Searching for a mean string theory description of the pure gauge theory in 4d

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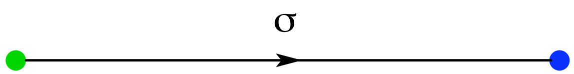
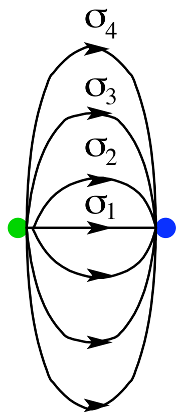
Landau Days, June 2010

Physical reason for the 5th "dimension"





Left: a typical string-like flux tube of tension σ. Right: a "fat string" as a collection of thin strings with different tensions $\{\sigma_n\}$.

• Continuous spectrum of string tensions. σ can be promoted to a new spacetime coordinate. So, strings effectively live in some warped 5-dimensional space.
Liouville field as the fifth dimension (*Polyakov* 1997)

• Discrete spectrum of string tensions.(*Andreev and Siegel* 2004)

• Both discrete and continues pieces.

The Model (soft wall metric model)

5-dimensional Euclidean background metric in string frame

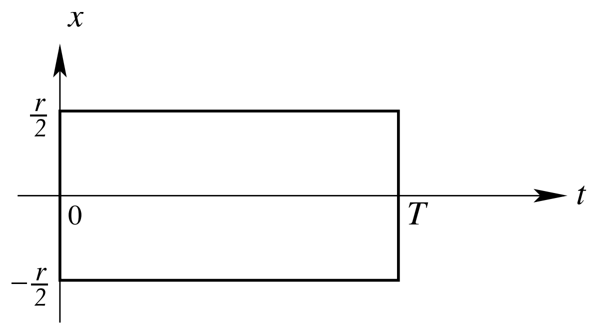
$$ds^2 = \frac{R^2}{z^2} h(z) \Big(dx^i dx^i + dz^2 \Big) \,, \quad h(z) = \exp \frac{1}{2} c z^2 \,, \quad i = 1, \ldots 4 \,.$$

We also take a constant dilaton ϕ. So, the gauge coupling doesn't run. *Andreev* (2006)

History

- *Hirn and Satz*, (2005). $h = \exp f z^4$
- *Son et al* (2006). $h = 1$, $\phi = cz^2$. Soft wall dilaton model
- *Metsaev* (2000). Regge like KK spectrum

Quark-Antiquark Potential



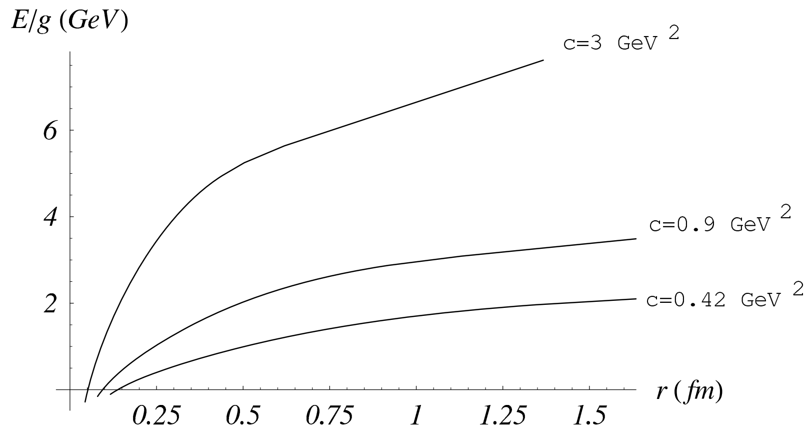
$$\langle\, W(C)\,\rangle \sim \exp\{-TE(r)\}\,, \quad T \to \infty\,.$$

Maldacena-Rey-Yee proposal: $\langle\, W(C)\,\rangle \sim \exp\{-S_{NG}\}\,.$

The potential is written in parametric form *Andreev and Zakharov* (2006)

$$r(\lambda) = 2\sqrt{\frac{\lambda}{c}}\int_0^1 dv\, v^2 e^{\frac{1}{2}\lambda(1-v^2)}\Big(1 - v^4 e^{\lambda(1-v^2)}\Big)^{-\frac{1}{2}}\,,$$

