

Primordial and present dark energy in $f(R)$ gravity

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Definition of DE

DE – unclustered component, seen by gravitational interaction only. Quantitative definition - through gravitational field equations conventionally written in the Einstein form:

$$\frac{1}{8\pi G} \left(R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = - \left(T^\nu_{\mu(vis)} + T^\nu_{\mu(DM)} + T^\nu_{\mu(DE)} \right) ,$$

$G = G_0 = \text{const}$ - the Newton gravitational constant measured in laboratory.

In the absence of direct interaction between DM and DE:

$$T^\nu_{\mu(DE); \nu} = 0 .$$

Possible forms of DE

► Physical DE

New non-gravitational field of matter. DE proper place – in the **rhs** of gravity equations.

► Geometrical DE

Modified gravity. DE proper place – in the **lhs** of gravity equations.

► Λ - intermediate case.

Two cases where DE shows itself:

- 1) inflation in the early Universe – **primordial DE**,
- 2) present accelerated expansion of the Universe – **present DE**.

$f(R)$ gravity

$$S = \frac{1}{16\pi G} \int f(R) \sqrt{-g} d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu{}_\mu .$$

The effective energy-momentum tensor of DE in $f(R)$ gravity:

$$8\pi G T_{\mu}^{\nu}{}_{(DE)} = F'(R) R^\nu{}_\mu - \frac{1}{2} F(R) \delta^\nu{}_\mu + (\nabla_\mu \nabla^\nu - \delta^\nu{}_\mu \nabla_\gamma \nabla^\gamma) F'(R) .$$

De Sitter solutions in the absence of matter: roots $R = R_{DS}$ of the algebraic equation

$$Rf'(R) = 2f(R) .$$

Conditions for viable $f(R)$ models

I. Conditions of classical and quantum stability:

$$f'(R) > 0, \quad f''(R) > 0.$$

Even the saturation of these inequalities should be avoided:

1. $f'(R_0) = 0$: a generic anisotropic space-like curvature singularity forms.
2. $f''(R_0) = 0$: a weak singularity forms, loss of predictability of the Cauchy evolution.

$$a(t) = a_0 + a_1(t - t_s) + a_2(t - t_s)^2 + a_3|t - t_s|^{5/2} + \dots$$

The metric is in C^2 , but not C^3 , continuous across this singularity, and there is no unambiguous relation between the coefficients a_3 for $t < t_s$ and $t > t_s$.

II. Conditions for the existence of the Newtonian limit:

$|F| \ll R$, $|F'| \ll 1$, $RF'' \ll 1$ for $R \gg R_{now}$ and up to some very large R .

The same conditions for smallness of deviations from GR.

III. Laboratory and Solar system tests.

No deviation from the Newton law up to 50μ .

No deviation from the Einstein values of the post-Newtonian coefficients β and γ up to 10^{-4} in the Solar system.

IV. Existence of a future stable (or at least metastable) de Sitter asymptote:

$$f'(R_{DS})/f''(R_{DS}) \geq R_{DS} .$$

Required since observed properties of DE are close to that of a cosmological constant.

V. Cosmological tests:

among them the anomalous growth of matter perturbations for recent redshifts

$$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}}$$

at the matter-dominated stage for $k \gg M(R)a$, where $M^2(R) = 1/3F''(R)$.

VI. $f(R)$ cosmology should not destroy previous successes of present and early Universe cosmology in the scope of GR, including the existence of the matter-dominated stage driven by non-relativistic matter preceded by the radiation-dominated stage with the correct BBN and, finally, inflation.

Inflationary $f(R)$ models

1. The simplest one (A.A. Starobinsky, 1980):

$$f(R) = R + \frac{R^2}{6M^2}$$

with small one-loop quantum gravitational corrections producing the scalaron decay. Remains viable:

$$n_s \approx 0.96, \quad r \approx 0.004.$$

2. Analogues of chaotic inflation: $F(R) \approx R^2 A(R)$ for $R \rightarrow \infty$ with $A(R)$ being a slowly varying function of R , namely

$$|A'(R)| \ll \frac{A(R)}{R}, \quad |A''(R)| \ll \frac{A(R)}{R^2}.$$

3. Analogues of new inflation, $R \approx R_1$:

$$F'(R_1) = \frac{2F(R_1)}{R_1}, \quad F''(R_1) \approx \frac{2F(R_1)}{R_1^2}.$$

Thus, all inflationary models in $f(R)$ gravity are close to the simplest one over some range of R .

