Dephasing at the integer quantum Hall transitions: Short-ranged interaction in the singlet channel

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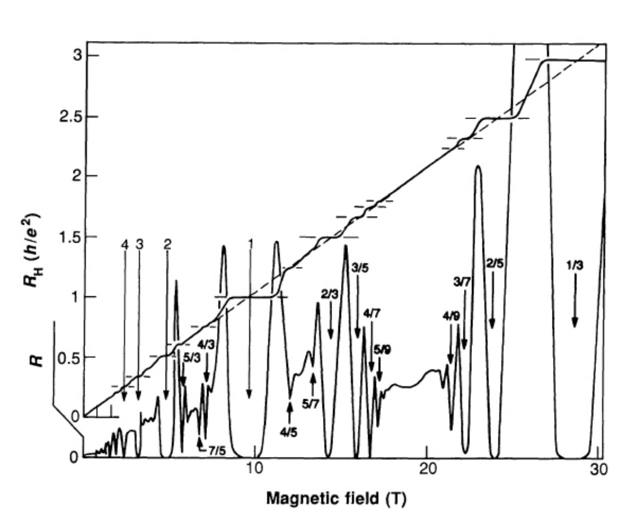
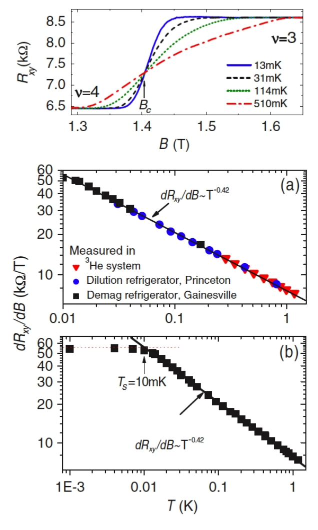
Integer quantum Hall transitions



Hall(R_H) and longitudinal (R) resistances as functions of the applied perpendicular magnetic field B at fixed temperature and electron concentration.

Adopted from Eisenstein and Stormer, Science 248, 1510 (1990)

Integer quantum Hall transitions

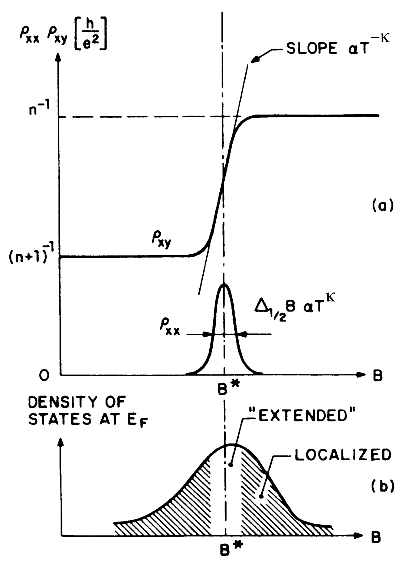


2DEG in $Al_xGa_{1-x}As/Al_{0.32}Ga_{0.68}As$ with $x = 0.085$.

Electron density $n = 1.2 \times 10^{11}\, cm^{-2}$, mobility $\mu = 8.9 \times 10^5\, cm^2/V\,s$.

Adopted from Wanli Li et al., Phys. Rev. Lett. 102, 216801 (2009)

Integer quantum Hall transitions



Adopted from Pruisken, Phys. Rev. Lett. 61, 1297 (1988)

IQHE – Scaling theory

- Finite system size L and $T = 0$:
  - Divergent correlation length

$$\xi \sim l|B - B_*|^{-\nu}$$

  - Conductivities

$$\sigma_{xx} = \mathscr{F}_{xx}(\xi/L), \qquad \sigma_{xy} = \mathscr{F}_{xy}(\xi/L)$$

