GENERALIZED (INVERSE) PARTICIPATION RATIO: CORNER STONE OF ANDERSON LOCALIZATION?

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- $$P^{(q)} = \int d^D x \, |\psi_E(x)|^{2q}$$
$$\propto \eta^{q-1} \langle \rho_\eta^q(x) \rangle$$

$$\rho_\eta(x) = \frac{1}{2\pi i} \langle x | \frac{1}{E-\mathcal{H}-i\eta} - \frac{1}{E-\mathcal{H}+i\eta} | x \rangle$$

- Sigma model representation (Wegner '79 AMMP '85)

$$P^{(q)} = \eta^{q-1} \langle [\mathrm{tr}\,\Lambda Q]^q \rangle \qquad \eta \to 0$$

$$S = S_0[Q] + \eta \rho \int \mathrm{tr}\,\Lambda Q$$

$$S_0 = -\frac{\sigma_0}{8} \int \mathrm{tr}\,\partial_\mu Q \partial_\mu Q - \frac{\sigma_H}{8} \int \mathrm{tr}\,\epsilon_{\mu\nu} Q \partial_\mu Q \partial_\nu Q$$

$$Q = T^{-1} \Lambda T \qquad T \in SU(m+n)$$

$$\Lambda = \begin{pmatrix} \mathbb{1}_m & \\ & -\mathbb{1}_n \end{pmatrix}$$

	Anderson loc. $(m=n=0)$	Heisenberg ferromagn. $(m,n\geq 1)$
$P^{(1)}$	$=1$ Ext. states $=1$ Loc. "	>0 Goldstone phase $=0$ Symm. "
$P^{(q>1)}$	$=0$ Ext. states >0 Loc. "	$=0$ Goldstone phase $=0$ Symm. "

- Approaching mob. edge from loc. side:

$$P^{(q)} = \xi^{-(q-1)D_q} F(\eta\, \xi^{D})$$

$$D_q = D - \frac{\gamma_q^*}{q-1} \qquad \gamma_q^* = \frac{q(q-1)}{2\pi\sigma_0^*} \qquad (2+\epsilon \text{ dim.})$$

$$\alpha_q = D_q - (q-1)\frac{dD_q}{dq} \qquad f(\alpha_q) = q\alpha_q - (q-1)D_q$$

- Parabolic approx:

$$f(\alpha) = D - \frac{(\alpha-\alpha_0)^2}{4(\alpha_0 - D)}$$

- QH plateau trn:

$$\alpha_0 = D + \frac{3}{8}\gamma_a^* \simeq 2.14 \qquad \alpha_0 = 2.25 - 2.30$$

(Burmistrov, AMMP '05) (Num. simul.)

- How to discuss Coul. int. ?

Problem: - No obvious def. GIPR
- Does MULTIFRACTALITY exist ?

Answer: Study Finkelstein as fieldtheory

- Start: free part. probl.

$$S'_{eff} = S_0 + \eta\rho\int \mathrm{tr}\,Q\Lambda + \sum_{q=2}^{\infty} \eta^q \zeta_q \underbrace{\int [\mathrm{tr}\,Q\Lambda]^q}_{\text{fluct. in local dens. of states } \rho(x)}$$

Complete set of operators:

$q=2$ $\quad O_1 = \operatorname{tr}\Lambda Q \operatorname{tr}\Lambda Q$

$O_2 = \operatorname{tr}[\Lambda, Q][\Lambda, Q]$

$q=3$ $\quad O_1 = \operatorname{tr}\Lambda Q \operatorname{tr}\Lambda Q \operatorname{tr}\Lambda Q$

$O_2 = \operatorname{tr}[\Lambda, Q][\Lambda, Q] \operatorname{tr}\Lambda Q$

$O_3 = \operatorname{tr}[\Lambda, Q][\Lambda, Q]\{\Lambda, Q\}$

$q=4$ $\quad O_1 = (\operatorname{tr} \Lambda Q)^4$

$$O_2 = \operatorname{tr}[\Lambda, Q]^2 \operatorname{tr}[\Lambda, Q]^2$$

$$O_3 = \operatorname{tr} \Lambda Q \operatorname{tr}[\Lambda, Q]^2 \{\Lambda, Q\}$$

$$O_4 = \operatorname{tr}[\Lambda, Q]^2 (\operatorname{tr} \Lambda Q)^2$$

$$O_5 = \operatorname{tr}[\Lambda, Q]^4$$

Anomalous dimension $\gamma_q^i = \frac{\lambda_q^i}{2\pi\sigma_0} + O(\sigma_0^{-2})$

q	λ_q^i
2	$2 \quad -2$
3	$6 \quad 0 \quad -6$
4	$12 \quad 4 \quad 0 \quad -4 \quad -12$
$\vdots$	$\vdots$
q	$q(q-1)$

- Coulomb int. (Skoric, Baranov, AMMP '99)

1) $Q = Q_{\alpha\beta}^{pp'} \to Q_{\alpha\beta}^{mn} \qquad m,n \in \mathbb{Z}$

2) S'_{eff} constrained by F-invariance

$$S'_{eff}[Q] = S'_{eff}[W^{-1} Q W]$$

W = global gauge trf.

$$S'_{eff}[Q] = S'_0[Q] + Tz \int O_F(x) + O(T^2)$$

$$O_F(x) = \sum_{n,\alpha}{}' \operatorname{tr} [I_n^\alpha, Q][I_{-n}^\alpha, Q]$$

I_n^α = generators $U(1)$ gauge trf

z = singlet int. amplitude

$$S_{eff}'[Q] = S_0'[Q] + Tz \int O_F(x) + \mathcal{O}(T^2)$$

$$O_F(x) = \sum_{n,\alpha}' \operatorname{tr}[I_n^\alpha, Q][I_{-n}^\alpha, Q]$$

I_n^α = generators U(1) gauge trf

z = singlet int. amplitude

- Anomalous dim. z ($2+\epsilon$ dim.)

$$\boxed{\gamma_z = -\frac{2}{\pi\sigma_0} - \frac{\pi^2+2}{6\pi^2\sigma_0^2} + \mathcal{O}(\sigma_0^{-3}) < 0}$$

→ vanishing (bosonic) q.p. density of states!

→ no multifractality!

• PROOF (lengthy, tentative)

Higher dim. operators $\propto T^2$

$q=2$ $\quad O_F$

$q=3$ $\quad O_1 = \sum'_{m,n,\alpha} \mathrm{tr}\, [I^\alpha_m, Q][I^\alpha_n, Q]\{I^\alpha_{-m-n}, Q\}$

$q=4$ $\quad O_1 = \sum' \mathrm{tr}\, [I^\alpha_m, Q][I^\alpha_{-m}, Q]\, \mathrm{tr}\, [I^\beta_n, Q][I^\beta_{-n}, Q]$

$O_2 = \sum' \mathrm{tr}\, [I^\alpha_m Q I^\alpha_{-m}, Q][I^\beta_n Q I^\beta_{-n}, Q]$

$O_3 = \sum' \mathrm{tr}\, [I^\alpha_m, Q][I^\beta_n, Q]\, \mathrm{tr}\, [I^\alpha_{-m}, Q][I^\beta_{-n}, Q]$

$\because$ NO POS. ANOMALOUS DIM.!

CONCLUSION

1) $\mathcal{F}$ algebra extended to $O(T^2)$

2) Theory renormalizable

3) (tentative) NO MULTIFRACTALITY !

4) OPEN QUESTION: TRIPLET INT.