Anderson (de)localization of Dirac fermions in graphene with resonant scatterers

P. Ostrovsky[1,2] M. Titov[3] I. Gornyi[1,4] A. Mirlin[1,5]

[1]Karlsruhe Institute of Technology [2]Landau ITP, Chernogolovka

[3]Edinburgh University [4]Ioffe Institute, St.Petersburg

[5]PNPI, St.Petersburg

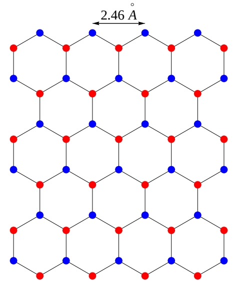
PRL **104**, 076802 (2010)
arXiv:1006:3299

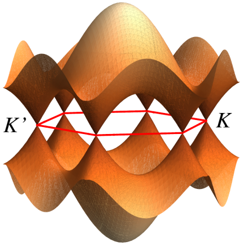
Landau Days, 21 June 2010

Outline

Outline

Clean graphene model

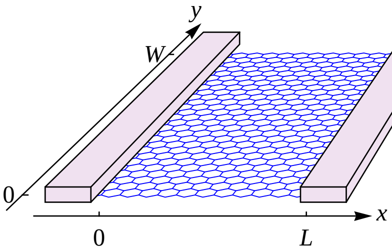




Tight-binding approximation

- two sublattices: A, B
- two valleys: K, K′
- linear dispersion: $\varepsilon = v_0|\mathbf{p}|$
- massless Dirac Hamiltonian:
  $H = v_0 \boldsymbol{\sigma}\mathbf{p} \qquad \boldsymbol{\sigma} = \{\sigma_x, \sigma_y\}$
- velocity: $v_0 \approx 10^8$ cm/s
- band width: $\Delta \sim 1$ eV

Ballistic setup



- rectangular sample with dimensions $L \times W$
- large aspect ratio: $W \gg L$

  $\Longrightarrow$ boundary conditions (edge modes) irrelevant
- zero energy (Dirac point)
- ideal contacts
- perfect metallic leads (highly doped regions of graphene)

Ballistic transport experiment

Danneau *et al.* '07

Setup

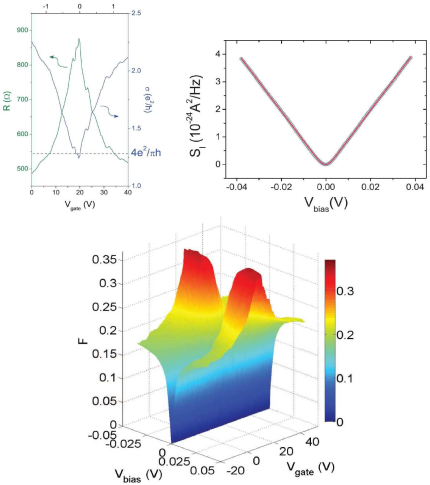
- Rectangular sample
- Temperature $4.2 \div 30$ K
- Large aspect ratio $W/L = 24$
- Ballistic limit $L \sim 200$ nm

At the Dirac point

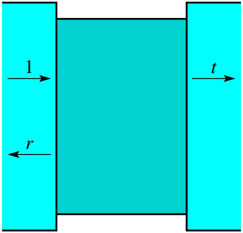
- Conductance

$$G(\epsilon = 0) \approx \frac{4e^2}{\pi h}\,\frac{W}{L}$$

- Fano factor $F(\epsilon = 0) \approx 1/3$



Electron transport in the clean sample



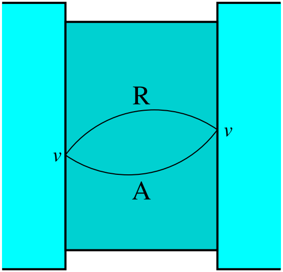
Transport properties

- vertical momentum p is conserved
- transmission probability $T = |t|^2 = \dfrac{1}{\cosh^2 pL}$
- conductance $G = \dfrac{4e^2 W}{h} \displaystyle\int \frac{dp}{2\pi} T = \frac{4e^2}{\pi h} \frac{W}{L}$
- Fano factor $F = 1 - \dfrac{\int dp\, T^2}{\int dp\, T} = \dfrac{1}{3}$
- Distribution of transmission probabilities
  $P(T) = \dfrac{W}{\pi} \left| \dfrac{dp}{dT} \right| = \dfrac{W}{2\pi L} \dfrac{1}{T\sqrt{1 - T}}$

