Emergent chirality

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Outline:

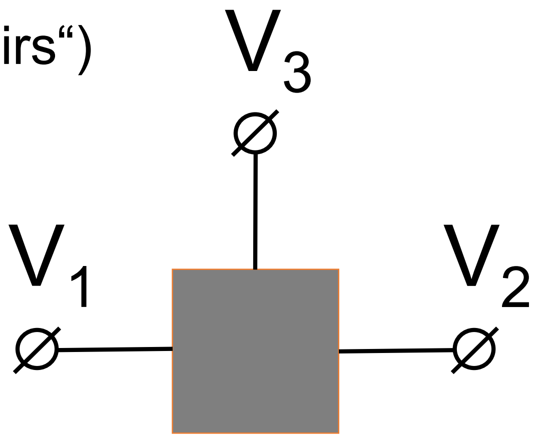
- **emergent chirality on the conceptual level**
- **emergent chirality for a Luttinger-liquid model on the microscopic level**

largely based on: D.Aristov, I.Gornyi, D.Polyakov, and P.Wölfle,

Phys. Rev. B **100**, 165410 (2019)

Main conceptual question for this talk :

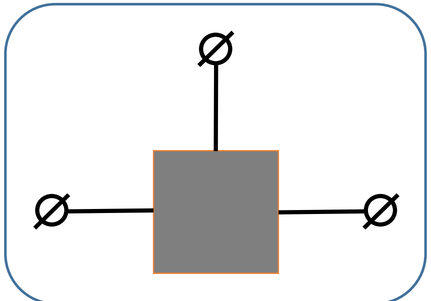
- Consider an electron system the **Hamiltonian** of which is **parity** (right-left) and **time-reversal symmetric**
- Connect the system to contacts („ideal reservoirs“) in a right-left symmetric way
- Apply static voltages $V_1, V_2, V_3 \ldots$ to contacts $1, 2, 3 \ldots$

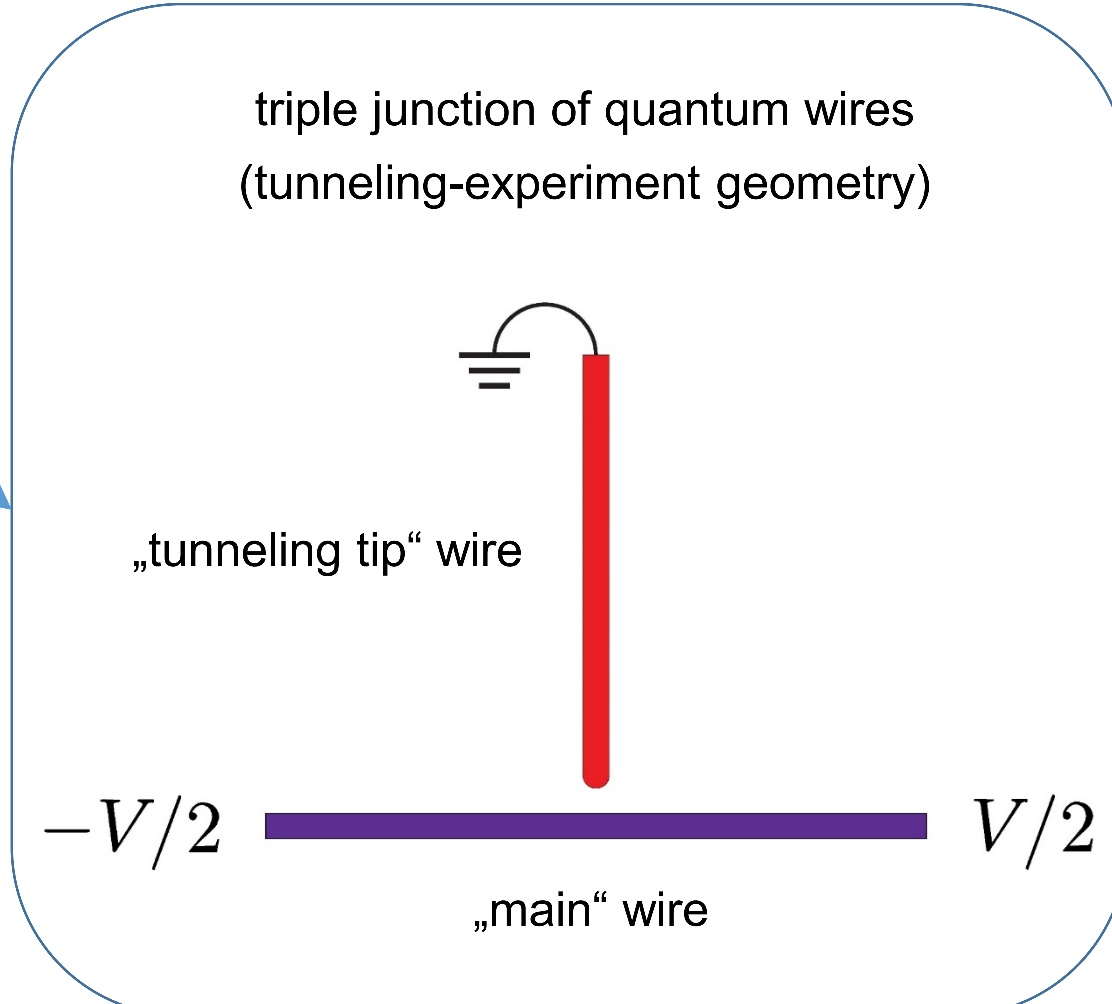


Can the current response break P and/or T symmetry :

„photogalvanic“ (ratchet) and/or Hall currents ?