An example of a viable DE model

A. A. Starobinsky, JETP Lett. **86**, 157 (2007)

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

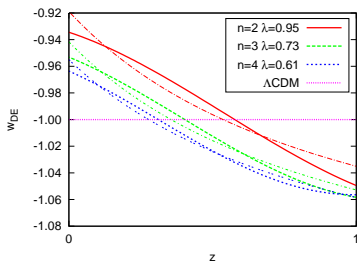
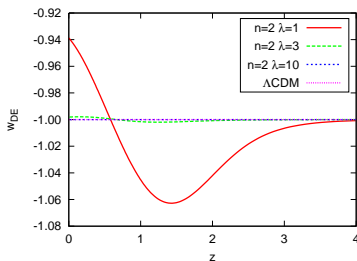
with $n \geq 2$. Similar models in:

1. W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
2. A. Appleby and R. Battye, Phys. Lett. B **654**, 7 (2007).

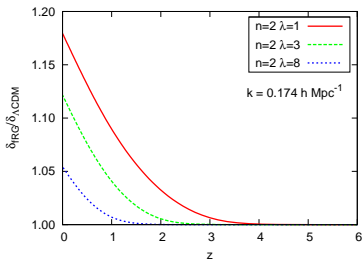
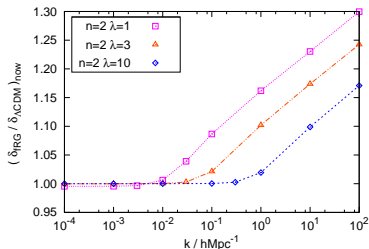
Phantom boundary crossing

H. Motohashi, A. A. Starobinsky, J. Yokoyama,
 Progr. Theor. Phys. **123**, 887 (2010).

Generic feature: phantom behaviour for $z > 1$,
 crossing of the phantom boundary $w_{DE} = -1$ for $z < 1$.



Anomalous growth of perturbations



Still not the end of the story!

Three new problems

In the early Universe:

- ▶ Unlimited growth of $M(R)$ for $t \rightarrow 0$: when $M(R)$ exceeds M_{PI} , quantum-gravitational loop corrections invalidate the use of an effective quasi-classical $f(R)$ gravity.
- ▶ Unlimited growth of the amplitude of δR oscillations for $t \rightarrow 0$ (the "scalaron overproduction" problem).
- ▶ "Big Boost" singularity before the Big Bang:
 $a(t) = a_0 + a_1(t - t_0) + a_2|t - t_0|^k + \dots$, $1 < k = \frac{2n+1}{n+1} < 2$,
 if $F(R) \propto R^{-2n}$ for $R \rightarrow \infty$.

Curing all three problems

S. A. Appleby, R. A. Battye and A. A. Starobinsky,
JCAP **1006**, 005 (2010).

Add $\frac{R^2}{6M^2}$ to $f(R)$ with M not less than the scale of inflation.
Then the first and third problems go away. The second
problem still remains, but (any) inflation can solve it.

However, in all known inflationary models R may be negative
during reheating after inflation (e.g. when $V(\phi) = 0$).

Necessity of an extension of $f(R)$ to $R < 0$ keeping $f''(R) > 0$.

As a result, a non-zero g -factor ($0 < g < 1/2$) arises:

$$g = \frac{f'(R) - f'(-R)}{2f'(R)}, \quad R_0 \ll R \ll M^2.$$

An example: g -extended R^2 -corrected AB model

$$f(R) = (1 - g)R + g\epsilon \log \left[\frac{\cosh(R/\epsilon - b)}{\cosh b} \right] + \frac{R^2}{6M^2} .$$

Combined inflationary–DE models

If $M \approx 3 \times 10^{-6} M_{Pl}$, the scalaron can play the role of an inflaton, too. Then the inflationary predictions are formally the same as for the pure $R + R^2/6M^2$ inflationary model which does not describe the present DE:

$$n_s = 1 - \frac{2}{N}, \quad r = \frac{12}{N^2}.$$

However, N is different, $N \sim 70$ for the unified model (versus $N \sim 55$ for the purely inflationary one) because the stage of reheating after inflation becomes completely different: it consists of unequal periods with $a \approx \text{const}$ and $a \propto t^{1/2}$.

Duration of the periods in terms of $\ln t$:

$-\ln(1 - 2g)$ and $-2\ln(1 - 2g)$ respectively.

So, $a(t) \propto t^{1/3}$ on average for a long time after the end of inflation.

Observable prediction which is, however, degenerate with other inflationary models in $f(R)$ gravity.