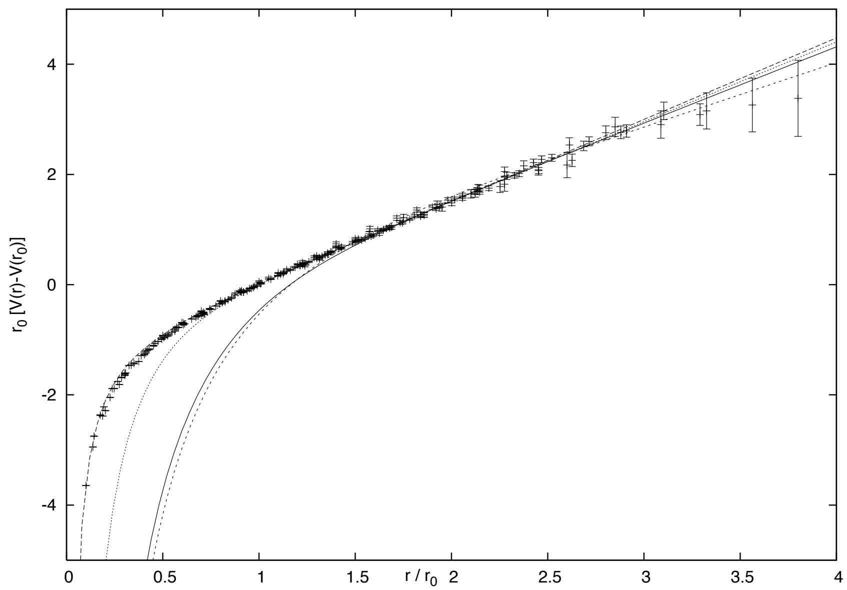
$$E(\lambda) = \frac{g}{\pi}\sqrt{\frac{c}{\lambda}}\left(-1 + \int_0^1 dv\, v^{-2}\Big[e^{\frac{1}{2}\lambda v^2}\Big(1 - v^4 e^{\lambda(1-v^2)}\Big)^{-\frac{1}{2}} - 1\Big]\right)\,.$$



There are two free parameters: c and $g = \frac{R^2}{\alpha'}$

Two options

- The Cornell model
- The lattice data



White (2007)

Two surprises

- c is of order $0.9\,GeV^2$ and g is of order 1
  The latter estimate looks like the end of phenomenology based on field theory (supergravity)
- Corrections to E are small

Meson spectrum via the Salpeter equation with $E(r)$ *Giannuzzi* (2008)

Particle	Th. mass (MeV)	Exp. mass (MeV) [7]	Decay const. (MeV)
η_c	3025.3	2980.3 $\pm$ 1.2	342
η_c'	3603.5	3637.0 $\pm$ 4	266
η_c''	4039.3		195
J/ψ	3079.8	3096.916 $\pm$ 0.011	356
ψ'	3624.3	3686.09 $\pm$ 0.04	237
ψ''	4057.0	4039$\pm$1	185
η_b	9433.9	9388.9 $^{+3.1}_{-2.3}$ (stat) $\pm$ 2.7 (syst) [1]	637
η_b'	9996.8		430
η_b''	10347.5		367
Υ	9438.3	9460.30$\pm$0.26	686
$\Upsilon(2S)$	9998.6	10023.26 $\pm$ 0.31	484
$\Upsilon(3S)$	10348.8	10355.2 $\pm$0.5	335
$\Upsilon(4S)$	10622.3	10579.4 $\pm$1.2	301

Gluon Condensate

$$G_2 = \langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle \,.$$

The goal is to compute the expectation value of a circular Wilson loop. Then

$$\ln W = -\sum_n c_n \alpha_s^n - \frac{\pi^2}{36} Z G_2 s^2 + O(s^3) \,,$$

s is a square of the loop.
For $c = 0$, it was done by *Gross* et al (1999) as well as *Maldacena* et al (1999)
For $c \neq 0$, the leading terms were analytically computed by *Andreev and Zakharov* (2007) So,