- Finite T and $L \to \infty$:
  - Temperature-induced (inelastic) length (caused by interactions)

$$L_T \sim T^{-p/2}$$

  - Conductivities

$$\sigma_{ab} = \mathscr{F}_{ab}(\xi/L_T) = \tilde{\mathscr{F}}_{ab}(T^{-\kappa}|B - B_*|)$$

$$\kappa = p/2\nu$$

Pruisken, Phys. Rev. Lett. 61, 1297 (1988)

IQHE – experiments vs numerics

- Laboratory experiments (electrons with Coulomb interaction)

$$\kappa = 0.42 \pm 0.01$$

Wei, Tsui, Paalanen, Pruisken, Phys. Rev. Lett. 61, 1294 (1988); Wanli Li et al. (2009)

- Numerical calculations (non-interacting electrons)
  - Localization length exponent

$$\nu = 2.35 \pm 0.03$$

Huckestein, Kramer, Schweitzer, Surf. Sci. 263, 125 (1992)

  - Anomalous dimension of short-ranged singlet interaction

$$\mu_2 = 0.66 \pm 0.04$$

Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996);

N.B.:

- Recently, $\nu = 2.593 \pm 0.006$ has been reported by Slevin and Ohtsuki, Phys. Rev. B 80, 041304 (2009)
- Only one paper discusses the exponent μ_2 numerically.

Subtlety: Two length scales

$\sigma_{ab}(T)$ is determined by temperature-induced length $L_T \propto T^{-p/2}$

- L_T is given by inelastic length L_{in} induced by RG (Finkelstein NLσM)

$$p = \frac{2}{2-\mu_2} \approx 1.50 \pm 0.04$$

Pruisken, Baranov, Europhys. Lett. 31, 543 (1995); Pruisken, Burmistrov, JETP Lett. 87, 252 (2008)

- L_T is given by the dephasing length L_ϕ with

$$p = 1 + \mu_2 \approx 1.66 \pm 0.04$$

Wang, Fisher, Girvin, Chalker, Phys. Rev. B 61, 8326 (2000)

N.B.:

- At low temperatures $L_\phi \gg L_{in}$
- $L_\phi \sim L_{in}$ for the Coulomb interaction

Goals

- Theory: (this talk)
  - To understand anomalous dimensions of which operator determine exponent p of L_ϕ
  - To compute p for Anderson transition in unitary symmetry class (A) in $d = 2 + \varepsilon$ dimensions
- Numerics:
  - To compute p for transition between quantum Hall plateau

Hamiltonian

- 2D electrons in a random potential, perpendicular magnetic field and with short-ranged e-e interaction

$$\mathscr{H} = \int d^2 r \psi^\dagger(r) \left[\frac{(-i\nabla - eA_{st})^2}{2m} + V_{\rm dis}(r) \right] \psi(r) + \frac{1}{2} \int d^2 r d^2 r' U(r-r') \psi^\dagger(r) \psi(r) \psi^\dagger(r') \psi(r')$$

  - A_{st} – static magnetic field
  - $V_{\rm dis}(r)$ – disorder potential (white noise)
  - $U(r-r')$ – electron-electron interaction (short-ranged)

N.B.: Short-ranged interaction (in singlet channel) is irrelevant

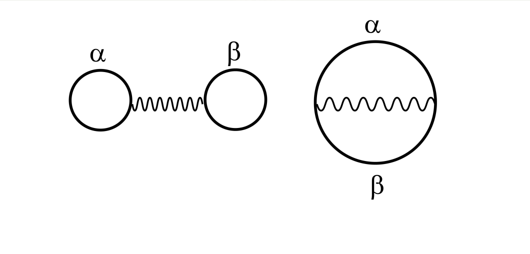
Castellani, Di Castro, Lee, Ma, Phys. Rev. B 30, 527 (1984)
Baranov, Pruisken, Europhys. Lett. 31, 543 (1995)
Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996)

1st order in U correction to Ω

- Convenient basis $\{\varepsilon_\alpha, \phi_\alpha(r)\}$:

$$\left[\frac{1}{2m}(-i\nabla - eA_{st})^2 + V_{\text{dis}}(r)\right]\phi_\alpha(r) = \varepsilon_\alpha \phi_\alpha(r)$$

