Paramagnetic Effects on the Vortex Lattice of Strongly Type-II Superconductors

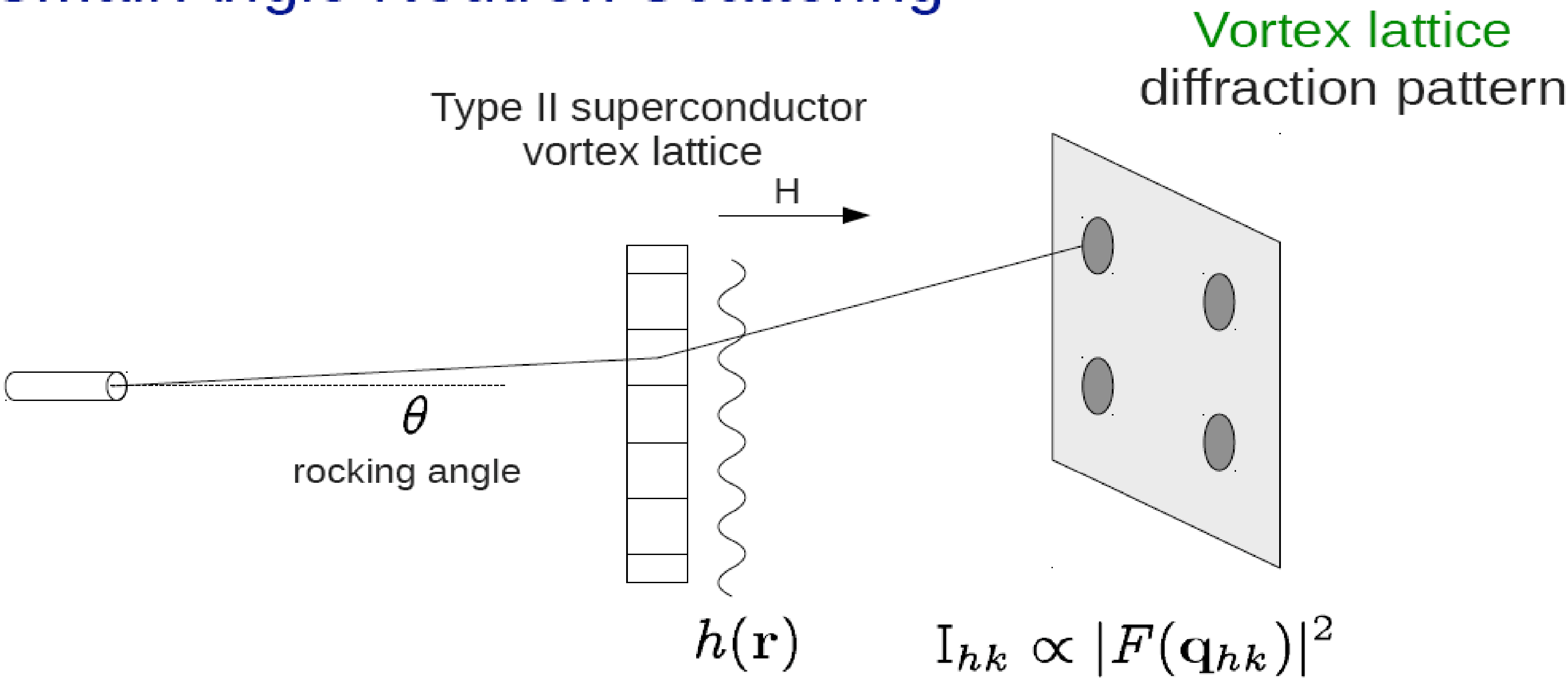
V.P.Michal & V.P.Mineev

CEA, Grenoble, France

Outline

- Vortex lattice form factor
- Clem variational solution
- $CeCoIn_5$
- Vortex lattice structure
- $CeCoIn_5$ formfactor
- Zeeman contribution
- Solution of GL and Maxwell equations
- Zeeman part of form factor
- Form factor
- Conclusion

