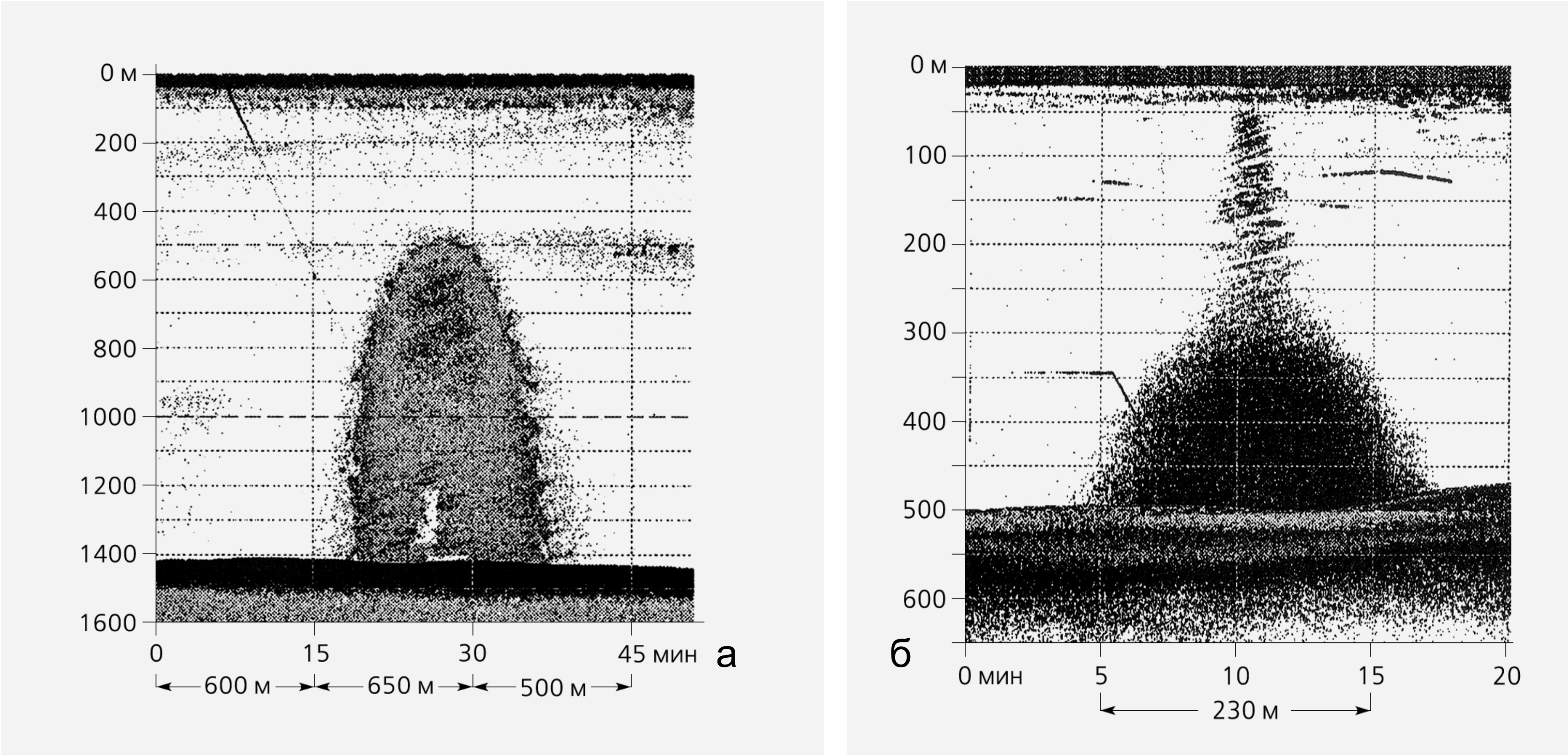
Hydrodynamics of laminar bubble chains

Byalko A.V.
Landau Institute for theoretical physics
& “Priroda” journal

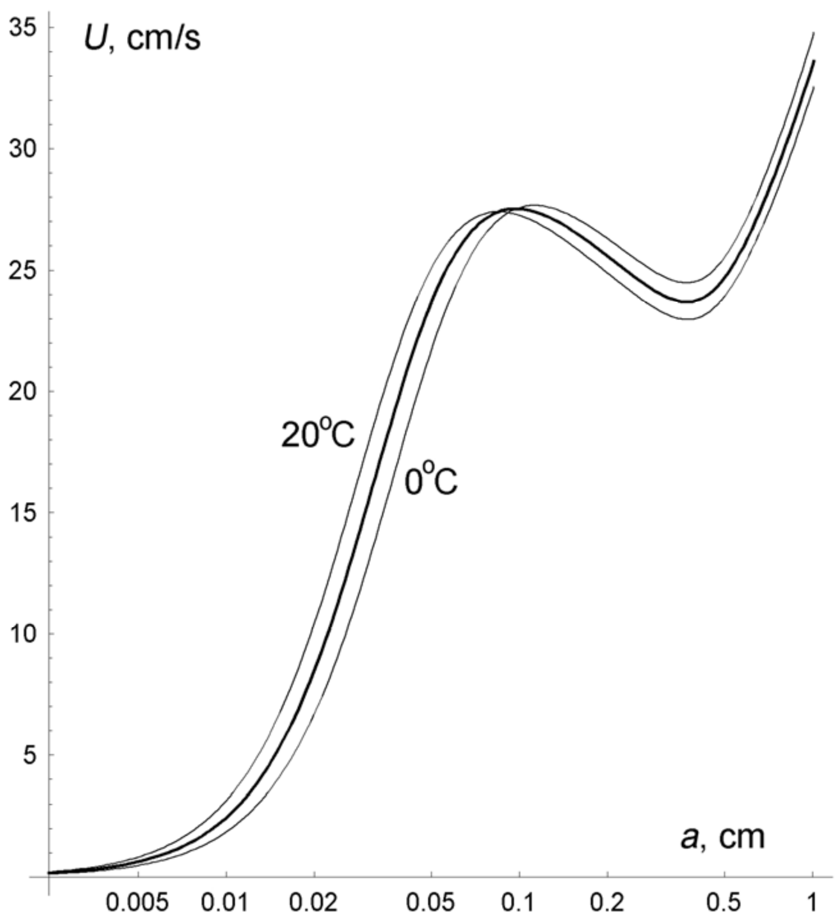
Abstract

A solution of hydrodynamic equations is obtained for the case of laminar bubble chains. Bubble sizes along the chain can either change both ways due to gas-liquid interactions or grow due to pressure decrease. The liquid velocity in region near chain axe occurs to be of the order of single bubble rise velocity.

Our final aim - explanation of methane flows
Exampls: ("Priroda", 2010, №3
Granin N.G, Makarov M.M.,Kucher K.M., Gnatovsky R.Yu.)



a) Methane flow "Sankt-Peterbug", lake Baikal, October 2005.
b) Methane flow «Stupa», Baikal, cap Kadilny, August 2007.

