How a hot electron gets through a cold disordered wire

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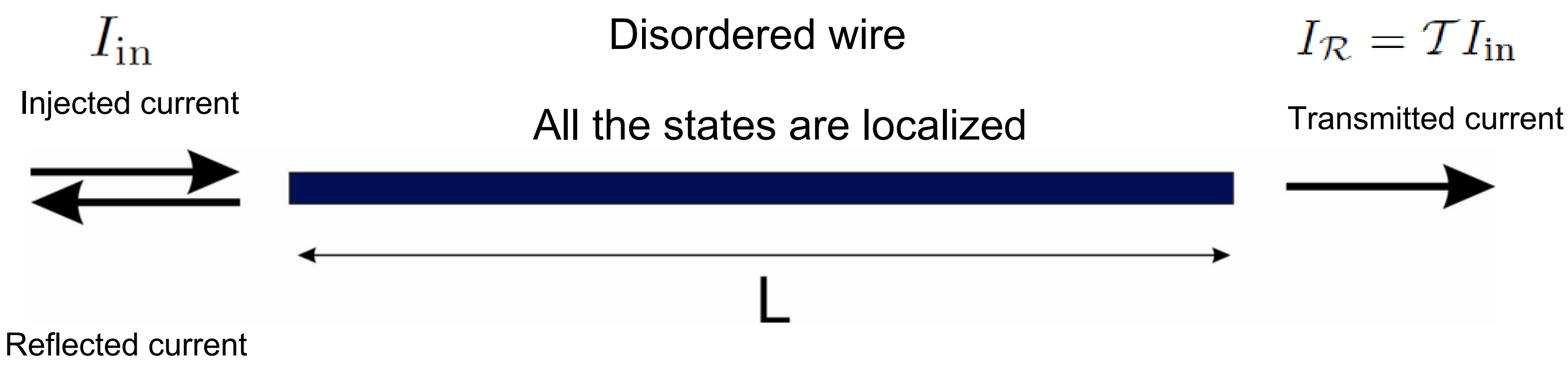
The schematic setup



- The wire is in equilibrium at temperature *T*
- Electrons are injected with the energy *E* >> *T*
- Electrons interact with a thermostat. They can only loose energy, not gain!

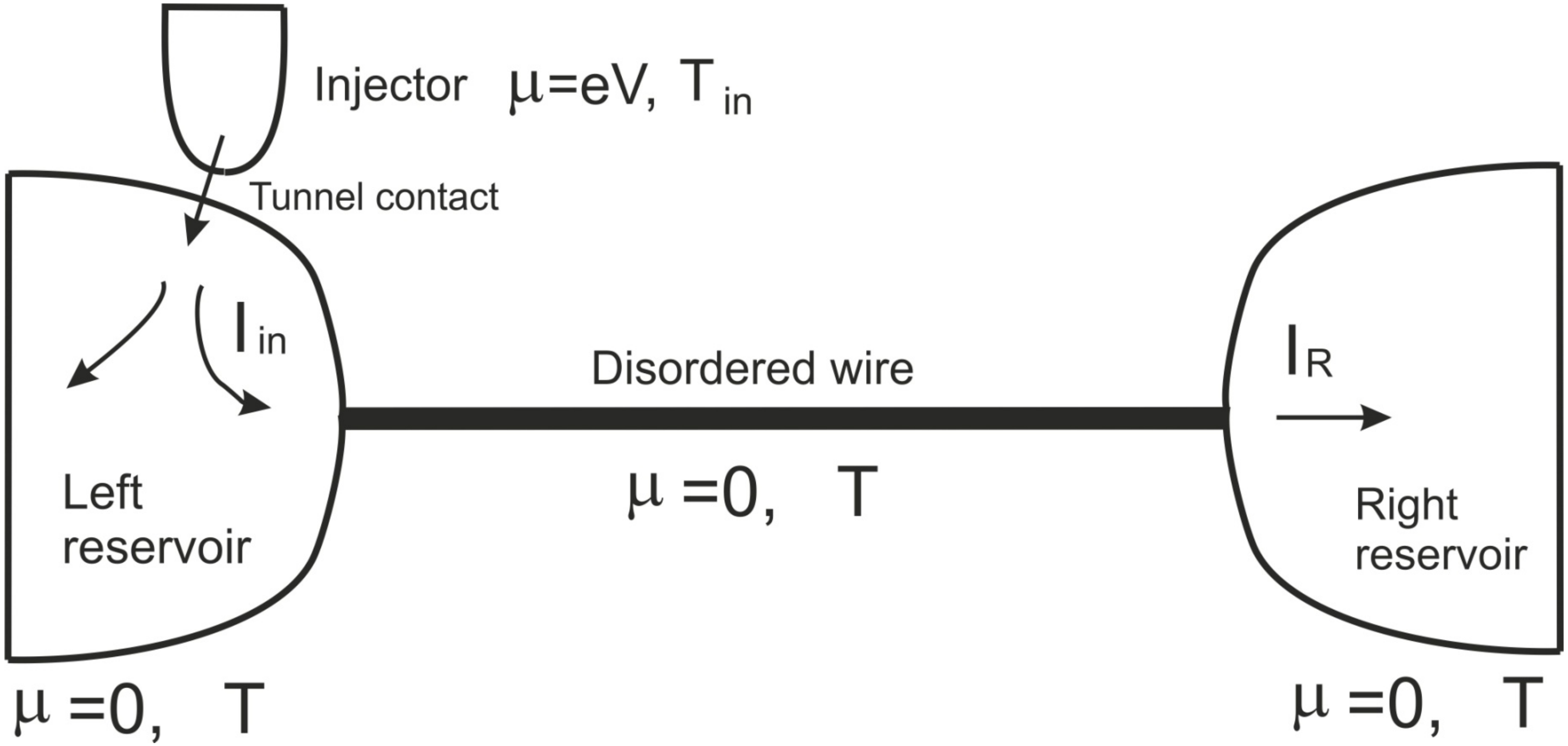
We are interested in the distribution function of the transmittance $\mathcal{T}$ **over the ensemble of disordered wires**

$$\mathcal{T} \propto \exp\{-\alpha s\}, \qquad \alpha = 2L/a \gg 1$$

For the direct elastic tunneling (short wires) *s* is sharply distributed near *s=1*

For the multi-hop inelastic tunneling (long wires) **the distribution of *s* is wide, with a strong tail at small *s*.**

