RUSSIAN ACADEMY OF SCIENCES
L.D Landau
INSTITUTE FOR
THEORETICAL
PHYSICS

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# Критические свойства роста двумерных фрактальных структур

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# Рост двумерных агрегатов (геометрическое “критическое” явление)

Кристаллы льда

$D_3$=2.2-2.6

$D_2$=1.4-1.8

# Рост двумерных агрегатов (геометрическое “критическое” явление)

## Дендриты

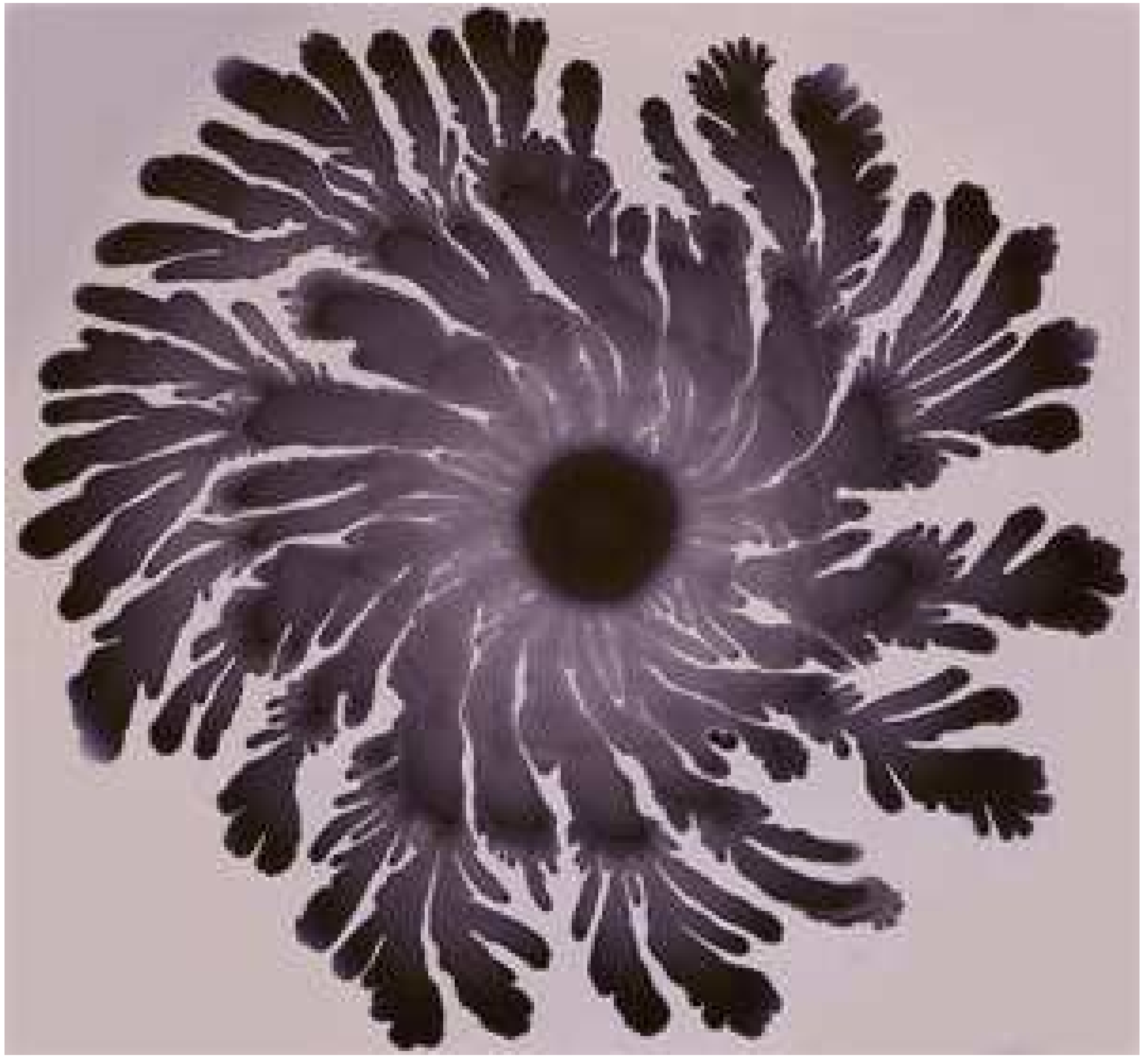
D=1.5-1.8

Самородная медь

Гётит в агате

Окислы марганца в халцедоне

# Рост двумерных агрегатов (геометрическое “критическое” явление)



D=1.7

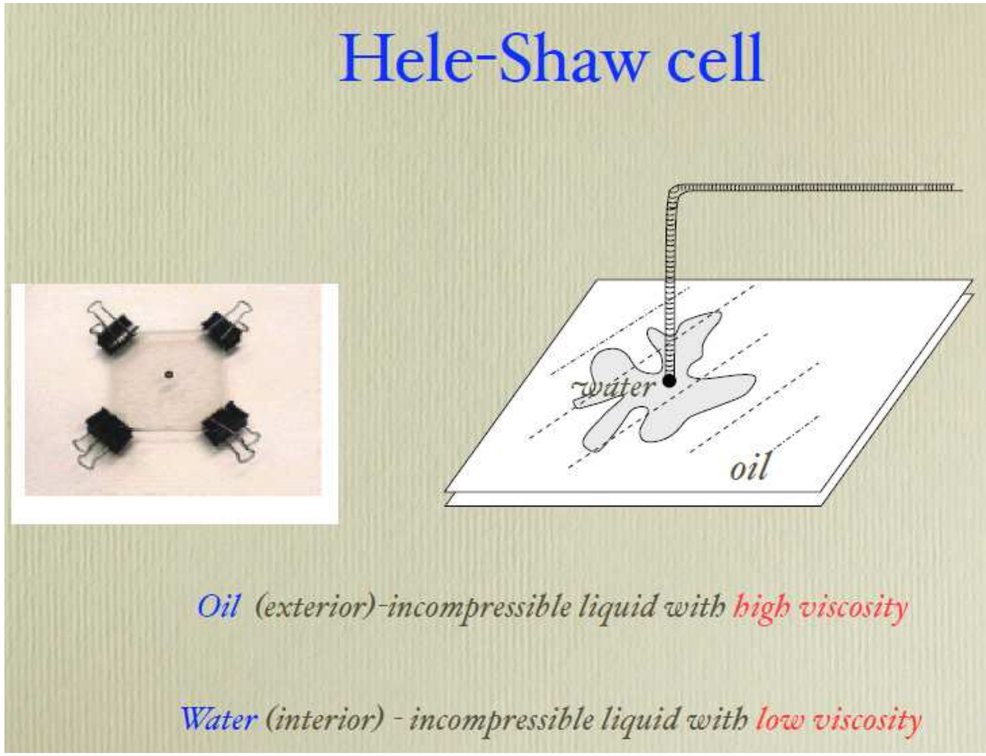
Колония бактерий Bacillus subtilis
фото с сайта www.igmors.u-psud.fr