- First order interaction correction to thermodynamic potential



1st order in U correction to Ω

- First order interaction correction to thermodynamic potential

$$\langle \beta \Omega_1 \rangle = \frac{1}{2\Delta^2} \int dE d\omega n_f(E) n_f(E+\omega) \int dr_1 dr_2 U(r_1 - r_2) \mathscr{K}_1(r_1, r_2, E, \omega)$$

where

$$\mathscr{K}_1 = \Delta^2 \left\langle \sum_{\alpha\beta} \Big| \phi_\alpha(r_1)\phi_\beta(r_2) - \phi_\alpha(r_2)\phi_\beta(r_1) \Big|^2 \delta(E+\omega-\varepsilon_\alpha)\delta(E-\varepsilon_\beta) \right\rangle$$

N.B.:

- $\Delta = 1/\nu_d L^d$ – single particle level spacing
- $n_f(E)$ – Fermi function

Scaling of $\mathscr{K}_1$ for $L \to \infty$

$$\mathscr{K}_1(r_1, r_2, E, \omega) = L^{-2d} \left(\frac{|r_1 - r_2|}{L_\omega} \right)^{\mu_2} \tilde{\mathscr{K}}_1 \left(\frac{|r_1 - r_2|}{L_\omega} \right),$$

$$\tilde{\mathscr{K}}_1(x) = \begin{cases} 1, & x \ll 1, \\ x^{-\mu_2}, & x \gg 1 \end{cases}$$

Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996)

where $L_\omega = L(\Delta/|\omega|)^{1/d}$

N.B.: Exponent $\mu_2 > 0$ contrary to the exponent of the inverse participation ratio

Scaling of the short-ranged interaction

- e-e interaction

$$U(R) = U_0 \left[1 + \left(\frac{R}{a}\right)^2\right]^{-\lambda/2}$$

- First order correction to thermodynamic potential

$$\langle \beta \Omega_1 \rangle = \nu_d \int \frac{dE d\omega}{\Delta} n_f(E) n_f(E+\omega) u(L_\omega)$$

where

$$u(L_\omega) = U_0 a^d \begin{cases} (a/L_\omega)^{\mu_2}, & d+\mu_2 < \lambda \\ (a/L_\omega)^{\mu_2} \ln \frac{L_\omega}{a}, & \lambda = d+\mu_2 \\ (a/L_\omega)^{\lambda-d}, & d < \lambda < d+\mu_2 \end{cases}$$

N.B.: Singlet interaction is irrelevant for $\lambda > d$

Baranov, Pruisken, Europhys. Lett. 31, 543 (1995)

Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996)

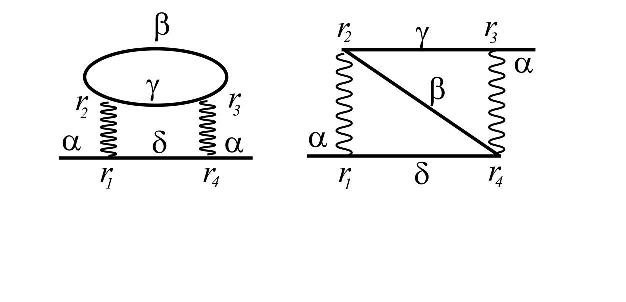
Self-energy – second order in U

- Averaged self-energy

$$\mathrm{Im}\,\Sigma^R(E,\omega) = \Delta \left\langle \sum_\alpha \mathrm{Im}\,\Sigma^R_\alpha(\omega)\delta(E-\varepsilon_\alpha) \right\rangle .$$

Abrahams, Anderson, Lee, Ramakrishnan (1981)

- Estimate for inverse dephasing time ($1/\tau_\phi = D/L_\phi^2$): $\quad \dfrac{1}{\tau_\varphi} \sim -\mathrm{Im}\,\Sigma^R(0,0)$