Distilled example of „emergent chirality“: Y junction



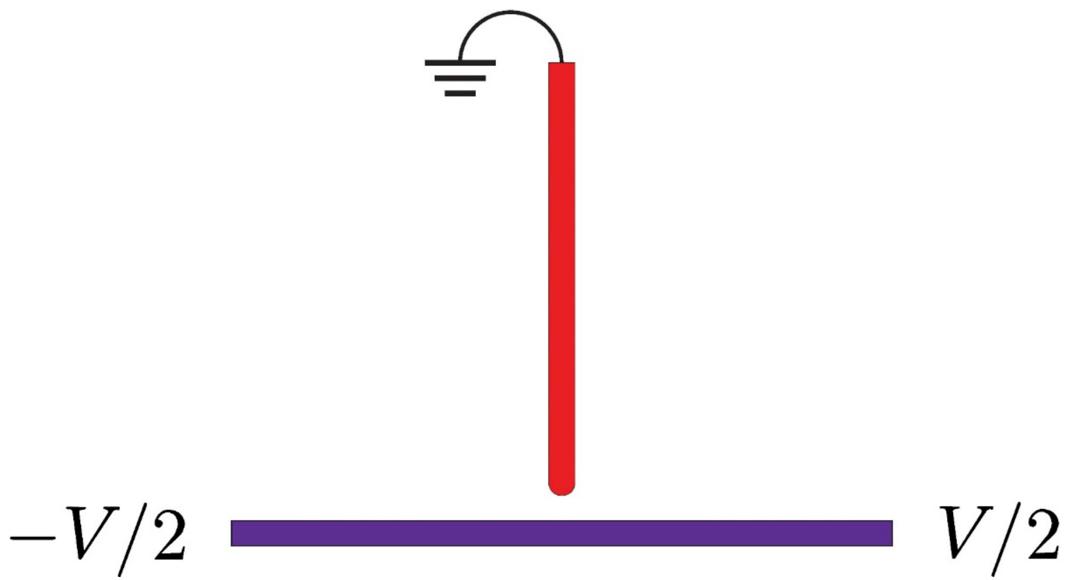


connected
symmetrically !

biased
symmetrically !

Distilled example of „emergent chirality“: Y junction

zero current in the grounded tip wire ?



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Distilled example of „emergent chirality“: Y junction

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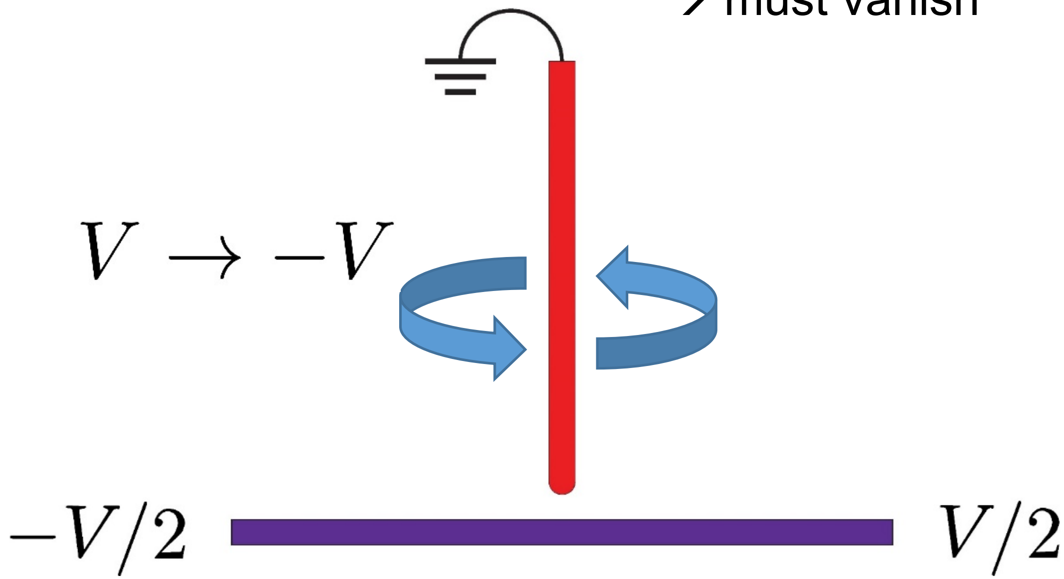
zero current in the grounded tip wire ?

linear response (FDT): current linear in V
→ must vanish



Distilled example of „emergent chirality“: Y junction

incl. *e-h* symmetry
wrt ground

connected
symmetrically !

