

Dynamics of self-sustained vacuum

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- * quantum vacuum as Lorentz invariant medium
- * dynamics of quantum vacuum and decay of cosmological constant
- * problem of remnant cosmological constant
- * remnant cosmological constant from quantum chromodynamics
- * response of vacuum energy to matter

$$\Lambda_{\rm exp} \sim 2-3 \ \epsilon_{\rm Dark \ Matter} \sim 10^{-123} \Lambda_{\rm bare}$$

$$\Lambda_{\rm bare} \sim \epsilon_{\rm zero \ point}$$



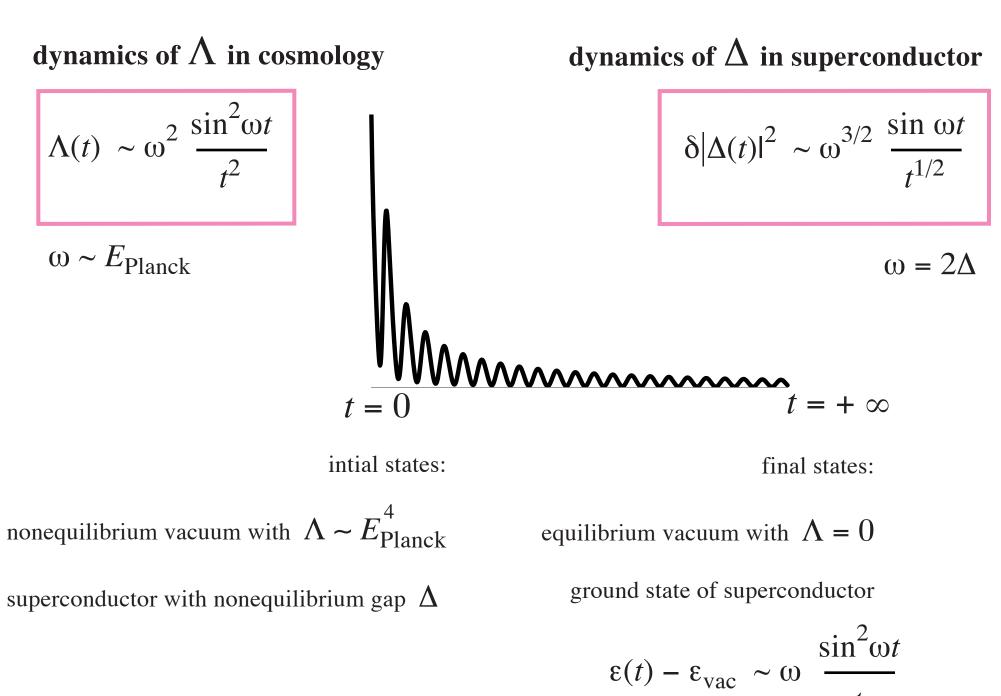
*it is easier to accept that $\Lambda = 0$ than 123 orders smaller

$$\frac{1}{0} = 0$$

*Polyakov conjecture: dynamical screeneng of Λ by infrared fluctuations of metric A.M. Polyakov Phase transitions and the Universe, UFN **136**, 538 (1982) De Sitter space and eternity, Nucl. Phys. **B 797**, 199 (2008)

*Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498 A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974) Barankov & Levitov, ... dynamic relaxation of vacuum to its equilibrium state

