. But the length scale of the vortex lattice is of the order of the magnetic length $l_B = \sqrt{\frac{c\hbar}{eB}}$ large compare to graphene interatomic scale. That is important because the electric field generated by SAW may have the wave length comparable to the size of unit cell of vortex lattice.

11)The electron chemical potential at the half filling of LI must be equal to the energy in the critical points $\vec{k}_0, \vec{k'}_0$. The energy of the excitations in their vicinity is given by Dirac like hamiltonian giving the spectrum $\epsilon(p) = \pm v_f p$

$$H_0 = v_f \int \psi^+(\vec{r})\sigma_l \hat{p}_l \psi(\vec{r}) d^2 r$$

where ψ is a two component column,ψ^+ is a two component line,σ_x, σ_y are 2×2 Pauli matrices,$p_l = \pi_l - k_{l0}$ or $\pi_l - k'l0$, π_l is the local quasimomentum.

12)In order to calculate the conductivity one must perform the second quantization putting

$$\psi(\vec{r},t)=\sum_{\vec{p}}\exp i(\vec{k_0}+\vec{p})\vec{r})\left[\psi_-(\vec{p})a_-(\vec{p},t)+\psi_+(\vec{p})a_+(\vec{p},t)\right]$$

$$\psi^+(\vec{r},t)=\sum_{\vec{p}}\exp -i(\vec{k_0}+\vec{p})\vec{r})\left[\psi_-^+(\vec{p})a_-^+(\vec{p},t)+\psi_+^+(\vec{p})a_+^+(\vec{p},t)\right]$$

13)Here two component $\psi_{\pm}(\vec{p})$,$\psi^{+}_{\pm}(\vec{p})$ are orthogonal and normalized, $a_{\pm}(\vec{p},t), a^{+}_{\pm}(\vec{p},t)$ are Fermi operators in Heisenberg representation with the undisturbed hamiltonian H_0. An analog representation is in the vicinity of $\vec{k'}_0$.

14)The current is induced by the electrical potential of the SAW in the form $\phi(y,t) = [\exp i(\kappa y - \omega t) + \exp i(\omega t - \kappa y)]\phi_0$ which gives a small term in the total hamiltonian

$$H = \int d^2 r \psi^+(\vec{r})[\sigma_l p_l) + e\phi(y,t)]\psi(\vec{r})$$

and the induced current can be found by Kubo formula

15)

$$< j_y(\vec{r},t) >= -\frac{ie^2}{\hbar}\int_{-\infty}^{t} dt_1 \int d^2 r_1 < [\hat{j}_y(\vec{r},t)\hat{\rho}(\vec{r}_1,t_1)] > \phi(y_1,t_1)$$

where $\hat{j}_y(\vec{r},t)$ is the current operator, $\hat{\rho}(\vec{r},t)$ is the density operator both in Heisenberg representation with the unperturbed hamiltonian H_0. The average is over the states with a negative energy. The external electric potential generate the transitions from the occupied states in the vicinity of $\vec{k'}_0$ to unoccupied states near $\vec{k}_0$ and opposite from $\vec{k}_0$ to $\vec{k'}_0$ both possible at large enough wave vector κ of SAW.

16) But this model suggests the infinite life time of the created electron-hole pairs and gives the divergent time integral in Kubo formula. The finite life time τ can be introduced phenomenologically by the exponentially decreasing factor $\exp \frac{t_1}{\tau}$ in accordance with the results of the "cross" technique.

17) Putting this factor in Kubo formula and using the known expressions for the current and the density operators it is easy to perform the required integration and obtain the compact result for the average current generated by an electron in a close vicinity of the critical point

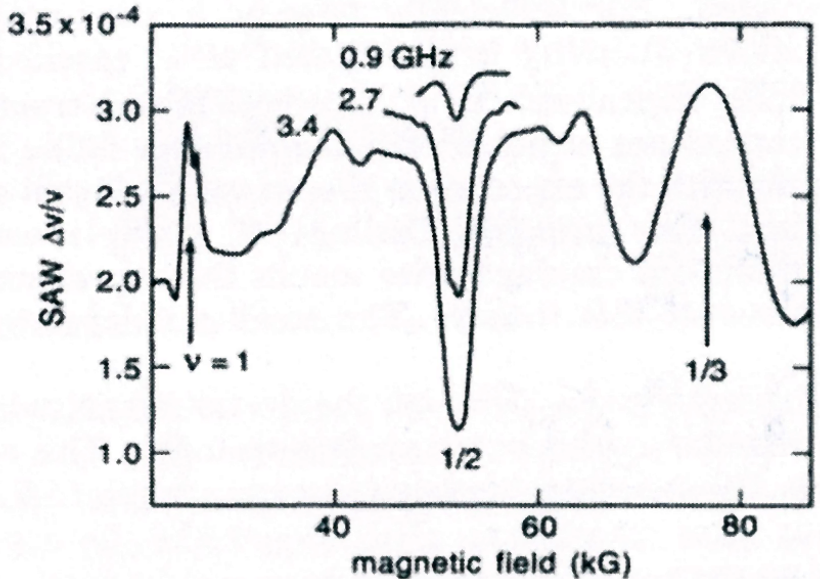
$$< j_y >= \frac{2e^2}{\hbar} v_f \phi_0 \frac{\omega(\kappa)\tau^2}{1+\omega^2\tau^2} |\psi_+^+(k'_{0y}+\kappa)\psi_-(k'_{0y})|^2 S$$