Small Angle Neutron Scattering

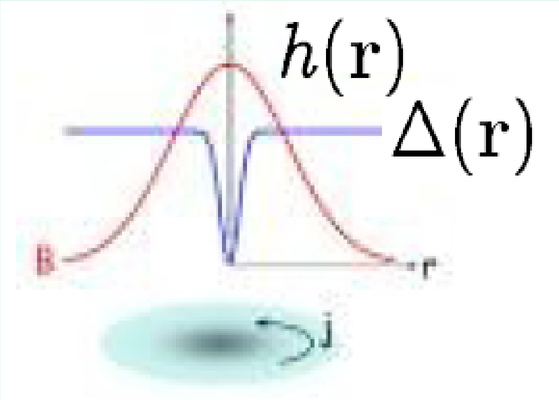


$$F(\mathbf{q}_{hk}) = \int \mathrm{d}^2\mathbf{r}\, h(\mathbf{r}) e^{-i\mathbf{q}_{hk}\cdot\mathbf{r}} = \frac{B}{\phi_0} h_v(\mathbf{q}_{hk})$$

$\mathbf{q}_{hk}$ gives the periodicity of the lattice

Fourier transform of the local magnetic field generated by a single vortex

John Clem solution 1975



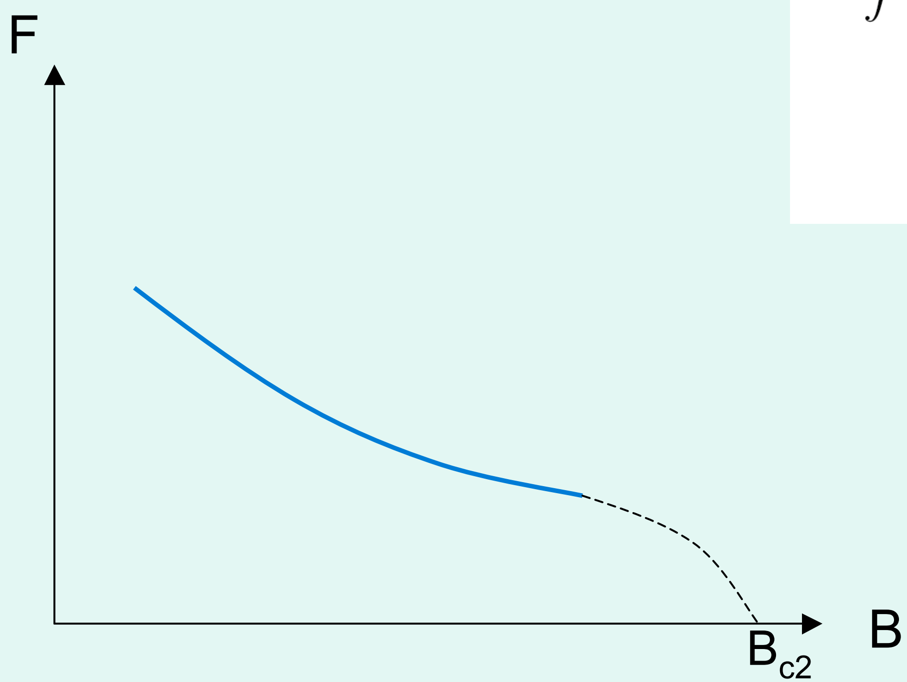
$$\begin{cases} \alpha\Delta + 2\beta\Delta^3 - \gamma\nabla^2\Delta + 4e^2 v_s^2 \Delta = 0 \\ \nabla\times\nabla\times\mathbf{A} = \nabla\times\mathbf{h} = 4\pi\mathbf{j} \\ \mathbf{j} = 8e^2\gamma\Delta^2 v_s \hat{\varphi} \qquad v_s = \frac{\phi_0}{2\pi r} - A(r) \end{cases}$$

$$\begin{cases} \Delta(r) = \Delta_\infty \frac{r}{R} = \Delta_\infty \frac{r}{\sqrt{r^2+\xi_v^2}} \\ \mathbf{A}(r) = \frac{\phi_0}{2\pi r}\left(1 - \frac{R K_1(R/\lambda)}{\xi_v K_1(\xi_v/\lambda}\right)\hat{\varphi} \end{cases}$$

$$\int\int \mathbf{h} d\mathbf{S} = \oint \mathbf{A} d\mathbf{r} = \phi_0 \qquad v_s(r \ll \lambda) = \frac{\phi_0}{2\pi r}$$

