Natural Optical activity of metals

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What is the natural optical activity?

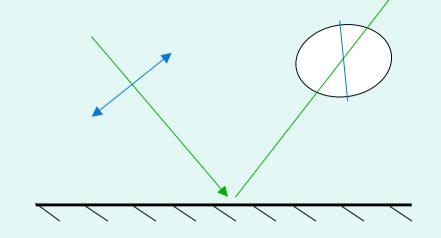
$$\varepsilon_{ij}(\omega, \mathbf{q}) = \varepsilon_{ij}(\omega, 0) + i\gamma_{ijl}q_l$$

$$\gamma_{ijk}(\omega, \mathbf{q}) = e_{ijk}\gamma(\omega, \mathbf{q})$$

Natural optical activity of metals

$$\mathbf{j} = \sigma \mathbf{E} + \lambda \operatorname{rot} \mathbf{E}$$

$$\lambda = \lambda' + i\lambda''$$



Kerr effect

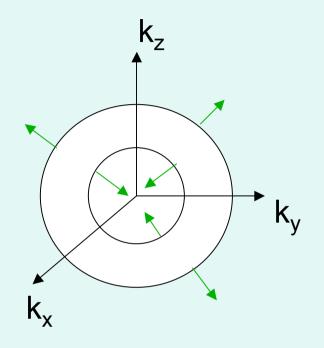
$$N = n + i\kappa$$

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 $n^2 - \kappa^2 = 1 - \frac{4\pi\sigma''}{\omega}$, $2n\kappa = \frac{4\pi\sigma'}{\omega}$

$$\theta = \frac{(1 - n^2 + \kappa^2)\Delta\kappa + 2n\kappa\Delta n}{(1 - n^2 + \kappa^2)^2 + (2n\kappa)^2}$$

$$\Delta n = n_{+} - n_{-} = \frac{4\pi\lambda''}{c} \qquad \Delta \kappa = \kappa_{+} - \kappa_{-} = \frac{4\pi\lambda'}{c}$$

Metals without inversion symmetry



$$\xi_{\alpha\beta}(\mathbf{k}) = \xi_0(\mathbf{k})\delta_{\alpha\beta} + \boldsymbol{\gamma}(\mathbf{k})\boldsymbol{\sigma}_{\alpha\beta}$$

$$G = O$$
, $\operatorname{Li}_2(\operatorname{Pd}_{1-x},\operatorname{Pt}_x)_3\operatorname{B}_3$, $\gamma(\mathbf{k}) = \gamma_0\mathbf{k}$

$$G = C_{4v}$$

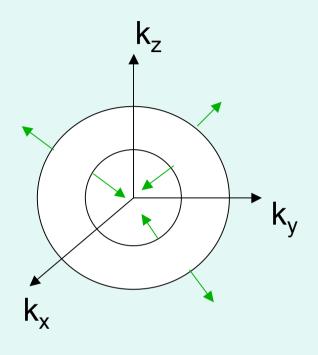
CePt₃Si, CeRhSi₃ and CeIrSi₃

$$\gamma(\mathbf{k}) = \gamma_{\perp}(k_y\hat{x} - k_x\hat{y}) + \gamma_{\parallel}k_xk_yk_z(k_x^2 - k_y^2)\hat{z}$$

$$H_0 = \sum_{\mathbf{k}} \xi_{\alpha\beta}(\mathbf{k}) a_{\mathbf{k}\alpha}^{\dagger} a_{\mathbf{k}\beta} = \sum_{\mathbf{k},\lambda=\pm} \xi_{\lambda}(\mathbf{k}) c_{\mathbf{k}\lambda}^{\dagger} c_{\mathbf{k}\lambda}$$

$$\xi_{\lambda}(\mathbf{k}) = \xi_0(\mathbf{k}) + \lambda |\gamma(\mathbf{k})|$$

Some properties originating from the band splitting



De Haas - van Alphen

$$M_{osc} = \sum_{\lambda} A_{\lambda} \cos \left(\frac{2\pi F_{\lambda}}{H} + \phi_{\lambda} \right)$$

$$F_{-} - F_{+} = \frac{2c}{\hbar e} |\gamma_{\perp}| k_F m_{\perp} \left(1 + \frac{\mu_B^2 H^2}{2\gamma_{\perp}^2 k_F^2} \right)$$