The more detailed setup

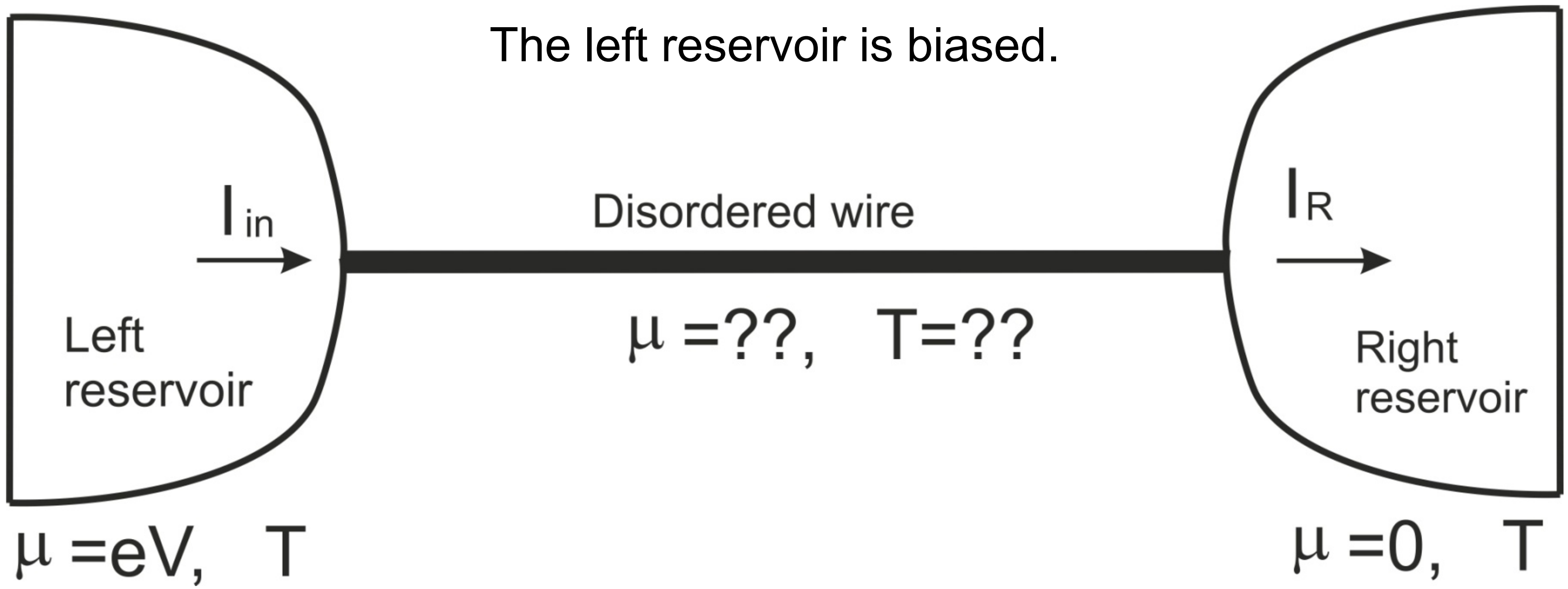


The injector is strongly biased ($eV >> T$) and/or hot ($T_{in} >> T$).

The rest of the system is in equilibrium and cold

Nonequilibrium probe particle in an equilibrium environment

Standard setup: the linear hopping conduction



$$I_{\mathcal{R}} \propto \exp\{-T_0/2T\} \qquad T_0 = 1/ga$$

- g is the density of states at the Fermi level in the wire
- a is the radius of the localized states

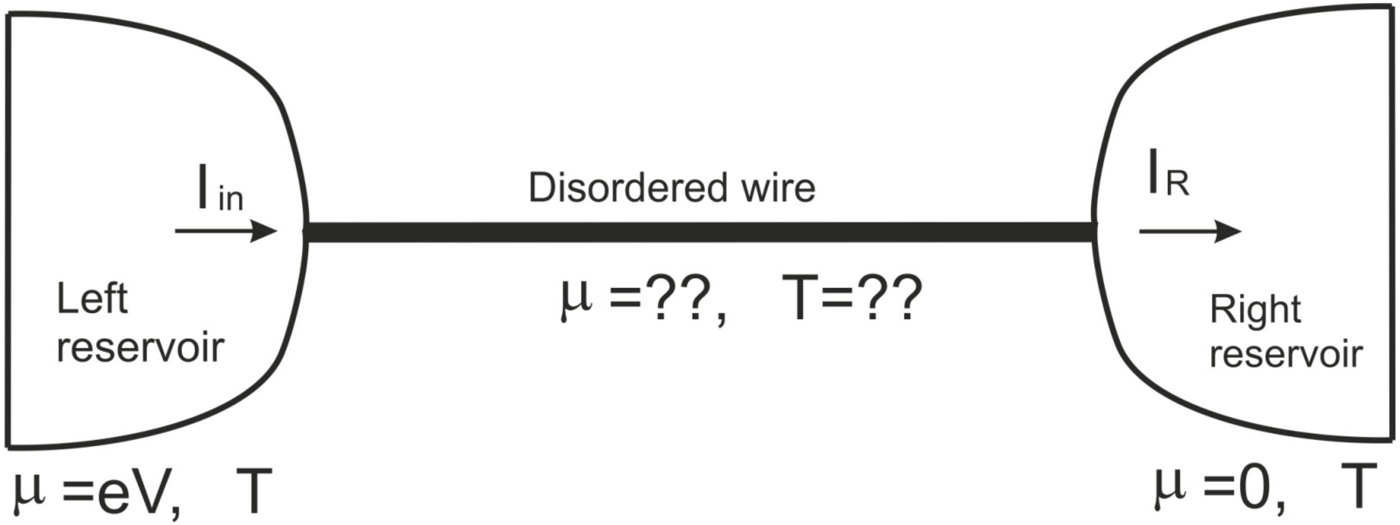
The result does not obey the Mott law: $I_{\mathcal{R}} \propto \exp\left\{-c\sqrt{T_0/T}\right\}$ (in 1D)

The reason: special role of local fluctuations of the density of states (1D specifics)

Kurkijarvi (1973); Raikh and Ruzin (1989)

Equilibrium probe particle in an equilibrium environment

Standard setup: the nonlinear hopping conduction



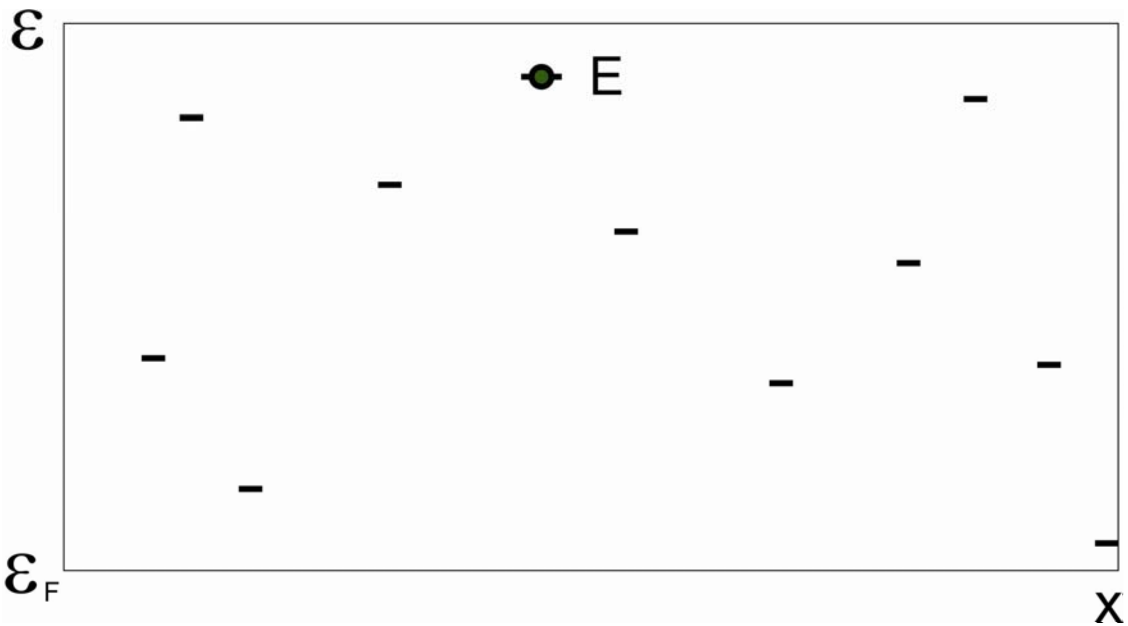
If *eV* is large, then the wire is far from equilibrium:

$$I_{\mathcal{R}} \propto \exp\left\{-\sqrt{8T_0/e\mathcal{E}a}\right\} \qquad \mathcal{E} = V/L$$

Nguen and Shklovskii (1981);
Natterman, Giamarchi, and Le Doussal (2003);
Fogler and Kelley (2005).