$$\mathbf{h} = \frac{\phi_0}{2\pi\lambda\xi_v}\frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)}\hat{z}$$

$$\xi_v \simeq \sqrt{2}\,\xi \qquad \kappa = \lambda/\xi \gg 1$$



$$F = B\frac{K_1(Q\xi_v)}{Q\lambda K_1(\xi_v/\lambda)}$$

$$Q = \sqrt{q^2+\lambda^{-2}} \qquad q = 2\pi/a \qquad a = \sqrt{\phi_0/B}$$

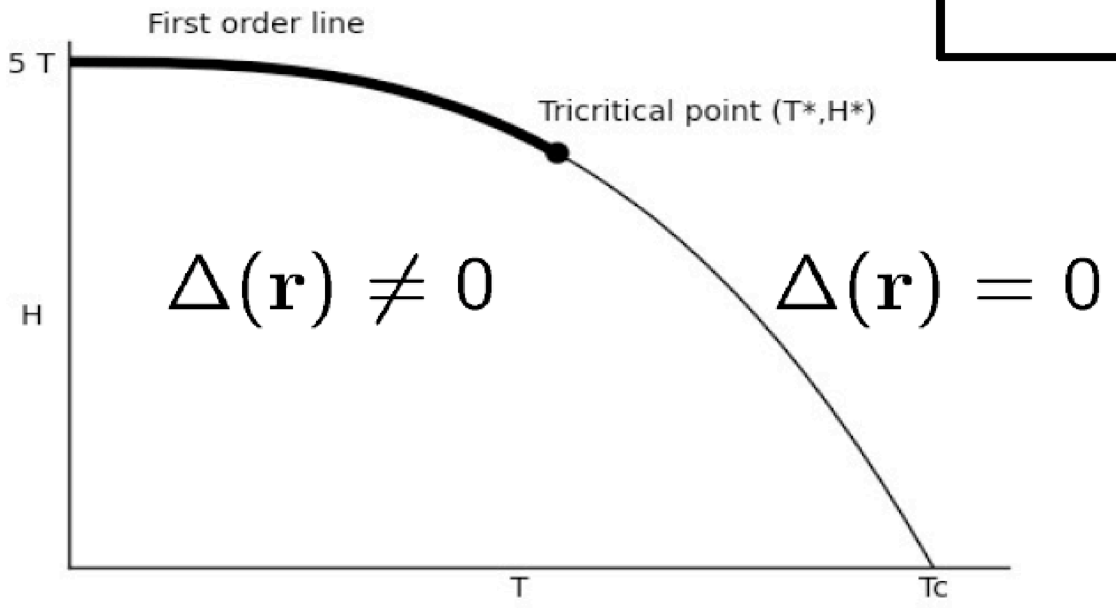
$CeCoIn_5$

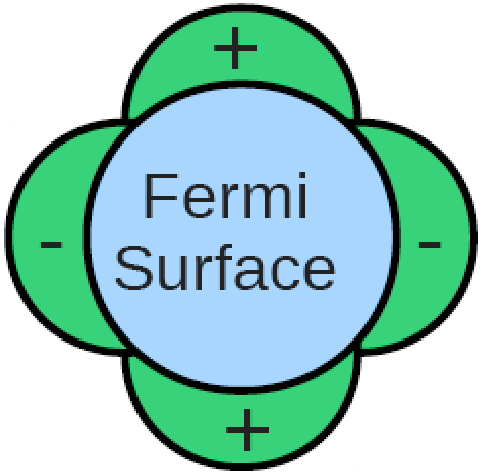
$CeCoIn_5$ Phase Diagram (H || c-axis)

Strongly Pauli-limited superconductor

$$1 \ll \alpha_M = \frac{\sqrt{2} H_{c2}}{H_p} \sim \frac{T_c}{E_F} \frac{m^*}{m}$$