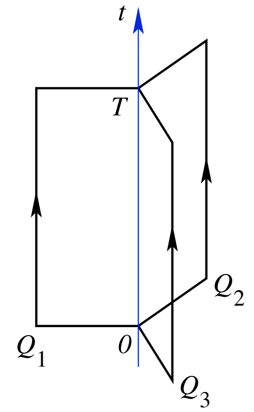
$$G_2 = \frac{9}{\pi^2} \kappa g Z^{-1} c^2 \,, \quad \kappa = \frac{7}{3} \Big(\frac{17}{6} - 4 \ln 2 \Big) \,.$$

A numerical estimate

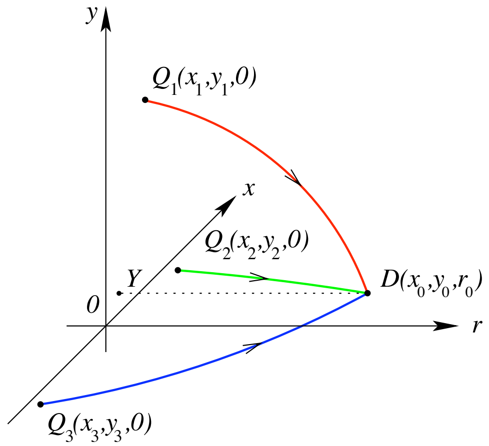
$$G_2 = 0.010 \pm 0.0023 \; GeV^2 \,.$$

Multi-Quark Potentials

Andreev (2008)



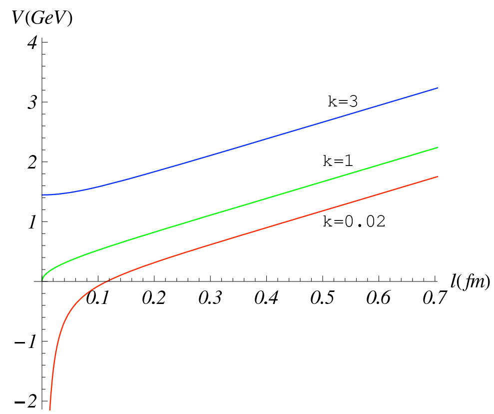
Top: Baryonic Wilson loop for $SU(3)$. Bottom: A configuration in 5 dimensions



$$S = \sum_{i=1}^{3} S_{NG}(i) + S_{vert} \,.$$

Simple example

- The triangle formed by the quarks is equilateral
- The action S_{vert} is that of a massive particle. So, we have a new free parameter.



For arbitrary number of colors

- The Y-law at large distances
- universality of the string tension

Spatial String Tension

At finite T we use another deformation (now 10-dimensional)

$$ds^2 = \frac{R^2}{z^2} h(z) \Big(f dt^2 + dx^i dx^i + f^{-1} dz^2 \Big) + h^{-1}(z) R^2 d\Omega_5^2 \,, \quad h(z) = \exp \frac{1}{2} c z^2 \,,$$

$$f = 1 - \frac{z^4}{z_T^4} \,, \quad T = \frac{1}{\pi z_T} \,, \qquad i = 1, \ldots 3 \,.$$

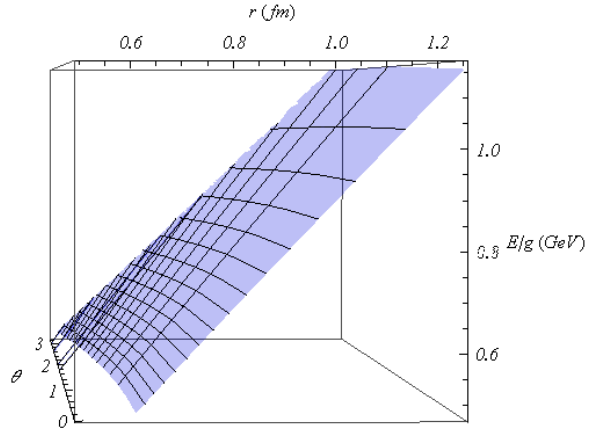
Andreev and Zakharov (2006), *Andreev* (2007)

The Wilson loop to be considered is spatial. Its expectation value provides the pseudo-potential.