Nonequilibrium probe particle in an nonequilibrium environment

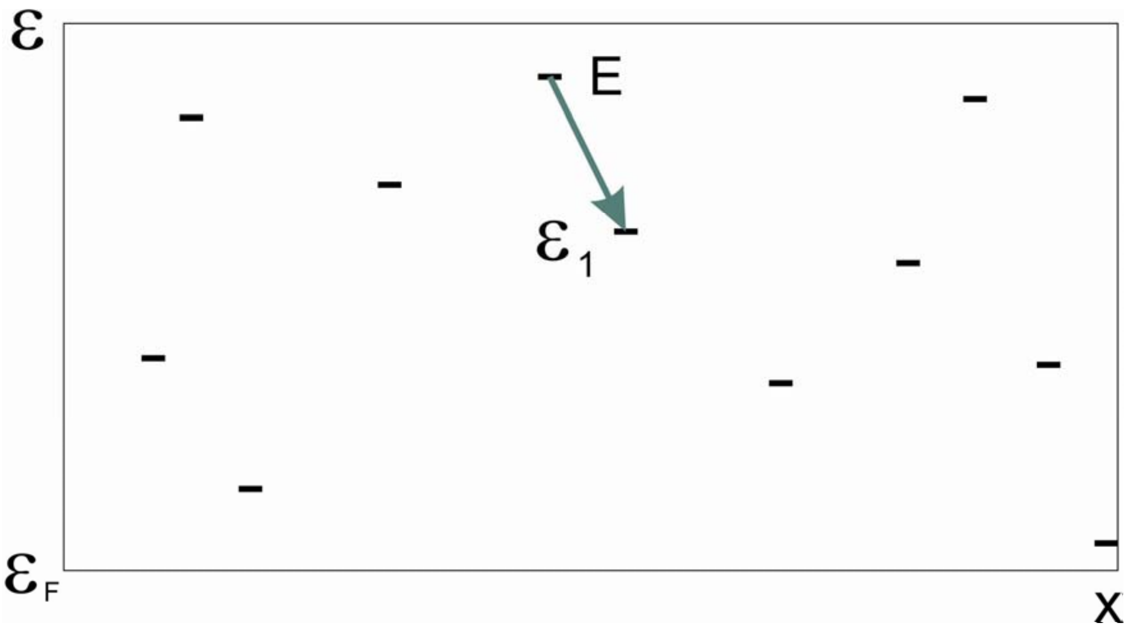
Diffusion of photo-excited carriers



A particle is created in a localized state with energy E >> T

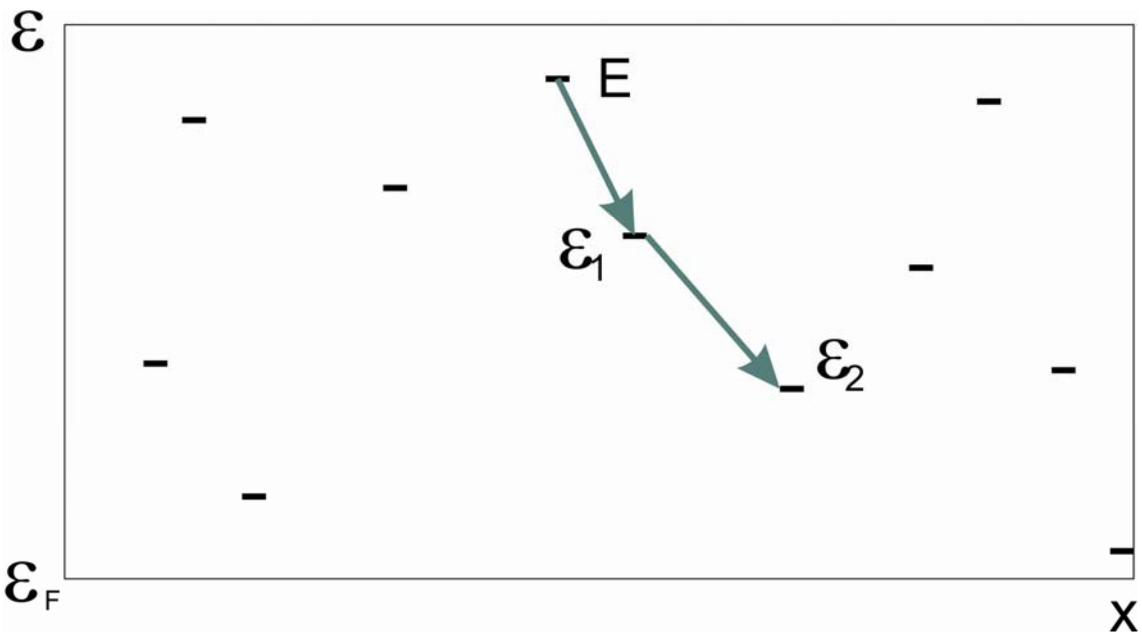
A closely related problem!
Here the particle also can hop from one localized state to another with a loss of energy.

Diffusion of photo-excited carriers



Then it hops to the closest (in space) localized state with energy $\epsilon_F < \epsilon_1 < E$