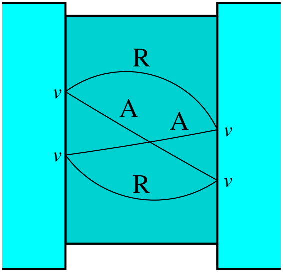
Green functions formalism

- Green functions: $G^{R,A} = (\epsilon \pm i0 - H)^{-1}$
- Velocity operator: $\mathbf{v} = \partial H / \partial \mathbf{p} = v_0 \boldsymbol{\sigma}$
- Transmission moments (generalized Kubo formula)

$$\mathrm{Tr}\, T^n = \mathrm{Tr}[v_x G^R(0, L) v_x G^A(L, 0)]^n$$



Conductance



Noise

Generating function

Matrix Green function [Nazarov '94]

$$\check{G} = \begin{pmatrix} \epsilon + i0 - H & -\delta(x) v_x \sin\frac{\phi}{2} \\ -\delta(x-L) v_x \sin\frac{\phi}{2} & \epsilon - i0 - H \end{pmatrix}^{-1}$$

Generating function (free energy)

$$\mathcal{F}(\phi) = \mathbf{Tr} \log \check{G}^{-1}(\phi)$$

Conductance: $G = -\frac{2e^2}{h} \frac{\partial^2 \mathcal{F}}{\partial \phi^2}\Big|_{\phi=0}$

Fano factor: $F = \frac{1}{3} - \frac{2}{3} \frac{\partial^4 \mathcal{F}/\partial \phi^4}{\partial^2 \mathcal{F}/\partial \phi^2}\Big|_{\phi=0}$

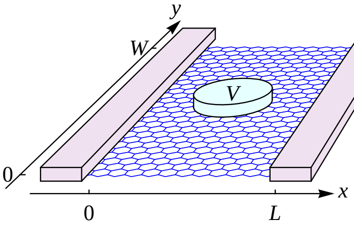
Clean graphene

$$\mathcal{F}_0(\phi) = -\frac{W\phi^2}{\pi L}, \qquad G = \frac{4e^2}{\pi h}\frac{W}{L}, \qquad F = \frac{1}{3}$$

Outline

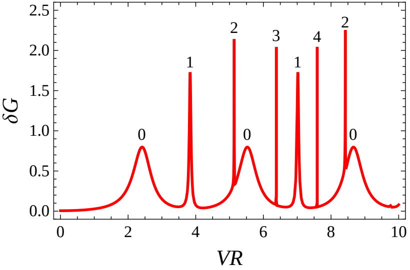
Single strong impurity: numerics

Bardarson, Titov, Brouwer '09



Resonances

$J_m(VR) = 0$



General formalism

Generating function

$$\mathcal{F}(\phi) = \mathbf{Tr}\log(\check{G}_0^{-1} - V) = \mathcal{F}_0 + \mathbf{Tr}\log(1 - \check{G}_0 V)$$

Small impurity $\Rightarrow$ G_0 at coincident points $\Rightarrow$ diverge!

How to resolve? Introduce T-matrix!

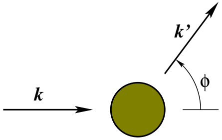
$T = V(1 - gV)^{-1}$ with $g(\mathbf{r}) = -\dfrac{i\boldsymbol{\sigma}\mathbf{r}}{2\pi r^2}$ $\leftarrow$ Green function of infinite graphene

$$\delta\mathcal{F} = \mathbf{Tr}\log[1 - gV - (\check{G}_0 - g)V] = \mathbf{Tr}\log[1 - (\check{G}_0 - g)T] + \cancel{\mathbf{Tr}\log[1 - gV]}$$

With regularized Green function $\check{G}_{\text{reg}}(\mathbf{r}) = \lim_{\mathbf{r}'\to\mathbf{r}}\left[\check{G}_0(\mathbf{r},\mathbf{r}') - g(\mathbf{r}-\mathbf{r}')\right]$

$\delta\mathcal{F} = \log\det[1 - \check{G}_{\text{reg}}(\mathbf{r})\,T]$ $\leftarrow$ just 2×2 determinant!

