MULTI-VORTEX DYNAMICS IN CHARGE DENSITY WAVES.

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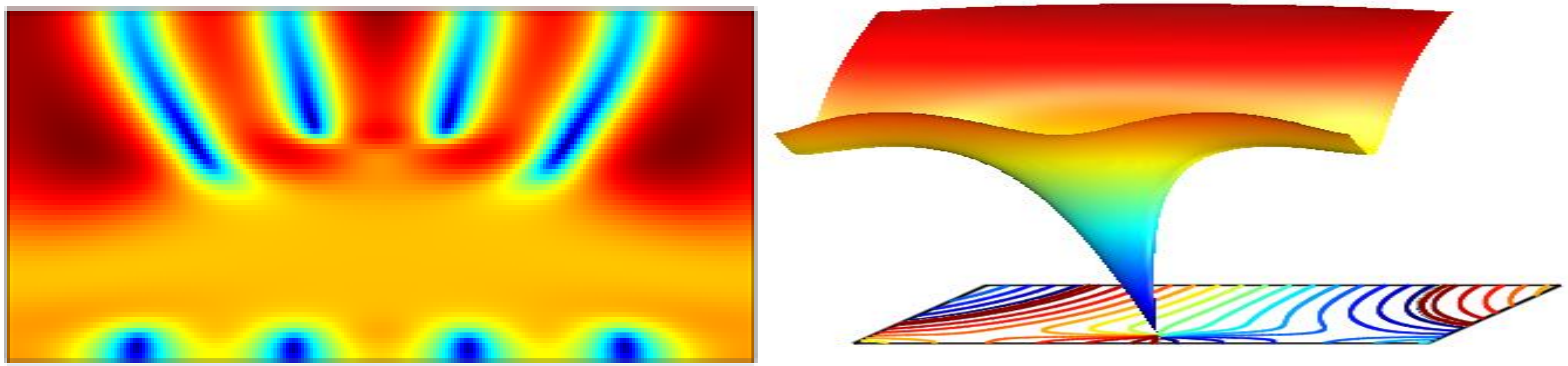
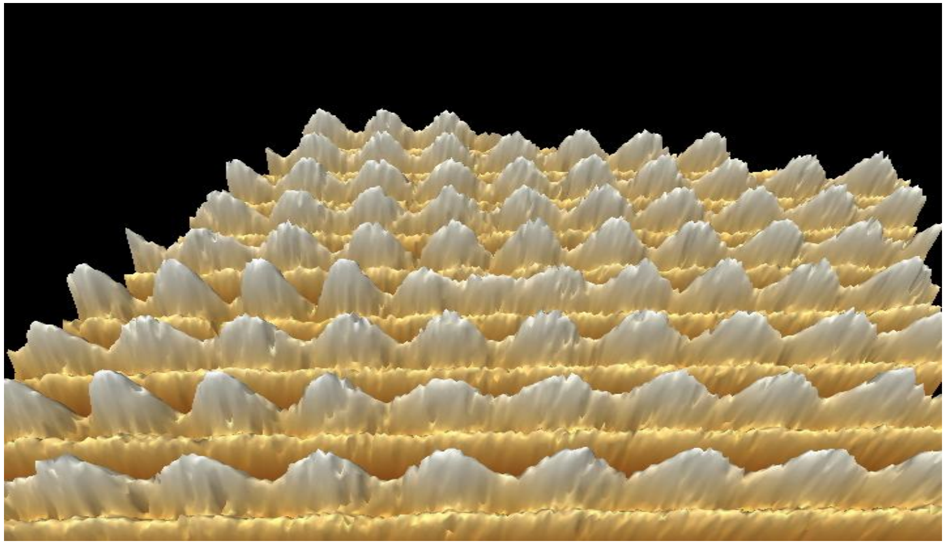
Inspirations: **Yuri. I. Latyshev** , A. Sinchenko, and the team of IREE

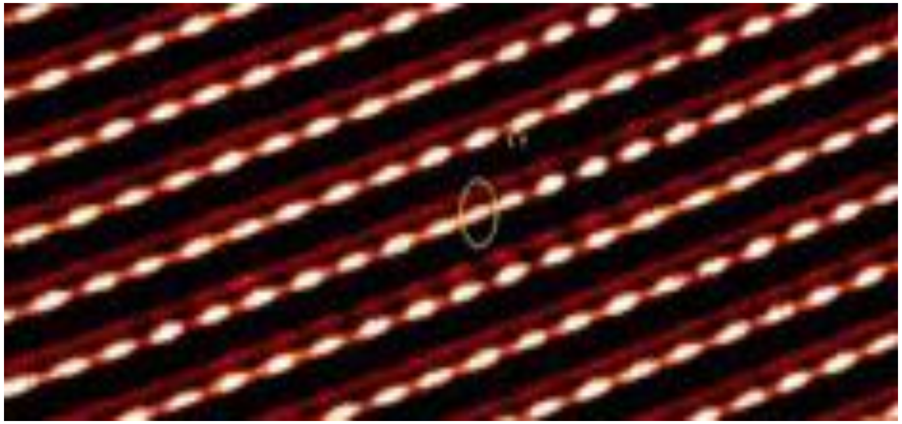
Graphical abstract



Nucleation of electronic vortices, their flash sweeping,
and the structure of the single remnant vortex in the junction



STM view of a CDW and of amplitude soliton, C. Brun



Microscopics of local and instantaneous electronic states in CDWs .

BCS-like Peierls-Fröhlich model for the CDW.

Exact static solutions – solitons of multi-electronic models.

Adiabatic generalization to dynamic processes – instantons.