# Рост двумерных агрегатов (геометрическое “критическое” явление)

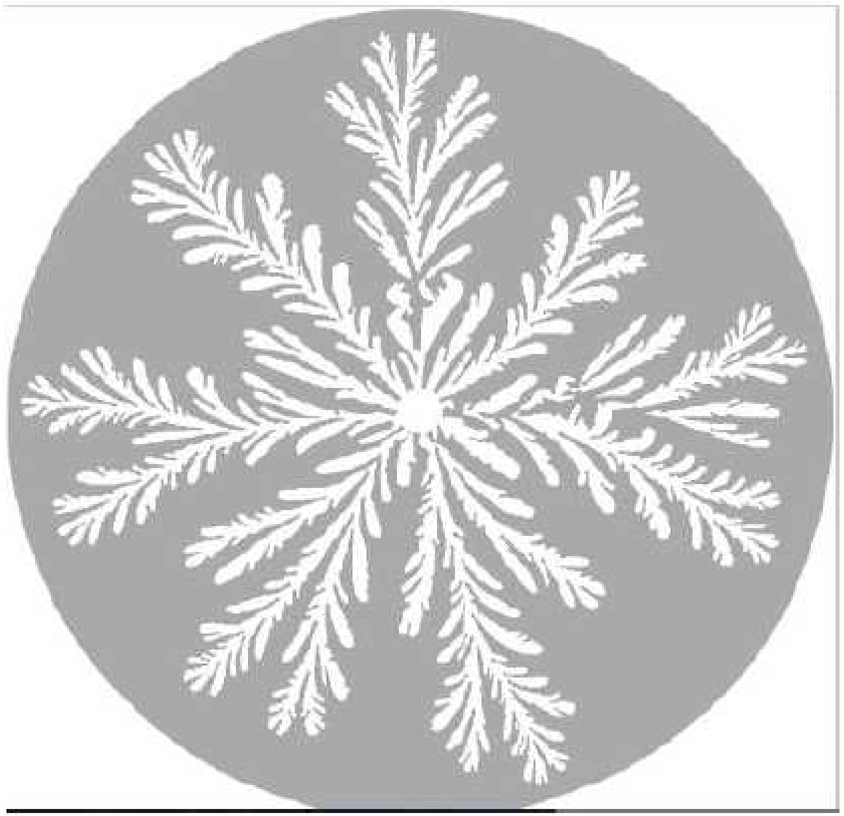

D=1.6-1.9

Нано-размерные кластеры загрязнения на чистой поверхности кристалла

# Рост двумерных агрегатов (геометрическое “критическое” явление)




after Sharon, Moore, McCormick, and Swinney, University of Texas at Austin



D=1.7

Viscosous fingering – interface of two liquids between two plates (Hele-Shaw)

# Рост двумерных агрегатов (геометрическое “критическое” явление)

Модели

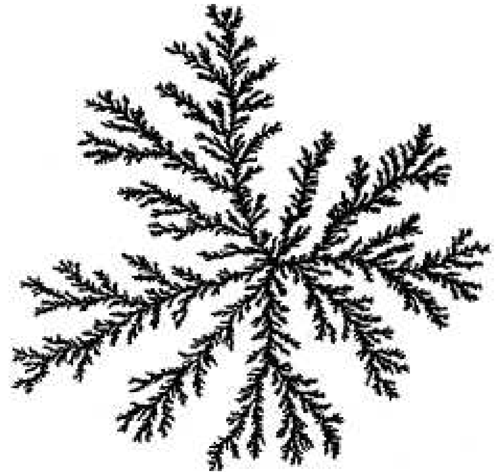
- Diffusion limited aggregation – DLA
- Dielectric breakdown model - DBM
- Laplacian growth
- Iterative conformal maps (Hastings-Levitov dynamics)

# Рост двумерных агрегатов (геометрическое “критическое” явление)

Diffusion limited aggregation – DLA
Witten and Sander, PRL, 1981

1. Place seed at origin (0,0), N=1
2. Particle starts at radius of birth Rbirth
3. Diffusion in space
4. If touch, it sticks, N=N+1
5. If particles goes out of the radius of death $R_{death}$ it is killed
6. New iteration – from step 2.

D=1.66



D=1.71

# Рост двумерных агрегатов (геометрическое “критическое” явление)

Dielectric Breadkdown Model – DBM

Niemeyer, Pietronero and  Wiesmann, PRL, 1984

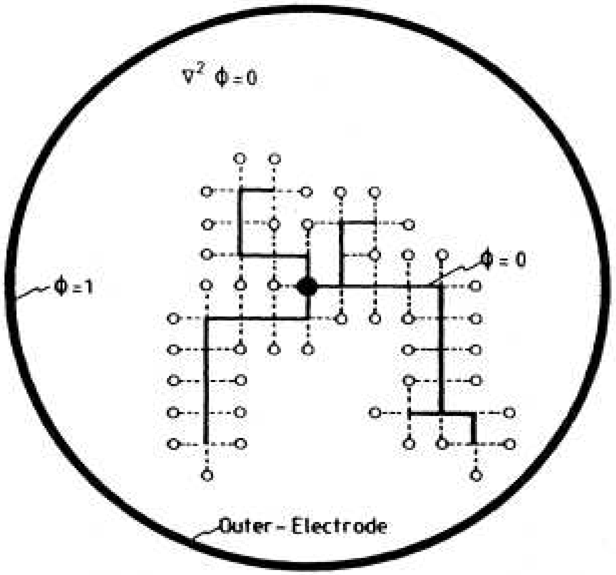



FIG. 2. Iterative mathematical nature of the DBM. The growth process corresponds to an irreversible dynamical process with long-range correlations in both space and time. No statistical weight can be assigned to a given configuration without taking into account its entire history. In the circle is a schematic of the DBM. The central point represents one of the electrodes ($\phi=0$), while the other electrode is given by a circle at large distance ($\phi=1$). The discharge pattern (black dots and bonds) is equipotential with the central electrode ($\phi=0$). The dashed bonds represent the candidates for the next growth processes, and their relative growth probabilities are proportional to the potential gradient (local field).

# Рост двумерных агрегатов (геометрическое “критическое” явление)

Iterative conformal maps

Hastings and Levitov, 1998

D=1.65 - 1.72

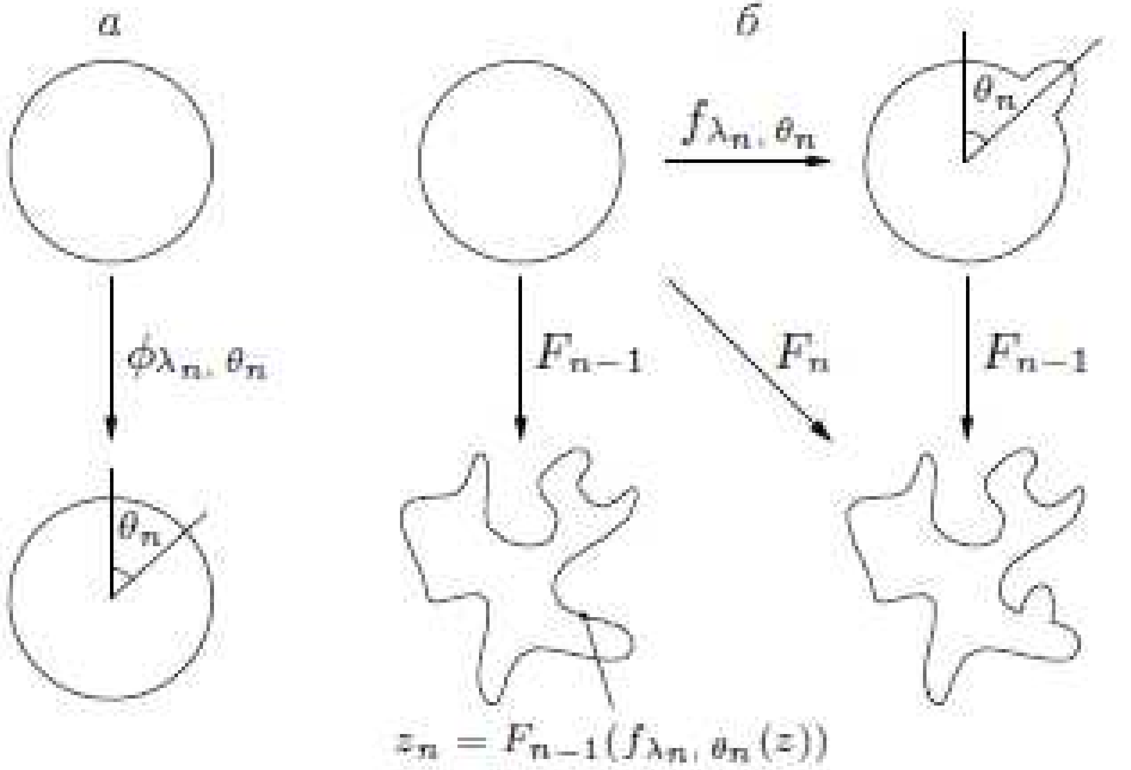



**Рис. 1.** Действие отображений $\phi_{\lambda_n,\theta_n}$, $f_{\lambda_n,\theta_n}$, $F_{n-1}$, $F_n$

# Рост двумерных агрегатов (геометрическое “критическое” явление)

Hele-Shaw dynamics

Wiegmann, et al., 2008

D=?