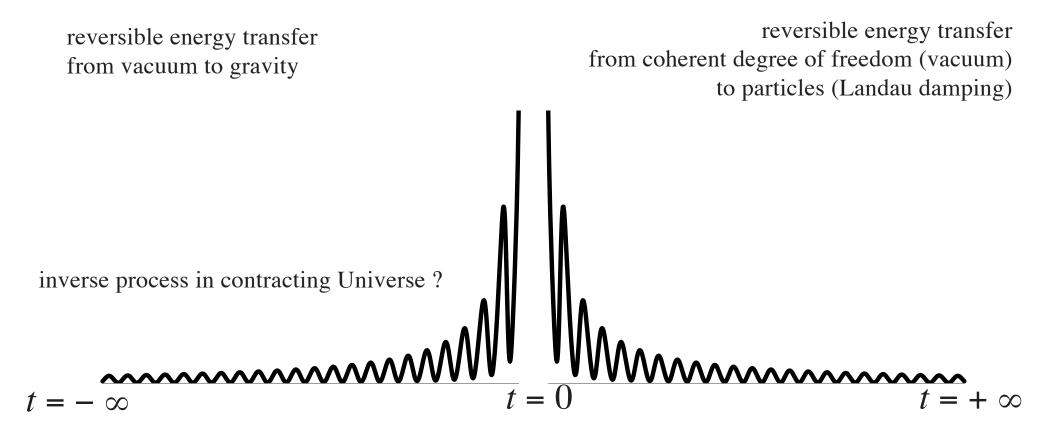


t

reversibility of the process

$$\delta |\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}$$

$$\omega = 2\Delta$$

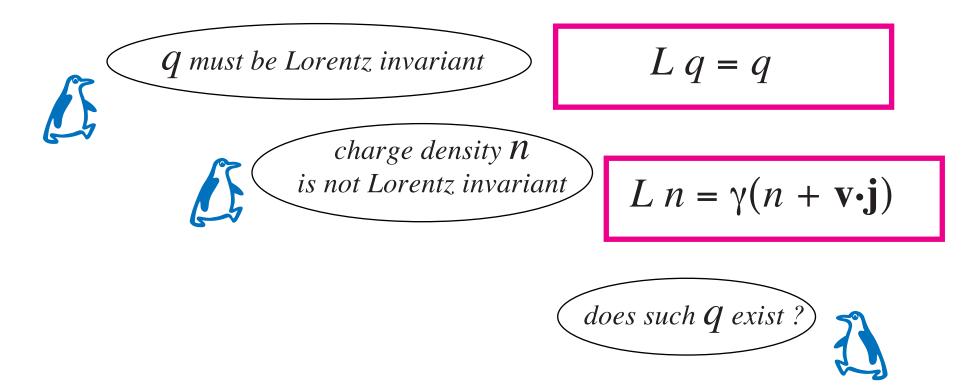


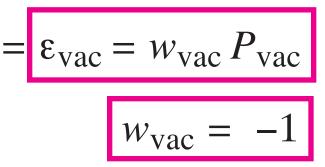
$$\Lambda(t) \sim \omega^2 \; \frac{\sin^2 \omega t}{t^2}$$

 $\omega \sim E_{\text{Planck}}$

how to describe quantum vacuum & vacuum energy Λ ?

- * quantum vacuum has equation of state w=-1
- * quantum vacuum is Lorentz-invariant
- * quantum vacuum is a self-sustained medium, which may exist in the absence of environment
- st for that, vacuum must be described by conserved charge q
 - Q is analog of particle density \mathcal{N} in liquids





relativistic invariant conserved charges q



$$\nabla_{\alpha} q^{\alpha\beta} = 0$$

$$\nabla_{\alpha} q^{\alpha\beta\mu\nu} = 0$$

$$q^{\alpha\beta} = q \ g^{\alpha\beta}$$

$$q^{\alpha\beta\mu\nu} = q e^{\alpha\beta\mu\nu}$$

Duff & van Nieuwenhuizen *Phys. Lett.* **B 94,** 179 (1980)

impossible

$$\nabla_{\alpha} q^{\alpha} = 0 \qquad \qquad q^{\alpha} = ?$$

examples of vacuum variable q

gluon condensates in QCD

do not depend on particular realization of q

thermodynamics in flat space the same as in cond-mat

$$\begin{array}{l} \begin{array}{l} \mbox{conserved}\\ \mbox{charge } Q \end{array} & \mathcal{Q} = \int dV \, q \end{array}$$

$$\begin{array}{l} \mbox{thermodynamic}\\ \mbox{potential} \end{array} & \Omega = E - \mu Q = \int dV \left(\varepsilon \left(q \right) - \mu q \right) \end{array} \begin{array}{l} \mbox{Lagrange multiplier}\\ \mbox{or chemical potential } \mu \end{array}$$

$$\begin{array}{l} \mbox{pressure}\\ \mbox{} E = - \frac{dE}{dV} = -\varepsilon + q \, d\varepsilon/dq \\ \mbox{} E = V \, \varepsilon (Q/V) \end{array}$$

$$d\Omega/dq = 0$$

equilibrium vacuum

$$d\epsilon/dq = \mu$$

equilibrium self-sustained vacuum

$$\frac{d\varepsilon}{dq} = \mu$$

$$\varepsilon - q \frac{d\varepsilon}{dq} = -P = 0$$

vacuum energy & cosmological constant

equilibrium self-sustained vacuum

$$d\varepsilon/dq = \mu$$

$$\varepsilon - q \ d\varepsilon/dq = -P = 0$$

$$equation of state$$

$$P = -\Omega$$

$$Cosmological constant$$

$$A = \Omega = \varepsilon - \mu \ q$$

$$Cosmological constant$$

$$A = \varepsilon - \mu \ q = 0$$

$$Cosmological constant$$

dynamics of q in flat space whatever is the origin of q the motion equation for q is the same

action $S = \int dV \, dt \, \varepsilon \, (q)$ motion equation $\nabla_{\kappa} \left(d\varepsilon/dq \right) = 0$ solution $d\varepsilon/dq = \mu$

integration constant μ in dynamics becomes chemical potential in thermodynamics

4-form field $F_{\kappa\lambda\mu\nu}$ as an example of conserved charge q in relativistic vacuum