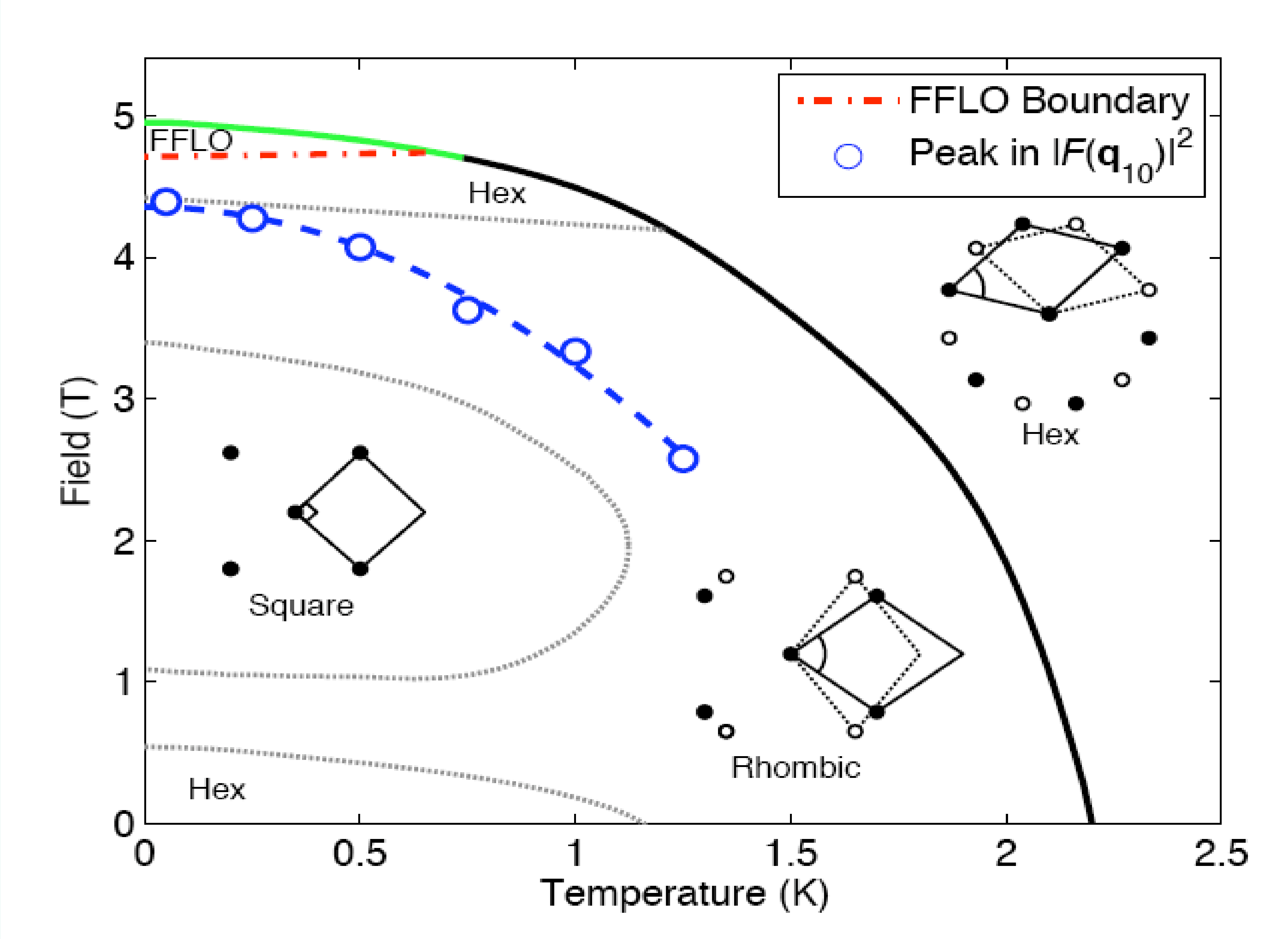
$$m^* \simeq 100 m_e$$



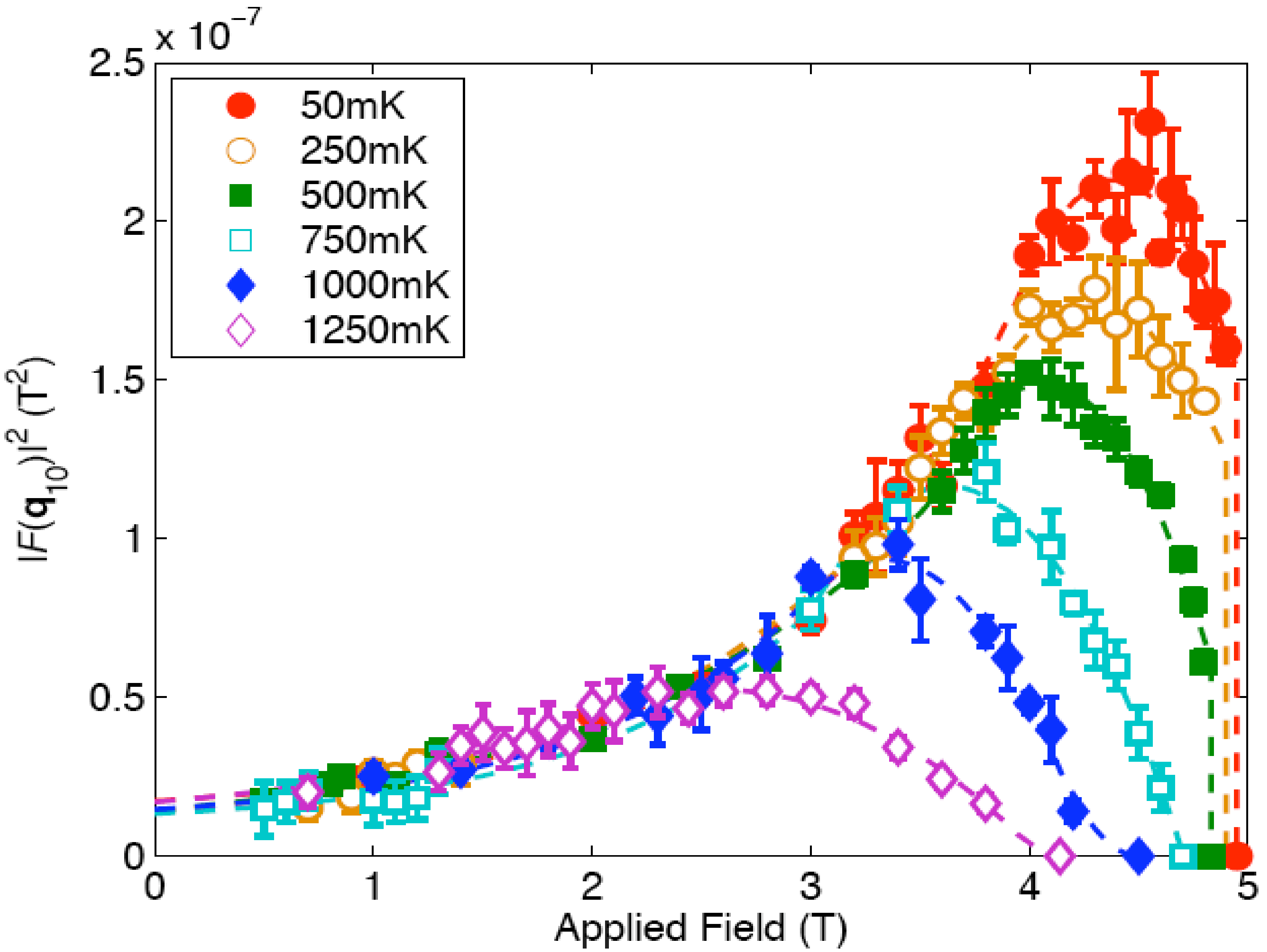


d-wave pairing $\Delta_{\mathbf{k}}(\mathbf{r}) = \sqrt{2}\cos(2\varphi)\Delta(\mathbf{r})$