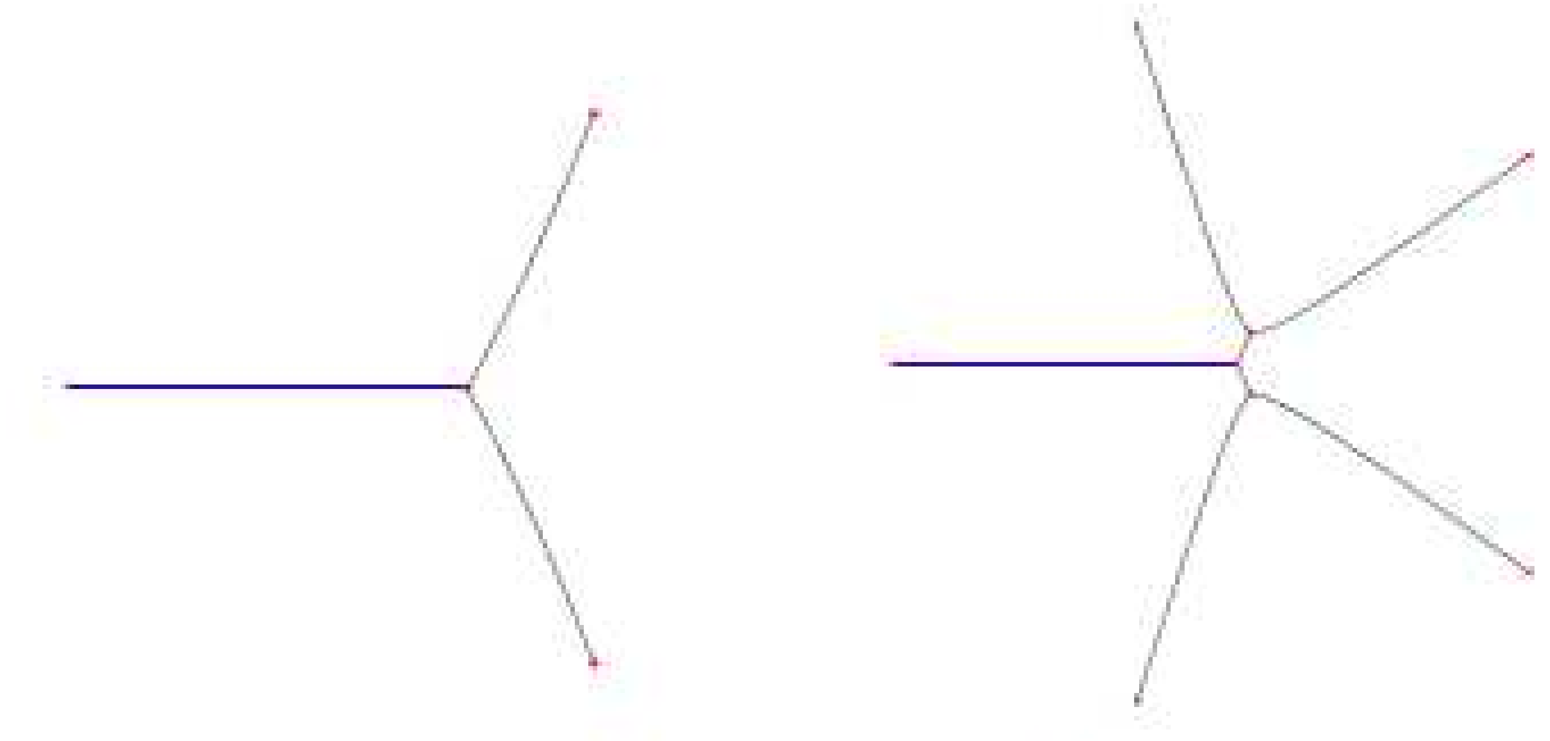


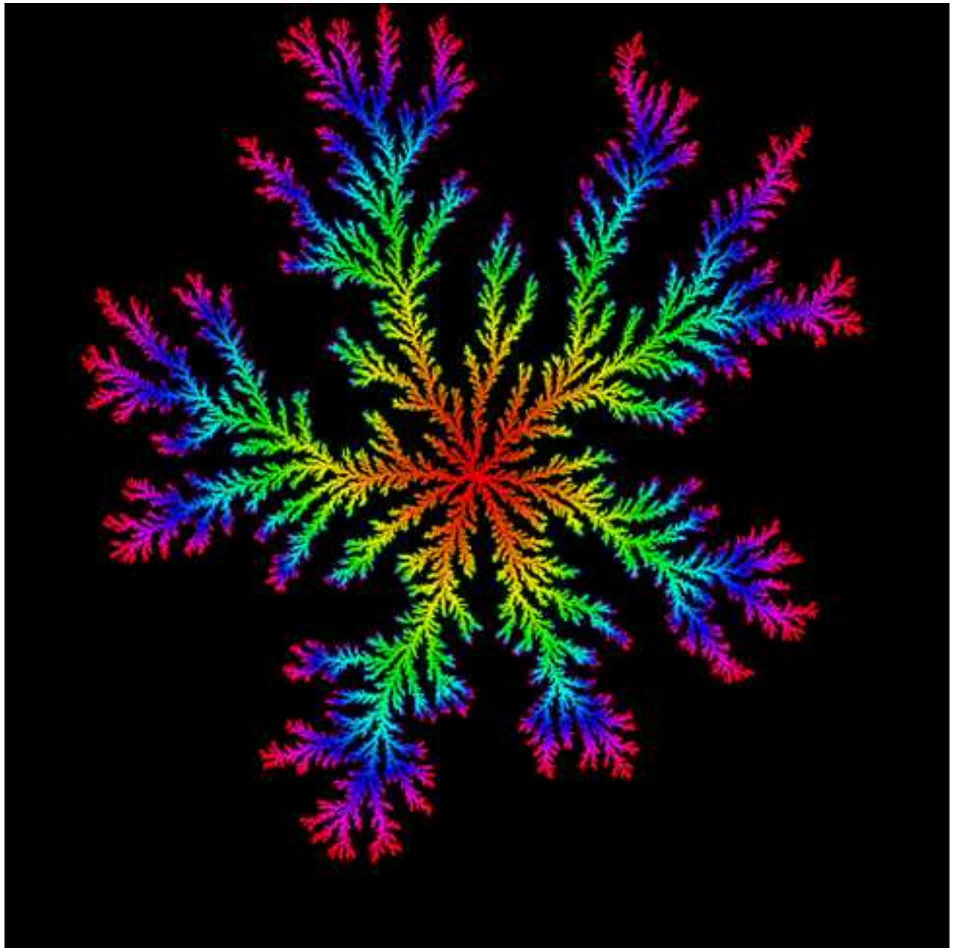
FIG. 2: A growing and branching shock pattern, with one (left) and two (right) generations of branchings. The bold line along the negative x-axis represents a narrow viscous finger (fluid). At this scale, the viscous finger is vanishingly narrow.