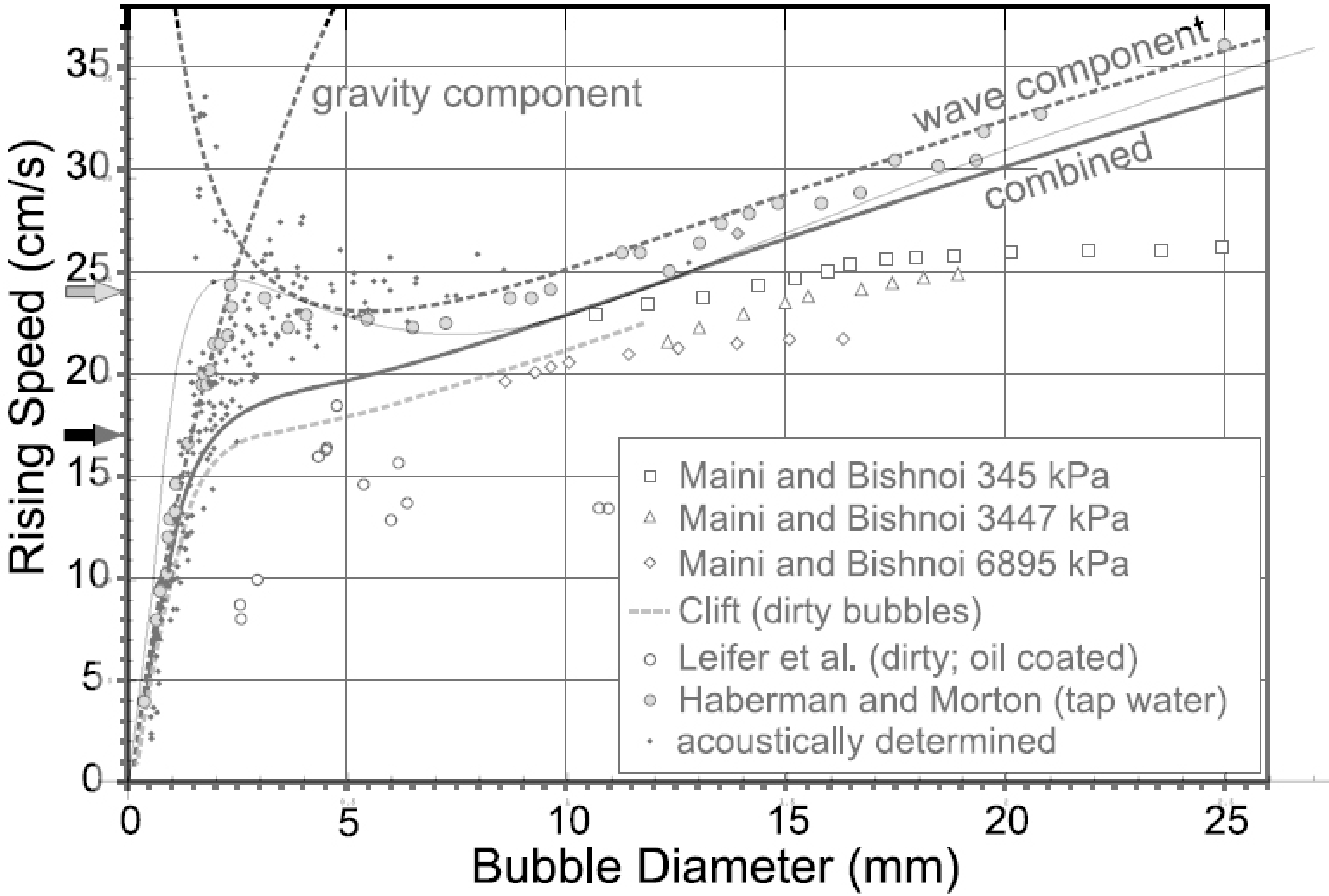


$$L_\nu = \nu^{2/3} g^{-1/3} = 0.0056 \text{ cm}$$

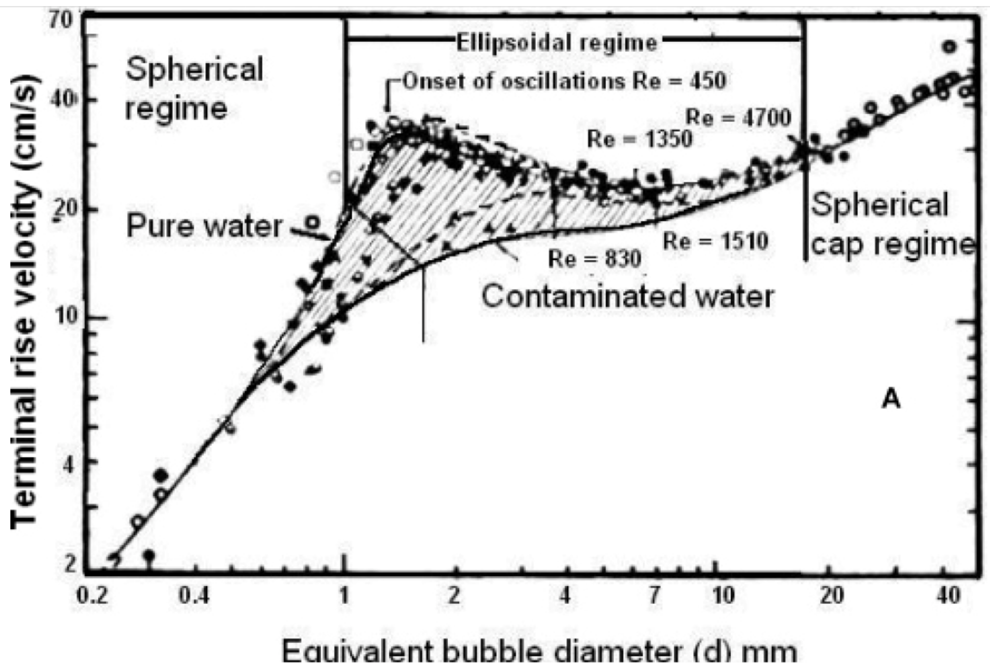
$$L_\sigma = \sqrt{\sigma/\varrho g} = 0.207 \text{ cm}$$

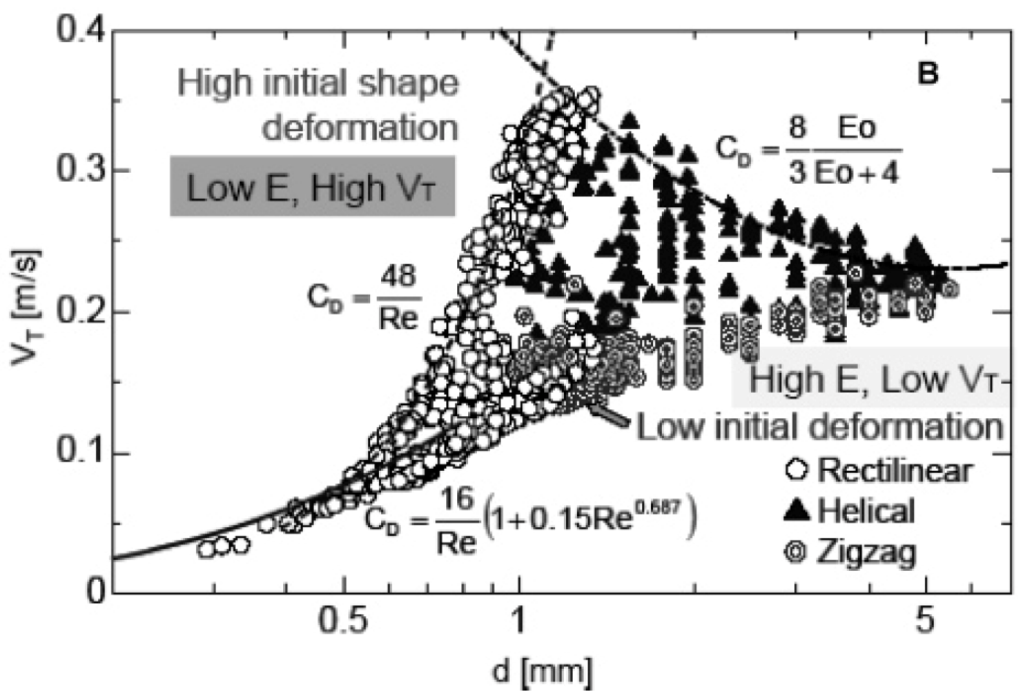
Rodrigue D. A General Correlation for the Rise Velocity of Single Gas Bubbles// The Canadian Journal of Chemical Engineering, 2004, V. 82, 382—386:

$$U = \frac{g a^2}{3\nu} \frac{\left[1 + 7.817\, 10^{-4} \left(\frac{L_\nu}{L_\sigma}\right)^{\frac{33}{10}+\frac{146}{99}} \left(\frac{a}{L_\nu}\right)^{8*\frac{73}{99}}\right]^{\frac{21}{176}}}{\left[1 + 0.1073 \left(\frac{L_\nu}{L_\sigma}\right)^{\frac{20}{33}} \left(\frac{a}{L_\nu}\right)^{\frac{80}{33}}\right]^{\frac{10}{11}}}$$



McGinnis D. F. et al. Fate of rising methane bubbles in stratified waters: How much methane reaches the atmosphere? Journal of geophysical research, Vol. 111, C09007.





Amol A. Kulkarni and Jyeshtharaj B. Joshi. Bubble Formation and Bubble Rise Velocity in Gas-Liquid Systems: A Review. Ind. Eng. Chem. Res. 2005, 44, 5873-5931

(A) Typical trends in rise velocity with bubble size for pure and contaminated liquids (reproduced with permission from Clift, R.; Grace, J. R.; Weber, M. E. Bubbles, Drops, and Particles; Academic Press: London, 1978).

(B) Effect of initial bubble deformation on terminal rise velocity in distilled water. (reproduced with permission from Tomiyama, A. Drag, lift and virtual mass forces acting on a Single Bubble, 3rd International Symposium on Two-Phase Flow Modelling and Experimentation, Pisa, 22-24 September 2004.

E - distortion factor, C_D - drag coefficient

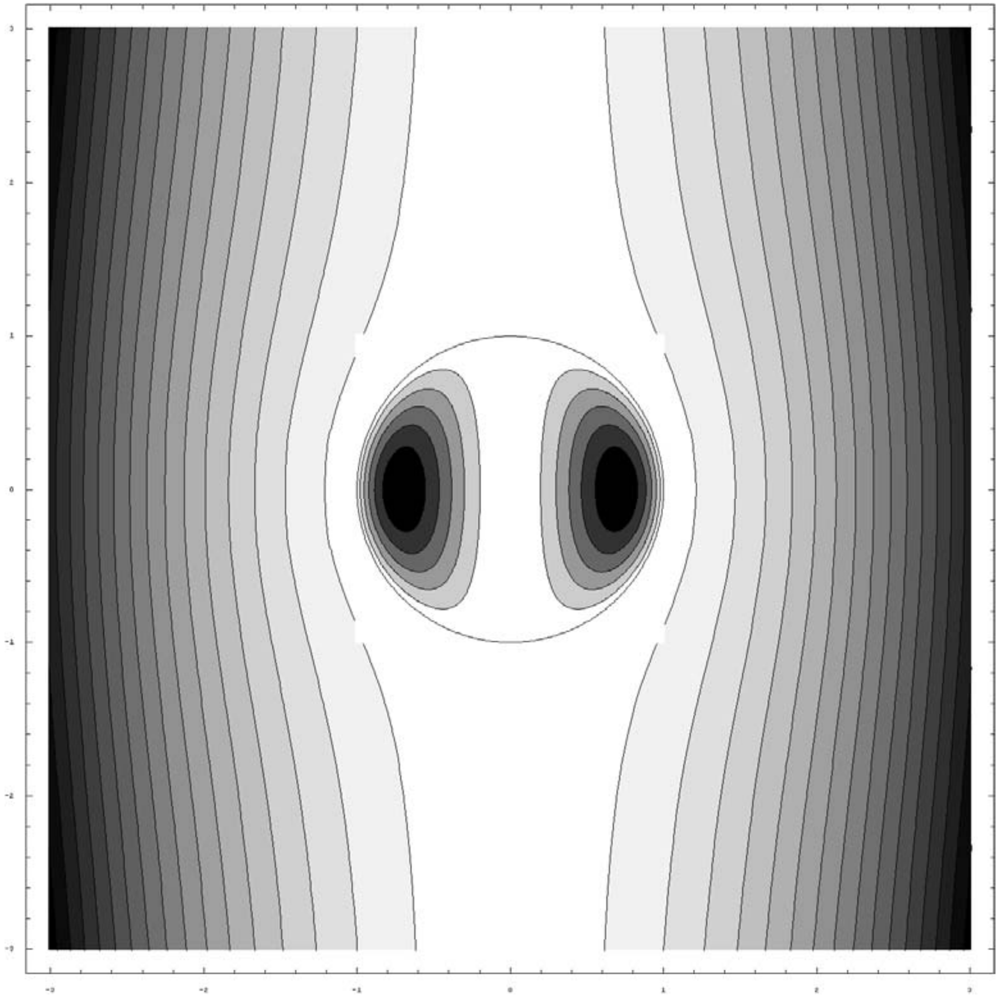
Potential ψ

[Van Dyke M. Perturbation methods in fluid dynamics. Academical Press, 1964.]

$$v_r = \frac{1}{r^2 \sin(\vartheta)} \frac{\partial \psi}{\partial \vartheta}; \qquad v_\vartheta = -\frac{1}{r \sin(\vartheta)} \frac{\partial \psi}{\partial r}.$$

The continuity equation fulfills automatically in such approach.
Dimension of potential coincides with dimension of flow i.e. cm^3/s.
Surfaces of constant potential correspond to lines of flow.
In the bubble system of reference the total flow thru removed surface is zero:

$$\lim_{r\to\infty} 2\pi r^2 \int_0^\pi v_r \sin\vartheta \, d\vartheta = \lim_{r\to\infty} [\psi(r,\pi) - \psi(r,0)] 2\pi = 0.$$