Incommensurate CDW : **$A\cos(Qx+\phi)$** **$Q=2K_f$**

Complex order parameter : **$\Delta(x,t) \sim A\exp(i\phi)$**

Electronic states **$\Psi = \Psi_+ \exp(iK_f x) + \Psi_- \exp(-iK_f x)$**

$$Tr\begin{vmatrix} -i\partial_x & \Delta^* \\ \Delta & i\partial_x \end{vmatrix} + \frac{K}{2}\int dx\left(|\Delta|^2 - \frac{1}{|\omega_{ph}|^2}|\partial_t \Delta|^2\right)$$

Justification of the mean-field BCS, and for co-observation of electrons and solitons: Small phonon frequency: experimentally **$\omega_{ph} < 0.1\Delta$**

John Bardeen's Grand Unification:

……

1940's - ***transistor***

1950's - ***Superconductivity***

1976 – 30/01/1991- - ***Charge Density Waves***

Common features of incommensurate CDWs and the superconductors:

O_{cdw} =Acos(2K_Fx+ ϕ) → complex order parameters $\mathbf{O_{cdw,sc}}$ **~ A exp[iφ]**

hence vortices ←→ dislocations, phase slips ←→ phase solitons

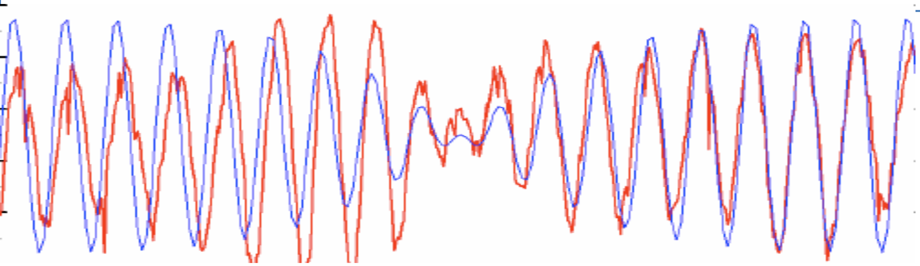
Similar microscopic theories: Peierls-Frohlich vs BSC

Pair-breaking gaps **2Δ** - hence tunneling, FFLO ←→ solitonic lattices

Tighter links at **D=1**: Spinons as amplitude solitons

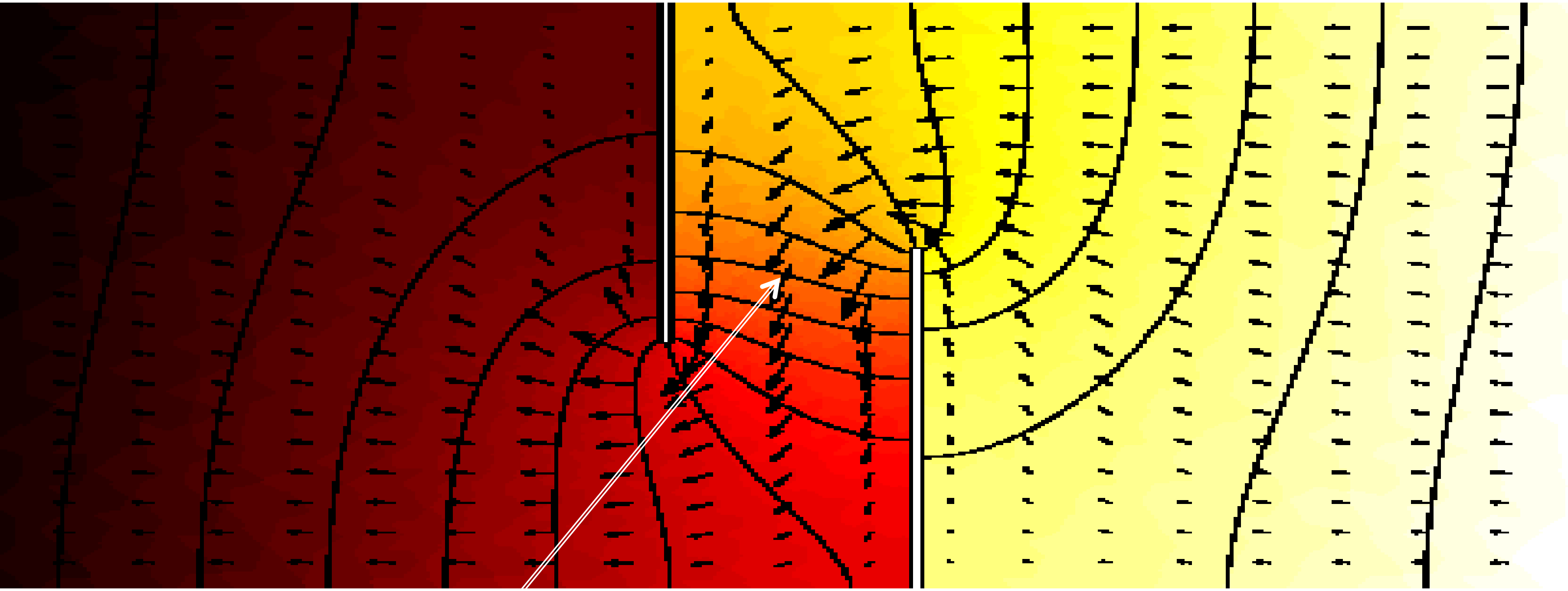
Phases φ_{cdw} , φ_{sc} are in conjugation: $[\varphi_i, \partial_x\varphi_j]\sim\delta(x)$

2Δ becomes the common spin gap; broken pair becomes 2 free spinons



Yurii Latyshev technology of mesa-structures

Distribution of potentials:
values in colours, equipotential lines in black and currents as arrows.