# *Off*-lattice killing-*free* algorithm

1. Place seed at origin (0,0), N=1
2. Particle starts at radius of birth Rbirth
3. Diffusion in space
4. If touch, it sticks, N=N+1
5. If particles goes out of the radius of death $R_{death}$ it is returned on $R_{birth}$ with probability

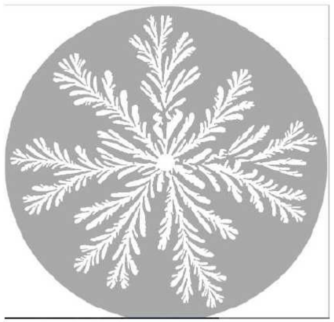
$$P(\varphi)=\frac{1}{2\pi}\frac{x^2-1}{x^2-2x\cos\varphi+1}$$

6. New iteration – from step 2.

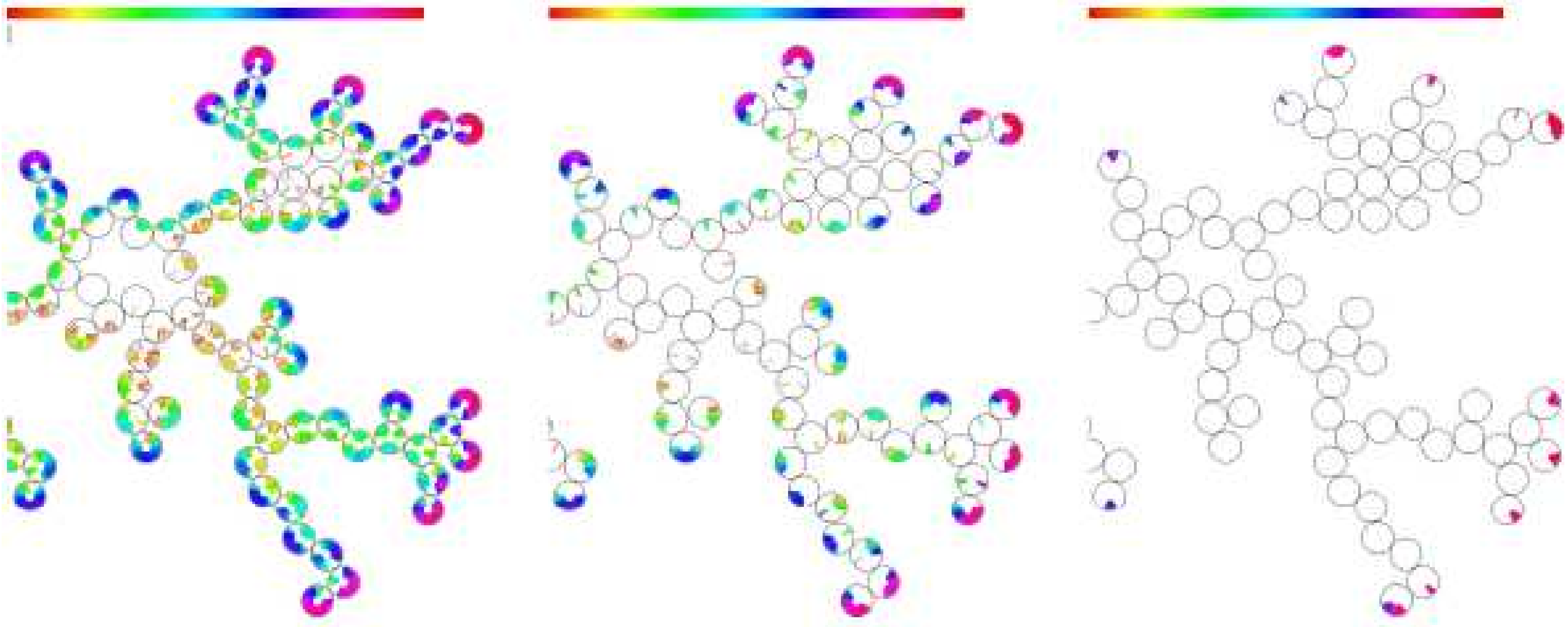


50 000 000 particles
Ensemble with 1000 clusters
D=1.710…

after Sharon, Moore, McCormick, and Swinney,
University of Texas at Austin

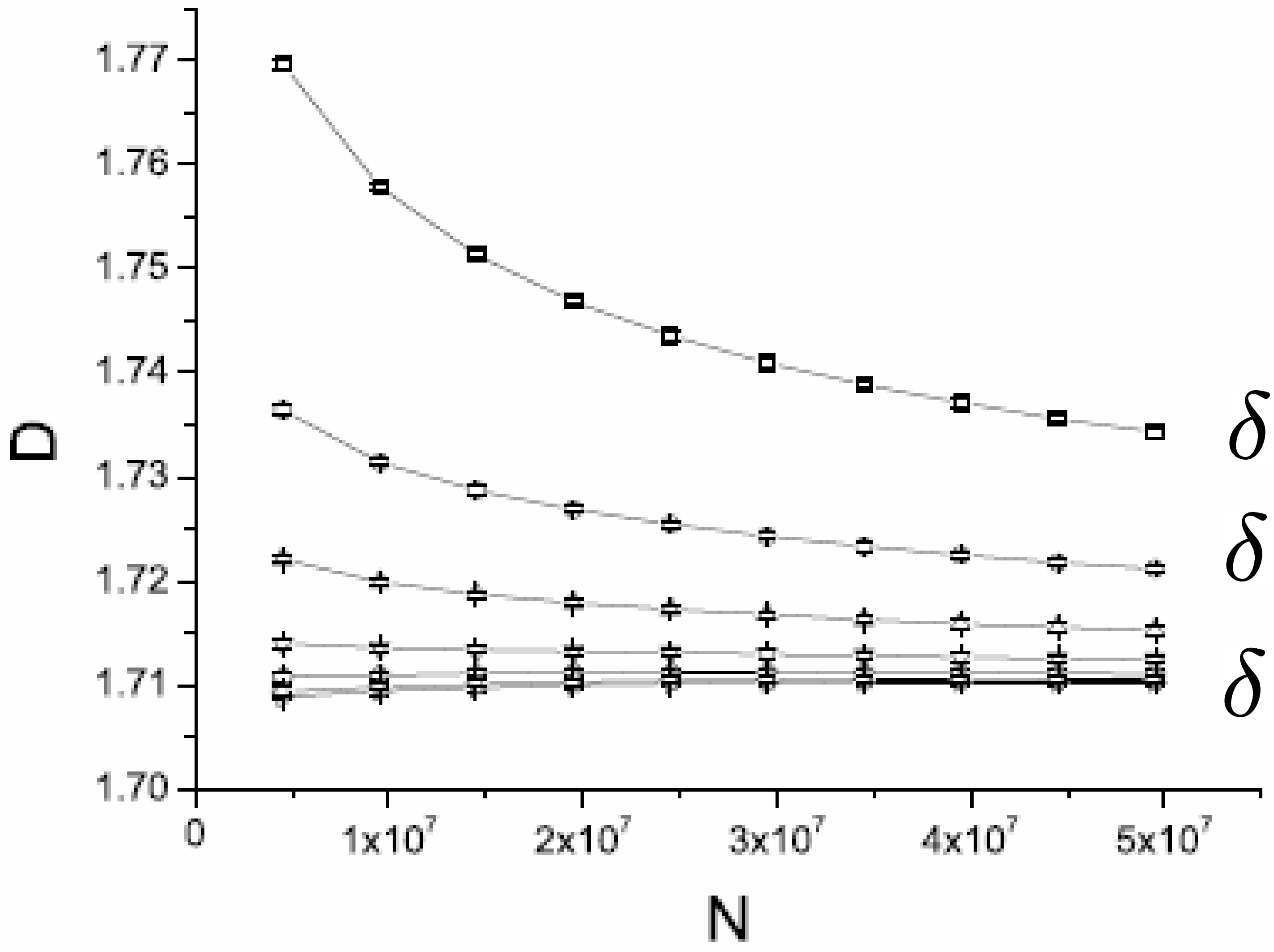


# Probing harmonic measure with particles of size $\delta$



$\delta = 0.1$ $\delta = 1$ $\delta = 10$

# *Effective* fractal dimension

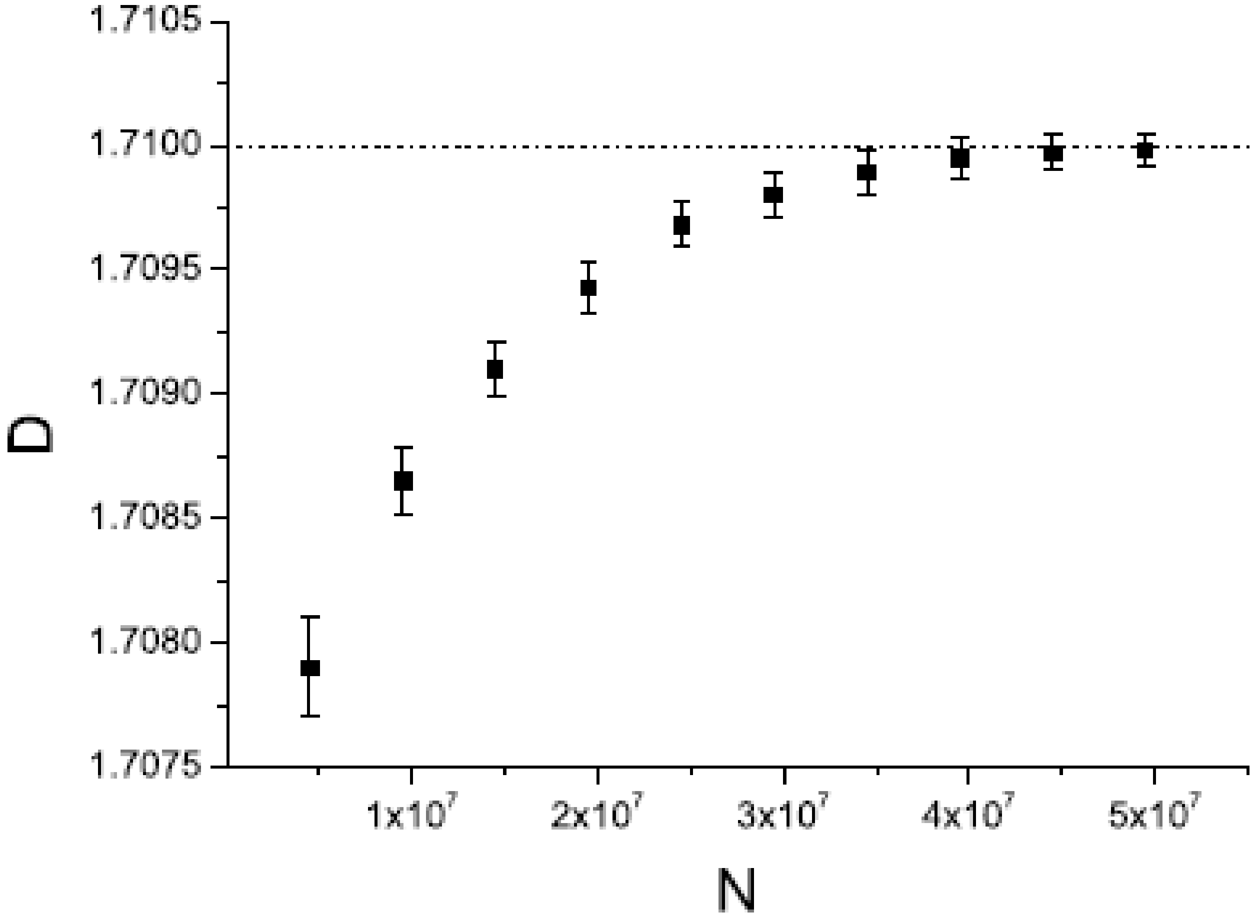