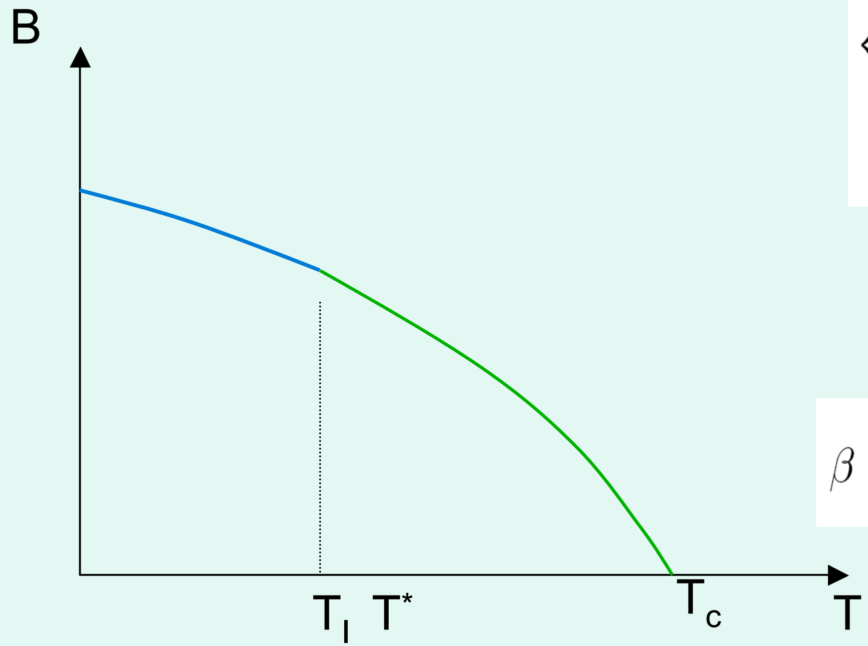
Vortex lattice structure



$CeCoIn_5$ Form Factor White et al arXiv:1001.2142



Zeeman contribution



$$\begin{cases} \alpha\Delta + 2\beta\Delta^3 - \gamma\nabla^2\Delta + 4e^2 v_s^2 \Delta = 0 \\ \nabla\times\nabla\times\mathbf{A} = \nabla\times\mathbf{h} = 4\pi\mathbf{j} \\ \mathbf{j} = 8e^2\gamma\Delta^2 v_s \hat{\varphi} + \varepsilon \frac{d}{dr}|\Delta|^2\hat{\varphi} \qquad v_s = \frac{\phi_0}{2\pi r} - A(r) \end{cases}$$

$$\alpha = N_0 \left[\ln\frac{T}{T_c} + 2\pi T Re \sum_{\omega>0} \left(\frac{1}{\omega} - \frac{1}{\omega + i\mu B} \right) \right]$$

$$\beta = \frac{\pi N_0}{4} \langle |\psi|^4 \rangle K_3 \qquad \gamma = \frac{\pi N_0}{4} \langle v_F^2 |\psi|^2 \rangle K_3 \qquad \varepsilon = \frac{d\alpha}{dB}$$

$$K_3 = 2T Re \sum_{\omega>0} \frac{1}{(\omega + i\mu B)^3}$$

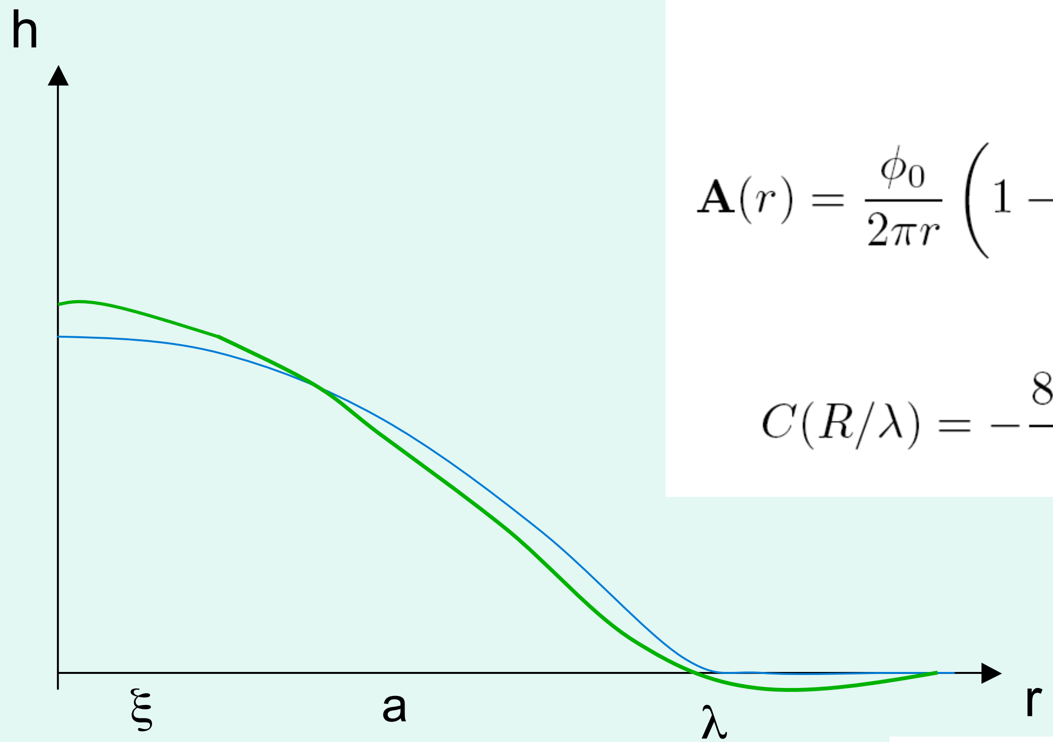
$$\Delta_{\mathbf{k}}(\mathbf{r}) = \psi(\mathbf{k})\Delta(\mathbf{r}) \qquad \psi_s(\mathbf{k}) = 1 \qquad \psi_d(\mathbf{k}) = \sqrt{2}\cos 2\varphi$$