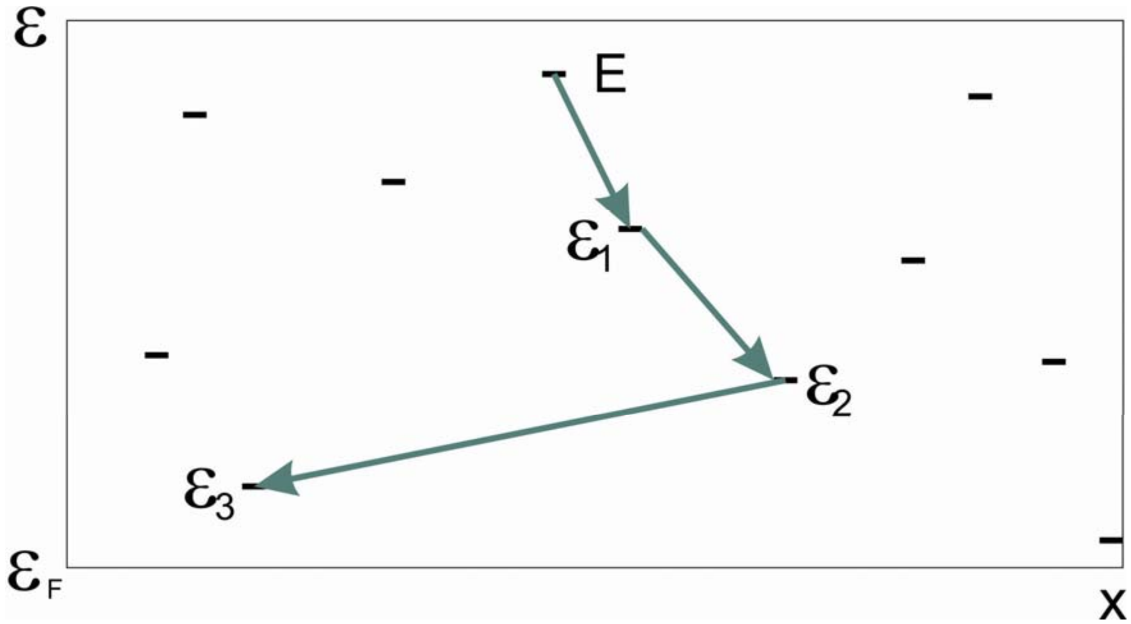
Diffusion of photo-excited carriers



Then -- to the closest (in space) localized state

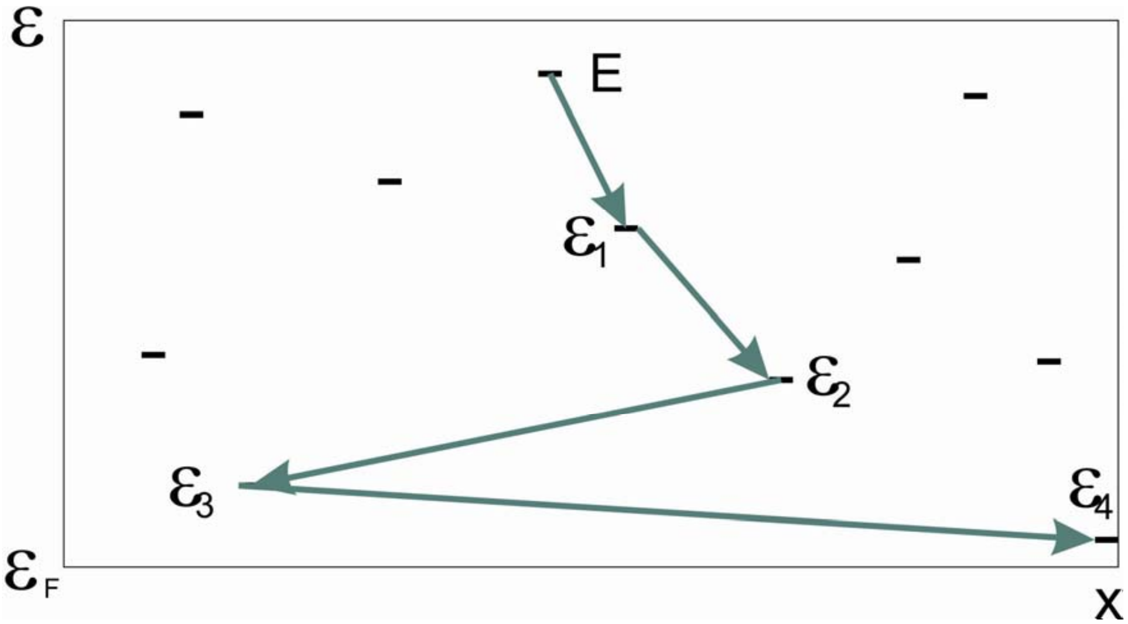
with energy $\epsilon_F < \epsilon_2 < \epsilon_1$

Diffusion of photo-excited carriers



Then -- to the closest (in space) localized state with energy $\epsilon_F < \epsilon_3 < \epsilon_2$

Diffusion of photo-excited carriers



et cetera…

This is a kind of ``diffusion'' with exponentially increasing steps:

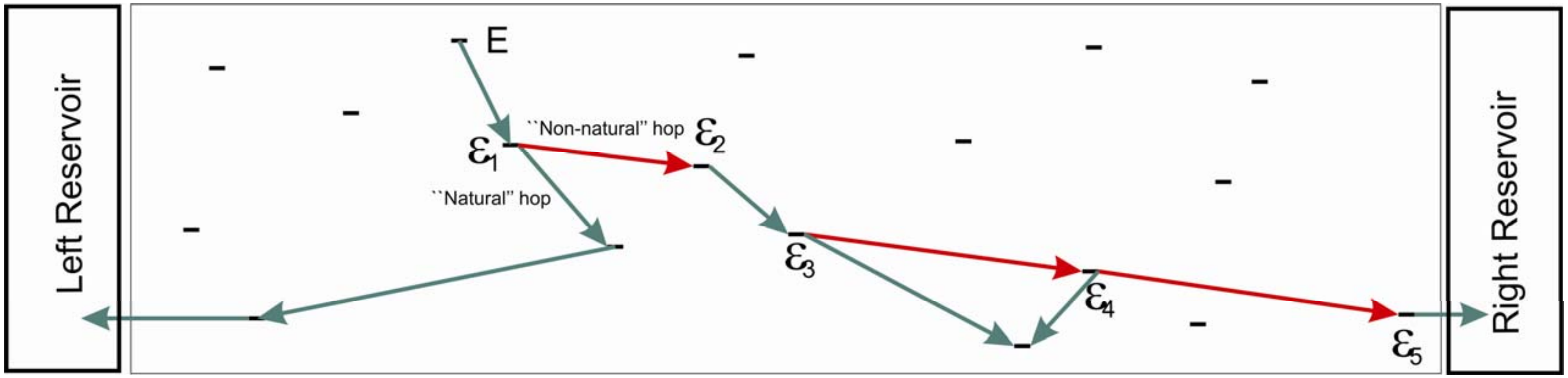
$$\Delta x_{i+1} \sim q \Delta x_i \qquad\qquad q > 1$$

The averaged (over configurations) density of particles decays as a power law:

$$n(x) \sim |x - x_0|^{\beta}$$

Shklovskii, Fritzsche, and Baranovskii (1989)

Why the ``diffusional'' approach is not applicable to our problem?



If the starting point is close to the left reservoir, then, in a typical configuration, the ``fully natural path'' definitely leads to the left reservoir.

To get to the right one, the particle should exercise **``non-natural'' hops to the non-closest neighbors.** The probability of such deviations is exponentially small.

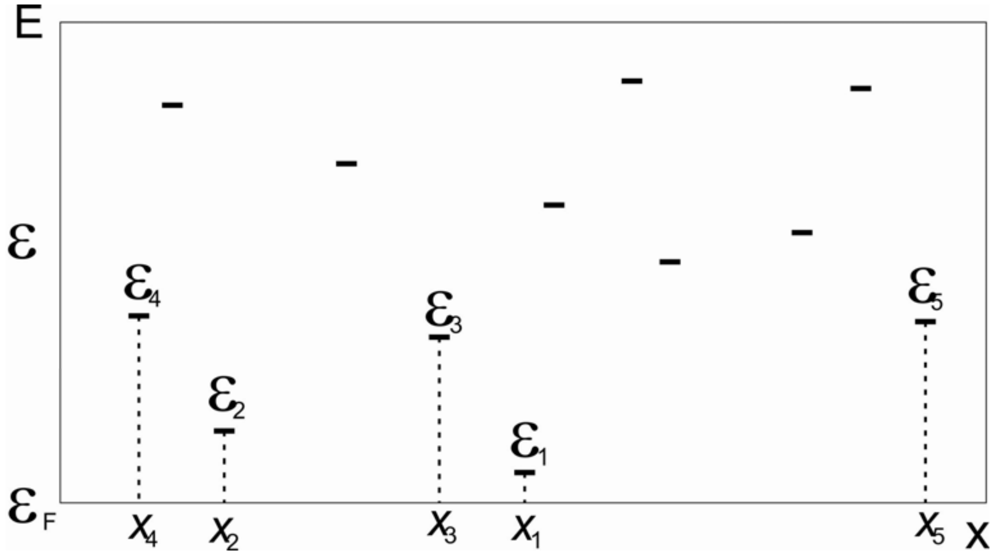
Thus, the probability $\mathcal{T}$ to get to the right reservoir from the left one is **exponentially small for a typical configuration**

