# Influence of membrane wrinkling on vesicle motion in strong stationary flow

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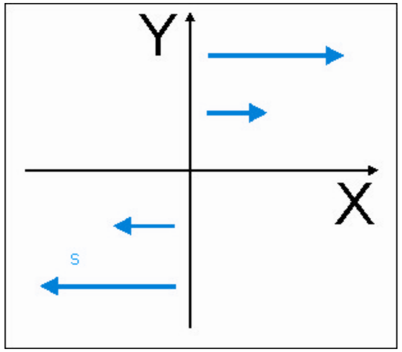
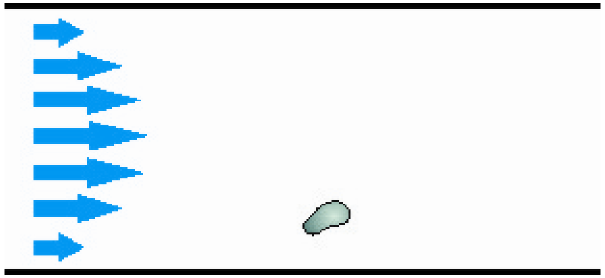
# Vesicle in shear flow

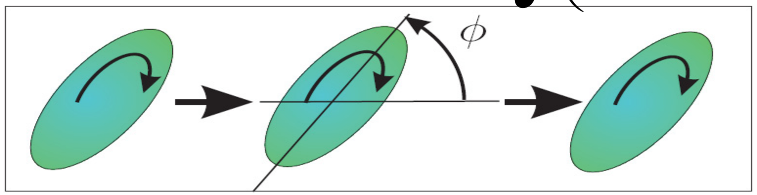



**Particular example of external flow:** shear

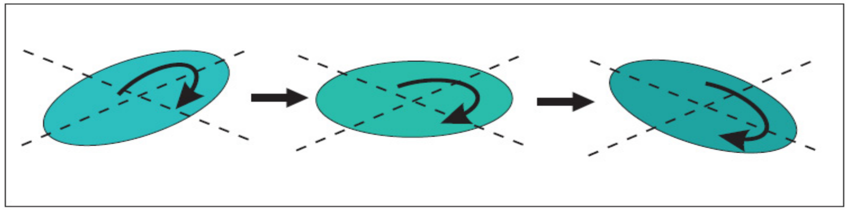
$$v_x = sy$$



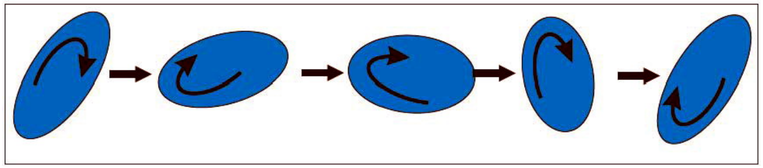
**Free energy** $\mathcal{F}_{\text{free}} = \int \left( \kappa \frac{H^2}{2} + \sigma \right) \mathrm{d}A,$



**Tank-Treading regime:**
the vesicle conserves its form and orientation



**Trembling (or swinging or vacillating-breathing ) regime:**
the vesicle does not perform full revolution



**Tumbling regime:**

# Characteristic times and transition from TT to TU

**Reynolds number is small at the scales**

$$\eta \Delta \mathbf{v} - \nabla p + \boldsymbol{f}_{\mathrm{membrane}} = 0$$

Far from the vesicle velocity of fluid tends to unperturbed V,

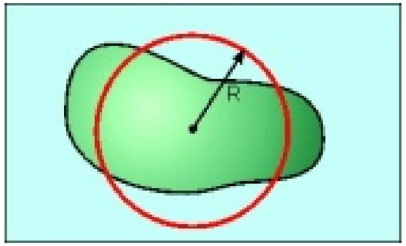
$$\partial_k V_i = \varsigma_{ik} = s_{ik} - \varepsilon_{ikl}\omega_l$$

$$\|\varsigma_{ik}\| = \begin{pmatrix} 0 & s+\omega & 0 \\ s-\omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