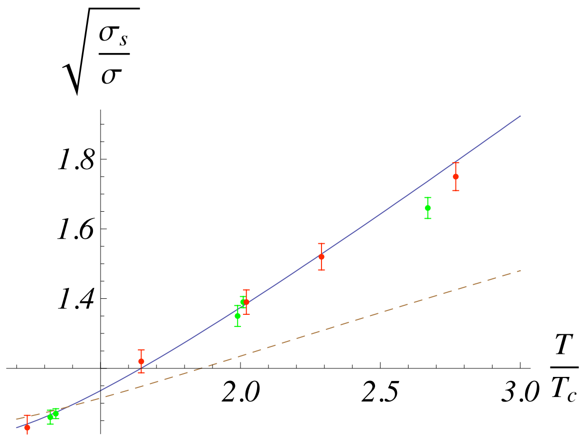
In addition, it is interesting to see what happens with internal degrees of freedom.

$$\sigma_s = \sigma \Big(\frac{T}{T_c} \Big)^2 \exp \Big\{ \Big(\frac{T_c}{T} \Big)^2 - 1 \Big\} \quad if \quad T \geq T_c \,,$$

with $T_c = \frac{1}{\pi} \sqrt{\frac{c}{2}}$ and $\sigma_s = \sigma$ if $T \leq T_c$.



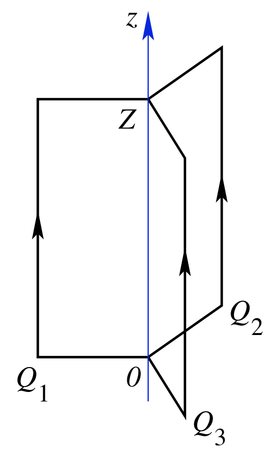
Top: $E(r, \theta)$. Bottom: A comparison with the lattice and a hadronic gas model.



Green bars from *Bali* et al (1993), red bars from *Karsch* et al (1995); hadronic model by *Agasian* (2003).

Baryonic Pseudo-Potential

Andreev (2008)



For arbitrary number of colors

- The Y-law at large distances
- universality of the spatial string tension

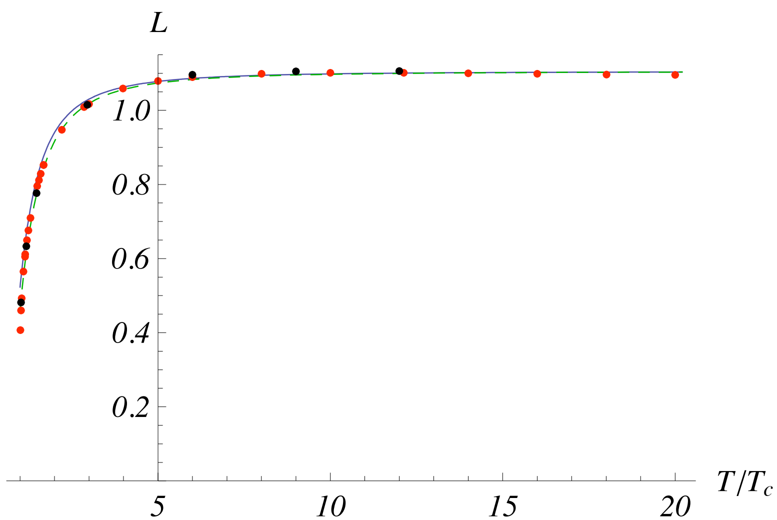
In addition

- To leading order, at large distances, the potential and pseudo-potential are described by **5**-dimensional theory.
- The discrepancy for $T > 3T_c$ is due to the running coupling.

Renormalized Polyakov Loop in the Deconfined Phase

$$L(T) = \exp\left[C - g\left(\sqrt{\pi}\frac{T_c}{T}\, Erfi\left(\frac{T_c}{T}\right) + 1 - e^{(T_c/T)^2}\right)\right],$$

Andreev (2009)



The dots are from lattice simulations by *Gupta* et al (2008). The normalization constant $C = 0.10$.