The Stokes—Rybchinsky solution. Lines of flow in around a bubble in the bubble reference system. The term

$$\frac{U}{2} r^2 \sin^2(\vartheta)$$

corresponds to the uniform flow with velocity

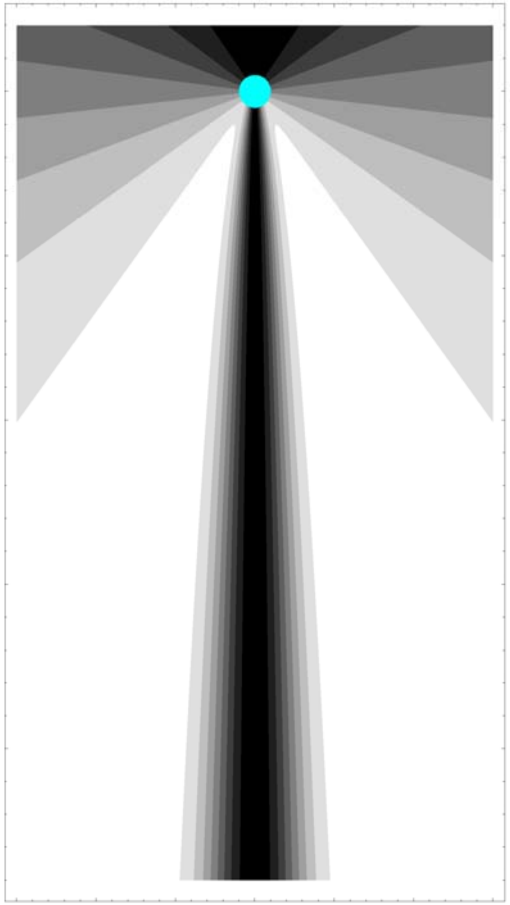
$$U(a) = \frac{ga^2}{3\nu}$$

The drag force equals to Archimedrs force:

$$F = 4\pi\rho\nu U a = \frac{4\pi}{3} g\rho a^3; \ U(a) = \frac{ga^2}{3\nu}.$$

$$\psi_{\text{in}}(r,\vartheta) = \frac{U}{4a^2} r^2 (r^2 - a^2) \sin^2(\vartheta);$$

$$\psi_{\text{out}}(r,\vartheta) = \frac{U}{2} r (r - a) \sin^2(\vartheta).$$



Oseen solution

$$\psi_a(r,\vartheta) = -a\nu(1+\cos\vartheta)\left\{1-\exp\left[-\frac{Ur}{2\nu}(1-\cos\vartheta)\right]\right\}$$

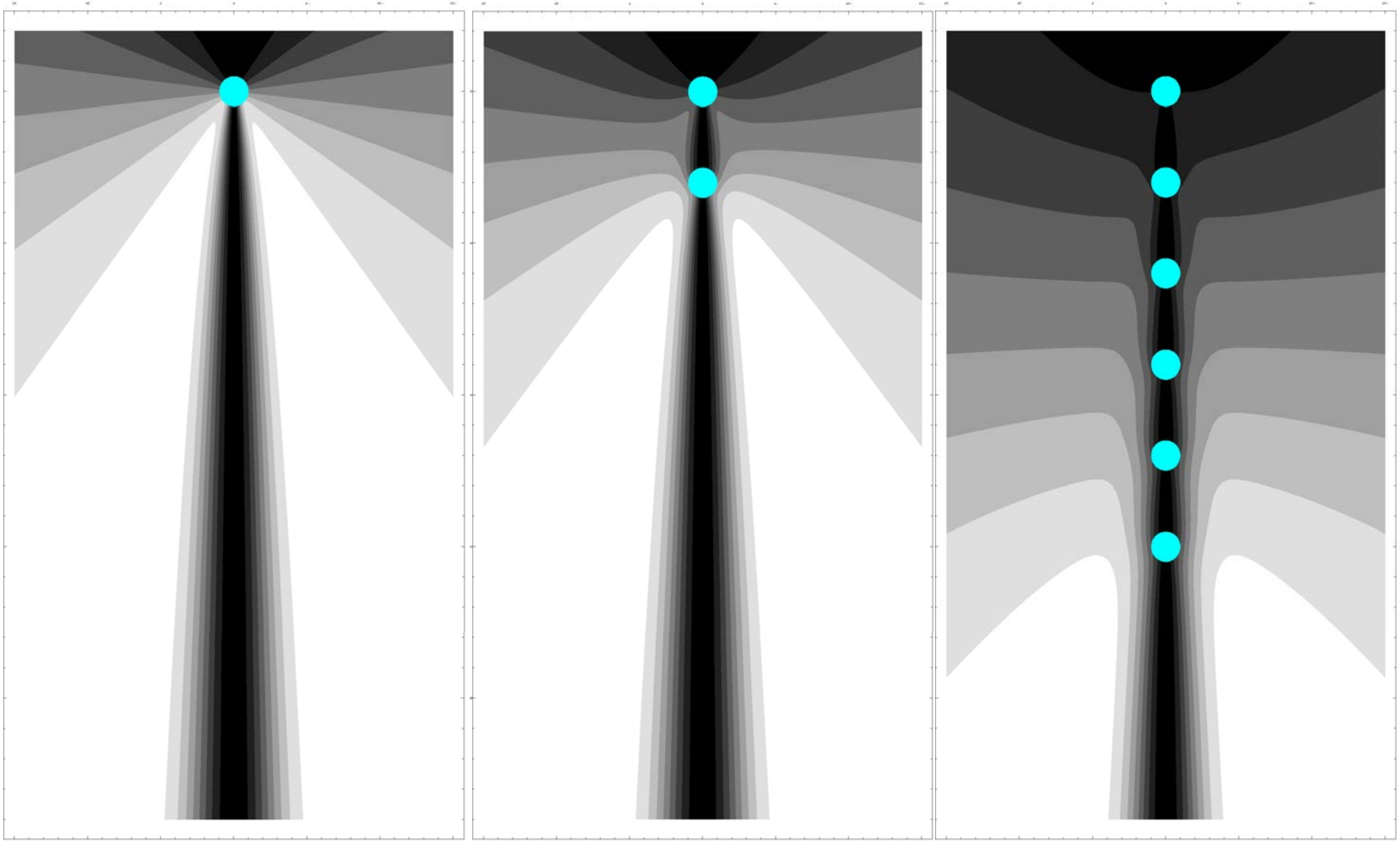
At small distances $r << \nu/U$ the Oseen and Rybchinsky expressions concise, but the Oseen solution does not satisfy boundary conditions at $r = a$.

Simultaneously the Oseen region of applicability is much wider, it can be applied in the region $r >> a$R for any Reynolds numbers R=Ua/ν.

An equivalent form of Oseen solution:

$$\psi_a(r,\vartheta) = -\frac{F(a)}{2\pi\rho U(a)}\cos^2\frac{\vartheta}{2}\left[1-\exp\left(-\frac{U(a)r}{\nu}\sin^2\frac{\vartheta}{2}\right)\right].$$

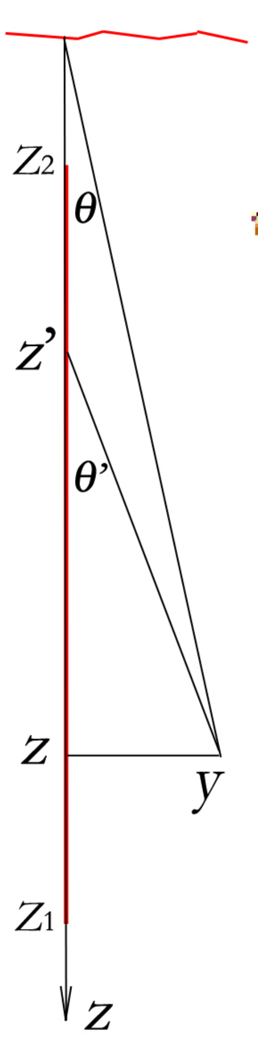
$$F(a) = \frac{4\pi}{3}g\rho a^3.$$



$$\psi(r,\vartheta) = \sum_a \psi_a(r,\vartheta) \approx \frac{1}{\tau}\int_{z_1}^{z_2} \frac{dz'}{U'+V'}\psi_a(r',\vartheta').$$