## Dynamics in strong external flow

## Case S=0, κ=0

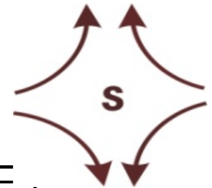

$$\tau_{\mathrm{rotation}} \sim \frac{1}{\omega}$$




$$r = R\left[1 + u(\theta,\phi)\right],$$

$$u \sim \sqrt{\Delta}$$

## Dynamics in strong external flow Case ω=0, κ=0

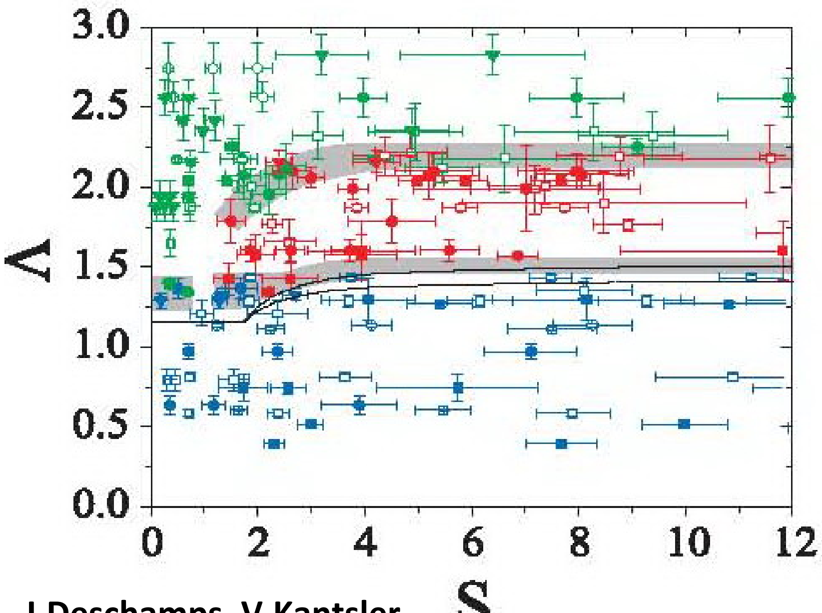



$$\tau_{\mathrm{stretch}} \sim \left(1 + \frac{\eta_{\mathrm{out}}}{\eta_{\mathrm{in}}}\right)\frac{\sqrt{\Delta}}{s}$$

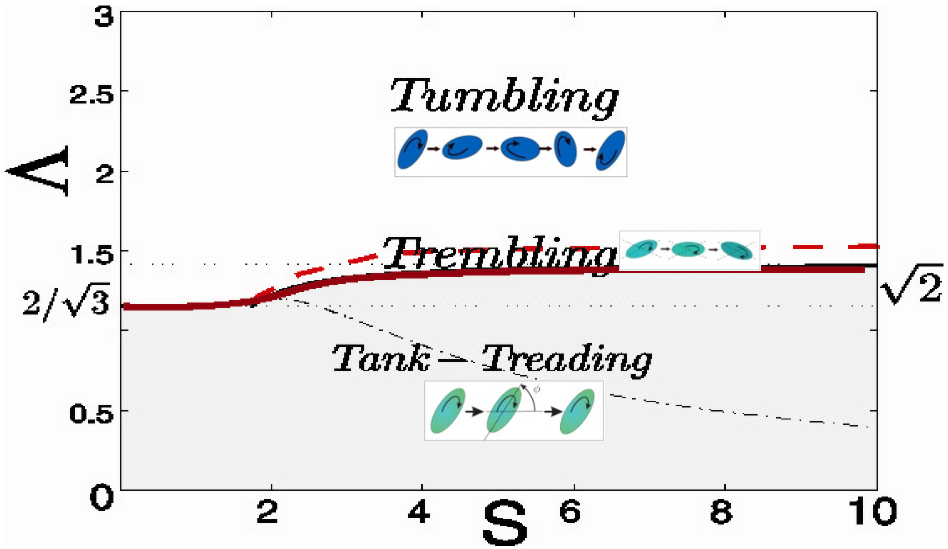
**Tank-treading to tumbling transition,**
**Δλ<<1**

$$\tau_\omega \sim \tau_s, \qquad \Lambda \sim \frac{\tau_{\mathrm{rotation}}}{\tau_{\mathrm{stretch}}} \sim \frac{s}{\omega}(\lambda+1)\sqrt{\Delta} \sim 1$$

# The phase (regime) diagram




J.Deschamps, V.Kantsler & V.Steinberg, PRL 102, 118105 (2009)