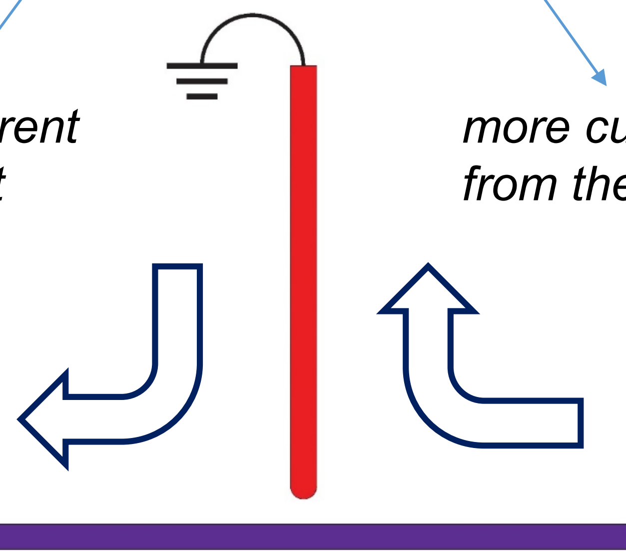
biased
symmetrically !

zero current in the grounded tip wire ?

exact compensation: for noninteracting electrons

more current
to the left

more current
from the right



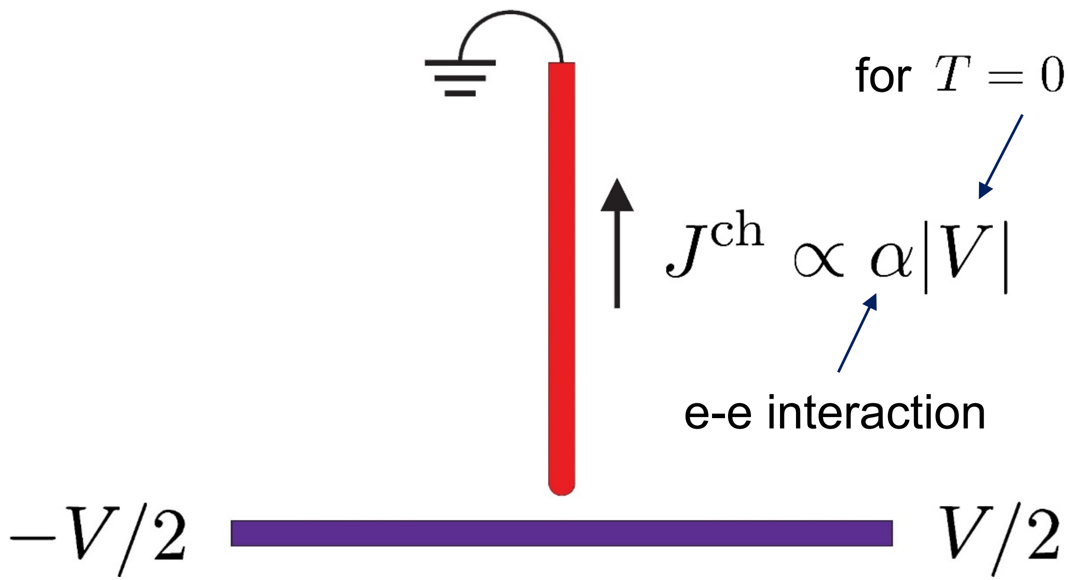
$-V/2$ $V/2$

Distilled example of „emergent chirality“: Y junction

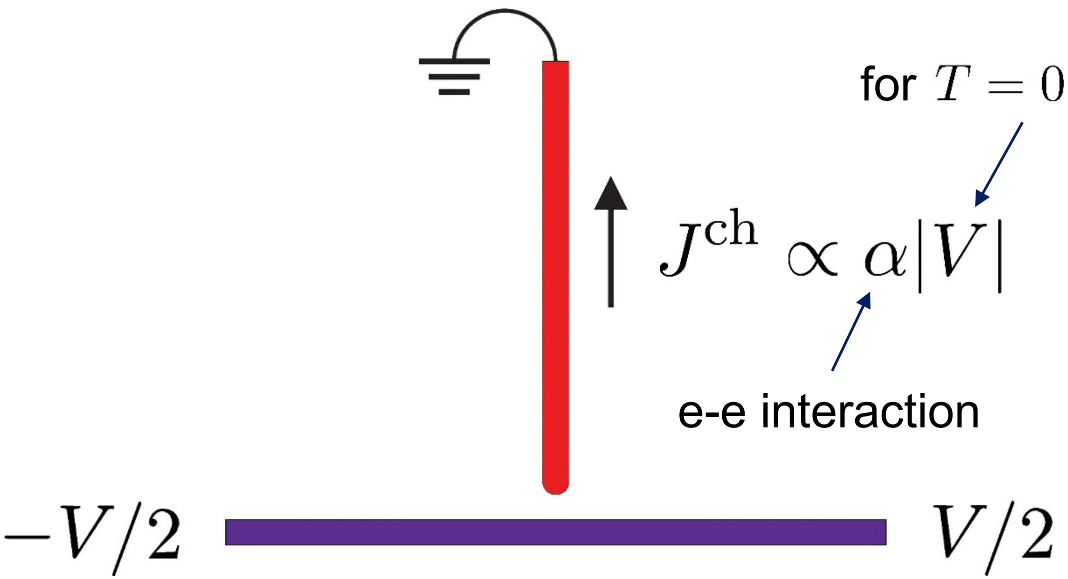
connected
symmetrically !

biased
symmetrically !