However, the diffusional approach gives a power law: $\overline{\mathcal{T}} \sim L^{-\beta}$ for the average $\mathcal{T}$

This paradox is due to **anomalous contribution of very rare configurations** with a fully natural path, crossing the wire

$\overline{\mathcal{T}}$ **is not a representative quantity!!**

The kinetic equation



N ``quasiresonances''

$$\overline{N} \sim gLE$$

$$\varepsilon_F < \varepsilon_i < E$$
$$0 < x_i < 1$$

$\varepsilon_i < \varepsilon_j$ for $i < j$

$$\dot{n}_i = P_{\mathcal{L}\to i}\tilde{I}_{\rm in} + \sum_{j>i} n_j P_{j\to i} - n_i P_{i\to \rm out},$$

The population numbers n_i

The transition rates $P_{j\to i}$

$$P_{i\to\rm out} = P_{i\to\mathcal{L}} + P_{i\to\mathcal{R}} + \sum_{j<i} P_{i\to j}.$$ -- the total escape rate.

$$(n_i/\tilde{I}_{\rm in}) = \frac{P_{\mathcal{L}\to i} + \sum_{j>i}(n_j/\tilde{I}_{\rm in})P_{j\to i}}{P_{i\to\rm out}}$$ -- the recursive solution.

$$\mathcal{T} \equiv I_R/\tilde{I}_{\rm in} = P_{\mathcal{L}\to\mathcal{R}} + \sum_i P_{i\to\mathcal{R}}(n_i/\tilde{I}_{\rm in}).$$ -- the transmittance

The ``exponential approximation''

$$P_{j\to i} = P^{(0)}_{j\to i}(\varepsilon_j - \varepsilon_i)\exp\{-2L|x_i - x_j|/a\}$$

If one keeps only the exponential factors, then

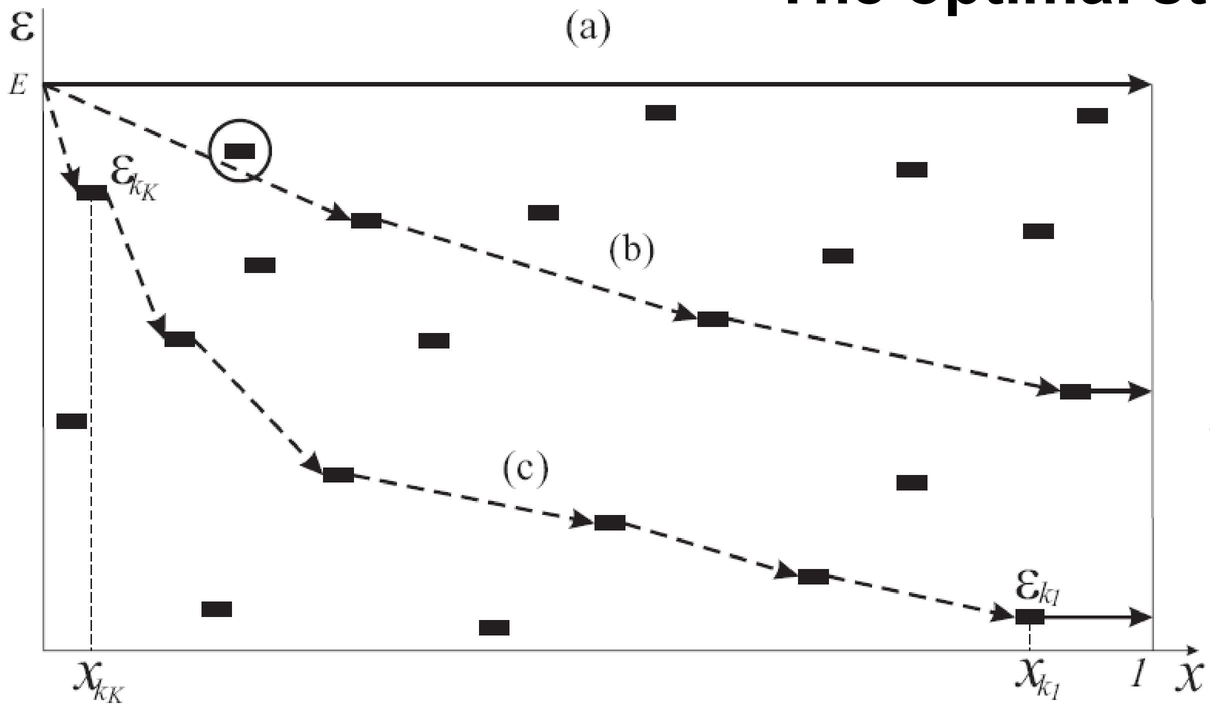
$$P_{j\to i} \propto \theta(\varepsilon_j - \varepsilon_i)\exp\{-\alpha|x_i - x_j|\} \qquad \alpha = 2L/a \gg 1;$$

● In the exponential approximation the sum in the r.h.s. of the kinetic equation Is dominated by single term with the largest exponent

● In the exponential approximation the details of the interaction with the thermostat ($P^{(0)}_{j\to i}(\varepsilon_j - \varepsilon_i)$) are irrelevant. Only the order in the sequence of levels matters, not their particular values.

● In the exponential approximation the transmittance $\mathcal{T}$ is dominated by a single path in the set of quasiresonances -- the optimal staircase.

The optimal staircase (I)



The configuration $\mathcal{C} \equiv (x_1, \ldots, x_N)$ is fixed

Different staircases $\mathcal{S}$ are shown:
(b) – reducible,
(c) – irreducible.

- The staircase $\mathcal{S}$ is a monotonically descending sequence of *K* quasiresonances:

$$\varepsilon_{k_1} < \varepsilon_{k_2} < \ldots < \varepsilon_{k_K}, \quad x_{k_1} > x_{k_2} > \ldots > x_{k_K}$$

- The optimal staircase $\mathcal{S}_{\rm opt}(\mathcal{C})$ can only be ``irreducible'':
  no additional quasiresonances can be incorporated.

The optimal staircase (II)

- The optimal staircase $\mathcal{S}_{\rm opt}(\mathcal{C})$ provides the maximum $X(\mathcal{C})$ to $X(\mathcal{S}|\mathcal{C})$ where

$$X(\mathcal{S}|\mathcal{C}) = \sum_{i_k \in \mathcal{S}} \chi_{i_k}(\mathcal{C}), \qquad \chi_i(\mathcal{C}) \equiv \min\left\{x_i, 1 - x_i, \min_{j<i}\{|x_i - x_j|\}\right\},$$

$\chi_{i_k}(\mathcal{C})$ is the distance from the quasiresonance *i* to the closest object with lower energy (other quasiresunance or a reservoir) , so that the escape rate

$$P_{i \to {\rm out}} \propto \exp\{-\alpha \chi_i\}$$

- The transmittance $\mathcal{T}(\mathcal{C}) \propto \exp\{-\alpha s(\mathcal{C})\}$ is given by $s(\mathcal{C}) = 1 - X(\mathcal{C})$

- The distribution function $\mathcal{F}_N(s) = \prod_{i=1}^{N} \int_0^1 dx_i \delta[s - s(\mathcal{C})].$