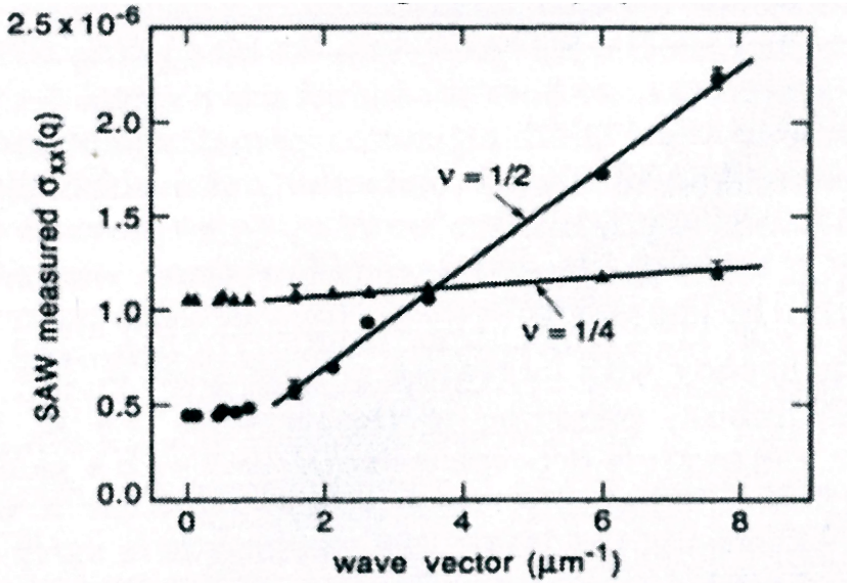
where S is the sample area.

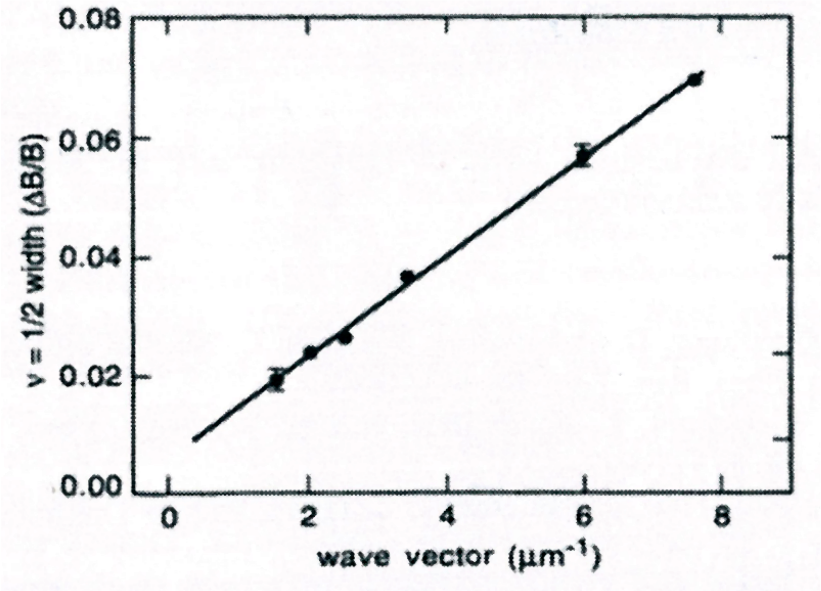
18) The product of the normalization factors is equal to S^{-2} and we get assuming $\omega\tau \ll 1$ a compact formula

$$< j_y >= \frac{2e^2}{\hbar\lambda^2} v_f c_0 \tau^2 \kappa\phi_0$$

here λ is the averaged wave length of the electrons in the "effective" domain near the critical points.
Thus we show that the conductivity at the half filling of LI is proportional to the wave vector of the SAW. That is confirmed by the experimental results giving an explicit linear dependence on the wave vector from some finite value of κ. Thus we have an essential difference when we have two Fermi points instead of Fermi surface. It must be noted that we neglect the thermal effects.


3.5 x 10^-4
SAW Δv/v
0.9 GHz
2.7
3.4
ν = 1
1/2
1/3
magnetic field (kG)


2.5 x 10^-6
SAW measured σxx(q)
ν = 1/2
ν = 1/4
wave vector (μm^-1)


ν = 1/2 width (ΔB/B)
wave vector (μm^-1)