Two band superconductivity

$$G_{\lambda}(\omega_{n}, \mathbf{k}) = -\frac{i\omega_{n} + \xi_{\lambda}}{\omega_{n}^{2} + \xi_{\lambda}^{2} + |\tilde{\Delta}_{\lambda}(\mathbf{k})|^{2}}$$
$$F_{\lambda}(\omega_{n}, \mathbf{k}) = \frac{t_{\lambda}(\mathbf{k})\tilde{\Delta}_{\lambda}(\mathbf{k})}{\omega_{n}^{2} + \xi_{\lambda}^{2} + |\tilde{\Delta}_{\lambda}(\mathbf{k})|^{2}}$$

In the simplest model with BCS pairing interaction $v_g(\mathbf{k}, \mathbf{k}') = -V_g$, the gap functions are the same in both bands: $\tilde{\Delta}_+(\mathbf{k}) = \tilde{\Delta}_-(\mathbf{k}) = \Delta$ and we deal with pure singlet pairing

Large residual spin susceptibility at T=0

Current

$$j_i(\omega_n, \mathbf{q}) =$$

$$-e^{2}Tr\left[T\sum_{m=-\infty}^{\infty}\int\frac{d^{3}k}{(2\pi)^{3}}\{\hat{v}_{i}(\mathbf{k})\hat{G}^{(0)}(K_{+})\hat{v}_{j}(\mathbf{k})\hat{G}^{(0)}(K_{-})+\hat{v}_{i}(\mathbf{k})\hat{F}^{(0)}(K_{+})\hat{v}_{j}^{t}(-\mathbf{k})\hat{F}^{+(0)}(K_{-})\}+\hat{m}_{ij}^{-1}\hat{n}_{e}\right]A_{j}(\omega_{n},\mathbf{q})$$

$$\mathbf{v}_{\alpha\beta}(\mathbf{k}) = \frac{\partial \xi_{\alpha\beta}(\mathbf{k})}{\partial \mathbf{k}} \qquad (m_{ij}^{-1})_{\alpha\beta} = \frac{\partial^2 \xi_{\alpha\beta}(\mathbf{k})}{\partial k_i \partial k_j}$$

$$K_{\pm} = (\Omega_m \pm \omega_n/2, \mathbf{k} \pm \mathbf{q}/2)$$

$$\Omega_m = \pi (2m + 1 - n)T \qquad \omega_n = 2\pi nT$$

Gyrotropy current

$$j_i^g(\omega_n, \mathbf{q}) = -ie_{ijl}e^2\gamma_0^2 T \sum_{m=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \hat{\gamma}_l [G_-(K_+)G_+(K_-) - G_+(K_+)G_-(K_-)]$$

$$-F_{-}(K_{+})F_{+}^{\dagger}(K_{-}) + F_{+}(K_{+})F_{-}^{\dagger}(K_{-})]A_{j}(\omega_{n},\mathbf{q}).$$

Gyrotropy conductivity and the Kerr angle $\Delta \ll \hbar\omega \ll \gamma_0 k_F \ll \varepsilon_F$

$$\Delta \ll \hbar\omega \ll \gamma_0 k_F \ll \varepsilon_F$$

$$\mathbf{j} = \sigma \mathbf{E} + \lambda \operatorname{rot} \mathbf{E}$$

$$\lambda = -i \frac{e^2 \omega}{8\pi^2 \gamma_0 k_F} \left(1 - \frac{1}{3} Y(T) \right) \qquad Y(T) = \int \frac{1}{2T} \frac{1}{\cosh^2(\sqrt{\xi^2 + \Delta^2}/2T)} d\xi$$

$$Y(T) = \int \frac{1}{2T} \frac{1}{\cosh^2(\sqrt{\xi^2 + \Delta^2/2T})} d\xi$$

$$\Delta n = n_+ - n_- = \frac{4\pi\lambda''}{c} = -\frac{\alpha}{2\pi} \frac{\hbar\omega}{\gamma_0 k_F} \left(1 - \frac{1}{3} Y(T) \right) \qquad \Delta \kappa = 0$$

$$\Delta \kappa = 0$$

$$\alpha = \frac{e^2}{\hbar c}$$

$$\theta = \frac{2n\kappa\Delta n}{(1-n^2+\kappa^2)^2+(2n\kappa)^2}$$

Conclusion

- There was found the current response to the electromagnetic field with finite frequency and wave vector in noncentrosymmetric metal in normal and in superconducting states.
- The conductivity tensor contains a gyrotropic part responsible for the natural optical activity.
- As an example the Kerr rotation for the polarized light reflected from the surface of noncentrosymmetric metal with cubic symmetry is calculated. The found value of the Kerr angle is expressed through the fine structure constant and the ratio of the light frequency to the spin-orbit band splitting. The result can be used for the direct experimental determination of the band splitting.