$$D = \tau(U+V) \gg a$$

Short uniform bubble chain



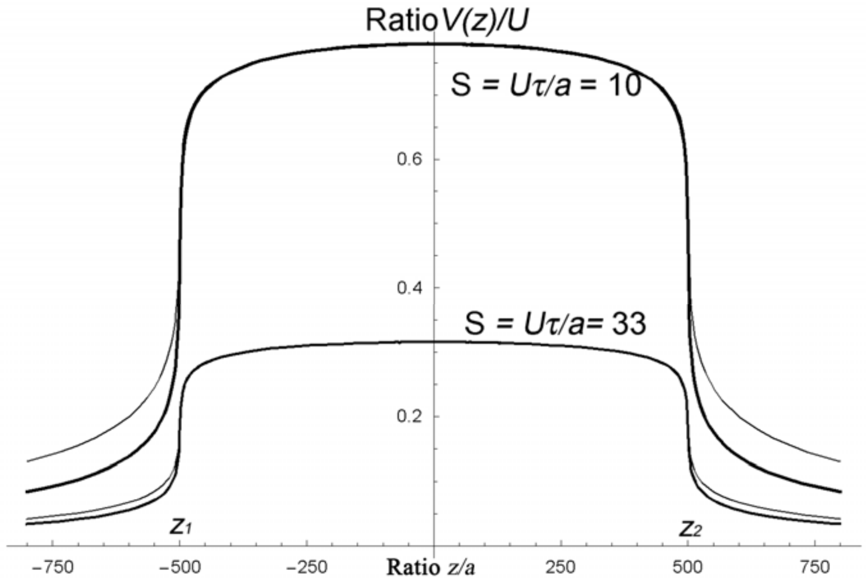
$$\psi(r,\vartheta) = -\frac{2g}{3\tau}\int_{z_2}^{z_1} \frac{a^3 dz'}{U[U+V(z')]}\cos^2\frac{\vartheta'}{2}\left[1-\exp\left(-\frac{Ur'}{\nu}\sin^2\frac{\vartheta'}{2}\right)\right].$$

$$y = r\,\sin\vartheta = r'\sin\vartheta' = (z-z')\,\mathrm{tg}\,\vartheta';$$

$$dz' = \frac{y}{\sin^2\vartheta'}d\vartheta';\;\; r' = \sqrt{y^2+(z-z')^2}.$$

$$V(z) = -\lim_{y\to 0}\frac{\partial\psi(y,z)}{y\partial y} = \frac{g}{3\tau\nu}\lim_{y\to 0}\int_{z_1}^{z_2} dz' \frac{a^3}{[U+V]\sqrt{y^2+(z-z')^2}}.$$

$$V^2 + UV - \frac{ga^3}{3\tau v}L(z) = 0; \quad L(z) = \ln\left(\frac{-z + z_1 + \sqrt{(-z+z_1)^2 + a^2}}{-z + z_2 + \sqrt{(-z+z_2)^2 + a^2}}\right).$$



Functions *V(z)/U* in two approximations
for S = *Uτ/a* = 10 and S = *Ut/a* = 33
z_1=-500; z_2=500 (in bubble sizes)

$$V(z,\tau) = -\frac{U}{2} + \sqrt{\frac{ga^3}{3\tau v}L(z) + \frac{U^2}{4}}$$

$$U = \frac{a^2 g}{3v} \ll \frac{aL}{\tau} \text{ then } V \cong \sqrt{\frac{aU}{\tau}L(z)} < U.$$

$$\frac{a^2}{\tau v} \ll 1, \qquad \text{then } V \cong U\sqrt{\frac{a^2}{3\tau v}L(z)} < U.$$

Laminar flow condition:

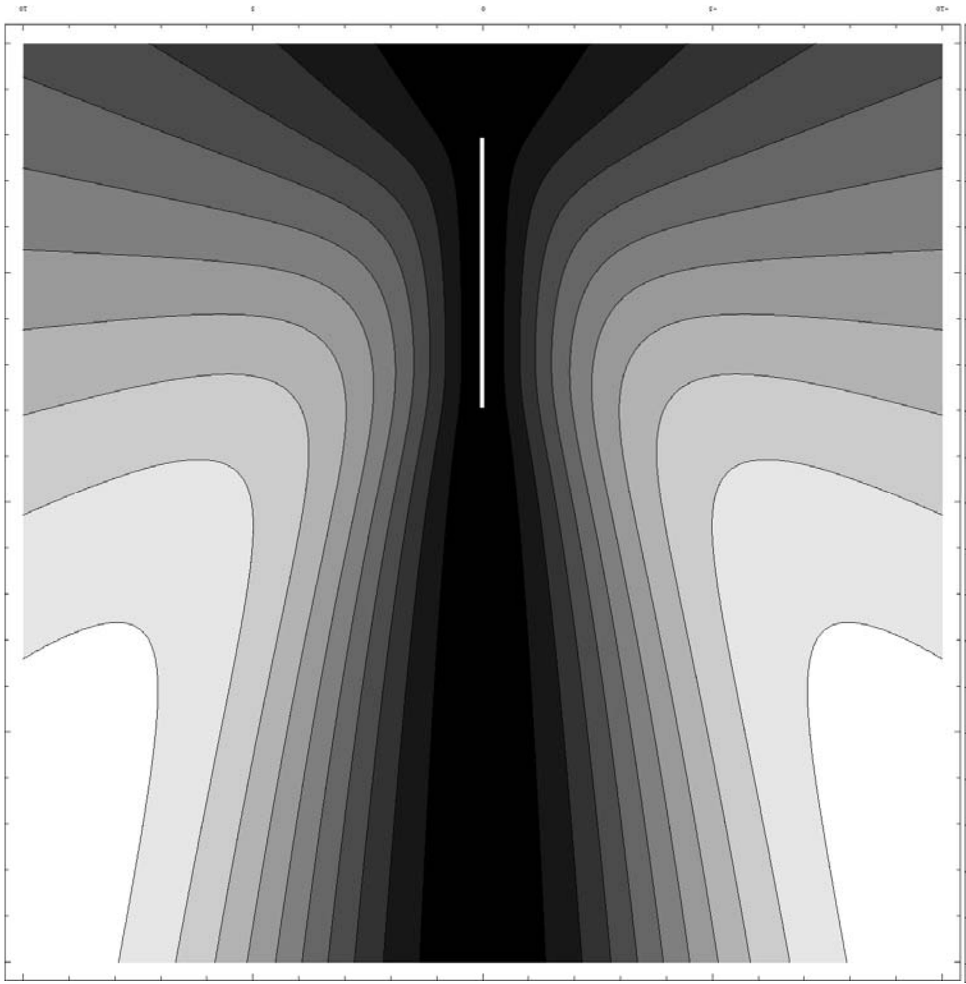
Short uniform bubble chain

$$\psi(r,\vartheta) = -\frac{2g}{3\tau}\int_{z_2}^{z_1} \frac{a^3 dz'}{U[U+V(z')]}\cos^2\frac{\vartheta'}{2}\left[1-\exp\left(-\frac{Ur'}{\nu}\sin^2\frac{\vartheta'}{2}\right)\right].$$