- At given *E* the number N of quasiresonances is not fixed, but obeys the Poissonian distribution with $\overline{N} \sim gLE$
The ``grand-canonical'' distribution for *s*

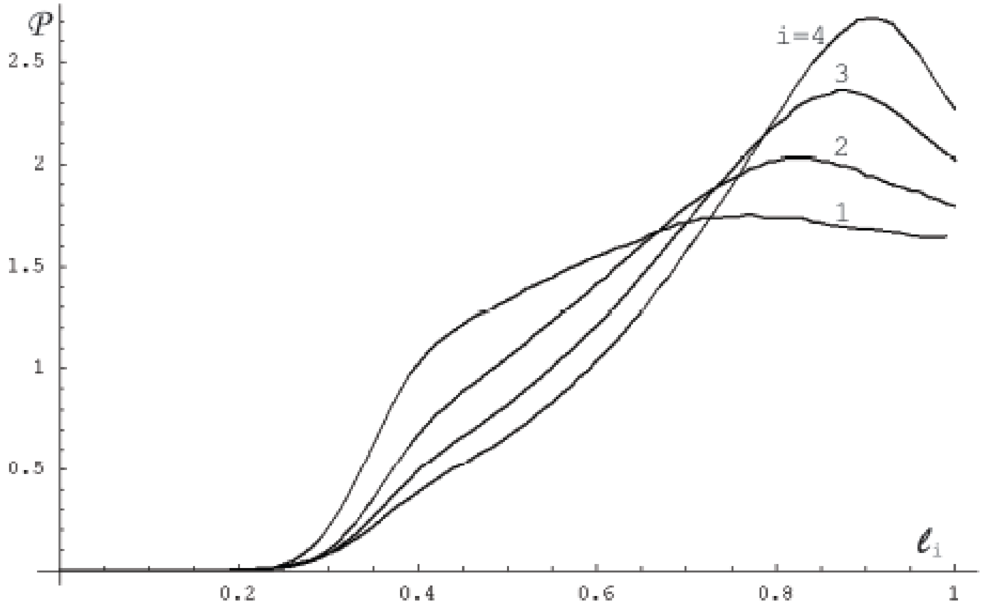
$$F_{\overline{N}}(s) = \sum_{N=0}^{\infty} \mathcal{F}_N(s) \overline{N}^N \exp(-\overline{N})/N!$$

The optimal staircase (III). Scaling?

An assumption about the self-similarity in the structure of the optimal staircase.

- Introduce the relative position $\ell_i = x_{k_i}/x_{k_{i-1}}$ of the i-th quasiresonance in the optimal staircase
- Let $\mathcal{P}_i(\ell_i)$ be the distribution function for ℓ_i
- The scaling assumption would mean $\mathcal{P}_i \equiv \mathcal{P}$
- Immediate consequence: the number of hops in the optimal staircase

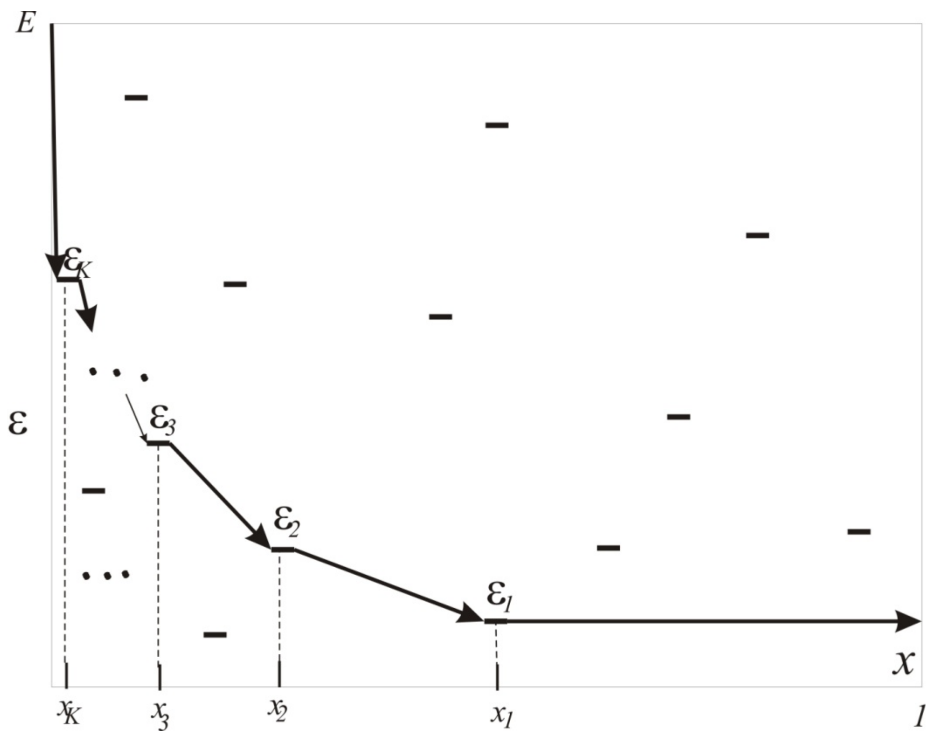
$$K(N) \sim \ln N$$



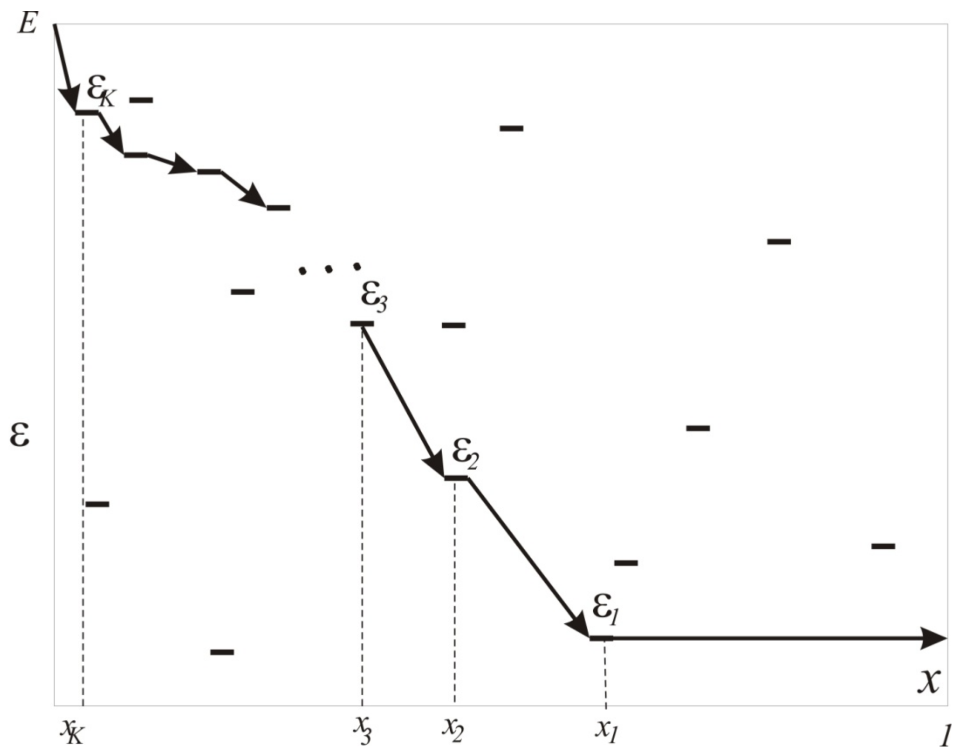
The check of scaling in numerical experiment

The scaling works only very roughly

The optimal staircase.   Scaling or not?