$$q^{2} = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}$$

$$F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]}$$

$$F^{\kappa\lambda\mu\nu} = q e^{\kappa\lambda\mu\nu}$$

$$Maxwell equation$$

$$\nabla_{\kappa} (F^{\kappa\lambda\mu\nu} q^{-1}d\epsilon/dq) = 0$$

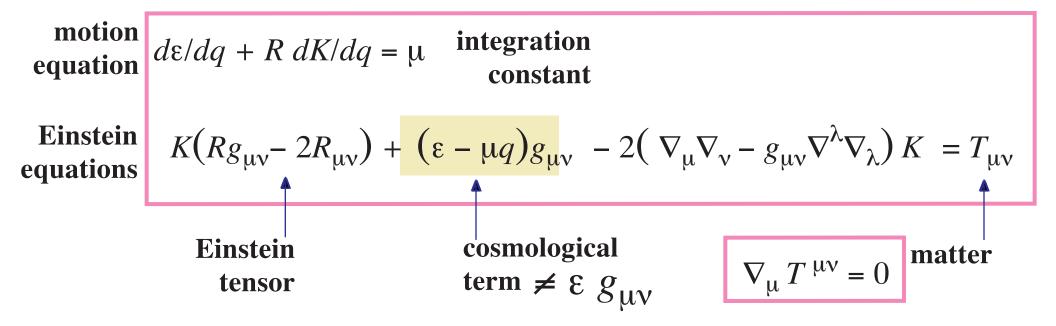
$$\nabla_{\kappa} (d\epsilon/dq) = 0$$

general dynamics of q in curved space

action

$$S = \int d^4 x \, (-g)^{1/2} \left[\epsilon (q) + K(q)R \right] + S_{\text{matter}}$$

gravitational coupling K(q) is determined by vacuum and thus depends on vacuum variable q



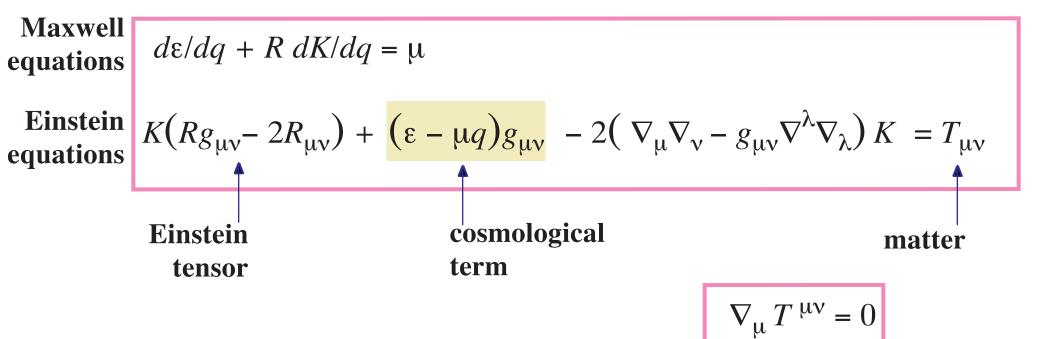
case of *K*=*const* **restores original Einstein equations**

$$K = \frac{1}{16\pi G} \qquad G - \text{Newton constant}$$

motion
equation
original
Einstein
equations
$$\frac{1}{16\pi G} (Rg_{\mu\nu} - 2R_{\mu\nu}) + \Lambda g_{\mu\nu} = T_{\mu\nu} \qquad \Lambda = \varepsilon - \mu q$$