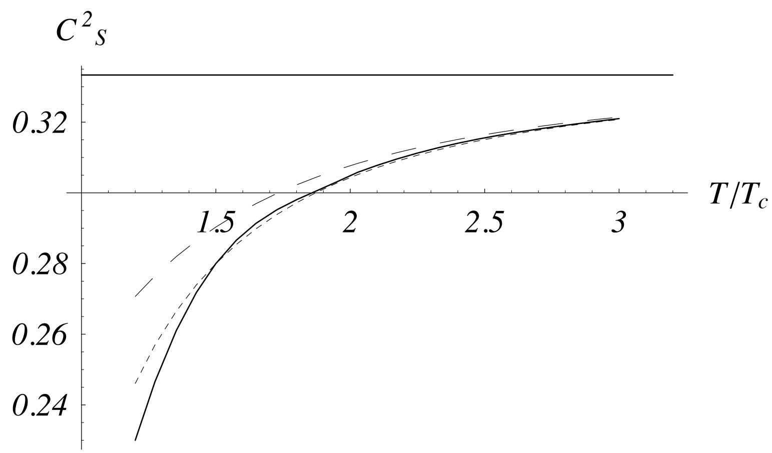
Thermodynamics

Andreev (2007)

The entropy density

$$s(T) = s_0 T^3 \exp\left(-\frac{1}{2}\frac{T_c^2}{T^2}\right).$$

The speed of sound



The solid curve represents the lattice result by *Karsch* et al (1995).

The preasure

$$p(T) \approx \frac{1}{4} s_0 T^4 \left(1 - \frac{T_c^2}{T^2}\right).$$

it is normalized at $T = T_c$ as $p(T_c) = 0$.

Rob Pisarski observation "Fuzzy bag" (2006) based on the data of *Karsch* (1995)

Field Strength Correlators

Andreev (2010)

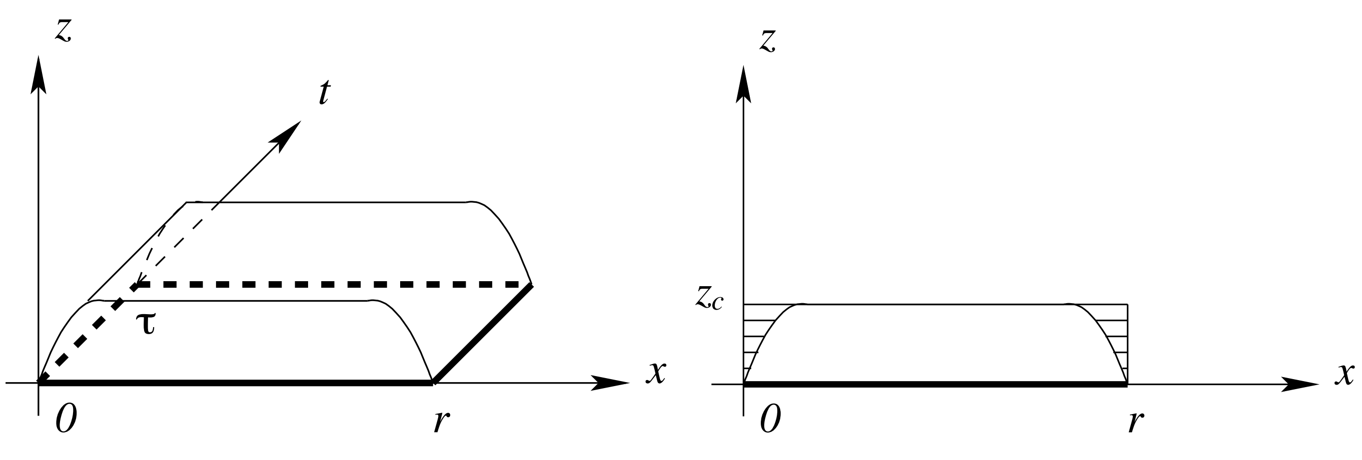
The two-point field strength correlator (in four-dimensional Euclidean space)

$$D_{\mu\nu,\rho\tau}(r) = \langle\, tr \left[G_{\mu\nu}(0) U_P(0,r) G_{\rho\tau}(r) U_P(r,0) \right] \rangle$$

For large r (separation of the field strength operators)

$$D_{\mu\nu,\rho\tau}(r) \sim P_{\mu\nu,\rho\tau}\, e^{-\frac{r}{\lambda}}$$

with λ the correlation length



Conclusions

• Black hole and string techniques in higher dimensions are useful

at energies below the GeV scale in $d = 4$

• QCD is a right place to try string theory.

Lattice and experimental data are available

• Mean string theory description

• No quarks. It is still far from the real world.

• Can string theory compete with the lattice?

(Strings vs Blue Genes)