Scattering of Dirac fermions



Scattering state

$$\psi = e^{ikx}\begin{pmatrix}1\\1\end{pmatrix} + \frac{f(\phi)}{\sqrt{-ir}} e^{ikr}\begin{pmatrix}1\\e^{i\phi}\end{pmatrix}$$

$$f(\phi) = -\sqrt{\frac{k}{2\pi}}\langle \mathbf{k}'|T|\mathbf{k}\rangle$$

s-wave scattering

$T = \ell \leftarrow$ scattering length

$$T = -\lim_{k\to 0}\sqrt{\frac{2}{\pi k}}\int_{-\pi}^{\pi} d\phi\, f(\phi)\begin{pmatrix}1 & 0\\0 & e^{i\phi}\end{pmatrix}$$

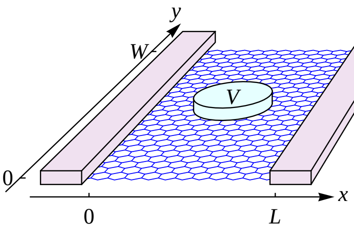
Cross section: $\Lambda = k\ell^2/2$

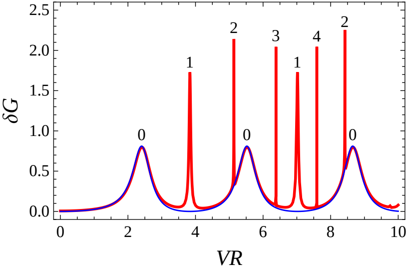
Sharp impurity [cf. Novikov '07]

$$U(r) = \begin{cases} V, & r < R\\ 0, & r > R\end{cases}$$

$$\ell = 2\pi R\frac{J_1(VR)}{J_0(VR)} \simeq 2\pi R\tan(VR - \pi/4)$$

Single strong impurity: s-wave approximation

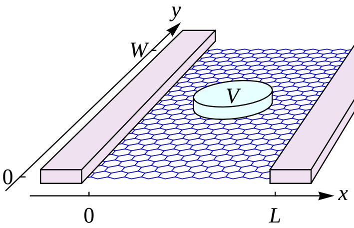


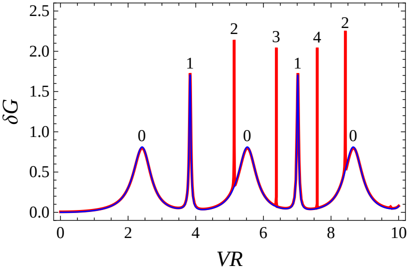


Correction to conductance

$$\delta G = \frac{32 e^2}{\pi^2 h} \left[\frac{16 L^2}{\ell^2} + \frac{1}{\sin^2(\pi x / L)} \right]^{-1}$$

Single strong impurity: s-wave + p-wave





Include two channels in the T-matrix

$$T = \begin{pmatrix} \ell & 0 \\ 0 & \ell_1 \end{pmatrix} \quad \text{with} \quad \ell = 2\pi R \frac{J_1(VR)}{J_0(VR)} \qquad \ell_1 = 2\pi R^3 \frac{J_2(VR)}{J_1(VR)}$$

Outline

General formalism

Generating function

$$\mathcal{F}(\phi) = \mathcal{F}_0 + \mathbf{Tr}\log(1 - \check{G}_0 V) \quad \text{with} \quad V = \sum_n V_n(\mathbf{r})$$

Unfolding

$$\mathcal{F}(\phi) = \mathcal{F}_0 + \mathbf{Tr}\log(1 - \hat{G}_0 \hat{V}) = \mathcal{F}_0 + \mathbf{Tr}\log[1 - (\hat{G}_0 - g)\hat{T}]$$

$\hat{G}_0 \quad \Longrightarrow \quad N \times N$ matrix with all elements equal to $\check{G}_0$

$\hat{V} = \mathrm{diag}\{V_1, V_2, \ldots, V_N\} \qquad \hat{T} = \mathrm{diag}\{T_1, T_2, \ldots, T_N\}$

Small impurities (s-wave scattering)

Regularized Green function $\hat{G}_{\mathrm{reg}} = \begin{cases} \check{G}_{\mathrm{reg}}(\mathbf{r_n}), & n = m, \\ \check{G}_0(\mathbf{r_n}, \mathbf{r_m}), & n \neq m \end{cases}$

$\delta\mathcal{F} = \log\det(1 - \hat{G}_{\mathrm{reg}}\hat{T}) \leftarrow 2N \times 2N$ determinant!

Resonant scalar impurities

At resonance

$$T = \ell \to \infty \quad \Rightarrow \quad \delta\mathcal{F} = \log\det G_{\text{reg}}$$