$\delta = 100$

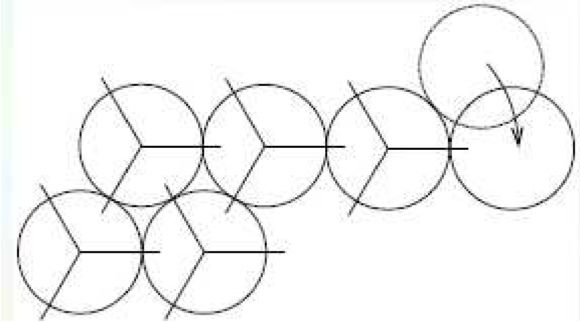
$\delta = 10$

$\delta = 0.1$

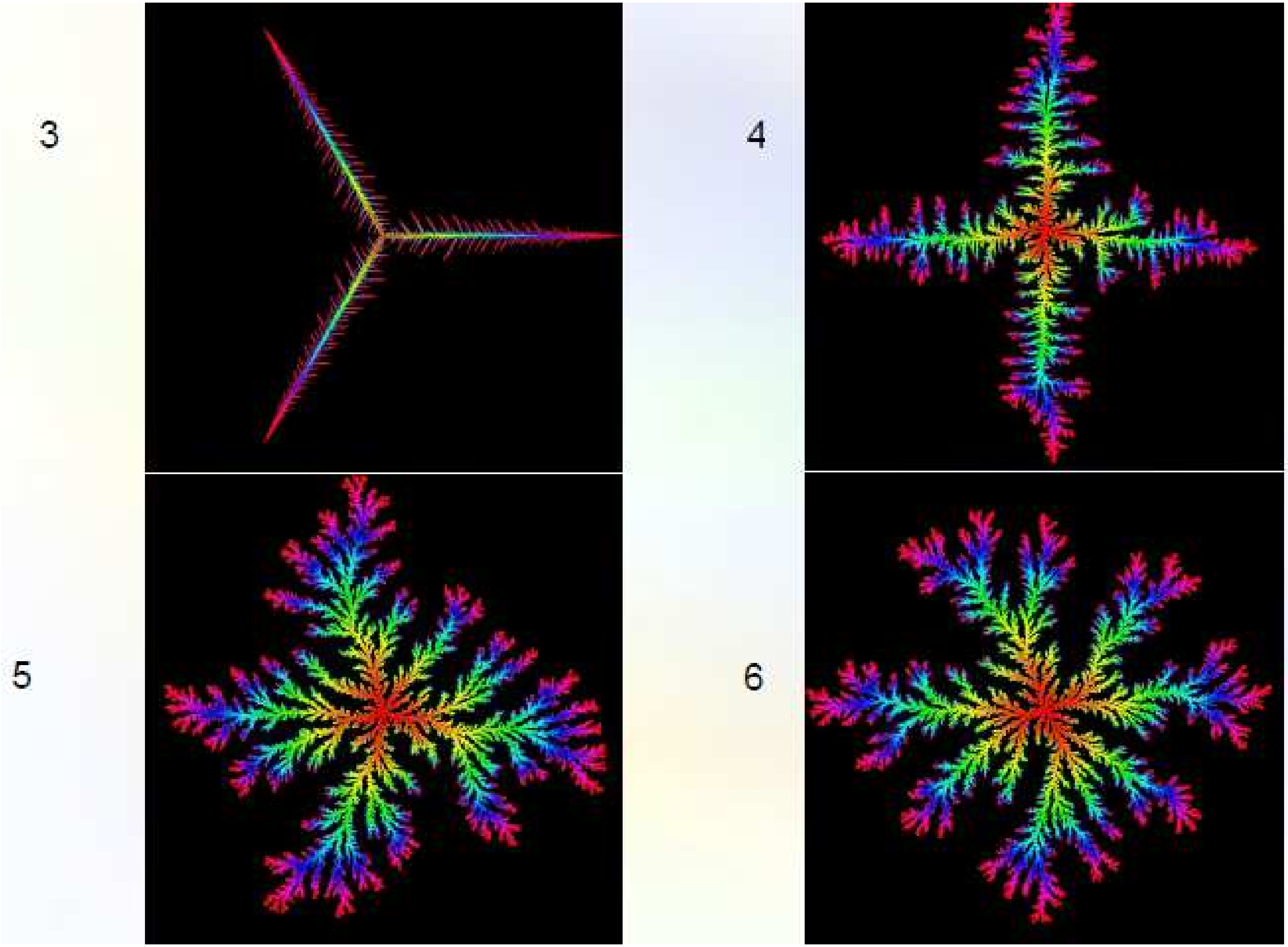
# *Effective* fractal dimension




Fractal dimension $D = 1.7100(2)$

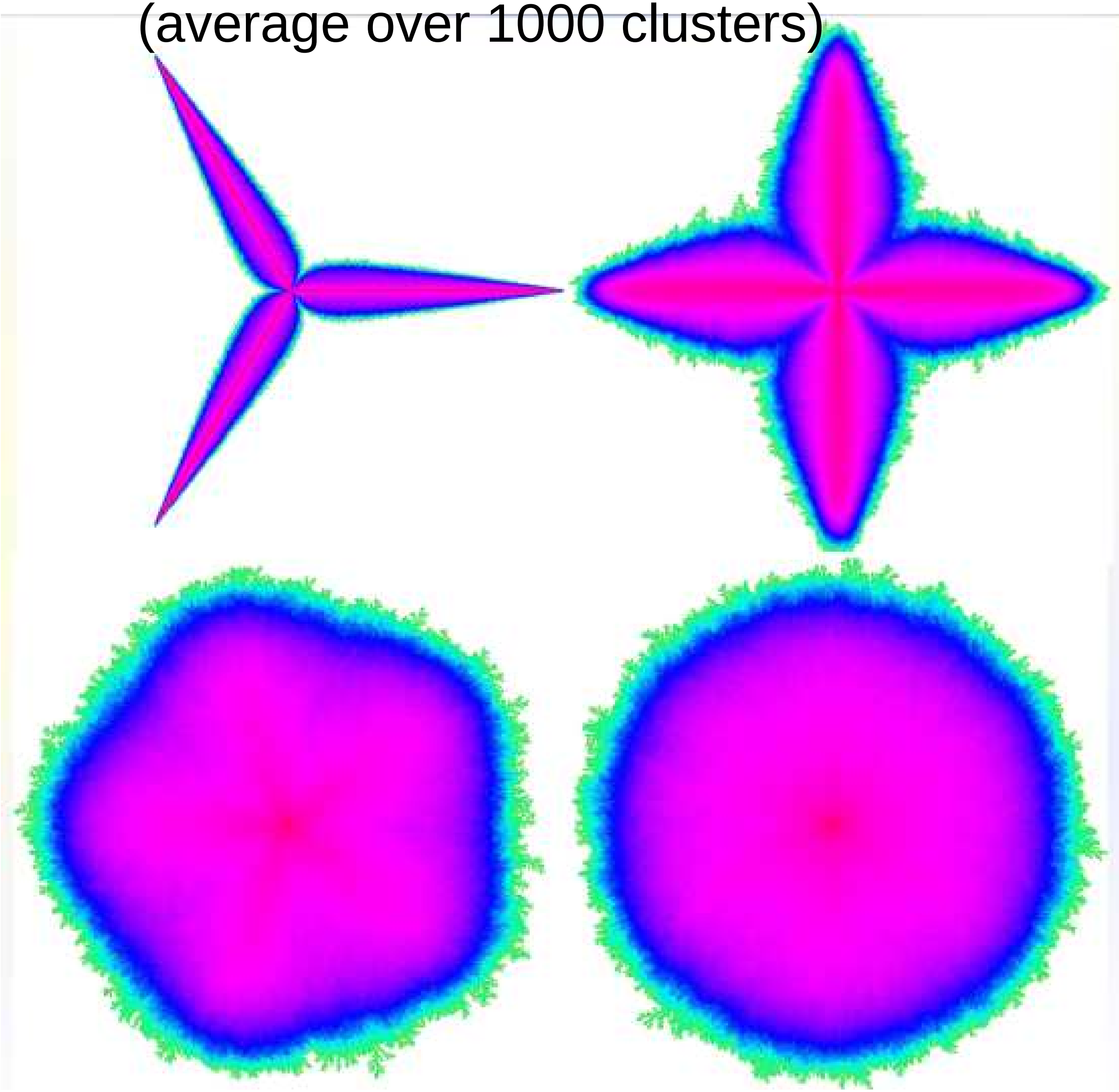


# Anisotropic clusters




Density of the particles

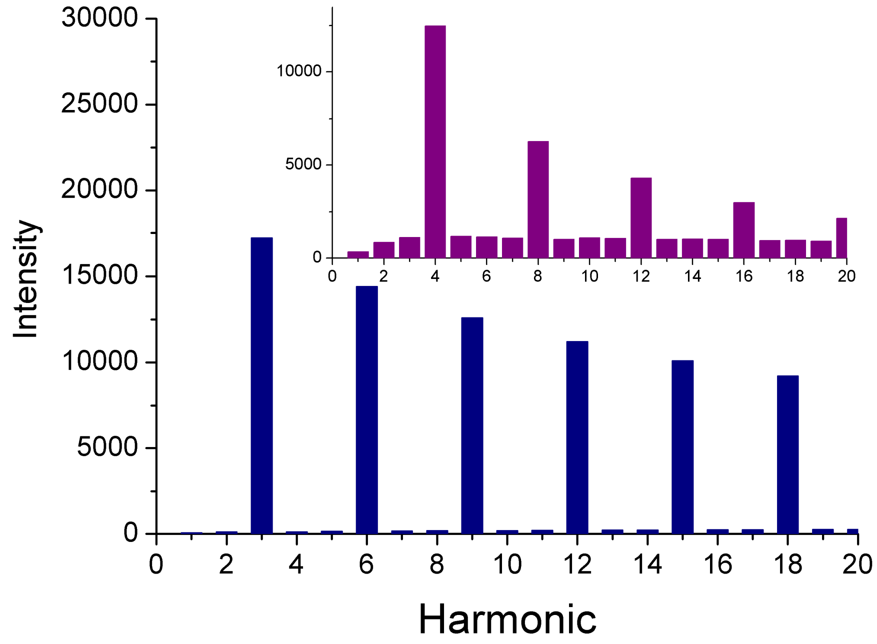
(average over 1000 clusters)