N.B.: $\mathrm{Im}\,\Sigma^R(0,0)$ is in fact out-scattering rate (Blanter, Phys. Rev. B 54, 12807 (1996))

Self-energy – second order in U

$$\operatorname{Im}\Sigma^R(E,\omega) = -\pi \int dr_1 dr_2 dr_3 dr_4 U(r_1-r_2)U(r_3-r_4) \int \frac{d\omega_1 d\omega_2}{\Delta^3}$$
$$\times \Big\{ n_f(\omega+\omega_1)[1-n_f(\omega+\omega_2)] \Big] + [n_f(\omega+\omega_2)-n_f(\omega+\omega_1)]n_f(\omega+\omega_1-\omega_2) \Big\}$$
$$\times \mathscr{K}_2(r_1,r_2,r_3,r_4,E,\omega,\omega_1,\omega_2)$$

where

$$\mathscr{K}_2 = \frac{\Delta^4}{8} \Big\langle \sum_{\alpha\beta\gamma\delta} \Big[\phi^*_\alpha(r_1)\phi^*_\beta(r_2) - \phi^*_\alpha(r_2)\phi^*_\beta(r_1)\Big] \Big[\phi_\delta(r_1)\phi_\gamma(r_2) - \phi_\delta(r_2)\phi_\gamma(r_1)\Big]$$
$$\times \Big[\phi^*_\gamma(r_3)\phi^*_\delta(r_4) - \phi^*_\gamma(r_4)\phi^*_\delta(r_3)\Big] \Big[\phi_\alpha(r_4)\phi_\beta(r_3) - \phi_\alpha(r_3)\phi_\beta(r_4)\Big]$$
$$\times \delta(E-\varepsilon_\alpha)\delta(\omega+\omega_1-\varepsilon_\beta)\delta(\omega+\omega_2-\varepsilon_\gamma)\delta(\omega+\omega_1-\omega_2-\varepsilon_\delta) \Big\rangle$$

Scaling of $\mathscr{K}_2$

- Parametrization of distances

$$\rho = |r_1 - r_2|, \qquad \rho' = |r_3 - r_4|,$$
$$R' = (r_1 + r_3)/2, \qquad R'' = (r_2 + r_4)/2,$$
$$R = R' - R''$$

- For $\rho, \rho' \ll R \ll L_\Omega \quad (\Omega = \omega_2 - \omega_1)$

$$\mathscr{K}_2(r_1, r_2, r_3, r_4, 0, 0, \omega_1, \omega_2) = L^{-4d} \left(\frac{\rho}{R} \frac{\rho'}{R} \right)^{\mu_2} \widetilde{\mathscr{K}_2} \left(\frac{R}{L_\Omega} \right)$$

where

$$\widetilde{\mathscr{K}_2}(x) = \begin{cases} x^\alpha, & x \ll 1 \\ 1, & x \gg 1 \end{cases}$$

Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996)

Self-energy – second order in U

$$\operatorname{Im}\Sigma^R(0,0) \sim v_d^3 \int d\Omega \frac{\Omega}{\sinh(\Omega/T)} \int d^d R\, u^2(R) \mathscr{F}\left(\frac{R}{L_\Omega}\right)$$

- Cases $\lambda > d+\mu_2$ or $d+\mu_2 > \lambda > d$:

$$\frac{1}{\tau_\phi} \sim \begin{cases} T^{1+2\varkappa/d}, & d+\alpha > 2\varkappa, \\ T^{1+2\varkappa/d} \ln T, & d+\alpha = 2\varkappa, \\ T^{2+\alpha/d}, & d+\alpha < 2\varkappa. \end{cases}$$

  where $\varkappa = \min\{\mu_2, \lambda - d\}$

- Case $\lambda = d+\mu_2$

$$\frac{1}{\tau_\phi} \sim \begin{cases} T^{1+2\mu_2/d} \ln^2 T, & d+\alpha > 2\mu_2, \\ T^{1+2\mu_2/d} \ln^3 T, & d+\alpha = 2\mu_2, \\ T^{2+\alpha/d}, & d+\alpha < 2\mu_2. \end{cases}$$