nonzero current in the tip wire



Chiral current



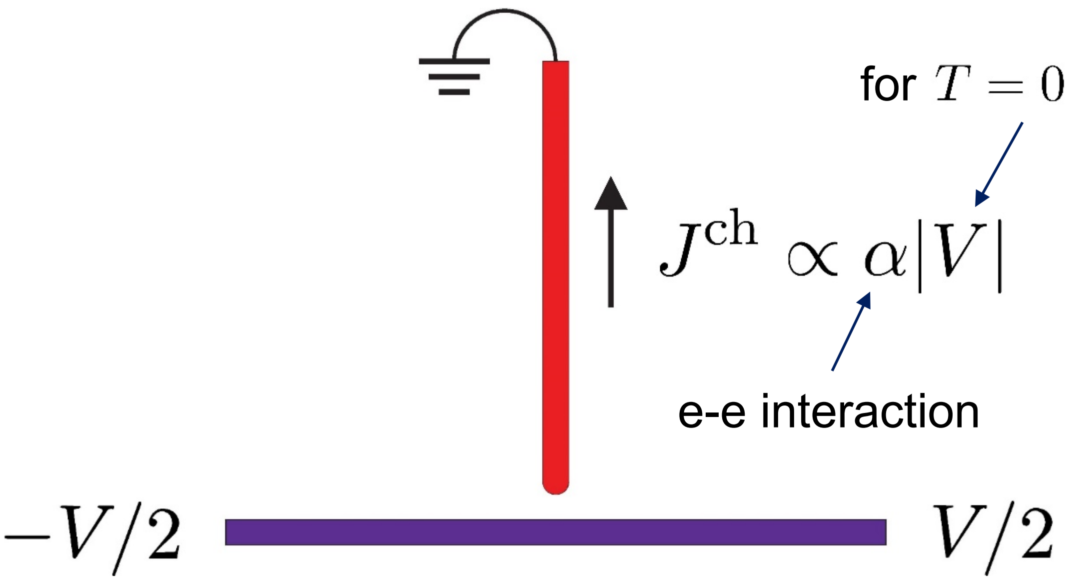
- **unidirectional** : $J_{\mathrm{ch}}(V_1, V_2, V_3) = J_{\mathrm{ch}}(-V_1, -V_2, -V_3)$

  ≡ chiral — flipping **all** voltages

- induced by e-e **interaction** : $J_{\mathrm{ch}} \propto \alpha$, depends on the sign of interaction

- essentially **nonequilibrium** phenomenon : $J_{\mathrm{ch}} \propto |V|$

Chiral current



noninteracting tip wire
(α - in the main wire):

← „**photogalvanic**“ chiral current

„**Hall**“ chiral current → later

- **unidirectional** : $J_{\mathrm{ch}}(V_1, V_2, V_3) = J_{\mathrm{ch}}(-V_1, -V_2, -V_3)$
  ≡ chiral   flipping **all** voltages

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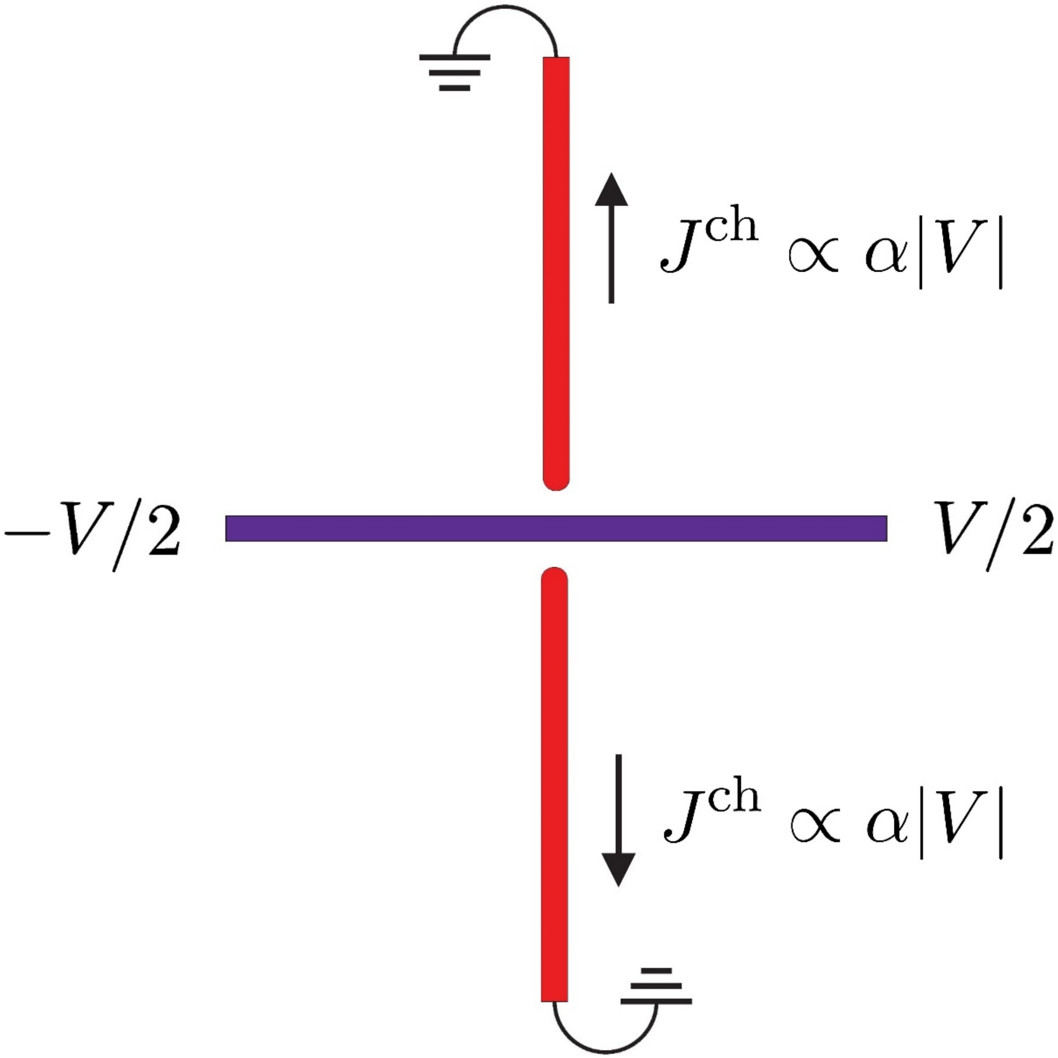
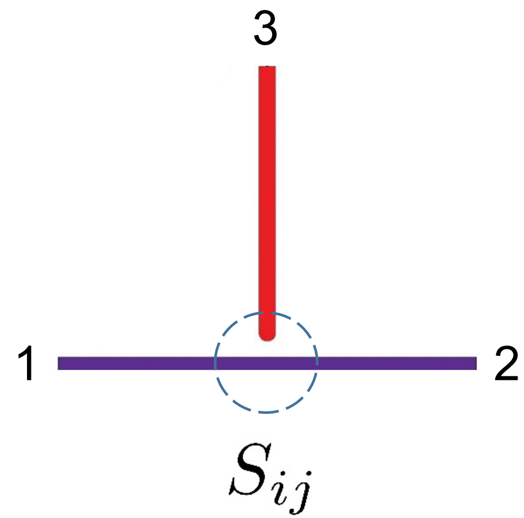
Chiral current