V.V.Lebedev, K.S.Turitsyn & S.S.Vergeles PRL (2007)

External flow strength (Cappilary number)

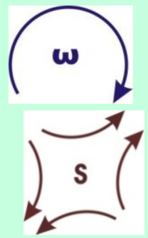
$$\mathbf{S} = \frac{\tau_{relax}}{\tau_{stretch}} =, \quad = * \frac{s\,\eta_{out}\,R^3}{\kappa\Delta}$$

Role of rotational part of external flow

$$\mathbf{\Lambda} = \frac{\tau_{stretch}}{\tau_{rotation}} = \quad =* \frac{\omega\sqrt{\Delta}}{s}\left(1 + *\frac{\eta_{in}}{\eta_{out}}\right)$$

External velocity gradient

$$\left\|\partial_i V^k\right\| = \begin{pmatrix} 0 & s+\omega & 0 \\ s-\omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

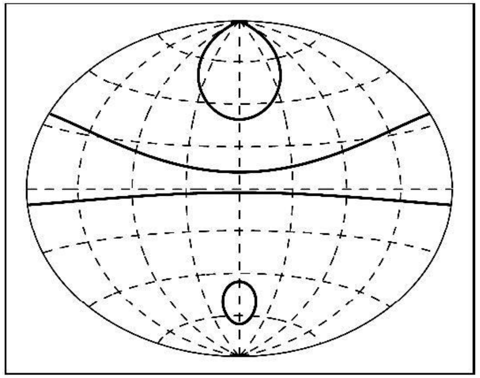


# Vesicle motion in strong external flow

**Vesicle form** $r = R\left[1 + u(\theta,\phi)\right],$

**Expansion over angular harmonics:** $u(\theta,\phi) = \sum_{l\geq 2} u^{(l)}(\theta,\phi)$

$$u^{(2)}(\theta,\phi;\,\Theta,\Phi) \propto \sqrt{\Delta}\left[\frac{\sin\Theta}{\sqrt{3}}(1-3\cos^2\theta) + \cos\Theta\sin^2\theta\cos(2\phi - 2\Phi)\right], \quad u^{(>2)} = 0$$



$\Theta - \Phi$ atlas

$$S^{-1}\tau_* \partial_t \Theta = -\sin\Theta\sin(2\Phi) - \frac{1}{S}\cos(3\Theta)$$

$$S^{-1}\tau_* \partial_t \Phi = \frac{1}{2}\left(\frac{\cos\Phi}{\cos\Theta} - \Lambda\right)$$

$S \gg 1$, **Time period** $\sim 1/(S\Lambda)$

**Parameter** $\Lambda > 1$

**Slow variable Υ**

$$\frac{\sin\Upsilon}{\Lambda - \cos\Upsilon} = \frac{\sin\Theta}{\Lambda - \cos\Theta\sin(2\Phi)}$$

**Tumbling:** $\frac{2\Lambda}{\Lambda^2+1} < \cos\Upsilon < 1$

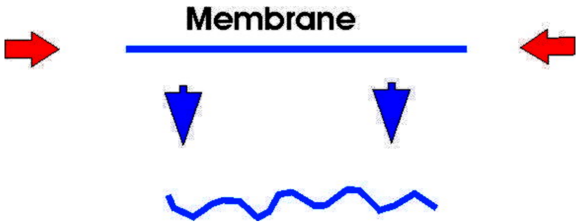
**Trembling:** $\frac{2\Lambda}{\Lambda^2+1} < \cos\Upsilon < \frac{1}{\Lambda}$

**Tunk-treading:** $\cos\Upsilon = \frac{1}{\Lambda}$

**Slow dynamics:**

$$\dot{\Upsilon} = \frac{1}{\tau_*} \times \Upsilon_d(\Upsilon)$$

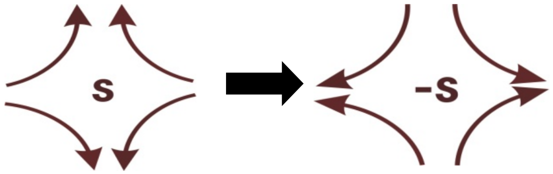
# Excitation of high order harmonics on the vesicle surface



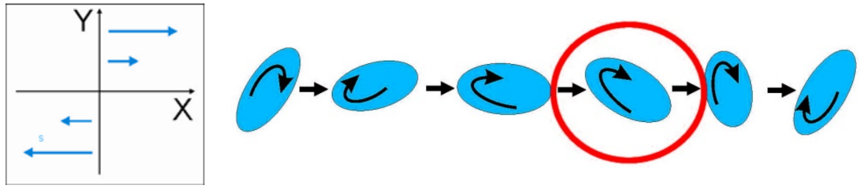

surface tension of the vesicle membrane becomes negative during some time interval.