What is the value of exponent α?
Lee&Wang conjecture: $\alpha = 0$ (based on numerics) for QH transitions

Results

- Anderson transition in $d = 2 + \varepsilon$:

$$\mu_2 = \sqrt{2\varepsilon}$$ Pruisken, Phys. Rev. B 31, 416 (1995)

$$\alpha = O(\varepsilon^{5/2})$$

- QHE transition:

$$\mu_2 = 0.66 \pm 0.04$$ Lee and Wang, Phys. Rev. Lett. 76, 4014 (1996)

$$\alpha \approx 0$$ (numerics by Bera, Evers)

Non-linear σ model (fermionic replica)

- Anderson transition in $d = 2+\varepsilon$

$$S_\sigma = \frac{\sigma_{xx}}{8}\int d^d r\, \mathrm{Tr}(\nabla Q)^2 - \frac{\nu_d \omega}{4}\int d^d r\, \mathrm{Tr}\,\Lambda Q$$

Wegner (1979), Efetov, Larkin, Khmelnitskii (1980), McKane, Stone (1981), Pruisken, Schaefer (1982)

- QHE transition

$$S_\sigma = \frac{\sigma_{xx}}{8}\int d^2 r\, \mathrm{Tr}(\nabla Q)^2 - \frac{\sigma_{xy}}{8}\int d^2 r \varepsilon_{ab}\, \mathrm{Tr}\, Q\nabla_a Q \nabla_b Q - \frac{\nu_d \omega}{4}\int d^d r\, \mathrm{Tr}\,\Lambda Q$$

Levine, Libby, Pruisken (1983), Pruisken (1984)

Dynamical matrix field $Q(r) \in U(2n)/U(n)\times U(n)$ obeys the constraint $Q^2(r) = 1$

$Q_{ab}^{p_1 p_2}$ $\quad$ $p_1, p_2 = \pm 1$ retarded/advanced indices; $a, b = 1, \ldots, n$ replica indices $\quad$ $\Lambda_{ab}^{p_1 p_2} = p_1 \delta^{p_1 p_2}\delta_{ab}$

$\mathscr{K}_2$ in terms of NLσM

$$\mathscr{K}_2 \sim \Delta^4 \left\langle \operatorname{Im} G^R_E(r_4,r_1) \operatorname{Im} G^R_{\omega+\omega_1}(r_3,r_2) \left[\operatorname{Im} G^R_{\omega+\omega_1-\omega_2}(r_1,r_4) \operatorname{Im} G^R_{\omega+\omega_2}(r_2,r_3) - \operatorname{Im} G^R_{\omega+\omega_1-\omega_2}(r_1,r_3) \operatorname{Im} G^R_{\omega+\omega_2}(r_2,r_4) \right] \right\rangle$$

$$\mathscr{K}_2 \sim \left\langle \operatorname{tr}\left[\Lambda Q_{ab}(r_1)\Lambda Q_{ba}(r_4)\right] \operatorname{tr}\left[\Lambda Q_{cd}(r_2)\Lambda Q_{dc}(r_3)\right] + \operatorname{tr}\left[\Lambda Q_{ab}(r_1)\Lambda Q_{bc}(r_4)\Lambda Q_{cd}(r_2)\Lambda Q_{da}(r_3)\right] \right\rangle$$

N.B.:

- Replica indices a, b, c and d are all different (different single particle states)
- Distances $|r_i - r_j| \gtrsim l$
- tr – trace over retarded/advance space only
- Non $U(n) \times U(n)$ invariant expression!