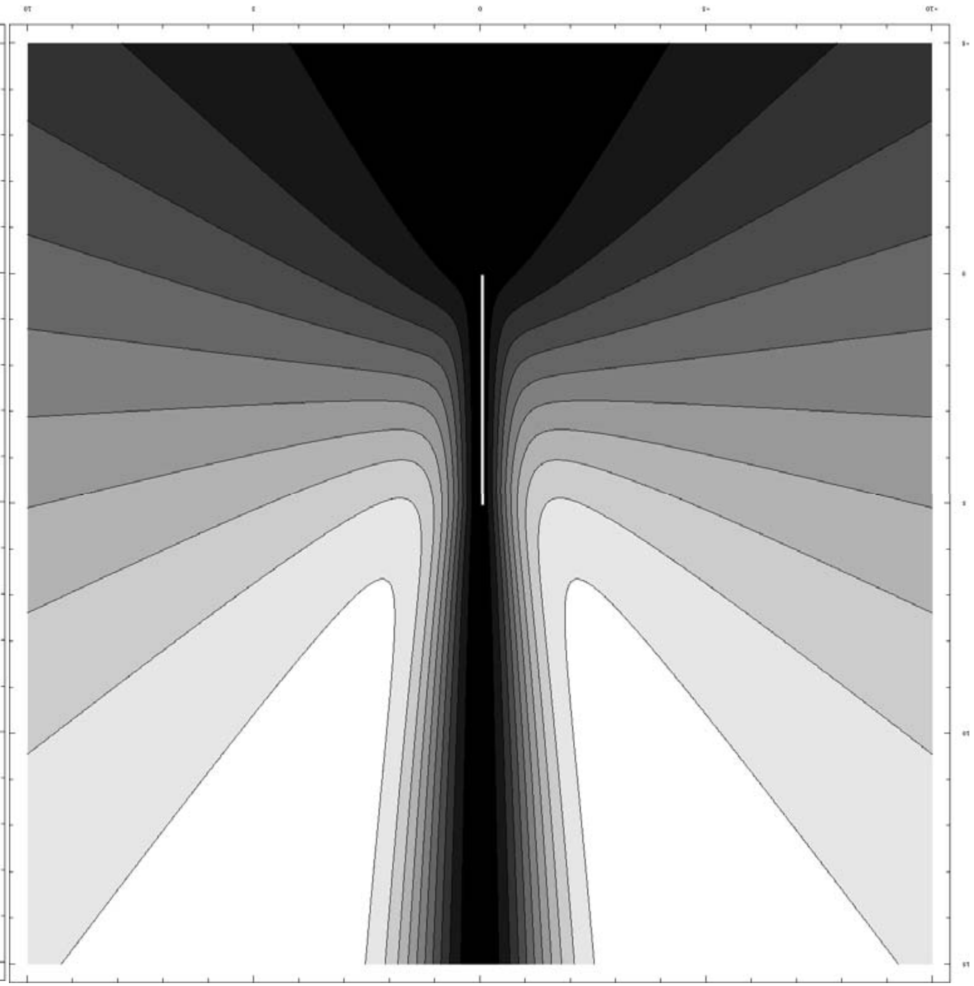
$$\xi = \frac{Uy}{2\nu}\mathrm{tg}\frac{\vartheta'}{2} = \frac{U}{2\nu}\left(\sqrt{(z-z')^2+y^2}-z+z'\right).$$

$$\psi(z,y) = \frac{g a^3 y^2}{6\nu\tau[U(a)+V(z)]}\left[\frac{1-\exp(\xi_1)}{\xi_1} \quad \mathrm{Ei}(\ \ \xi_1) \quad \frac{1-\exp(\xi_2)}{\xi_2} + \mathrm{Ei}(\ \ \xi_2)\right],$$

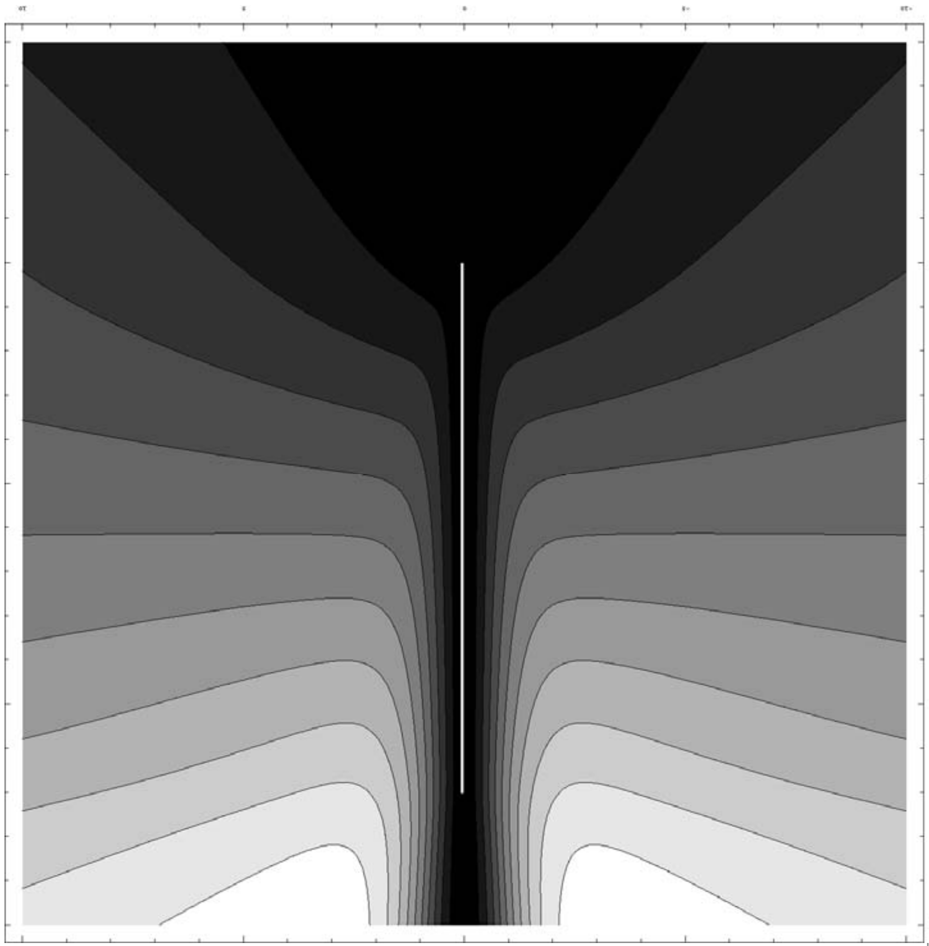
where $\xi_{1,2} = \frac{u}{2\nu}(\sqrt{(z-z_{1,2})^2+y^2}-z+z_{1,2}).$