full „inversion symmetry“

current in the main wire ejects (or absorbs, depending on the sign of α) electrons on both „sides“

Hamiltonian



scattering channels
= wire indices
= **contact** indices

Model: **Y junction of ballistic Luttinger liquids** (spinless)

$$H_j = \sum_{\eta_j=\pm} \int_0^L dx\, v_j \left(-i\eta_j \Psi^\dagger_{\eta_j} \partial_x \Psi_{\eta_j} + \pi \alpha_j n_{\eta_j} n_{-\eta_j} \right)$$

wire index

chirality (right- / left-movers)

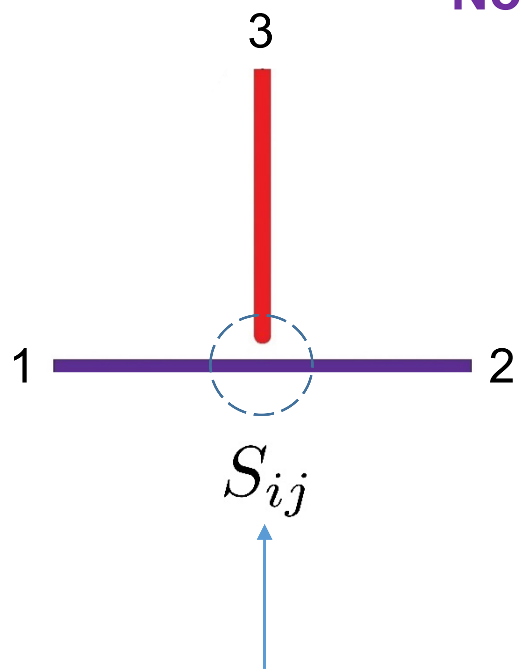
linear dispersion
(incl. forward e-e scattering)

interaction
(short-ranged)

- single-particle scattering matrix $\hat{S}$ at the junction
- boundary condition (on the Keldysh G) at the contact:

emitted electrons → $f_j(\epsilon) = \dfrac{1}{e^{(\epsilon-\mu_j)/T} + 1}$

Noninteracting scattering matrix



scattering channels
= wire indices
= **contact** indices

$$\hat{S} = \begin{pmatrix} r & t & t_3 \\ t & r & t_3 \\ t_3 & t_3 & r_3 \end{pmatrix}$$

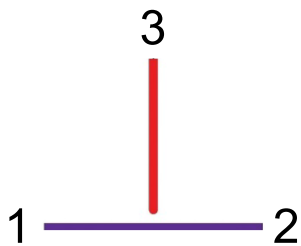
- symmetric (time-reversal symmetric): $S_{ij} = S_{ji}$
- right-left parity symmetric: $S_{13} = S_{23}$

unitarity → 3 independent parameters
up to an unobservable global phase

ballistic wires: one of the phases drops out
from the junction conductances

noninteracting Y junction: characterized by **two** conductances

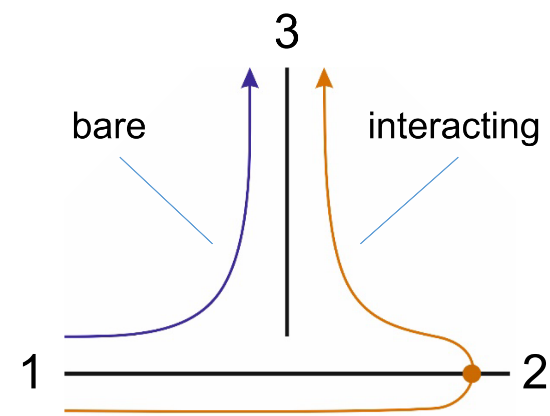
Scattering off nonequilibrium Friedel oscillations