$U(n) \times U(n)$ invariant expression for $\mathscr{K}_2$

- NLσM action is invariant under global unitary transformation:

$$Q(r) \to U^{-1} Q(r) U, \qquad U_{ab}^{p_1 p_2} = \delta^{p_1 p_2} U_{ab}^{p_1}$$

- For $r_1 = r_2 = r_3 = r_4$ averaging over unitary group $U(n) \times U(n)$ leads to

$$\mathscr{K}_2 \sim \sum_j R_j O_j$$

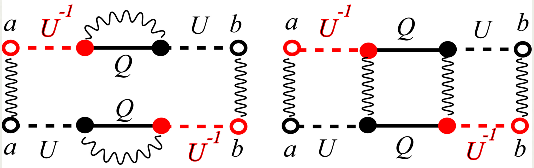
Averaging over unitary group – example

$$Q^{++}_{ab} Q^{++}_{ba} \to \langle U^{-1}_{a\alpha} Q^{++}_{\alpha\beta} U_{\beta b} U^{-1}_{b\beta'} Q^{++}_{\beta'\alpha'} U_{\alpha' a} \rangle_U$$

$$\begin{aligned}
\langle U^{-1}_{a\alpha} U_{\beta b} \rangle_U &= V_1 \delta_{ab} \delta_{\alpha\beta}, \\
\langle U^{-1}_{a\alpha} U_{\beta b} U^{-1}_{c\gamma} U_{\delta d} \rangle_U &= V_{1,1} \left[\delta_{ab} \delta_{\alpha\beta} \delta_{cd} \delta_{\gamma\delta} + \delta_{ad} \delta_{\alpha\delta} \delta_{bc} \delta_{\beta\gamma} \right] \\
&+ V_2 \left[\delta_{bc} \delta_{\alpha\beta} \delta_{da} \delta_{\gamma\delta} + \delta_{ab} \delta_{\alpha\delta} \delta_{cd} \delta_{\beta\gamma} \right]
\end{aligned}$$

where

$$V_1 = \frac{1}{n}, \qquad V_{1,1} = \frac{1}{n^2 - 1}, \qquad V_2 = -\frac{1}{n(n^2-1)}$$



$V_2 (\mathrm{tr}\, Q^{++})^2$ $\qquad$ $V_{1,1} \mathrm{tr} (Q^{++})^2$

$U(n) \times U(n)$ invariant expression for $\mathcal{K}_2$

The eight operators invoved are

$$
\begin{aligned}
O_4 &= \operatorname{Tr}[\Lambda, Q]^4, & R_4 &= \frac{-3+13n+16n^2+4n^3}{8n^2(1+n)^2(-6+n+4n^2+n^3)}, \\
O_{2,1,1} &= \operatorname{Tr}[\Lambda, Q]^2(\operatorname{Tr}\Lambda Q)^2, & R_{2,1,1} &= -\frac{3+2n}{4(-1+n)n^2(1+n)^2(3+n))}, \\
O_{3,1} &= \operatorname{Tr}[\Lambda, Q]^2\{\Lambda, Q\}\operatorname{Tr}\Lambda Q, & R_{3,1} &= -\frac{-7+3n+12n^2+4n^3}{4n^2(-1+n^2)^2(6+5n+n^2))}, \\
O_{2,2} &= \operatorname{Tr}[\Lambda, Q]^2\operatorname{Tr}[\Lambda, Q]^2, & R_{2,2} &= \frac{-3-21n+20n^2+32n^3+8n^4}{32n^2(-1+n^2)^2(6+5n+n^2)}, \\
O_{1,1,1,1} &= (\operatorname{Tr}\Lambda Q)^4, & R_{1,1,1,1} &= \frac{5+5n+2n^2}{8n^2(-1+n^2)^2(6+5n+n^2)}, \\
O_2 &= \operatorname{Tr}[\Lambda, Q]^2, & R_2 &= \frac{2(-2+2n+n^2)}{(-1+n)^2n(3+4n+n^2)}, \\
O_{1,1} &= (\operatorname{Tr}\Lambda Q)^2, & R_{1,1} &= \frac{2+2n-7n^2-7n^3-2n^4}{n^2(-1+n^2)^2(6+5n+n^2)}, \\
O_0 &= \operatorname{Tr}\Lambda^2, & R_0 &= \frac{(-2-n+5n^2+2n^3)}{n(3+n)(-1+n^2)^2}
\end{aligned}
$$