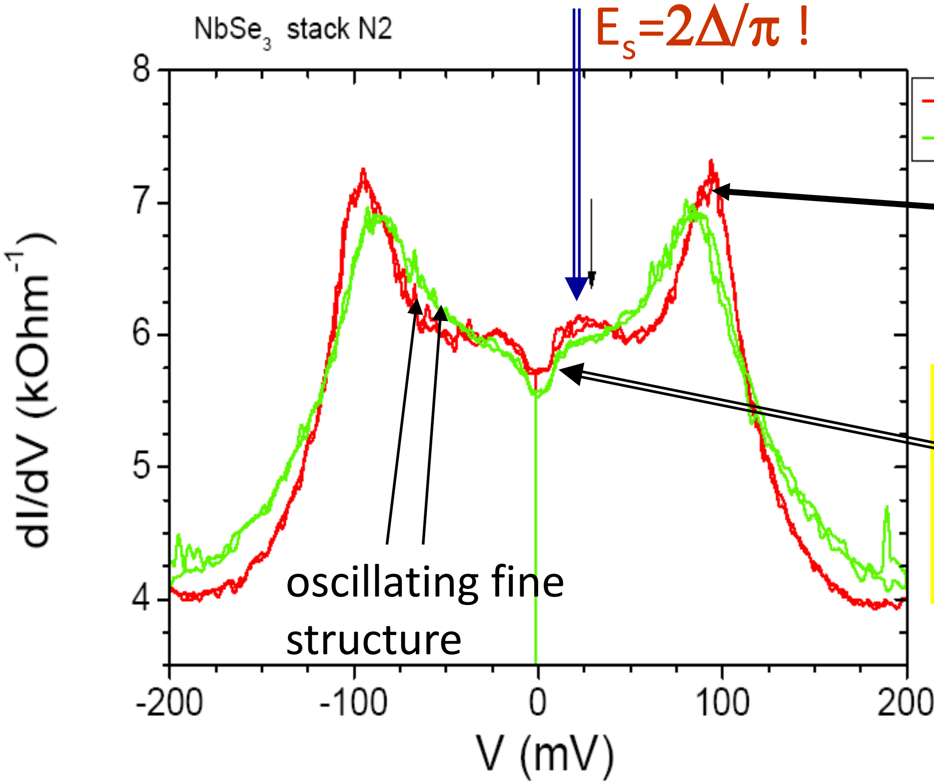
Junction with N=20-30 atomic layers.
Voltage drop is made by the normal current.
The problem is never static, at least it is stationary.

Direct observation of solitons and their arrays in tunneling on NbSe3

Y. Latyshev, P. Monceau, A. Orlov, S.B., et al, PRLs 2005 and 2006

creation of solitons at ≈$2\Delta/3$:

$E_s=2\Delta/\pi$!



peak 2Δ for inter-gap creation of e-h

absolute threshold at low $V_t \approx 0.2\Delta$

All features scale with the gap $\Delta(T)$!

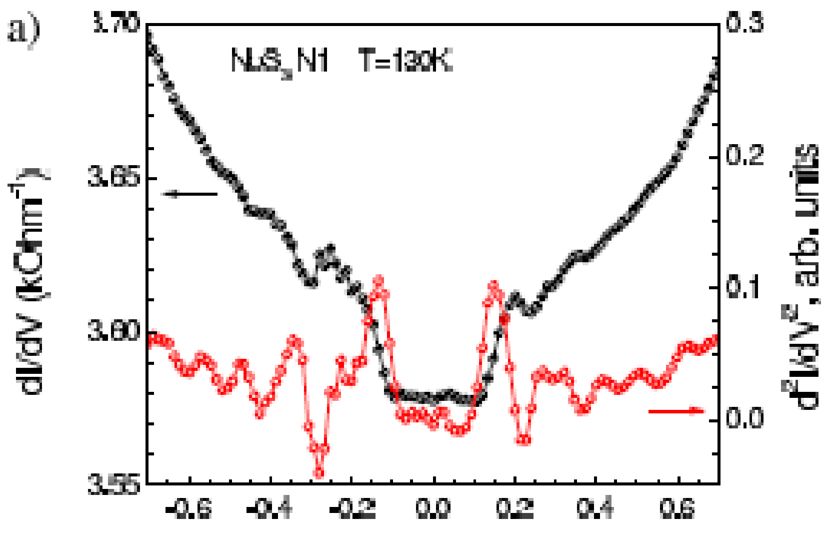
***Main puzzles:* 1. What is the low threshold?**