Friedel osc´s in wire *j* ← distribution function from contact *j* :

$$U_H(x) = \alpha_H \, 2\mathrm{Re} \int_0^\infty \frac{d\epsilon}{2\pi} \, e^{-2i\epsilon x/v} \times \begin{cases} r \;\; f_1(\epsilon), & x<0 \\ r^* f_2(\epsilon), & x>0 \end{cases}$$

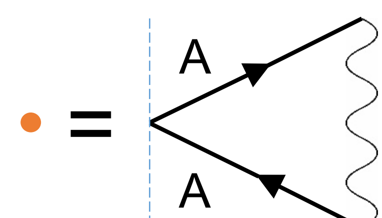
Hartree potential in the main (1+2) wire



interfering waves for scattering 1→3 at $\mathcal{O}(\alpha_H)$,

with interaction in wire 2 :

$$\delta t_3(\epsilon) = \frac{\alpha_H}{2} r^* t t_3 \int_0^\infty d\epsilon' \, f_2(\epsilon') \, \frac{1}{\epsilon - \epsilon' + i0}$$



Mechanism of emergent chirality

e-e interaction →

- **virtual** transitions (screening)
- **real** transitions

$$\delta t_3^a(\epsilon) \propto \int d\epsilon' \, f_2(\epsilon') \, \frac{1}{\epsilon - \epsilon' + i0} \rightarrow \ln \frac{\Lambda}{\max\{T, |\epsilon - \mu_2|\}} - \boxed{i\pi f_2(\epsilon)}$$

„Luttinger liquid renormalization"
(zero-bias anomaly,
orthogonality catastrophe, etc.)

elastic real transition

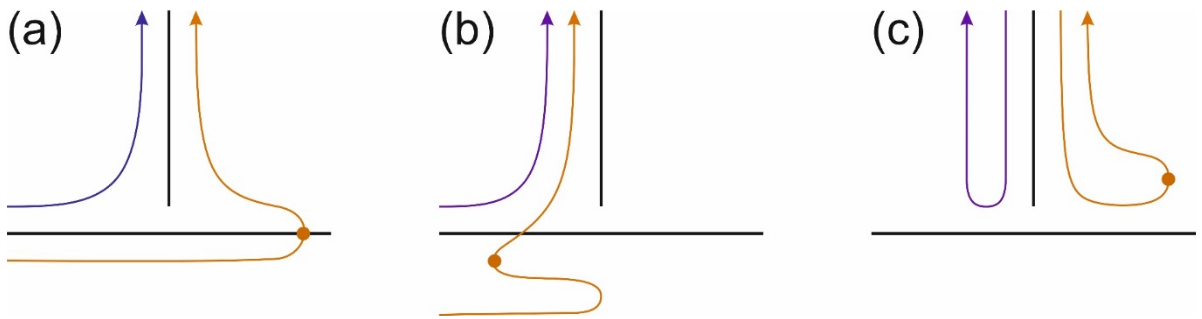
currents **odd** in voltages

currents **even** in voltages

Mechanism of emergent chirality

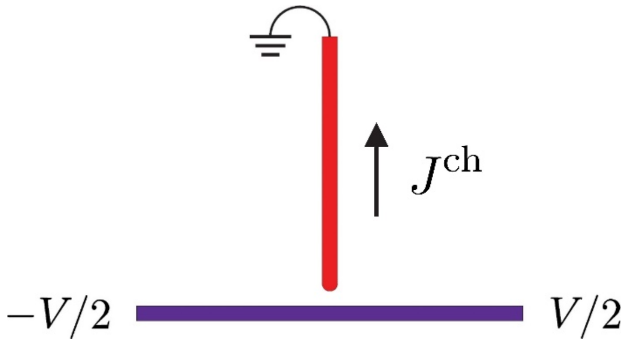
interfering waves for scattering into the tip wire at $\mathcal{O}(\alpha)$,

with interaction in the main wire :



+ mirror (right-left) images

pole terms in (a)+(b)+(c) for the symmetrically biased junction : $J^{\mathrm{ch}} = -\frac{e\alpha|V|}{4}\mathcal{A}$



„chirality coefficient“ $\mathcal{A} = \mathrm{Im}\left\{|t_3|^2 t r^*\right\}$

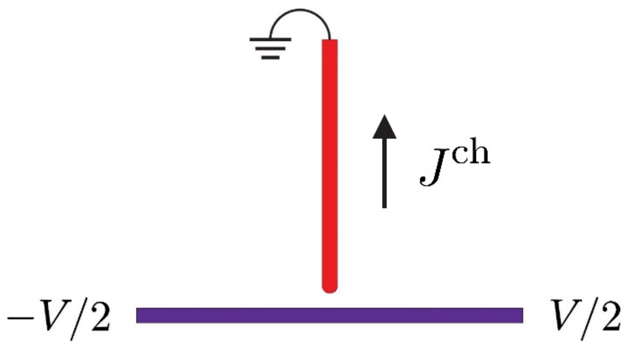
$$T \gg |V| : \quad J^{\mathrm{ch}} \simeq -\frac{1}{8} e\alpha\,\mathcal{A}\frac{V^2}{T}$$

Mechanism of emergent chirality

chirality coefficient $\mathcal{A} = \mathrm{Im}\left\{ |t_3|^2 t r^* \right\} = \frac{1}{4} \sin^2\theta \cos\theta \sin\psi$

generically nonzero ■

for a junction of N (half-)wires with N ≥ 3



Y junction → $\mathcal{A} = 0$:

- decoupled main wire
- perfectly absorbing main wire
- pointlike coupling between the wires

„Fundamental" conductance matrix and emergent chirality

∀ (not necessarily symmetric) voltages : $G_{jk} = e\,\partial J_j / \partial \mu_k$

redundancy : $\sum_j G_{jk} = 0$, $\sum_k G_{jk} = 0$ (Kirchhoff´s laws)

$3 \times 3\ \hat{G}$ → rank 2 → generically **four** independent conductances

→ 2×2 „fundamental" conductance matrix $\hat{\tilde{G}} = \begin{pmatrix} G_a & G_c + G_d \\ -G_c + G_d & G_b \end{pmatrix}$

in the (a, b) basis of independent currents/voltages :

$$J_a = (J_1 - J_2)/2$$
$$J_b = -J_3$$
$$V_a = \mu_1 - \mu_2$$
$$V_b = (\mu_1 + \mu_2)/2 - \mu_3$$

interactions change the structure of $\hat{\tilde{G}}$:

off-diagonal $G_c = G_d = 0$
for P and T symmetric
noninteracting electrons

„Fundamental“ conductance matrix and emergent chirality

$$\hat{\tilde{G}} = \begin{pmatrix} G_a & G_c + G_d \\ -G_c + G_d & G_b \end{pmatrix}$$

off-diagonal elements ← **induced by e-e interaction**
break P $\vee$ T symmetry

$$G_d = \frac{1}{2}(G_{11} - G_{22})$$

breaks P symmetry
(side diversion *aka* „**photogalvanic**“ response)

$$G_c = \frac{1}{2}(G_{12} - G_{21})$$

breaks T symmetry (**Hall** response)

def: emergent chirality ≡ $G_c \vee G_d$ nonzero **and** chiral

$$G_d^{\mathrm{ch}} = \frac{1}{8} e^2 \mathcal{A} \left[(\alpha_1 + \alpha_2) s_{12} - \alpha_3 (s_{23} + s_{31}) \right]$$

$$G_c = -\frac{1}{8} e^2 \mathcal{A} \left[\alpha_1 (s_{12} + s_{31}) + \alpha_2 (s_{12} + s_{23}) + \alpha_3 (s_{23} + s_{31}) \right]$$

$$s_{jk} = \mathrm{sgn}(\mu_j - \mu_k)$$

Higher-order renormalization

chiral conductances: RG near the fixed points
at symmetric bias of the main wire ($V_b = 0$)

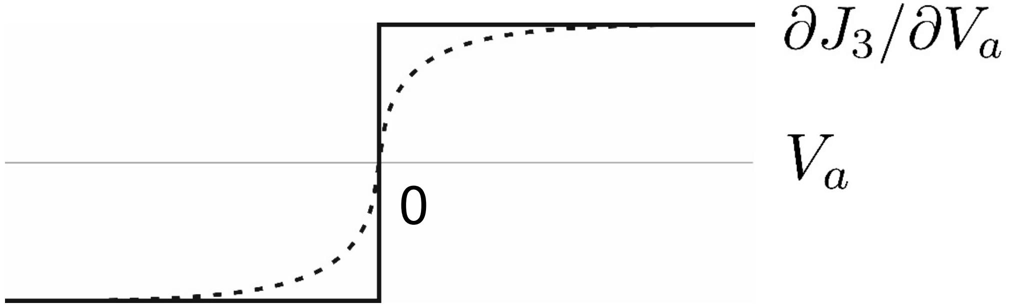
stable fixed point **N** (all wires decoupled): $J_3 \propto |V_a|^{1+2\alpha+\alpha_3}$

unstable fixed point **A** (tunneling tip decoupled): $J_3 \propto |V_a|^{1-\alpha+\alpha_3}$

N : steplike jump (at T=0) of the differential chiral conductance

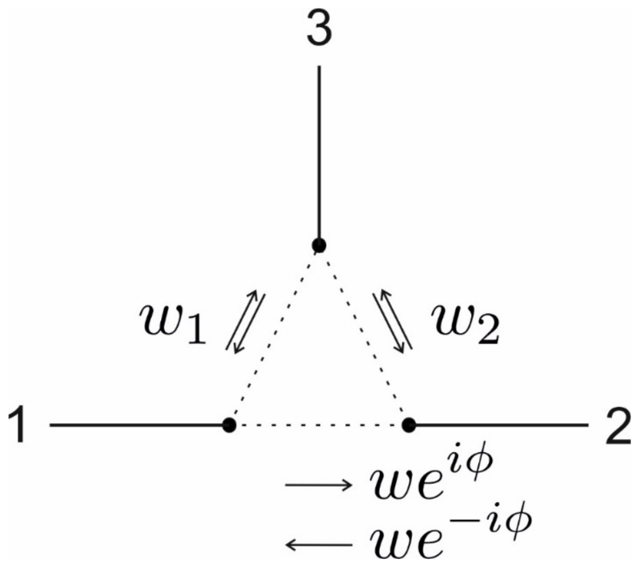
$$e\,\partial J_3/\partial V_a = G_c - G_d$$

„smoothed“ by renormalization for repulsive interaction



Effective chiral model

Y junction → three end points (of wires 1,2,3) connected by hopping



Y junction pierced by a magnetic flux ϕ

(model to study the interplay of flux and e-e interaction, attractive interaction-induced „ideal circulator")