N.B.: Operators $O_j[Q]$ are not eigen operators of RG

Construction of eigen operators

Brezin, Zinn Justin, Le Guillou, (1976); Wegner (1979)

- One-loop background field renormalization, $Q \to T_0^{-1} Q T_0$, yields

$$\langle O_j[T_0^{-1} Q T_0] \rangle = O_j[T_0^{-1} \Lambda T_0] - 2Y \left[\sum_k S_{jk} O_k[T_0^{-1} \Lambda T_0] + c_j \right]$$

where

$$S = \begin{pmatrix} 4n & 0 & 4 & 1 & 0 & 16n & 0 \\ 0 & 4n & 4 & 1/2 & 2 & 0 & 8n^2 \\ 3 & 3 & 4n & 0 & 0 & 8 & 16n \\ 8 & 4 & 0 & 4n & 0 & 32+16n^2 & 0 \\ 0 & 3 & 0 & 0 & 4n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2n & 2 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 2n \end{pmatrix}, \qquad c = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8n^2 \\ 0 \end{pmatrix},$$

and

$$Y = \frac{1}{\sigma_{xx}} \int \frac{d^d p}{p^2}$$

Pruisken, (1985) and to be published

Construction of eigen operators

- Eigen operators

$$
\begin{pmatrix} E_{-12+8n} \\ E_{-4+8n} \\ E_{8n} \\ E_{4+8n} \\ E_{12+8n} \\ E_{-2+4n} \\ E_{2+4n} \end{pmatrix}
=
\begin{pmatrix}
-\frac{1}{8} & -\frac{1}{8} & \frac{1}{6} & \frac{1}{32} & \frac{1}{24} & \frac{(-2+n)^2}{(-10+4n)} & \frac{2-8n+3n^2}{15-6n} & \frac{2(-2+n)^2 n^2}{15-16n+4n^2} \\
\frac{3}{8} & -\frac{3}{8} & 0 & -\frac{3}{32} & \frac{3}{8} & -\frac{3(-2+n)^2}{(-6+4n)} & -\frac{3(-2+n^2)}{(-3+2n)} & -\frac{6(-2+n)^2 n^2}{3-8n+4n^2} \\
0 & 0 & -\frac{1}{3} & \frac{1}{8} & \frac{1}{6} & \frac{4n^3}{-1+4n^2} & \frac{8-20n^2}{-3+12n^2} & \frac{8n^4}{-1+4n^2} \\
-\frac{3}{8} & \frac{3}{8} & 0 & -\frac{3}{32} & \frac{3}{8} & -\frac{3(2+n)^2}{6+4m} & \frac{3(-2+n^2)}{3+2n} & -\frac{6n^2(2+n)^2}{3+8n+4n^2} \\
\frac{1}{8} & \frac{1}{8} & \frac{1}{6} & \frac{1}{32} & \frac{1}{24} & \frac{(2+n)^2}{10+4n} & \frac{2+8n+3n^2}{15+6m} & \frac{2n^2(2+n)^2}{15+16n+4n^2} \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & \frac{1}{2} & \frac{2n^2}{1-2n} \\
0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & \frac{2n^2}{1+2n}
\end{pmatrix}
\begin{pmatrix} O_4 \\ O_{2,1,1} \\ O_{3,1} \\ O_{2,2} \\ O_{1,1,1,1} \\ O_2 \\ O_{1,1} \\ O_0 \end{pmatrix}
$$

Pruisken (1985) and unpublished

- One-loop renormalization of eigen operators

$$
\langle E_\lambda[Q] \rangle = (1 - \lambda Y) E_\lambda[\Lambda], \qquad Y = \frac{1}{\sigma_{xx}} \int \frac{d^d p}{p^2}
$$