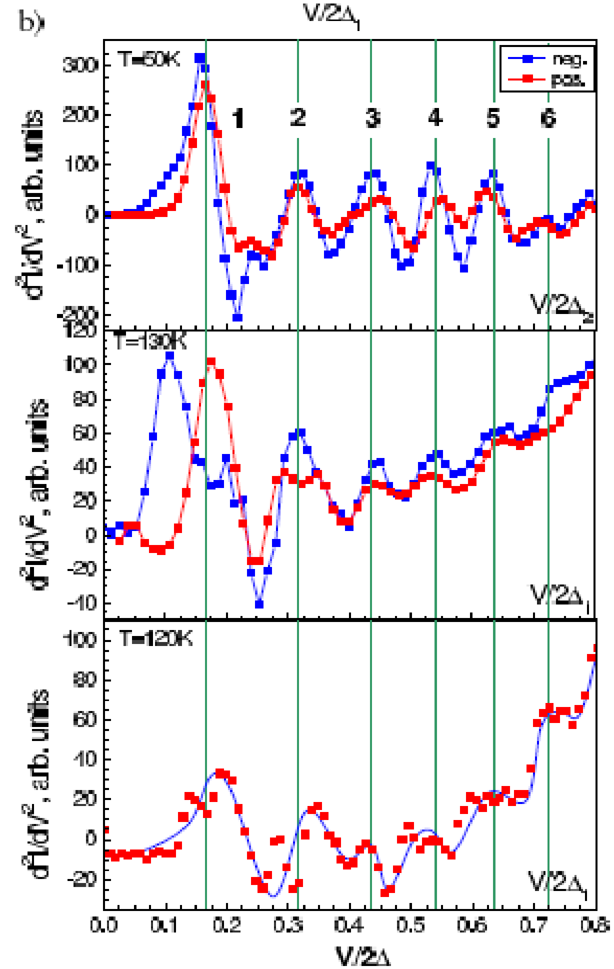
2. Why the voltage is not multiplied by N~20-30 - number of layers in the junction

- It seems to be concentrated at just one elementary interval – DISLOCATION CORE

In similar devices for superconductors the peak appears at V=2□×N

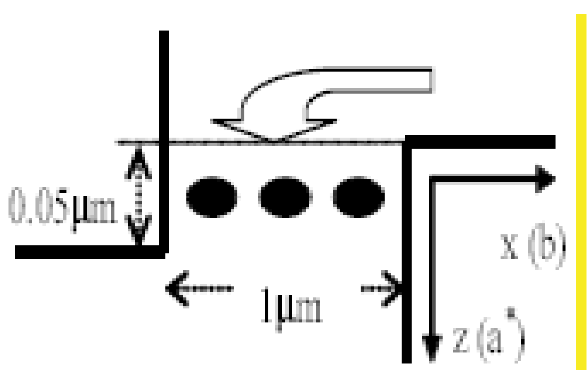
Junction reconstruction by entering of dislocations





Fine structure is not a noise !

It is : sequential entering into the junction area of dislocation lines = CDW vortices = solitons‘ aggregates.



Dislocation in CDW versus vortex in SC

$$\psi_{CDW} = A\exp(i\varphi)$$

$$\psi_{SC} = C\exp(i\theta)$$

$$j_{CDW} = -A^2 \frac{e}{\pi}\frac{\partial\varphi}{\partial t} \qquad n_{CDW} = A^2 \frac{e}{\pi}\frac{\partial\varphi}{\partial x}$$

$$j_{SC} \propto C^2 e v_F \frac{\partial\theta}{\partial x} \qquad n_{SC} \propto -C^2 \frac{e}{v_F}\frac{\partial\theta}{\partial t}$$

$$(\nabla\theta)^2 - 2\vec{A}\nabla\theta + \vec{A}^2 + [\vec{\nabla}\times\vec{A}]^2 + \vec{j}_{ext}\cdot\vec{A}$$

$$\Phi \equiv A_x$$

$$(\nabla\varphi)^2 + (\Phi + n)\partial_x\varphi + n\Phi - (\nabla\Phi)^2 + n^2\lambda_{scr}^2$$

$$-\nabla_y\Phi = E_y \equiv H_z = -\nabla_y A_x$$

No analogies: $\nabla_x\Phi = -E_x \Rightarrow \nabla_x A_x$ $\quad$ n like A_x but with A_x^2 $\quad$ like external supercurrent

Equivalence of actions of **E_y** and **H_z** upon the order parameters.

Reverse effect of order parameters upon the fields are opposite:

CDW – transverse electric field **E_y** is screened only via dislocations.

SC - magnetic field enters via vortices.

Unlike **H≈cnst** in SC, **E** in CDW is always self-consistent

Ginzburg-Landau – like model

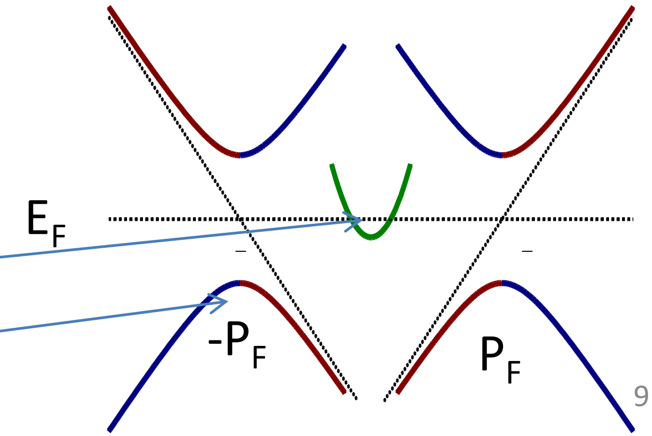
$$\psi = A\exp(i\varphi)$$

$$H = H_{CDW} + H_{el}$$

$$H_{CDW} = \int d^3r \left\{ \left[\left| \frac{\partial \psi}{\partial x} \right|^2 + \alpha \left| \frac{\partial \psi}{\partial y} \right|^2 \right] + |\psi|^2 \ln \frac{|\psi|^2}{e} \right\}$$

$$H_{\text{int}} = \int d^3r \left[\Phi A^2 \partial_x \varphi / \pi + \Phi n(\varsigma) + F(n) - |\nabla\Phi|^2 \varepsilon / 8\pi \right]$$

$$n(\varsigma) = \frac{n_0 T}{\varepsilon_F} \ln\left(1 + \exp\left(\frac{\varepsilon_F + \varsigma}{T}\right)\right)$$



Only extrinsic carriers *n* are taken explicitly. Intrinsic ones, in the gap region, are hidden in the CDW amplitude A.