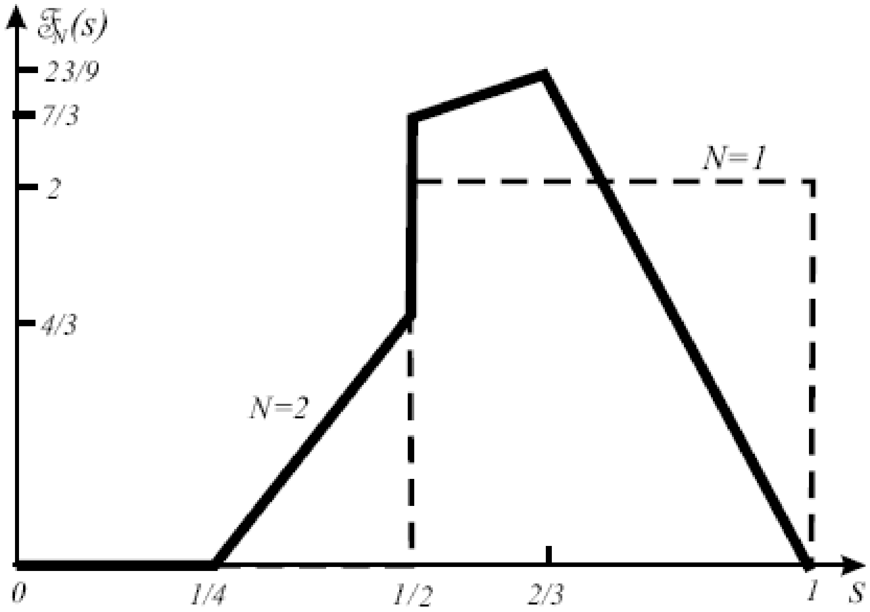
The scaling-like optimal staircase



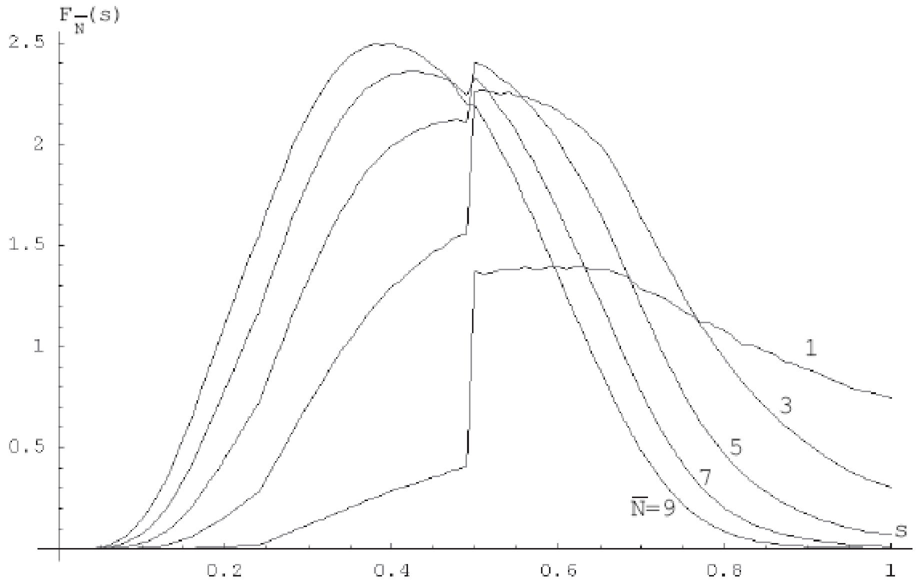
The realistic optimal staircase

- The few last hops are scaling-like (they are most important for *s*)
- Many short hops at the left end on the staircase are roughly equidistant.
  They determine the large-N corrections

The results: short wires.

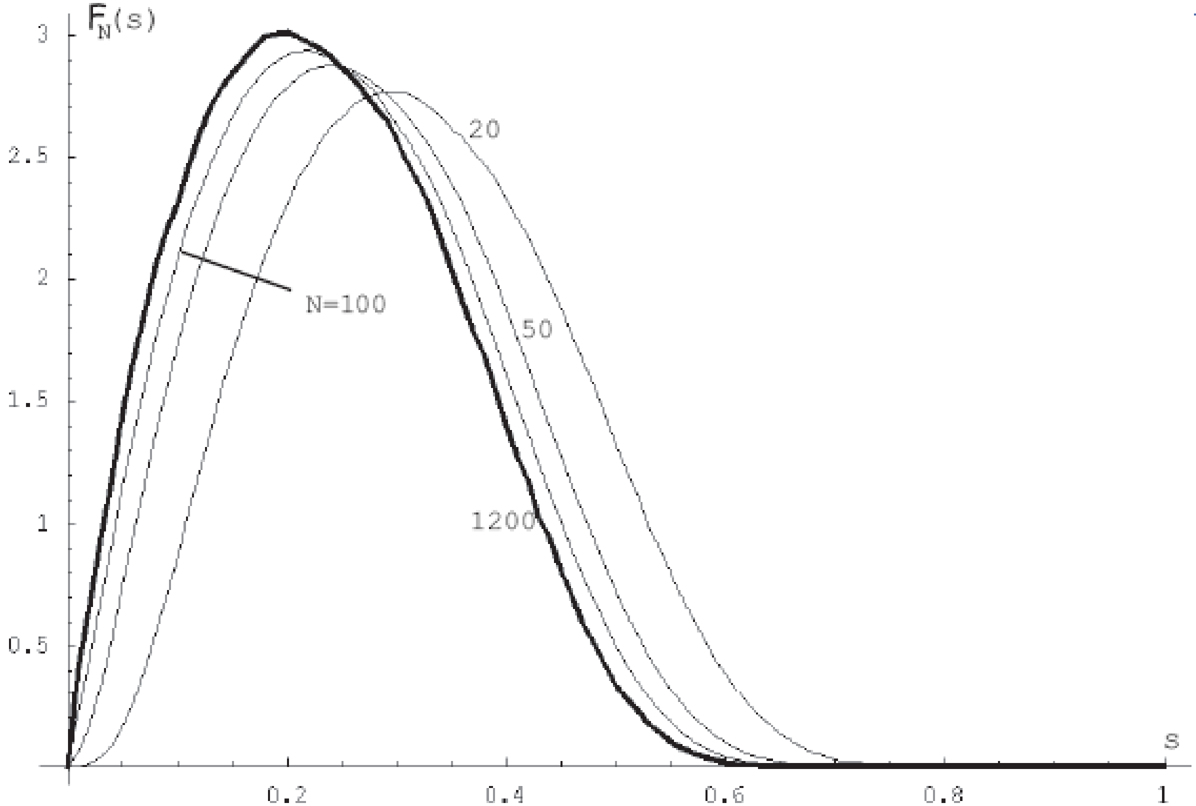


Distribution func $\mathcal{F}_N(s)$
with fixed N for $N=1$ and $N=2$
(analytical)



Grand-canonical
distribution functions $F_{\overline{N}}(s)$
for moderate $1 < \overline{N} < 10$ (numerical)

The results: long wires.