 Λ - cosmological constant

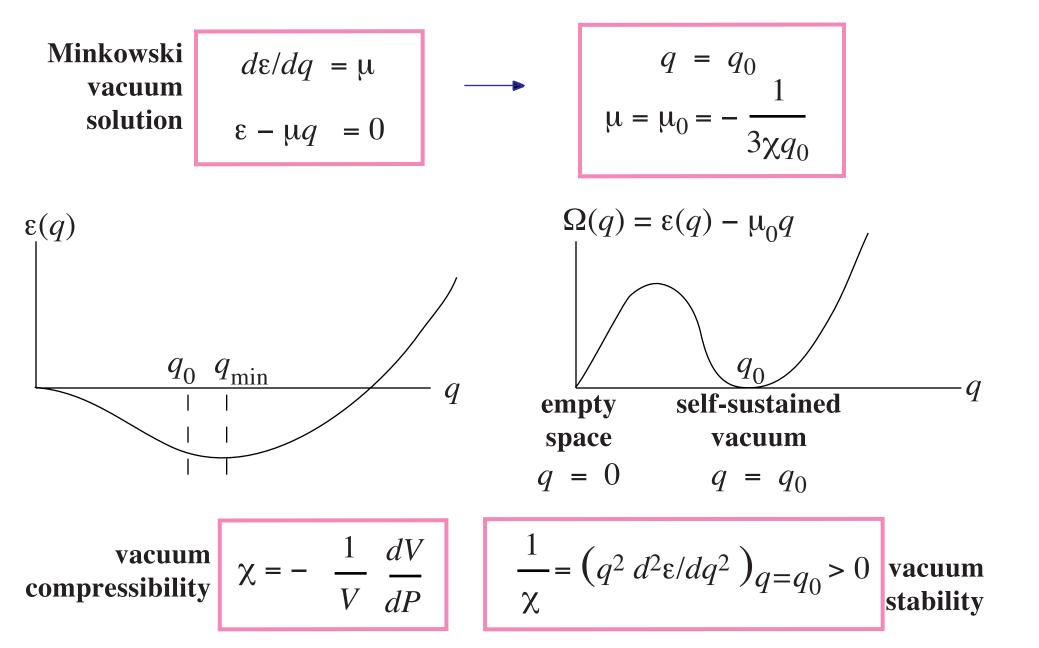
Minkowski solution



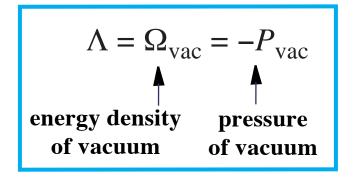
Minkowski
vacuum
solution
$$R = 0$$
 $d\varepsilon/dq = \mu$ vacuum energy in action: $\varepsilon (q) \sim E_{Planck}^4$ $\Lambda = \varepsilon(q) - \mu q = 0$ thermodynamic vacuum energy: $\varepsilon - \mu q = 0$

Model vacuum energy

$$\varepsilon(q) = \frac{1}{2\chi} \left(-\frac{q^2}{q_0^2} + \frac{q^4}{3q_0^4} \right)$$



Minkowski vacuum (q-independent properties)



$$P_{\text{vac}} = - \frac{dE}{dV} = - \Omega_{\text{vac}}$$

 $\chi_{\text{vac}} = -(1/V) \frac{dV}{dP}$
compressibility of vacuum

$$<(\Delta P_{\rm vac})^2 > = T/(V\chi_{\rm vac})$$

 $<(\Delta\Lambda)^2 > = <(\Delta P)^2 >$
pressure fluctuations

natural value of Λ determined by macroscopic physics

$$\Lambda = 0$$

natural value of χ_{vac} determined by microscopic physics

 $\chi_{\rm vac} \sim E^{-4}$

Planck

 $V > T_{\rm CMB} / (\Lambda^2 \chi_{\rm vac})$

 $V > 10^{28} V_{\rm hor}$





dynamics of q in curved space: relaxation of Λ

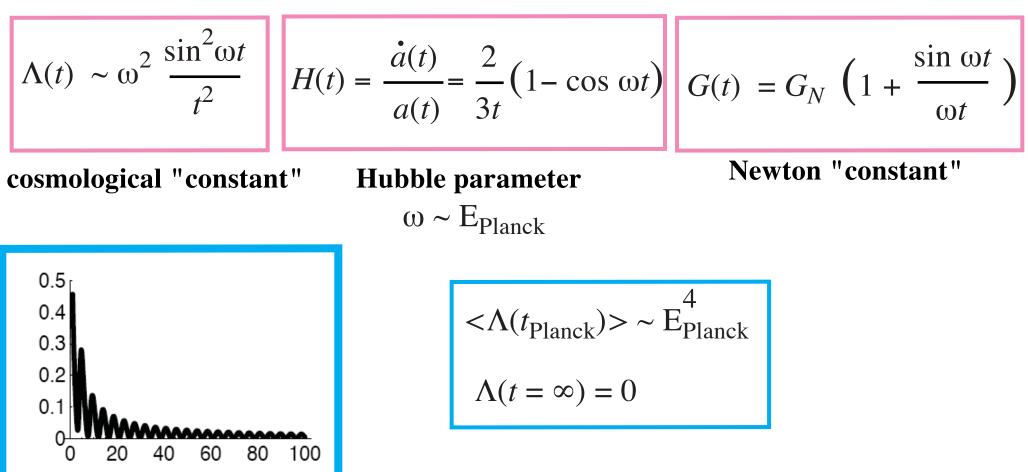
$$\begin{array}{l} \textbf{motion} \\ \textbf{equation} \\ \textbf{equation} \\ \end{array} \quad d\varepsilon/dq + R \ dK/dq = \mu \\ \\ \textbf{Einstein} \\ \textbf{equations} \\ \end{array} \quad K(Rg_{\mu\nu} - 2R_{\mu\nu}) + g_{\mu\nu} \Lambda(q) - 2(\nabla_{\mu}\nabla_{\nu} - g_{\mu\nu}\nabla^{\lambda}\nabla_{\lambda}) K = T_{\mu\nu}^{\text{matter}} \\ \\ \Lambda(q) = \varepsilon(q) - \mu_0 q \end{array}$$

dynamic solution: approach to equilibrium vacuum

$$q(t) - q_0 \sim q_0 \frac{\sin \omega t}{t} \qquad \Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2} \qquad H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t} \left(1 - \cos \omega t\right)$$
$$\omega \sim E_{\text{Planck}}$$

similar to scalar field with mass $M \sim E_{\text{Planck}}$ A.A. Starobinsky, Phys. Lett. **B 91**, 99 (1980)