Equations

$$\nabla(A^2\nabla\varphi) + \frac{\partial}{\partial x}(A^2\Phi) = -\gamma_\varphi A^2 \frac{\partial\varphi}{\partial t}$$

$$\left(\nabla^2 A + A(\nabla\varphi)^2\right) + |A| \ln |A|^2 = -\gamma_A \frac{\partial A}{\partial t}$$

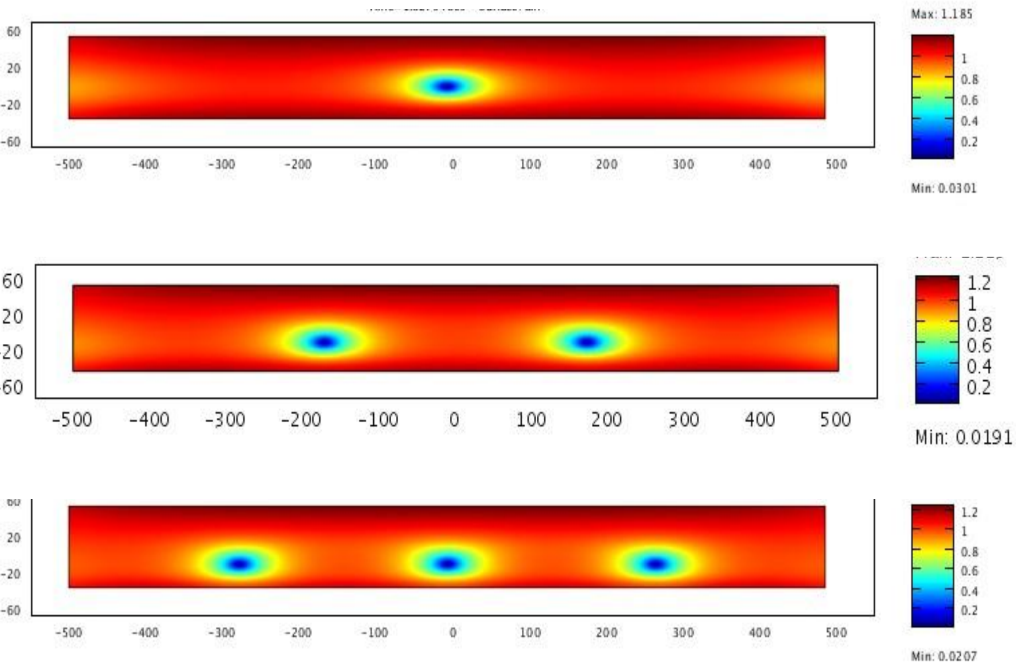
$$-\frac{\varepsilon}{4\pi e^2}\nabla^2\Phi = \frac{1}{\pi}\frac{\partial\varphi}{\partial x} + n(\varsigma)$$

$$\frac{dn}{dt} = -\nabla\left[\sigma\nabla\left(\varsigma + \Phi\right)\right] + \frac{\partial n}{\partial t} = 0$$