$$\alpha(B,T) = 0 \quad \rightarrow \quad B_c(T) = 0 \qquad B_{c2} = B_c(1 - \mathcal{O}(\alpha_M^{-1}))$$

$$\beta(T, B_c(T)) = 0 \quad \rightarrow \quad T^* = 0.56\ T_c \qquad T_I = T^*(1 - \mathcal{O}(\alpha_M^{-1}))$$

$$\gamma(T, B_c(T)) = 0 \quad \rightarrow \quad T^* = 0.56\ T_c \qquad T_{FFLO} = T^*(1 - \mathcal{O}(\alpha_M^{-1}))$$

Solution



$$\Delta(r) = \Delta_\infty \frac{r}{R} = \Delta_\infty \frac{r}{\sqrt{r^2+\xi_v^2}}$$

$$\mathbf{A}(r) = \frac{\phi_0}{2\pi r}\left(1 - \frac{RK_1(R/\lambda)}{\xi_v K_1(\xi_v/\lambda)}\right)\hat{\varphi} + \frac{R}{r}K_1(R/\lambda)C(R/\lambda)\hat{\varphi}$$

$$C(R/\lambda) = -\frac{8\pi\varepsilon\Delta_\infty^2\xi_v^2}{\lambda}\int_{\xi_v/\lambda}^{R/\lambda}\frac{dz}{zK_1^2(z)}\int^z\frac{K_1(z)}{z^2}dz$$

$$\xi_v \simeq \sqrt{2}\,\xi \qquad \kappa = \lambda/\xi \gg 1$$

$$\int\!\!\int \mathbf{h}d\mathbf{S} = \oint \mathbf{A}d\mathbf{r} = \phi_0 \qquad v_s(r \ll \lambda) \approx \frac{\phi_0}{2\pi r}$$

$$\mathbf{h} = \frac{\phi_0}{2\pi\lambda\xi_v}\frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)}\hat{z}$$

$$\mathbf{h} = \frac{\phi_0}{2\pi\lambda\xi_v}\frac{K_0(R/\lambda)}{K_1(\xi_v/\lambda)}\hat{z} + \frac{1}{\lambda}[-K_0(R/\lambda)C(R/\lambda) + K_1(R/\lambda)C'(R/\lambda)]\hat{z}$$

Zeeman form factor

$$F_Z \propto B^2 \ln \frac{\phi_0}{(2\pi\xi_v)^2 B}$$

Why this effect is unobservable in ordinary superconductors ?

$$\frac{F_Z}{F_{orb}} \propto \left(\frac{\mu B}{T}\right)^2_{T\approx T_c/2} \propto \left(\frac{\mu B}{T_c}\right)^2 \propto \left(\frac{B}{B_p}\right)^2 < \alpha_M^2, \quad \alpha_M \ll 1$$