Relaxation of Λ (generic q-independent properties)

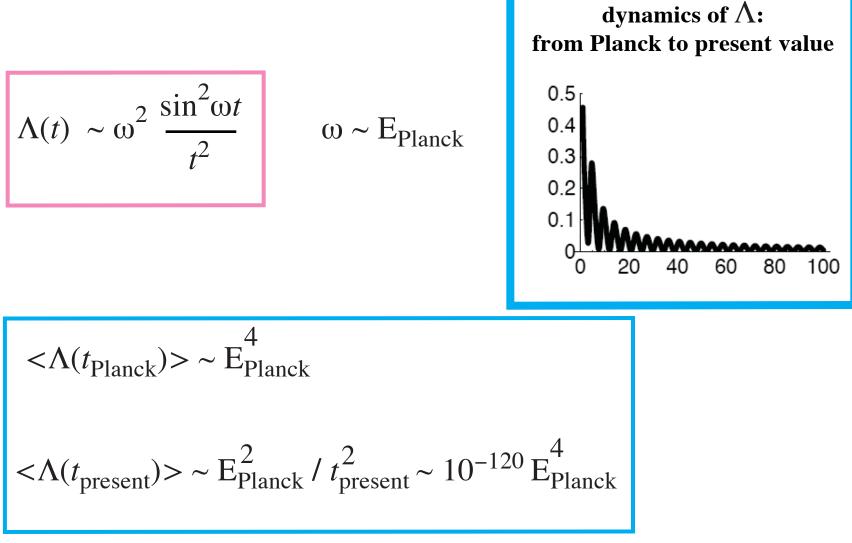


natural solution of the main cosmological problem ?

A relaxes from natural Planck scale value to natural zero value ∽



present value of Λ



coincides with present value of dark energy *something to do with coincidence problem ?*



cold matter simulation

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2} \quad H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t} \left(1 - \cos \omega t \right) \quad = \frac{2}{3t} \quad \sim t^{2/3}$$

$$<\Lambda(t)> \sim \frac{E^2_{\text{Planck}}}{t^2} \sim E^4_{\text{Planck}} \frac{a^3(t_{\text{Planck}})}{a^3(t)}$$

relaxation of vacuum energy mimics expansion of cold dark matter

$$\rho(t) a^3(t) = \text{const}$$

though equation of state corresponds to Λ

$$\Lambda = \Omega = -P$$

$$\Omega = \varepsilon(q) - \mu q$$

another example of vacuum variable: from 4-vector

version of Ted Jacobson's Einstein-Aether theory

energy density $\epsilon_{vac} \left(u^{\mu}_{\ v} \right)$ of vacuum is function of

$$\boldsymbol{\mu}^{\mu}_{\nu} = \boldsymbol{\nabla}_{\nu} \boldsymbol{\mu}^{\mu}$$

equilibrium vacuum is obtained from equation

$$\delta \varepsilon_{\rm vac} / \delta u^{\mu} = \nabla_{\nu} (\delta \varepsilon_{\rm vac} / \delta u^{\mu}_{\nu}) = 0$$

equilibrium solution:

$$u_{\mu\nu} = qg_{\mu\nu}$$

q = constvacuum variable

macroscopic thermodynamic vacuum energy: from energy momentum tensor

$$T_{\mu\nu} = \delta S / \delta g^{\mu\nu} = (\varepsilon_{\text{vac}}(q) - q \, \mathrm{d}\varepsilon_{\text{vac}}/\mathrm{d}q) g_{\mu\nu}$$