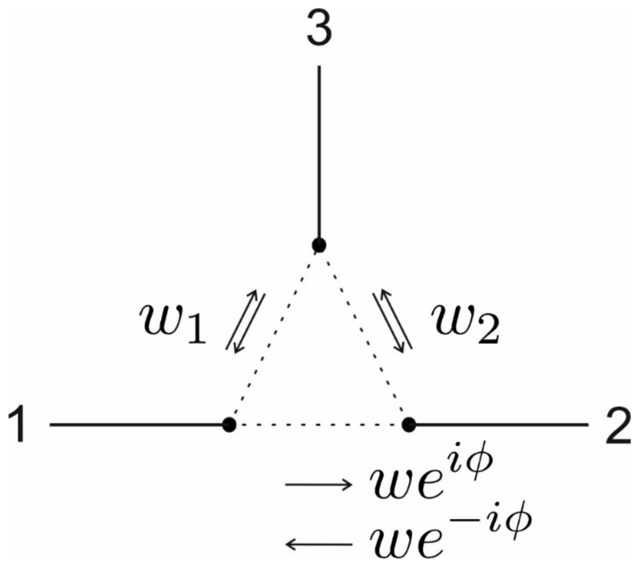
Oshikawa, Chamon, Affleck ´06

consider the **noninteracting** model **with flux** that mimics (same conductance matrix) the **interacting** model **without flux**

$$\hat{S} = (1 - i\hat{W})^{-1}(1 + i\hat{W})$$

matrix of hopping amplitudes $\hat{W} = \begin{pmatrix} 0 & we^{-i\phi} & w_1 \\ we^{i\phi} & 0 & w_2 \\ w_1 & w_2 & 0 \end{pmatrix}$

Effective chiral model



Y junction pierced by a magnetic flux ϕ

let $w_1 = w_2 = w$

hopping amplitudes $\hat{W} = w \begin{pmatrix} 0 & e^{-i\phi} & 1 \\ e^{i\phi} & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

eff. flux ← interaction strength,
changes sign when flipping the sign
of voltage between terminals 1 and 2

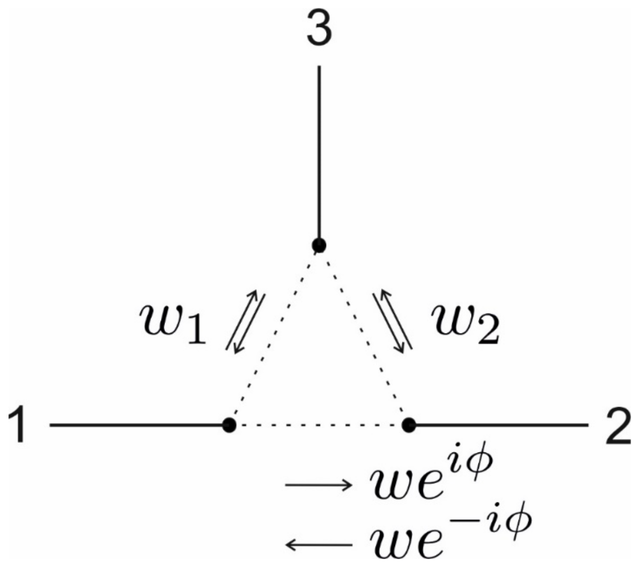
near point **N** : $\phi \rightarrow \frac{\pi}{2}\alpha_3 \mathsf{s}$

$$\mathsf{s} = \mathrm{sgn} \mathrm{V}_a \times s \times \mathrm{sgn}\, w$$

$$s = \mathrm{sgn}(\cos\theta \sin\psi)$$

bare scattering phases

Effective chiral model



Y junction with built-in asymmetry between wires 1 and 2

let $w_1 \neq w_2$

eff. anisotropy ← interaction strength, changes sign when flipping the sign of voltage between terminals 1 and 2

$$\frac{w_1 - w_2}{w_1 + w_2} \to -\frac{s\pi}{4}(\alpha + \alpha_3)|t| \,\mathrm{sgn} V_a$$

s $= \mathrm{sgnV}_a \times s \times \mathrm{sgn}\, w$

$$s = \mathrm{sgn}(\cos\theta \sin\psi)$$

Relation to other works on „rectification“

- currents in a Y junction that are even wrt one voltage, but change sign when all voltages flip sign → not the chiral currents

e.g., Wang, Feldman ´11

$$\mathbf{J}^{\mathrm{ch}}(\{\mathbf{V}\}) = \mathbf{J}^{\mathrm{ch}}(-\{\mathbf{V}\})$$

- Bernoulli effect in fluid-filled pipes

Bernoulli 1738, Euler 1752, Venturi 1797

rehau.com

$$\rho v^2/2 + p = \mathrm{const}$$ (no gravity)

conservation of energy in isentropic incompressible fluid
v hydrodynamic velocity, ρ mass density, p pressure

fluid is „sucked in“ irrespective of the direction of flow in the „main pipe“

requires fast equilibration in the moving frame (hydrodynamics) at each point of the junction

- quantum dot capacitively coupled to the leads + inelastic cotunneling

Jordan, Büttiker ´08

requires charge quantization on the junction

Conclusion:

- *emergent chirality*
  in a system with P and T symmetric Hamiltonian :
  emergence of currents that break P and/or T symmetry and do not change when all voltages flip signs

- *emergent chirality*
  - induced by interaction between particles
  - essentially nonequilibrium phenomenon