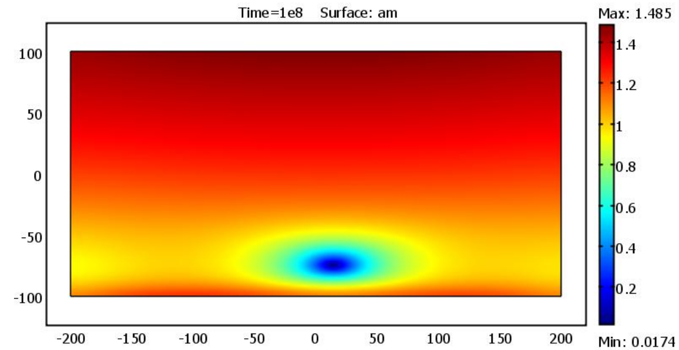
Near the vortex core

∂φ□∝, hence A□0

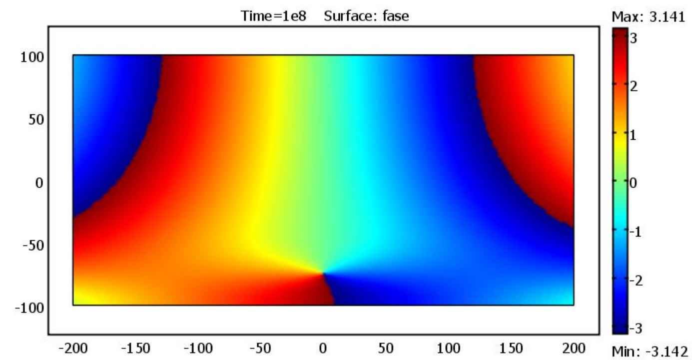
Amplitude



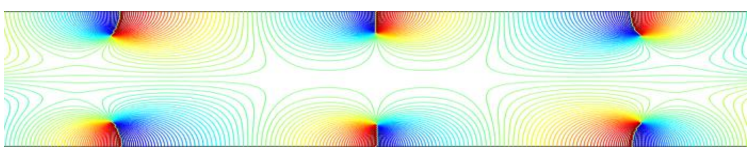
amplitude



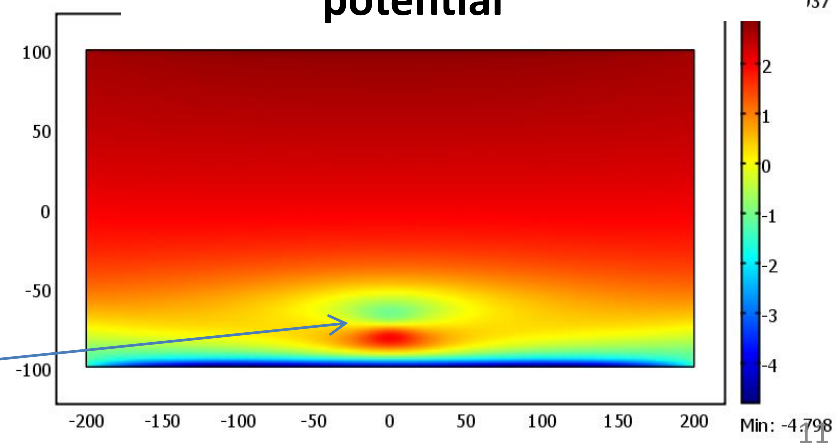
phase



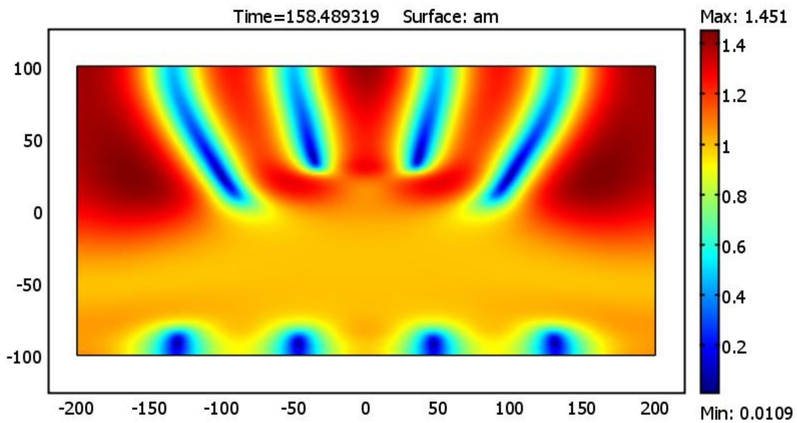
Phase: wider sample, higher V



potential



Strong drop of electric potential and the high current density are concentrated near the vortex core – location of tunneling processes.



traces of A=0

nucleations

Many vortices appear temporarily in the course of the evolution. For that run, only one will be left.

Unexpected result: long living traces of the amplitude reduction following fleshes of vortices.

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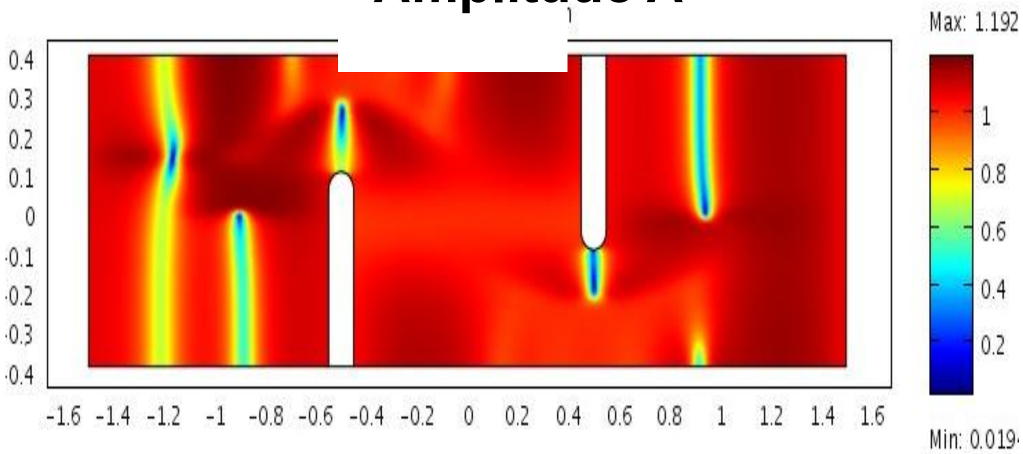
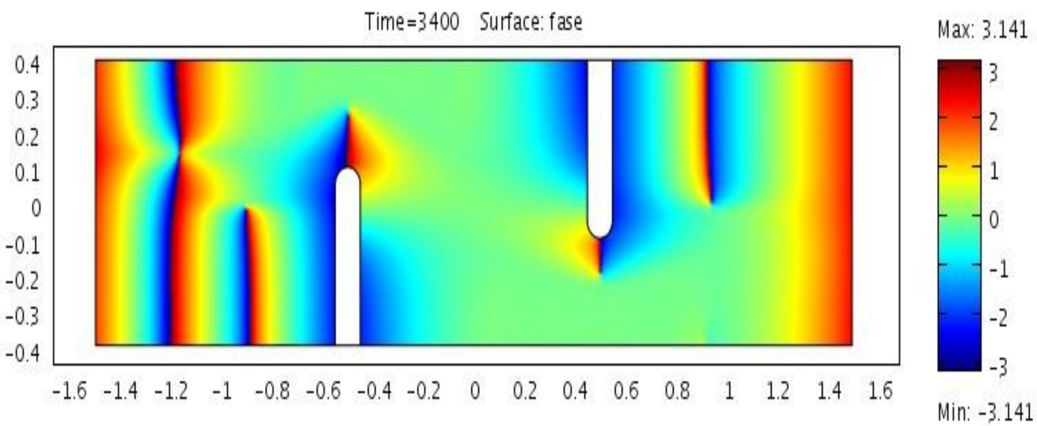
Real geometry: initial short time fast dynamics