Definition and forms of DE

$f(R)$ gravity

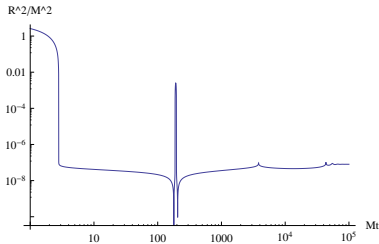
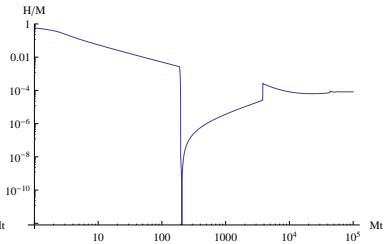
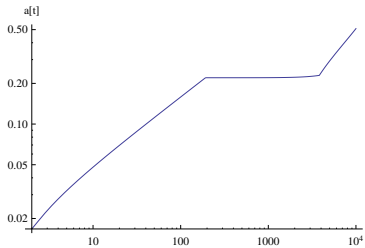
Viability conditions

New results on $f(R)$ DE models

Combined inflationary-DE models

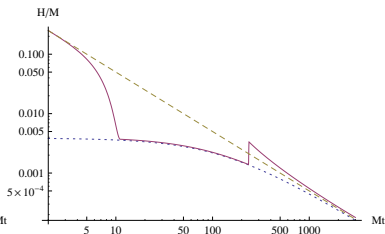
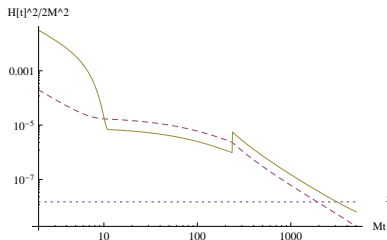
Massive neutrinos in $f(R)$ gravity

Conclusions about $f(R)$ models



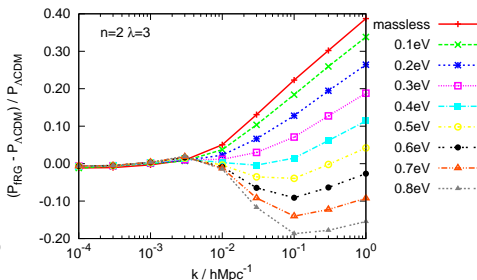
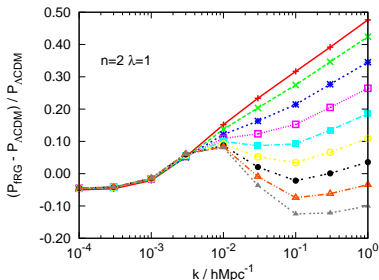
Reheating – due to gravitational particle creation which occurs mainly at the end of inflation. Less efficient than in the pure $f(R) = R + R^2/6M^2$ inflationary model,

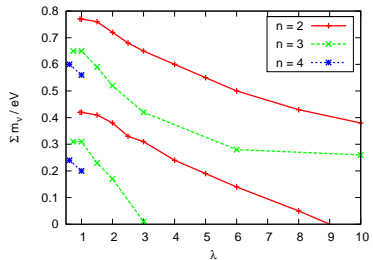
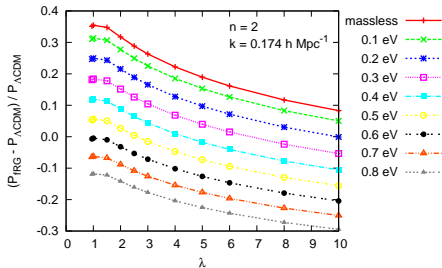
$$t = t_{\text{reh}} \sim M^{-4} M_{\text{Pl}}^3 \sim 10^{-18} \text{ s} .$$



Massive neutrinos in $f(R)$ gravity

The anomalous growth of perturbations may be partially compensated by an increase of $\sum_\nu m_\nu$, as compared to the standard Λ CDM, up to $\mathcal{O}(0.5 \text{ eV})$.





H. Motohashi, A. A. Starobinsky, J. Yokoyama,
 arXiv:1005.1171.

Conclusions about $f(R)$ models

- ▶ Though a very narrow class among all $f(R)$ models of present DE still remains viable, it is not empty: it is possible to construct models satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from Λ CDM.
- ▶ To achieve this, previously constructed viable $f(R)$ DE models should be extended to large R with the $\sim R^2$ asymptotic behaviour and to negative R keeping $f'(R) > 0$, $f''(R) > 0$ at least up to the scale of inflation.

- ▶ Unified description of primordial DE producing inflation and present DE in the scope of $f(R)$ gravity is possible for the specific choice of M : $M \approx 3 \times 10^{-6} M_{Pl}$.
- ▶ Combined inflationary – DE $f(R)$ models have a significantly different reheating stage after inflation as compared to pure inflationary $f(R)$ models, with strongly non-linear oscillations of the scale factor $a(t)$.
- ▶ The most critical test for all $f(R)$ models of present dark energy: anomalous growth of density perturbations in the matter component at recent redshifts $z \sim 1 - 3$.
- ▶ The sum of neutrino rest-masses may be increased up to $\sim (0.5 - 0.8)$ eV.