Form factor

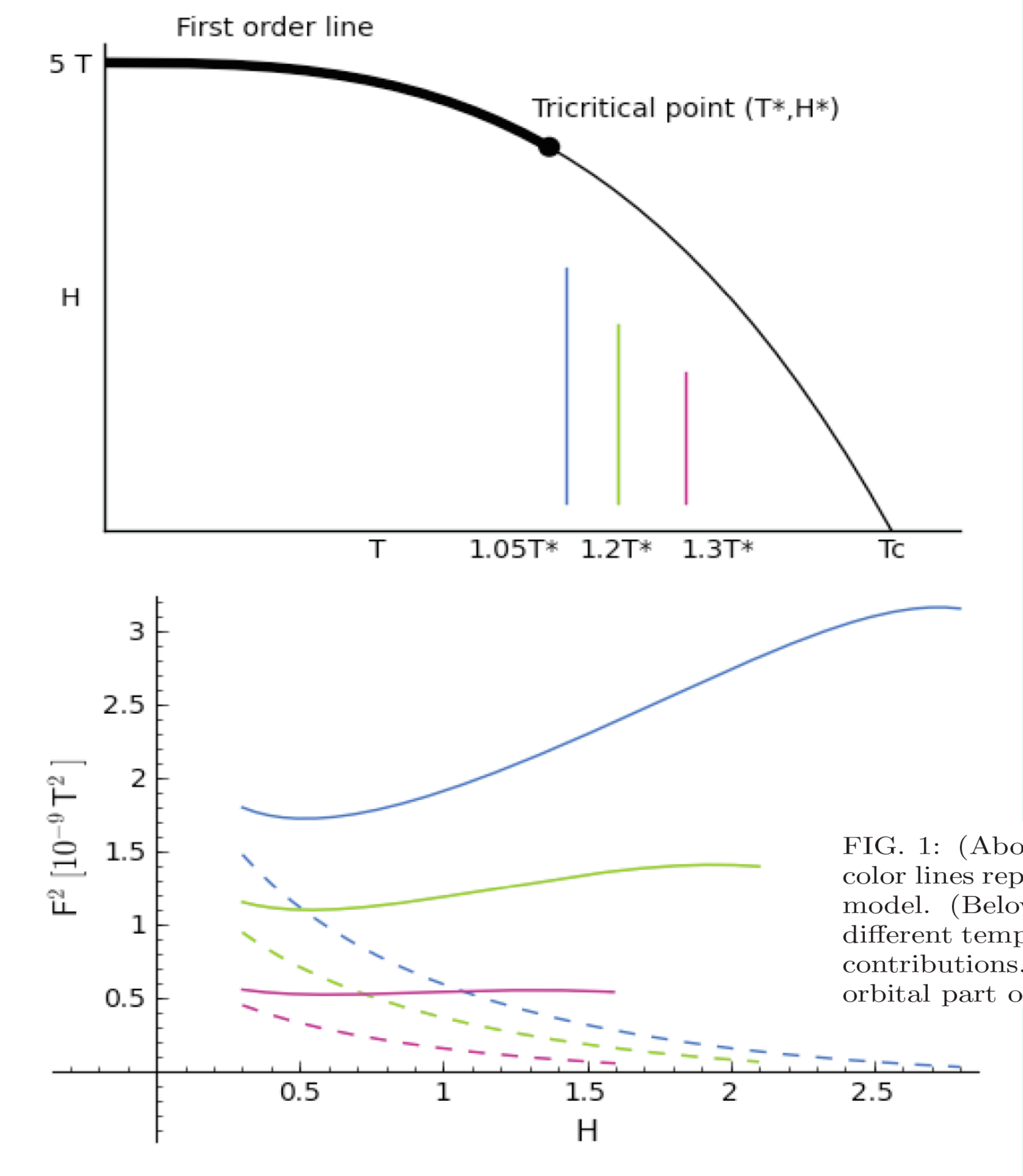


FIG. 1: (Above) $CeCoIn_5$ phase diagram for $H||$c-axis. The color lines represent the temperatures at which we applied the model. (Below) Variations of the squared form factor F^2 at different temperatures including both the orbital and Zeeman contributions. The dashed lines represent the variations of the orbital part only.

Conclusion

- The interaction of the electron spins with space inhomogeneous magnetic field inside superconductor in the mixed state produces diamagnetic currents of pure paramagnetic origin.
- The corresponding field concentrated around the vortex cores yields a dominant contribution to the vortex lattice form factor at high enough average magnetic field.
- The effect is observable in a superconductor with singlet pairing and small enough Fermi velocity (large effective mass of charge carriers).