$\mathscr{K}_2$ in terms of eigen operators

$$\mathscr{K}_2 \sim \langle a_{8n} E_{8n}[Q] + a_{4+8n} E_{4+8n}[Q] + a_{12+8n} E_{12+8n}[Q] \rangle$$

where

$$\begin{aligned} a_{8n} &= \frac{4n^2}{4n^2(n^2-1)^2} \\ a_{4+8n} &= -\frac{4n^2+8n+3}{6n^2(n+1)^2(n^2+n-2)} \\ a_{12+8n} &= \frac{4n^2+16n+15}{2n^2(n+1)^2(n+2)(n+3)} \end{aligned}$$

- Only eigen operators (in replica limit $n=0$) with $\lambda \geqslant 0$ are involved
- Exponent α is determined by the anomalous dimension of eigen operator with the smalest λ

Anderson transition in $d = 2 + \varepsilon$

- β-function

$$-\frac{dt}{d\ln L} = \beta(t) = \varepsilon t - 2nt^2 - 2(n^2+1)t^3 - (3n^2+7)nt^4 - \dots$$

where $t = 1/(2\pi\sigma_{xx})$.

Brézin, Hikami, Zinn Justin (1980), Hikami (1983)

- γ-functions (anomalous dimensions)

$$\gamma_{E_\lambda}(t) = -\lambda\left[t + \frac{3}{2}(n^2+1)t^3 + \left(\frac{n(n^2+7)}{3} + c_\lambda\zeta(3)\right)t^4 + \dots\right],$$

Pruisken (1985), Höf, Wegner (1986), Wegner (1986)

where

λ	$\mp 12 + 8n$	$\mp 4 + 8n$	$8n$	$\mp 2 + 4n$
c_λ	$\pm 3(2 \mp n)(3 \mp n)$	$\pm(6 \mp 13n + n^2)$	$\frac{21n}{2}$	$\pm\frac{1}{2}(1 \mp n)(3 \mp n)$

Anderson transition in $d = 2 + \varepsilon$ in replica limit ($n = 0$)

- β-function

$$-\frac{dt}{d\ln L} = \beta(t) = \varepsilon t - 2t^3 - 6t^5 - \dots$$

Brézin, Hikami, Zinn Justin (1980), Hikami (1983), Bernreuter, Wegner (1986)

- γ-functions (anomalous dimensions)

$$\begin{aligned}
\gamma_{E_0}(t) &= O(t^5) \\
\gamma_{E_4}(t) &= -4\left[t + \frac{3}{2}t^3 - 6\zeta(3)t^4 + \dots\right] \\
\gamma_{E_{12}}(t) &= -12\left[t + \frac{3}{2}t^3 - 18\zeta(3)t^4 + \dots\right] \\
\gamma_{E_2}(t) &= -2\left[t + \frac{3}{2}t^3 - 3\zeta(3)t^4 + \dots\right]
\end{aligned}$$

Pruisken (1985), Höf, Wegner (1986), Wegner (1986)

$$t_* = \sqrt{\varepsilon/2}\left(1 - \frac{3\varepsilon}{4}\right), \qquad \mu_2 = \sqrt{2\varepsilon}, \qquad \alpha = O(\varepsilon^{5/2})$$

Transitions between quantum Hall plateau

Numerics by Bera and Evers:

- exponent $\mu_2 \approx 0.8$
- exponent $\alpha \approx 0$

to be continued ...

Conclusions

- In general, T dependence of $1/\tau_\varphi$ in the presence of the short-ranged (singlet) e-e interaction is determined by anomalous dimensions μ_2 and α of the eigen operators E_{2+4n} and E_{8n}, respectively.

- Since $\alpha \approx 0$ for i) Anderson transition in $d = 2 + \varepsilon$ (theoretical evidence) and ii) for QH transition (numerical evidence) T dependence of $1/\tau_\varphi$ is determined by the exponent μ_2 alone:

$$\frac{1}{\tau_\varphi} \sim T^{1+2\mu_2/d}$$