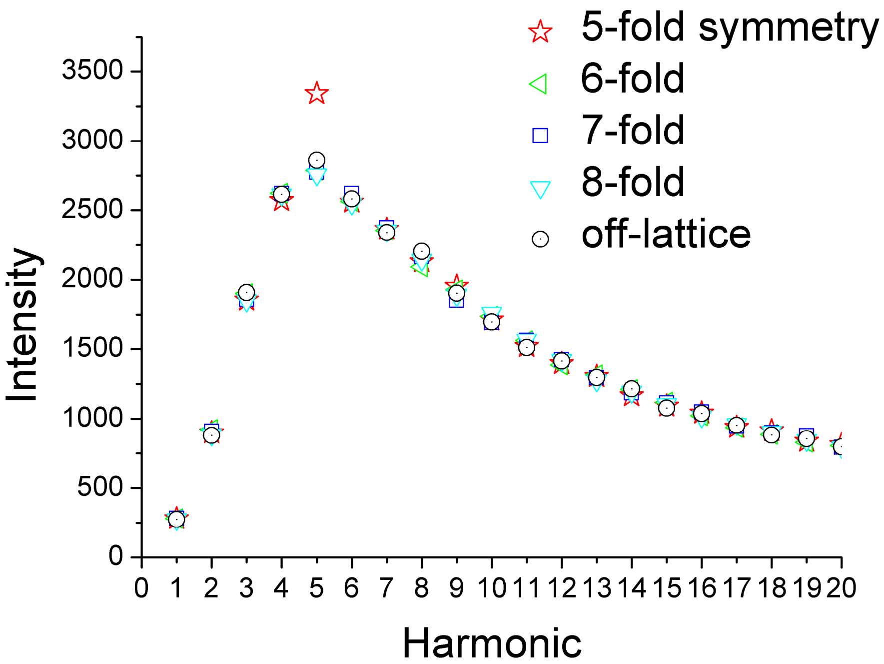
# Fourier analysis of the spectrum

$$P(\phi)=a_0+\sum a_k \sin(kx)+b_k \cos(kx)$$

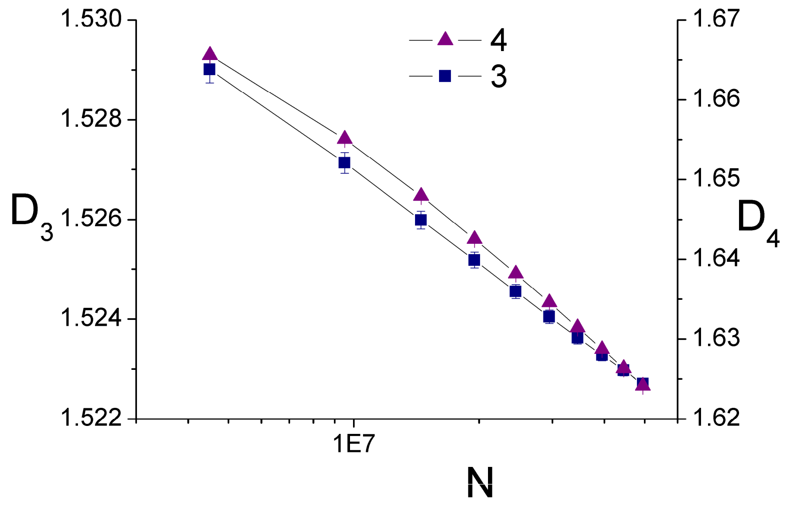
$$I_k=\sqrt{a_k^2+b_k^2}$$

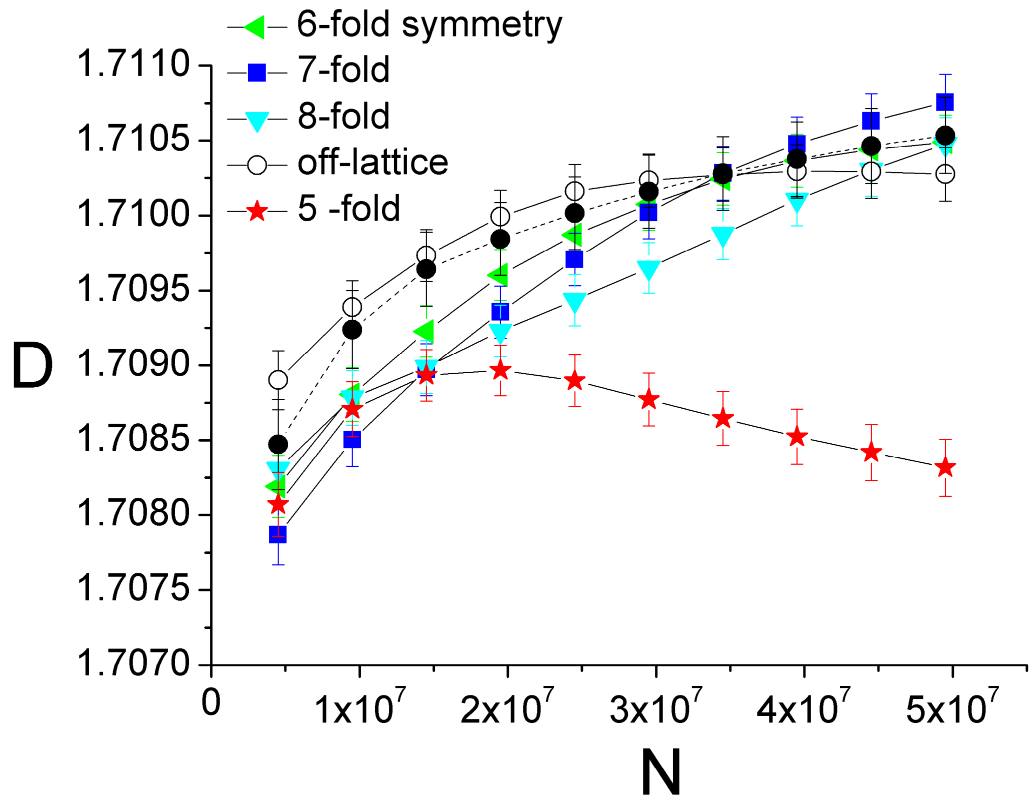


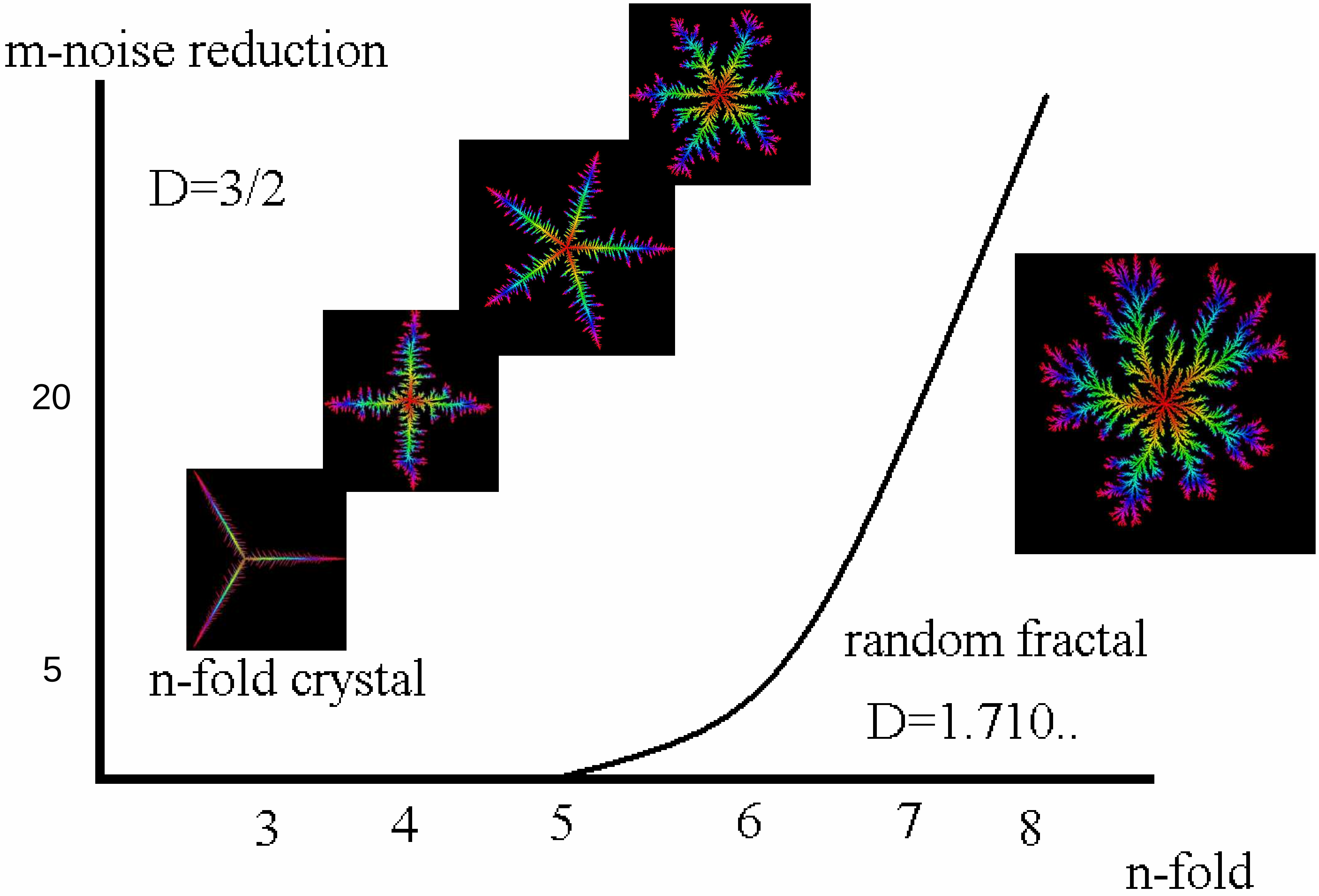

4-fold symmetry
3-fold symmetry



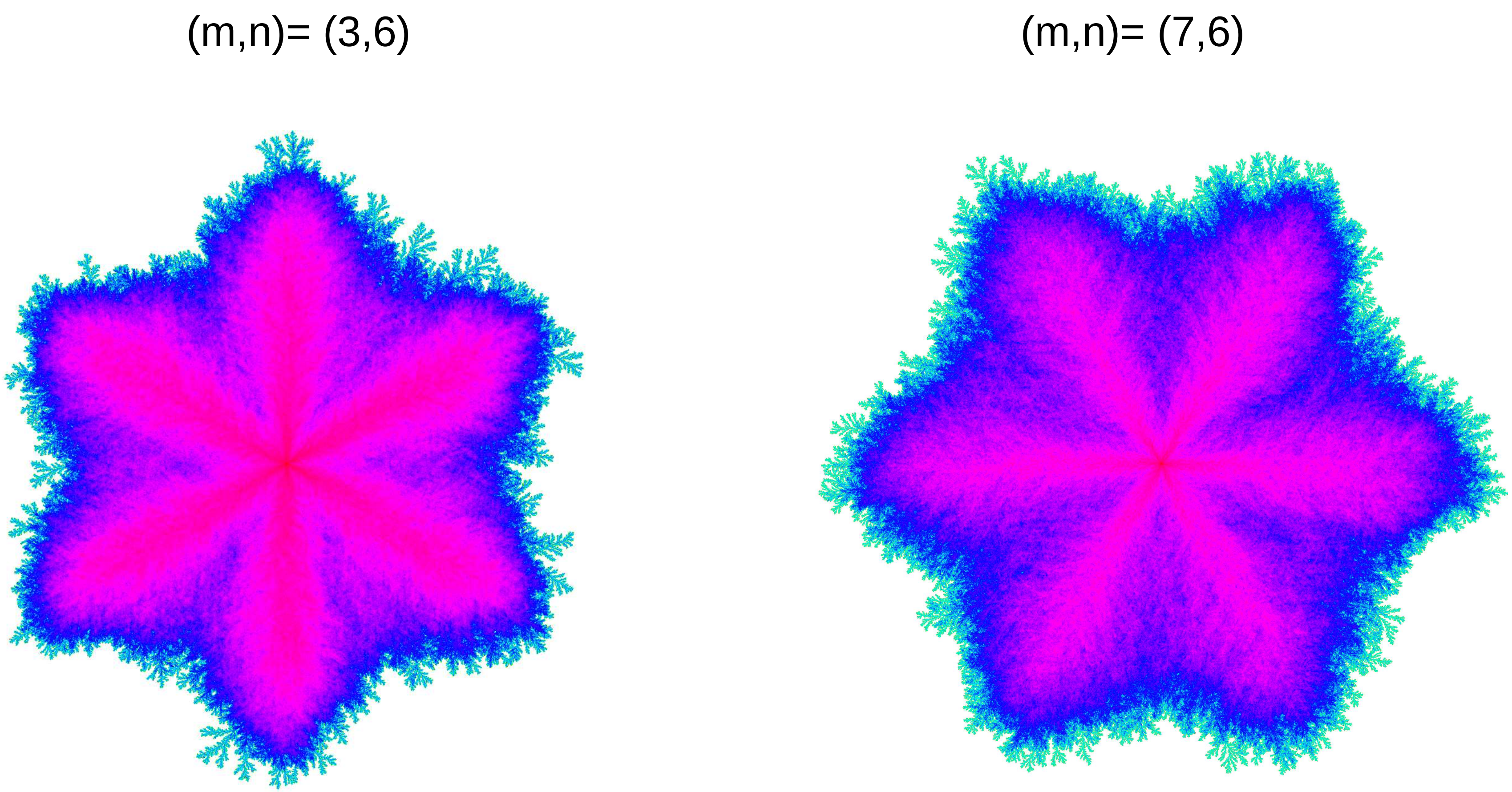
# Fractal dimension estimation




m-noise reduction
D=3/2
20
5
n-fold crystal
random fractal