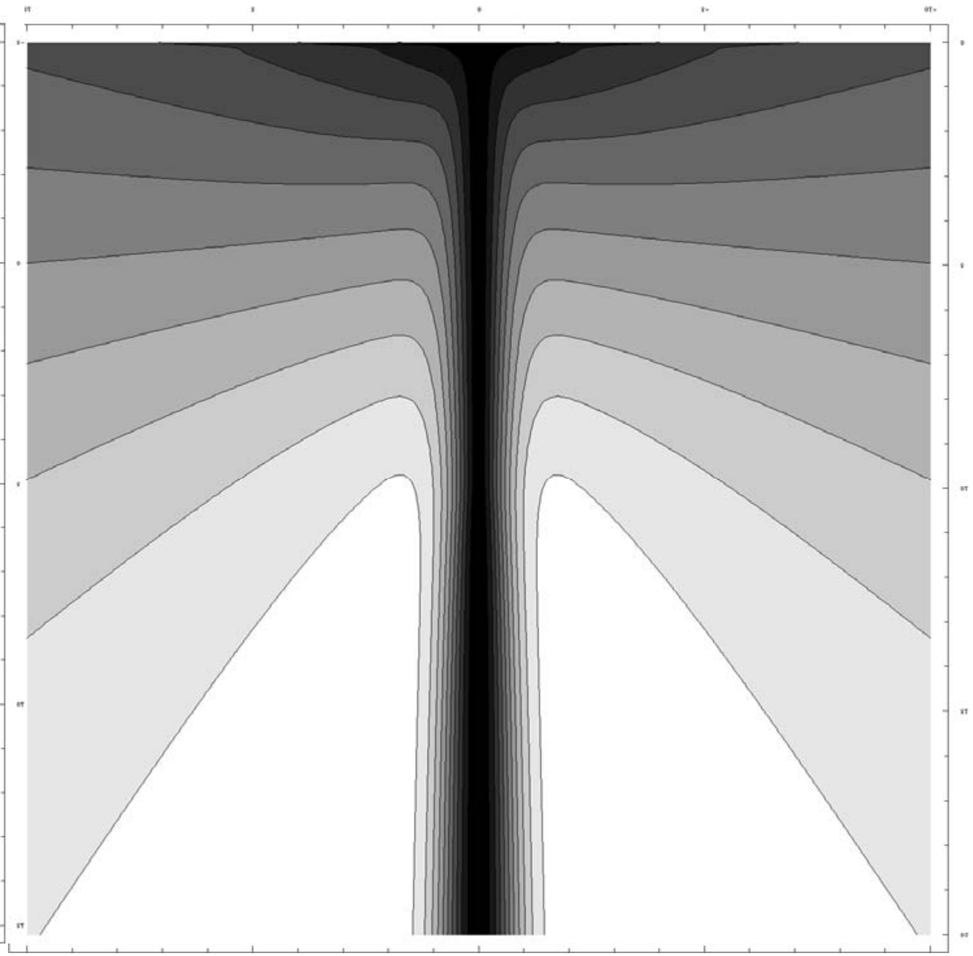
Flow around a short chain (white in center, R=1)



•Flow around a short chain (white in center, R=10, $\Delta z=5$)



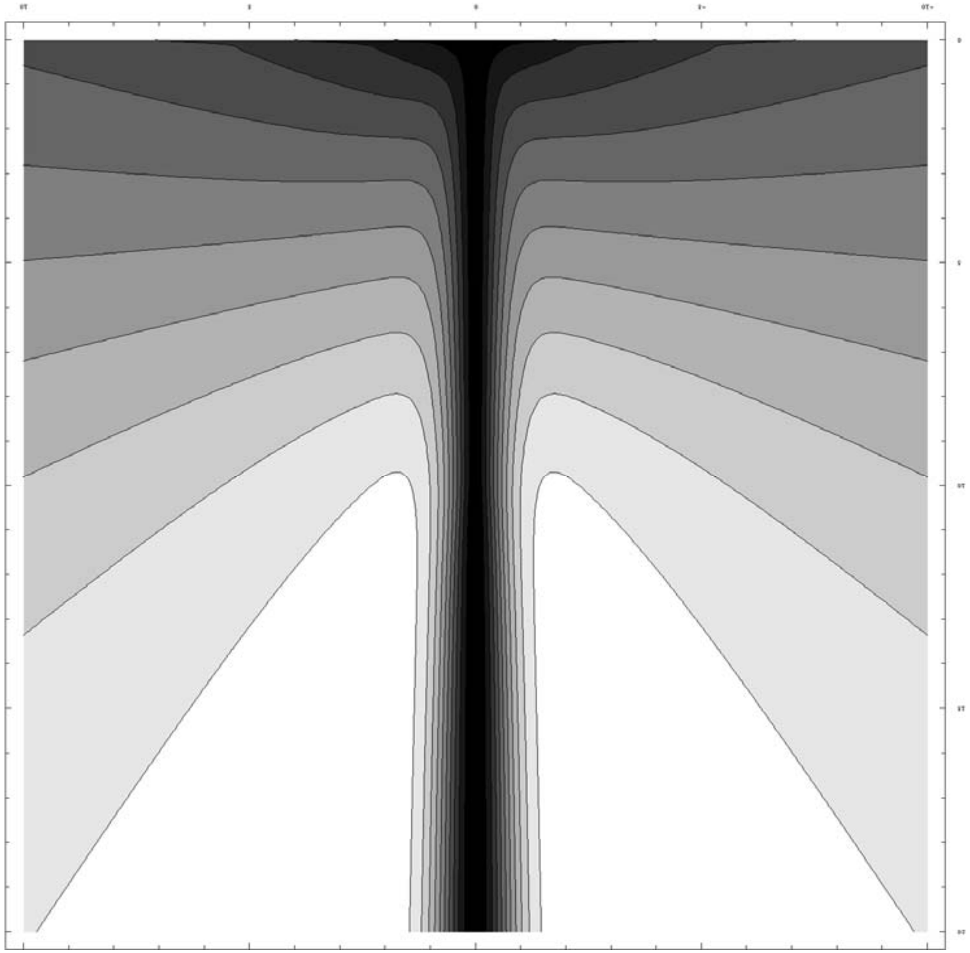
Lines of flow around a chain (white in center, R=10, Δz=12)



Lines of flow for “dying” chain real sizes (10x10 cm);

a_0 = 0.005 cm. z_1 = 10 cm.