It is $T_{\mu\nu}$ which is gravitating, thus cosmological constant is

$$\Lambda = \Omega(q) = \varepsilon_{\rm vac} \left(q\right) - q \, \mathrm{d}\varepsilon_{\rm vac}/\mathrm{d}q$$

microscopic vacuum energy

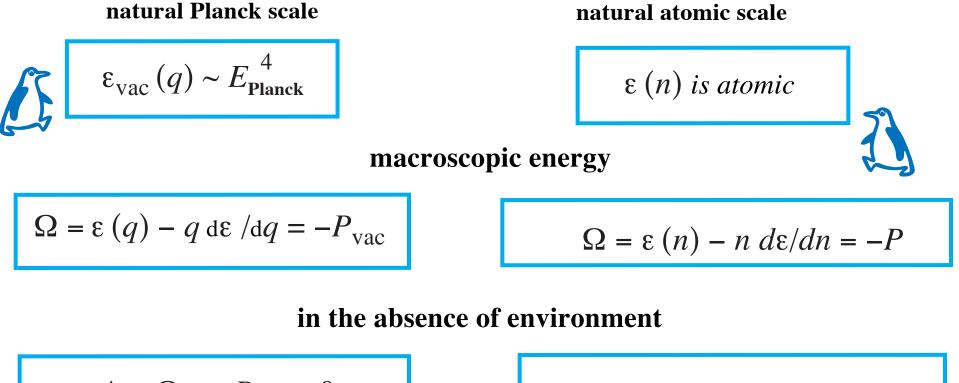
$$\varepsilon_{\rm vac}\left(q\right) \sim E_{\rm Planck}^4$$

cosmological constant

$$\Lambda = \Omega(q_0) = 0$$



relativistic quantum vacuum vs cond-mat microscopic energy



 $\Lambda = \Omega = -P_{\rm vac} = 0$

 $\Omega=-P=0$

two microscopic quantities cancel each other without fine tuning

self tuning due to **thermodynamics**



Why is the present Λ nonzero ?

remnant cosmological constant from infrared QCD (highly speculative)

Klinkhamer -Volovik Gluonic vacuum, *q*-theory, and the cosmological constant Phys. Rev. **D 79**, 063527 (2009)

Klinkhamer Gluon condensate, modified gravity, and the accelerating Universe arXiv:0904.3276

de Sitter expansion and nonzero Λ are induced by QCD anomaly

$$\Lambda \sim G^2 \lambda_{\rm QCD}^6$$

 $\lambda_{QCD} \sim 100 \text{ MeV}$ is QCD energy scale

close to present cosmological constant

supports suggestion by Zeldovich

 $\Lambda \sim G^2 m_{\rm pr}^6$ JETP Lett. 6, 316 (1967)

Lifshitz point

$$\omega^{2} = k^{2} + m(k) \qquad m(k) = \frac{k^{z}}{\lambda^{z-1}}$$

Lifshitz point in QCD

effective gluon mass diverges in k -> 0 limit

$$z = -1$$

 $m(k) \sim \frac{\lambda_{QCD}^2}{k}$

Gribov picture of confinement

$$z = -2$$

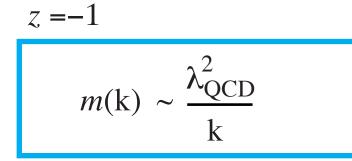
$$m(k) \sim \frac{\lambda_{QCD}^3}{k^2}$$

alternative picture of confinement Cabo et al. arXiv:0906.0494

remnant cosmological constant from QCD

estimation of vacuum energy in expanding Universe

$$\mathbf{E} = \frac{1}{2} \sum_{k} \left[\omega(\mathbf{k}, \mathbf{H}) - \omega(\mathbf{k}, 0) \right] \sim \frac{1}{2} \sum_{k} \left[m(\mathbf{k}, \mathbf{H}) - m(\mathbf{k}, 0) \right]$$



Gribov scenario

$$m(\mathbf{k}, \mathbf{H}) \sim \frac{\lambda_{\text{QCD}}^2}{(\mathbf{k}^2 + \mathbf{H}^2)^{1/2}}$$
$$\mathbf{E} \sim \lambda_{\text{QCD}}^2 \mathbf{H}^2$$