Amplitude A

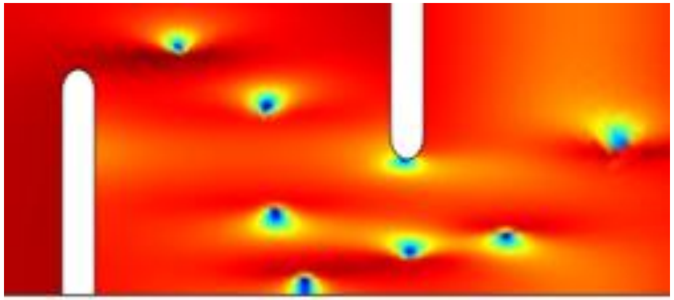


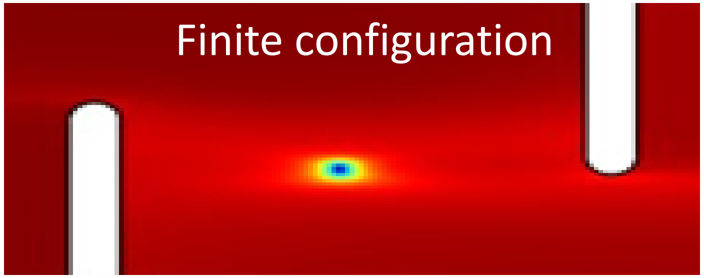
Phase □



All these 5 flashes are the phase-slip processes serving to redistribute the CDW collective charge

Intermediate configuration with many vortices originating both inside and outside of the junction.





Deadly problems with the TDGL model for CDW

*W*orks well for a stationary state and as a tool to reach it.
Takes explicitly the extrinsic carriers (not interacting with CDW).

Restrictions:

Intrinsic carriers have been integrated out, as always in a GL,

they come into the model only via the order parameter amplitude **A**.

Major problem:
Violation of the local charge conservation for the condensate .

automatically if A = const

$$n_c = \frac{A^2}{\pi}\frac{\partial\varphi}{\partial x} \qquad j_c = -\frac{A^2}{\pi}\frac{\partial\varphi}{\partial t} \quad \Longrightarrow \quad \frac{dn}{dt} = \frac{\partial n_c}{\partial t} + \frac{\partial j_c}{\partial x} = 0$$

In our case A(x,y,t) :

$$\pi\frac{dn_c}{dt} = \frac{\partial A^2}{\partial x}\frac{\partial\varphi}{\partial t} - \frac{\partial\varphi}{\partial x}\frac{\partial A^2}{\partial t} \neq 0$$

Way of resolution: keep carries in hand and decompose $\mathbf{A^2}$

Chiral transformation $\psi_{\pm} \rightarrow \psi_{\pm} e^{\pm i\varphi/2}$ $\Delta \rightarrow \Delta_{real} e^{i\varphi}$

$\hbar v_F = 1$

$$\begin{pmatrix} i\partial_t - i\partial_x + e\Phi + \frac{e}{c}A_x + \partial_x\varphi/2 + \partial_t\varphi/2 & \Delta \\ \Delta & i\partial_t + i\partial_x + e\Phi - \frac{e}{c}A_x + \partial_x\varphi/2 - \partial_t\varphi/2 \end{pmatrix}$$

$$\boxed{e\delta\Phi = \partial_x\varphi/2} \qquad \boxed{\frac{e}{c}\delta A_x = \partial_t\varphi/2} \qquad \delta E = \left(v_F^{-2}\partial_t^2\varphi - \partial_X^2\varphi\right)/2$$

Condition: Interactions depends only on derivatives of φ, i.e:
no Umklapp processes , electrons scatter on each other and on CDW phonons

Local energy functional

$$\psi = A\exp(i\varphi)$$

$$\left[(\partial_x A)^2 + \alpha(\partial_y A)^2\right] + \left[1\times(\partial_x\varphi)^2 + \alpha A^2(\partial_y\varphi)^2\right] +$$

Expect **A²** **-** actually 1.

Non analytic in Ψ

Both terms are **not** derivable perturbatively – the chiral anomaly.

$$+ e\Phi\partial_x\varphi/\pi + e\Phi n_{ex} + \left(\Phi + \partial_x\varphi/2\right)n_{in}$$

$$+ F(A, n_{in}, n_{ex}) \quad - \quad (\nabla\Phi)^2\varepsilon/8\pi$$

n_{in}, n_{ex} concentrations of intrinsic and extrinsic free carriers

$F(A,n_{in},n_{ex})$ static free energy, not additive any more

Its minimum at **A≠0** is erased at ***n_{ex}*** above a critical value (vortec core)

$$n = \frac{1}{\pi}\partial_x\varphi + n_{in}\ ,\ j = -\frac{1}{\pi}\partial_t\varphi + j_{in} \Rightarrow \frac{dn}{dt} = 0$$

\+ need a mechanism for **n_{in}** to compensate **∂φ** at **A→0**

True equations are not analytic in Ψ:
phase gradients are not multiplied by **A^2**