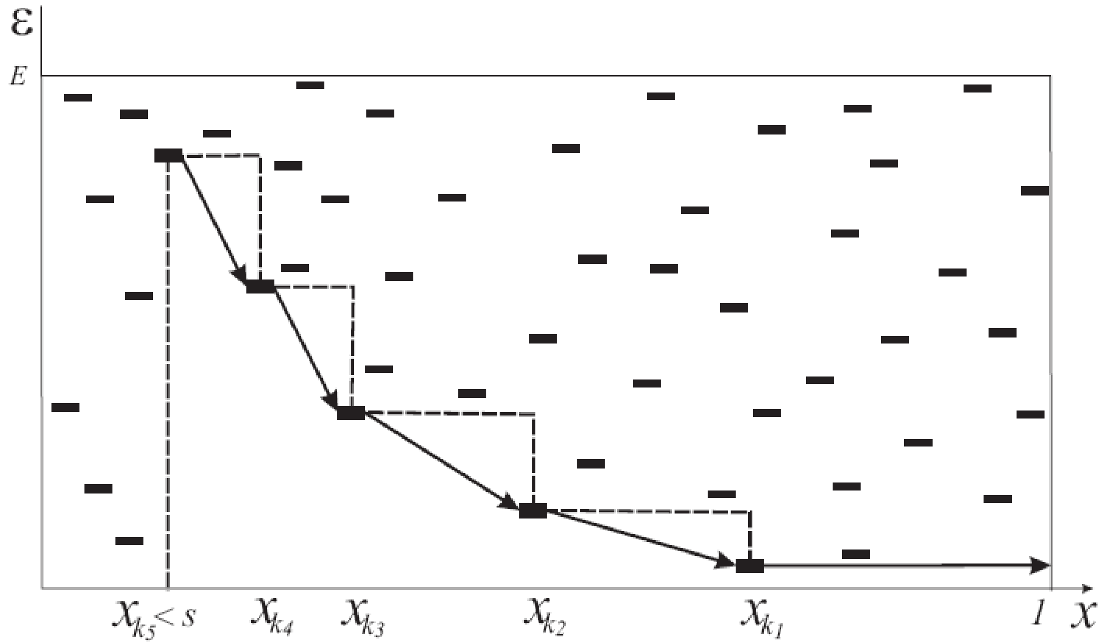


Distribution functions for large *N* (numerical)

For large *N* there is no difference between $\mathcal{F}_N(s)$ and grand-canonical $F_{\overline{N}}(s)$

- The distribution is wide
- $F_N(s) \approx F_\infty(s)$ for *N>200*
- For small *s* $F_\infty(s) \approx bs, \quad (b \approx 29).$

The origin of the small-*s* asymptotics

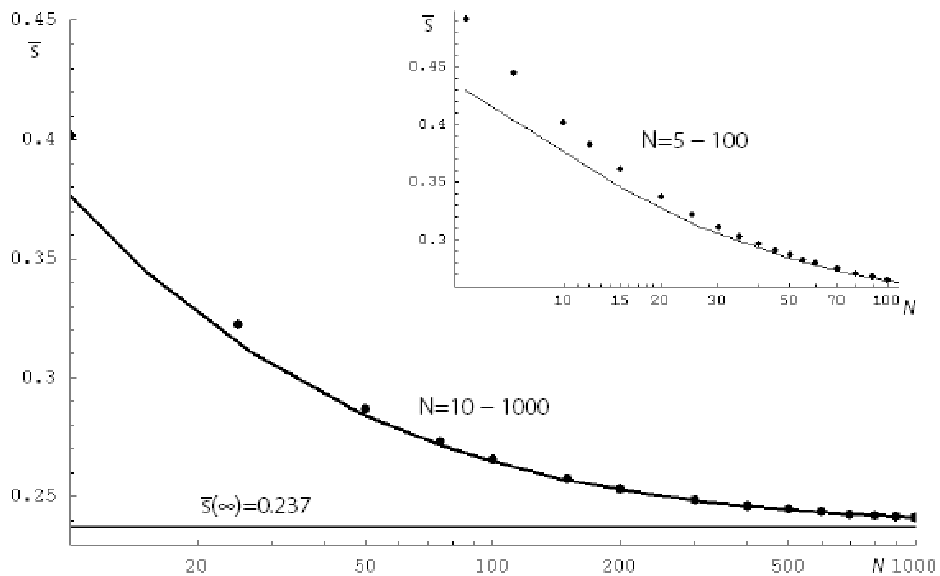


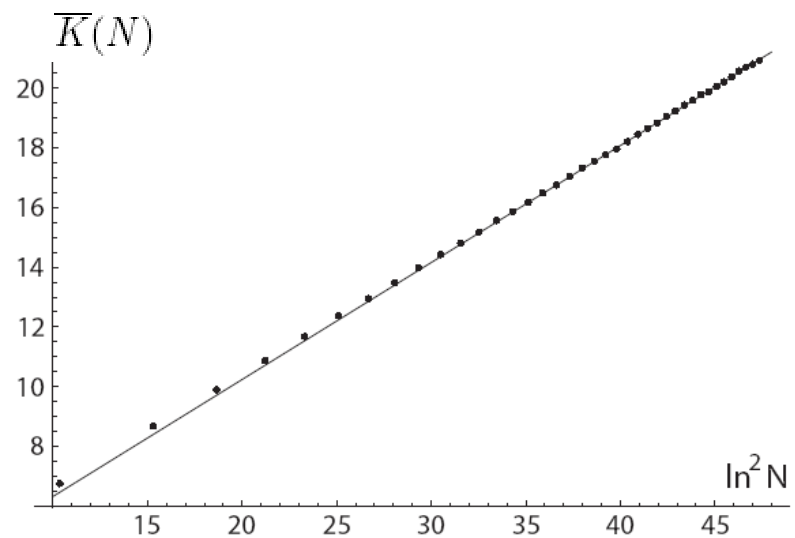
``Fortunate configuration'':
n last hops are the natural ones .

- It can be analytically shown that the probability to have such a configuration is proportional to *s*
- In such a configuration the optimal staircase is very close to the natural path.
- The average transmittance is dominated just by these rare fortunate configurations

$$\overline{T} \approx \int F(s) e^{-\alpha s} ds \approx b/\alpha^2 \propto L^{-2}$$

Large N corrections: the open question





- Average *s* as a function of *N*
- Average number of hops in the optimal staircase as a function of *N*

Empirical formulas: no analytical derivation.

$$\overline{s}_N \approx 0.237 + 0.598 \ln N/N, \qquad \text{for } N \gg 1.$$

$$\overline{K}(N) \approx 0.39 \ln^2 N + 2.4, \qquad \text{for } N \gg 1$$

These results contradict the scaling assumption. No alternative theory so far!

The summary

- **The distribution function for *s*** (the logarithm of the transmittance $\mathcal{T}$) is wide, the typical $\mathcal{T}$ is exponentially small in *L*.
- The average *s* is suppressed compared to the elastic case by the factor 0.237
- There is a huge linear tail in the distribution at small *s*, that dominates the average transmittance $\overline{\mathcal{T}}$ which decays only as L^{-2}
- **Open question**: the structure of the optimal staircase and large-*N* corrections.
- **Physical manifestations**:

  Biased injector: jumps in the current.

  Hot injector: no electron-hole symmetry --- large thermocurrent.

  Multi-wire setup: sharpening of the distribution

  Current-current correlations for the same sample at different parameters:

  - The injector voltage
  - The chemical potential in the wire
  - The spatial positions of contacts