negligible correction to Einstein action

$$z = -2$$

 $m(k) \sim \frac{\lambda_{QCD}^3}{k^2}$

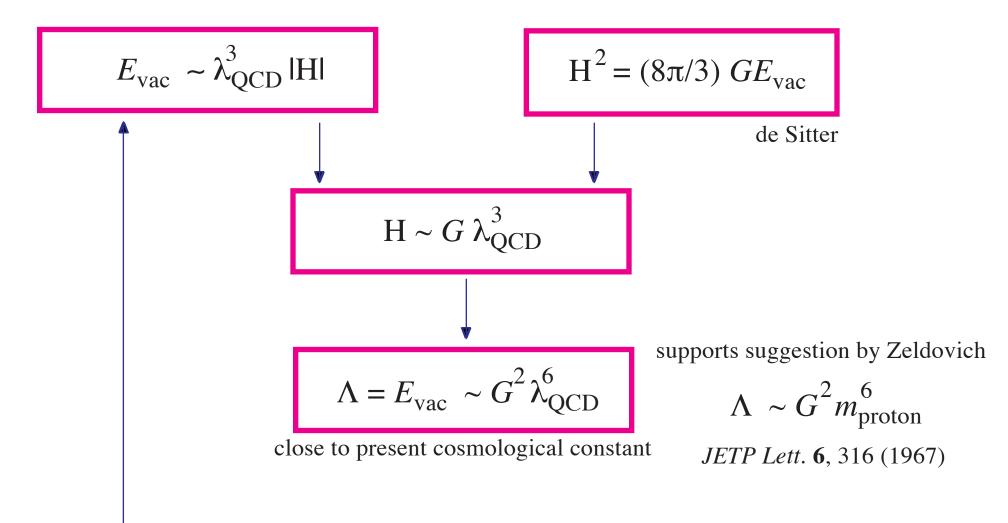
Cabo et al. arXiv:0906.0494

$$m(k, H) \sim \frac{\lambda_{QCD}^3}{k^2 + H^2}$$

$$E \sim \lambda_{QCD}^3 |H|$$

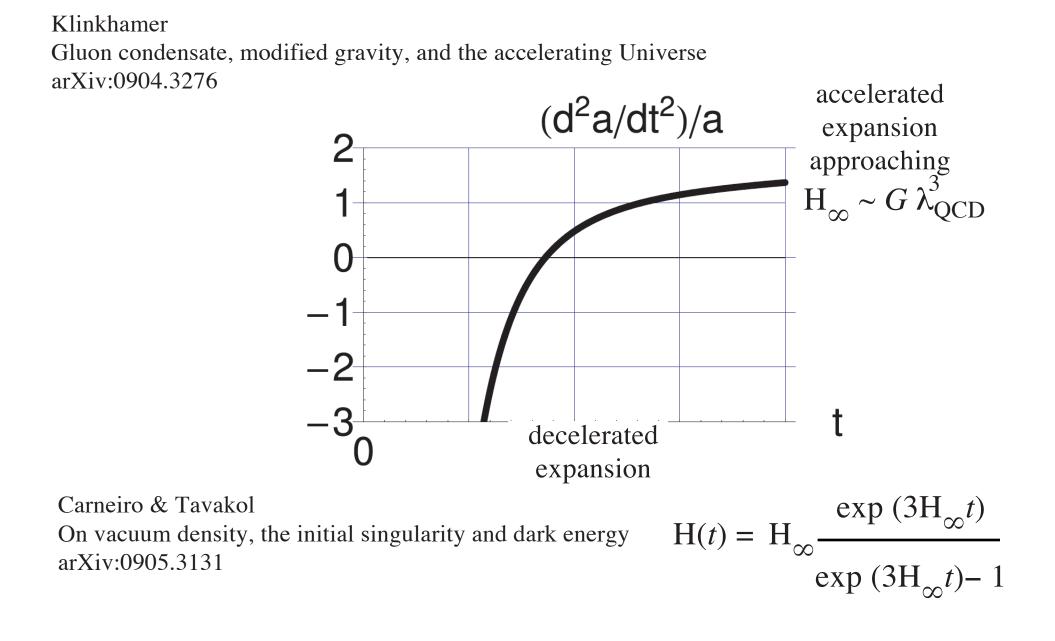
dominates at small Hubble parameter H

asymptotic de Sitter state due to infrared QCD



J. Bjorken, The classification of universes, astro-ph/0404233
R. Schutzhold, *PRL* 89, 081302 (2002)
Klinkhamer & Volovik, *Phys. Rev.* D 79, 063527 (2009)
Urban & Zhitnitsky,
The cosmological constant from the Veneziano ghost which solves the U(1) problem in QCD, 0906.2162

asymptotic de Sitter state from q-theory with QCD



q-theory with QCD and f(R) theory

at small curvature R approaches particular f(R) theory with $f(R) = R + |R|^{1/2}$

$$\lambda_{\text{QCD}}^{3}$$
 |H| -> λ_{QCD}^{3} |R|^{1/2}

$$q = \langle \mathbf{G}_{\alpha\beta} \mathbf{G}^{\alpha\beta} \rangle \sim \lambda_{\rm QCD}^4$$

action

$$S = \int d^{4}x \, (-g)^{1/2} \left[\epsilon (q) - \mu q + KR + q^{3/4} |R|^{1/2} \right] + S_{\text{matter}}$$
$$\epsilon (q) - \mu q = q \ln \frac{q}{q_{0}} \qquad q_{0} = \lambda_{\text{QCD}}^{4}$$

here q_0 is equilbrium value of gluon condensate in equilibrium vacuum with $\Omega = \Lambda = 0$

however instead of equilibrium vacuum the Universe approaches de Sitter state with