Former G-L like equations

$$\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial x}+2\Phi+\pi(n_e-n_h)\right)+\alpha\frac{\partial}{\partial y}\left(A^2\frac{\partial\varphi}{\partial y}\right)=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla A^2\nabla\varphi+\frac{\partial}{\partial x}A^2\Phi=\gamma_\varphi A^2\frac{\partial\varphi}{\partial t}$$

$$\nabla^2 A+\alpha A\left(\frac{\partial\varphi}{\partial y}\right)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$\nabla^2 A+A(\nabla\varphi)^2+\frac{\partial F}{\partial A}=-\gamma_A\frac{\partial A}{\partial t}$$

$$-r_0^2\nabla^2\Phi=\frac{\partial\varphi}{\partial x}+\pi\left(n_e-n_h+n_{ex}\right)$$

$$-r_0^2\nabla^2\Phi=A^2\frac{\partial\varphi}{\partial x}+n_{ex}$$

$$\nabla\hat{\sigma}\nabla\mu=\partial_t n=\frac{1}{4\pi}\frac{1}{r_0^2}\rho_n\partial_t\zeta$$

$$-\nabla\left[\sigma\nabla(\varsigma+\Phi)\right]+\frac{\partial n}{\partial t}=0$$

All energies are in units of Δ

$$\mu=\zeta+\Phi+\partial_x\varphi/2$$

In the metallic phase $\boldsymbol{\rho_n=1}$ then $\boldsymbol{\rho_c=0}$;
approaching from the CDW phase as $\boldsymbol{\rho_c}\sim\Delta^2$

Possible simplifications and explitness

Infinite conductivity: **-** a bridge to the naive GL eqs. $\mu = \zeta + \Phi + \partial_x \varphi / 2 \equiv 0$

$$\zeta = \frac{\partial F}{\partial n}, \; \rho_n = N_F^{-1} \frac{\partial n}{\partial \zeta} ; \; \rho_c = 1 - \rho_n$$

$$\rho_c \partial_x^2 \varphi = r_0^2 \nabla^2 E_x - \rho_n E_x$$

Poisson eq.

LHS resembles the static effective charge

$n_c = A^2 \partial_x \varphi / \pi$ - identifying **ρ_c** and **A^2**

But instead: **$\partial_x n_c = \rho_c \partial_x^2 \varphi / \pi$**

Never a closed expression for j

screening of **E_x** with a standard local screening length $l^2 = r_0^2 / \rho_n$

$$-\rho_c E_x + \left(\rho_c \partial_x^2 + \partial_y (\alpha A^2 \partial_y) - \gamma_\varphi \partial_t \right) \varphi = 0$$

Phase eq.

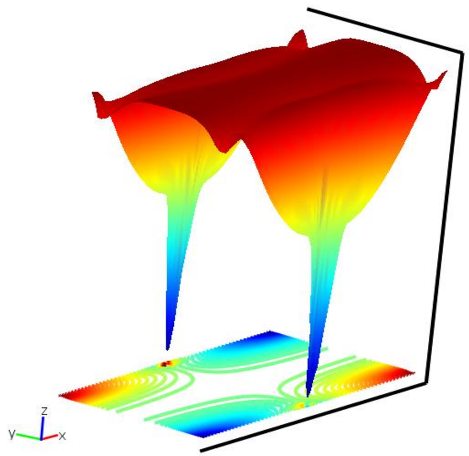
Resembles GL with **ρ_c** as **A^2** but with no differentiation of the amplitude :

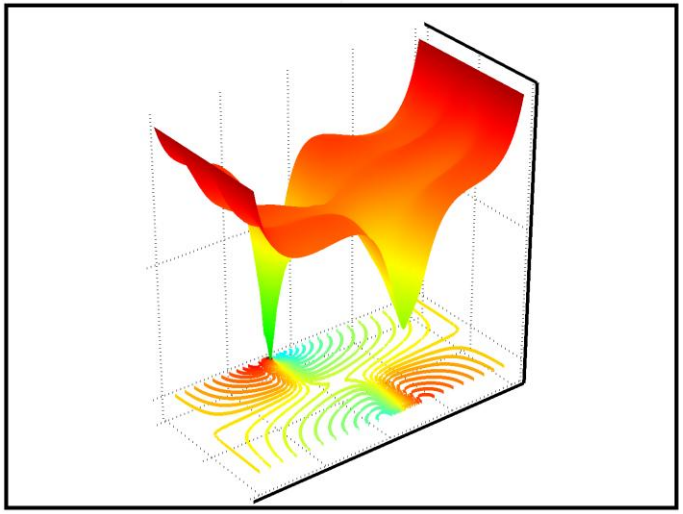
$\rho_c \partial_x \Phi$ instead of **$\partial_x (A^2 \Phi)$**

$\rho_c \partial_x^2 \varphi$ instead of **$\partial_x (A^2 \partial_x \varphi)$**

Not like variational eqs.

Collective current and density following the phase deformations are given by the total number of electrons independent on the temperature and the magnitude of the gap.
Nonanalytic dependence on the amplitude requires new more complicated numerical studies.





We still can run up to nucleation of vortices at a surface.
But then the program crashes and
we cannot trace proliferation of vortices as before.
A price for no explicit compensation of diverging $\boldsymbol{\partial\varphi}$ by vanishing $\mathbf{A^2}$

Conclusion and perspective.

➢We have performed a program of modeling of stationary states and of their transient dynamic for the CDW in restricted geometries taking into account multiple fields in mutual nonlinear interactions: the complex order parameter **Aexp(i)** of the CDW, the electric field, the density and the current of normal carriers.

➢Vortices are formed in the junction when the voltage across, or the current through, exceed a threshold; the number of vortexes increases step-wise
- in agreement with experiments.

➢The vortex core concentrates the total voltage drop,
working as a self-tuned microscopic tunnelling junction,
which might give rise to observed peaks of the inter-layer tunneling .

➢The studied reconstruction in junctions of the CDW can be relevant to modern efforts of the field-effect transformations in strongly correlated material which also show a spontaneous symmetry breaking.

➢The numeric procedure needs to be stabilized for the nonanalytic eqs.

➢The problems of glide and climb should be better considered.