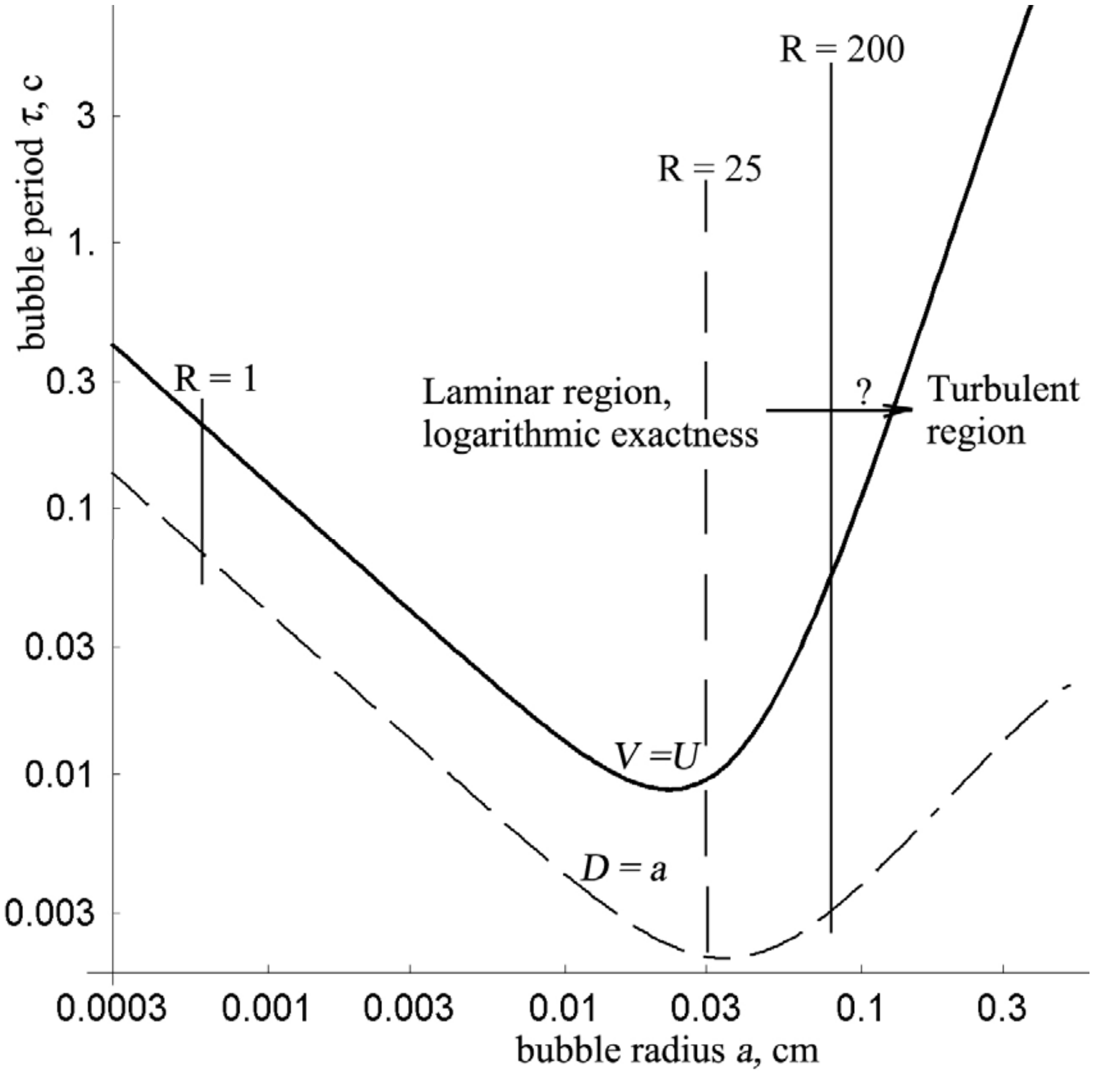
Dying chain



$$a = \left(\frac{\nu K z}{g}\right)^{1/3} ;\ U(z) = \frac{1}{3}\left(\frac{K^2 g z^2}{\nu}\right)^{1/3}$$

The absorption coefficient
K = (1.0—1.05)10^{-3} cm/s.

$$\psi(z,y) = \frac{3\nu^{\frac{5}{3}}}{K^{\frac{1}{3}} g^{\frac{2}{3}} \tau} \int_0^{z_1} \frac{dz'}{z'^{\frac{1}{3}}} \left(1 + \frac{(z-z')}{\sqrt{y^2+(z-z')^2}}\right) \left\{1 - \exp\left[-\frac{1}{6}\left(\frac{K^2 g z^2}{\nu^4}\right)^{\frac{1}{3}} \left[\sqrt{y^2+(z-z')^2} - (z-z')\right]\right]\right\}.$$



Limits of chain solution applicability:

$D = U\tau >> a$: Chain region; $V <= U$: Region of logarithmic exactness.

R = $aU/\nu < 25$: Laminar region; *U(a)* well defined;

25 < R < 200: Laminar region; *U* depends on bubble distortion

Applicability to turbulent bubble rise (R > 200) is questionable.

Bubble “pump”

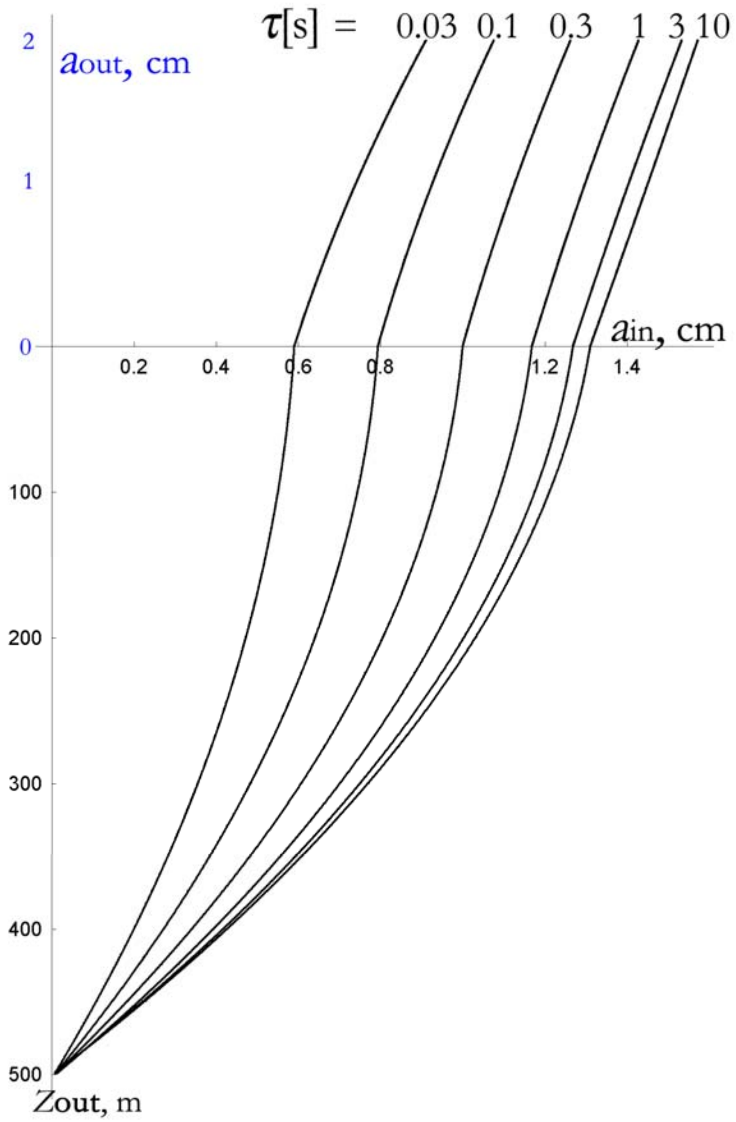
$$\lim_{r\to\infty} 2\pi r^2 \int_0^{\pi/2} v_r \sin\vartheta \, d\vartheta = \lim_{r\to\infty} \left[\psi_a(r,\pi) - \psi_a\left(r,\frac{\pi}{2}\right)\right] 2\pi = -\frac{2\pi g a^3}{3U(a)}.$$

$$\Phi_{\text{liq}} = \frac{2\pi g}{3\tau} \int_0^{z_1} dz \frac{a^3}{U(U+V)}, \quad \Phi_{\text{gas}} = \frac{4\pi g a_1^3}{3\tau} \quad \text{where} \quad a_1 = a(z_1)$$

$$a^3 (z + z_0) = \text{const}$$

$$\frac{\Phi_{\text{liq}}}{\Phi_{\text{gas}}} = \frac{g z_0}{2} \int_0^{z_1} \frac{dz}{(z+z_0)U(U+V)} \sim \frac{g z_0}{< U^2 >} \sim 10^2.$$

An attempt to use laminar results in the turbulent region.



The absorption coefficient
K = (1.0—1.05)10^{-3} cm/s.
p_0 is atmospheric pressure,
$p_0/\rho g$ = 10 m.

$$\frac{da}{dz} = \frac{K}{U(a) + V(z,\tau)} - \frac{a}{3(z + p_0/\rho g)}.$$

Long methane bubble chains starting at depth 500 m.
Final depth (lower scale) or size after reaching the sea surface (upper scale) in dependence on bubble period τ and their initial size a_{in}.