$$\mathbf{H} \sim G \; q_0^{3/4}$$

$$\Lambda \sim G^2 q_0^{3/2}$$

two regimes of vacuum dynamics

decay with oscillations

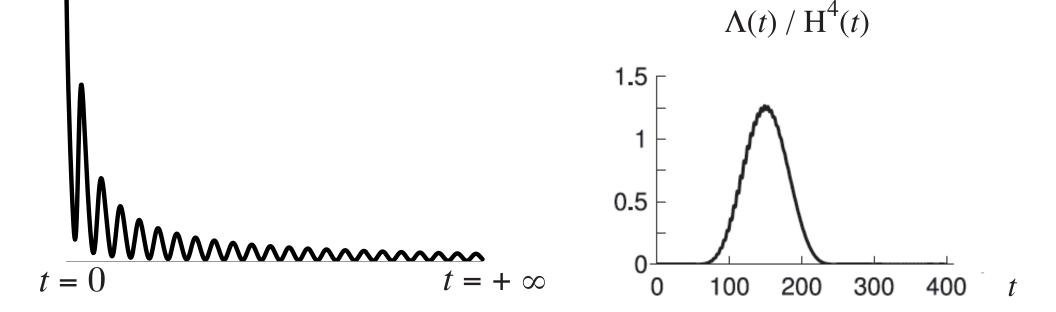
$$\Lambda(t) \sim \omega^2 \; \frac{\sin^2 \omega t}{t^2}$$

 $\omega \sim E_{\rm Planck}$

response to perturbation of matter

 $\Lambda(t) \sim (w^2(t) - 1/3)^2 H^4(t) + \text{small oscillations}$

w(t) matter equation of state



Klinkhamer & Volovik, arXiv:0905.1919

remnant cosmological constant from electroweak crossover

response to perturbation of matter without dissipation

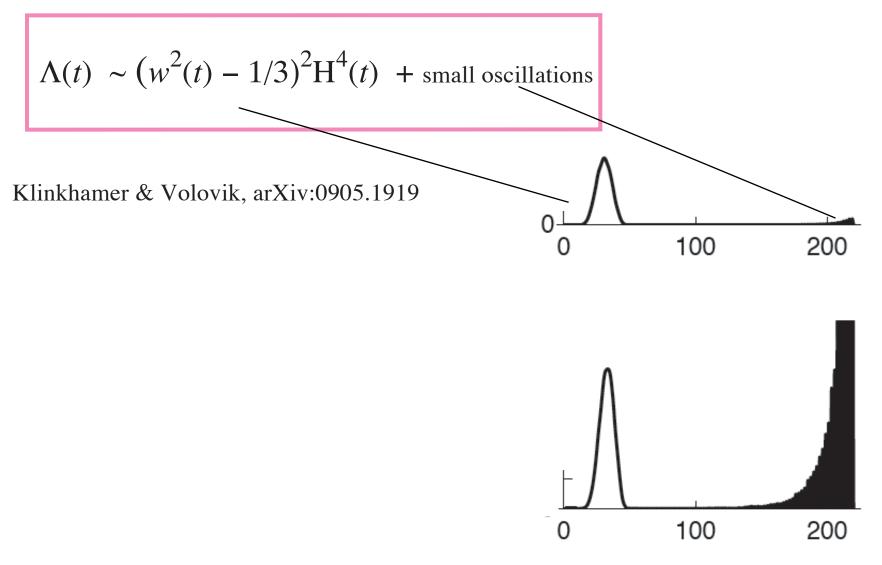
$$\Lambda_0(t) \sim (w^2(t) - 1/3)^2 \text{H}^4(t)$$

 $w(t) - 1/3 \neq 0$ during electroweak crossover, when $H(t_{ew}) \sim E_{ew}^2 / E_{Planck}$

Klinkhamer & Volovik, arXiv:0905.1919

vacuum instability in contracting universe

response to perturbation of matter



catastrophic growth of oscillating vacuum energy & collapse

conclusion

properties of relativistic quantum vacuum as a self-sustained system

st quantum vacuum is characterized by conserved charge q

 ${\it q}\,$ has Planck scale value in equilibrium

* vacuum energy has Planck scale value in equilibrium but this energy is not gravitating

* gravitating energy which enters Einstein equations is thermodynamic vacuum energy

$$\Omega(q) = \varepsilon - q \, d\varepsilon/dq$$

* thermodynamic energy of equilibrium vacuum

$$\Omega(q_0) = \varepsilon(q_0) - q_0 \, d\varepsilon/dq_0 = 0$$

$$\epsilon(q) \sim {\rm E}_{\rm Planck}^4$$

$$T_{\mu\nu} = \Lambda g_{\mu\nu} \neq \varepsilon(q) g_{\mu\nu}$$

$$T_{\mu\nu} = \Lambda g_{\mu\nu} = \Omega(q) g_{\mu\nu}$$

* non-equilibrium vacuum with large initial Λ relaxes with fast oscillations

* small remnant cosmological constant may come from QCD