D=1.710..
3
4
5
6
7
8
n-fold

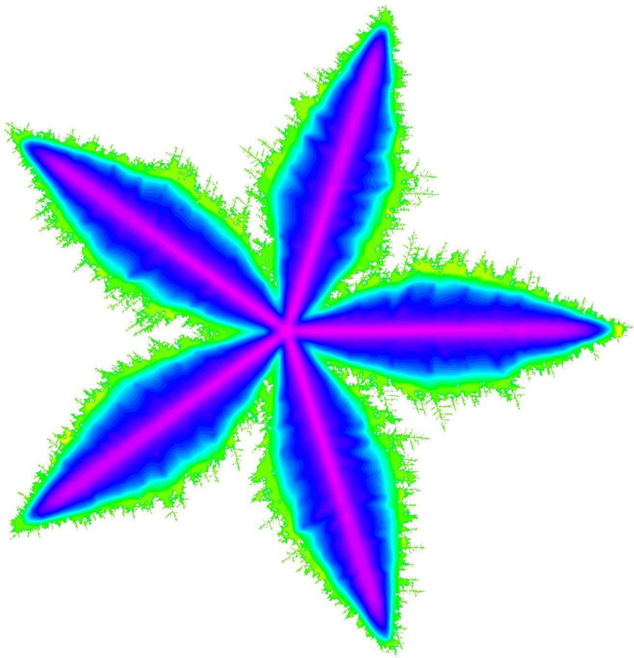
Density of the particles



# Density of the particles

# (average over 1000 clusters)

(m,n)= (7,5)



# Предложение

При двумерном росте DLA кластера (асимптотически) есть лишь два режима:

1. n-fold фрактальный кристалл, D=3/2
2. случайный кристалл, D=1.710…

- Dynamical phase transition in DLA model.
- Critical line in the (n-m) plane (n-fold symmetry and m-noise reduction).