Conductance

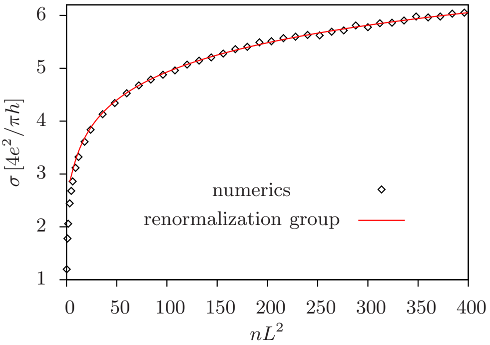
$$G = \frac{4e^2}{\pi h}\left[\frac{W}{L} + \frac{2}{\pi}\,\mathrm{Tr}\,M^{-1}M^{-T}\right] \qquad M = \begin{pmatrix} A & B \\ B^\dagger & -A^T \end{pmatrix}$$

$$A_{mn} = \frac{1}{\sin\frac{\pi}{2L}\left[x_m + x_n + i(y_m - y_n)\right]} \qquad A = A^\dagger$$

$$B_{mn} = \frac{1 - \delta_{mn}}{\sin\frac{\pi}{2L}\left[x_m - x_n + i(y_m - y_n)\right]} \qquad B = -B^T$$

Symmetry class DIII

Resonant scalar impurities: numerics

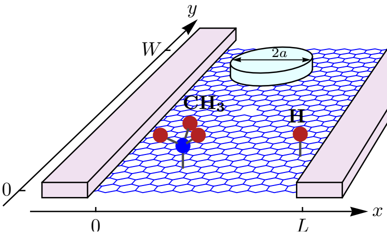


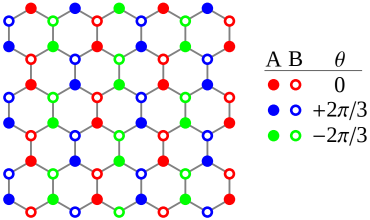
Weak antilocalization in class DIII

$$\frac{d\bar{\sigma}}{d\log L} = 2 - \frac{2}{\bar{\sigma}} + O(\bar{\sigma}^{-2}) \qquad G = \frac{\sigma W}{L} \qquad \sigma = \frac{4e^2}{\pi h}\bar{\sigma}$$

$$\sigma = \frac{4e^2}{\pi h}(\log nL^2 - \log\log nL^2)$$

Vacancies





Strong on-site potential ⇔ vacancy

$$T_A = \frac{\ell}{2}\begin{pmatrix} 1 & 0 & 0 & -e^{i\theta} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -e^{-i\theta} & 0 & 0 & 1 \end{pmatrix} \qquad T_B = \frac{\ell}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & e^{i\theta} & 0 \\ 0 & e^{-i\theta} & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

T matrix projects on sublattice (A or B) and on direction θ in valleys

Resonance: $\ell \to \infty$

Conductance with vacancies

$$G = \frac{4e^2}{\pi h}\left\{\frac{W}{L} + \pi\,\mathrm{Tr}[K, Y](K+K^T)^{-1}[K^T, Y](K+K^T)^{-1}\right\}$$

$$K_{mn} = \frac{e^{\frac{i\pi}{4}(\zeta_m-\zeta_n)+\frac{i}{2}(\theta_m-\theta_n)}}{\sin\frac{\pi}{2L}\left[\zeta_m x_m + \zeta_n x_n + i(y_m - y_n)\right]} \qquad K = K^\dagger$$

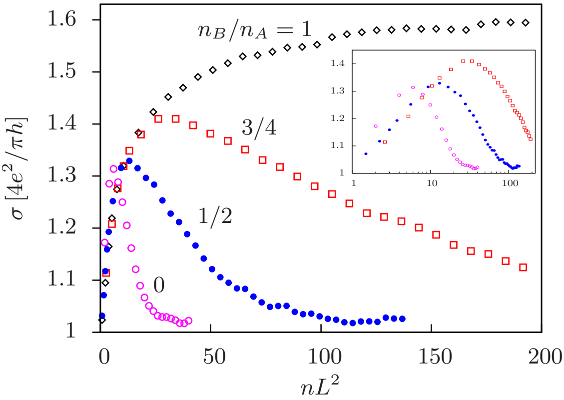
$$Y = L^{-1}\,\mathrm{diag}\{y_1, y_2, \ldots, y_N\}$$

$\zeta_i = \pm 1$ and θ_i are sublattice and color of ith vacancy

Symmetry class BDI [Gade & Wegner '91]

$$\frac{d\bar{\sigma}}{d\log L} = 0$$ perturbatively in ALL loop orders!

Vacancies: preliminary numerics (only one color)



- Unstable fixed point for $n_B = n_A$ (probably conductivity saturates)
- Stable fixed point for $n_B \neq n_A$ with $\sigma \approx \dfrac{4e^2}{\pi h}$

Summary

1. Novel efficient approach to studying transport in strongly disordered systems is developed
2. The theory is applied to graphene with resonant scatterers
3. Resonant scalar impurities lead to antilocalization
4. Vacancies establish various critical regimes
5. Results agree with the symmetry analysis based on the nonlinear sigma model but extend beyond its applicability

PRL **104**, 076802 (2010); arXiv:1006.3299