Negative surface tension leads to instability of short-wave harmonics develops.

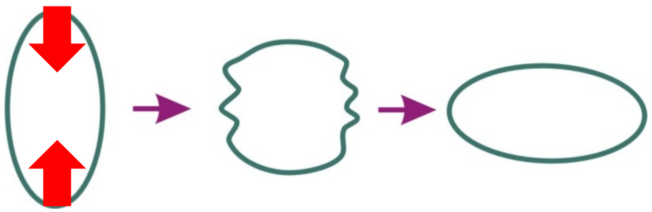
## Nonstationaty flows




## Stationaty flows

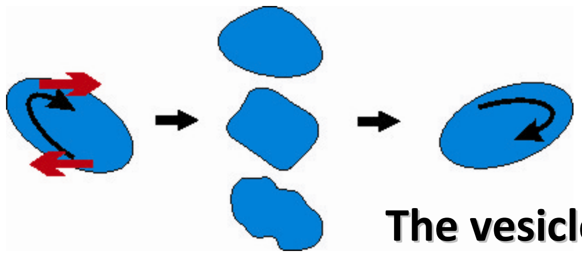



## In strong external flow



Wrinkling

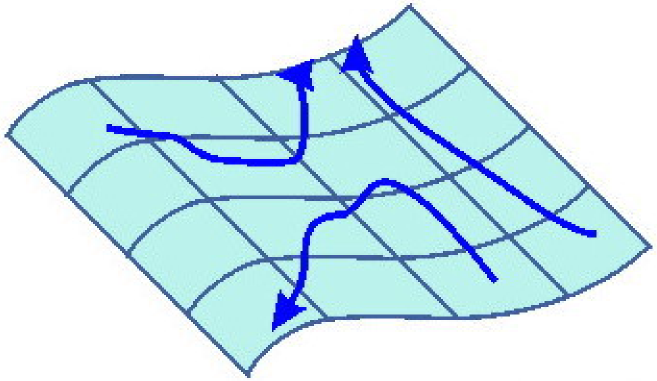
V.Kantsler, E.Serge & V.Steinberg, PRL 99, 178102 (2007)
Theory: K.S.Turitsyn & S.S.Vergeles, PRL 100, 028103 (2008)



The vesicle loses elliptical shape

J.Deschamps, V.Kantsler & V.Steinberg, PRL 102, 118105 (2009)

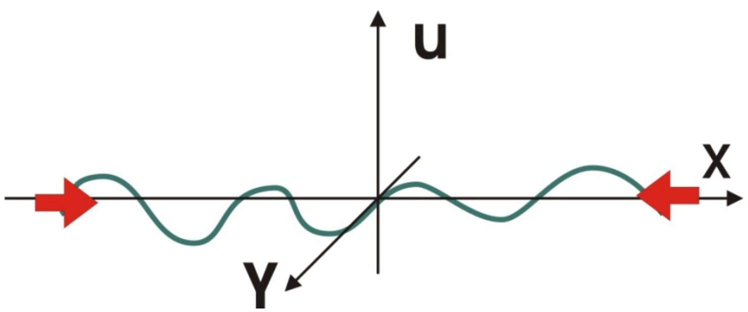
# Membrane wrinkling



Membrane (with possible surface flow)

Free energy $$\mathcal{F}_{\text{free}} = \int \left( \kappa \frac{H^2}{2} + \sigma \right) dA,$$

H -- mean curvature
κ -- bending modulus
dA -- area of surface element
σ -- surface tension, which guarantees local surface area conservation




$$\mathcal{F}_{free} = \sum_{\vec{k}} \left( \kappa k^4 + \sigma k^2 \right) | u_{\vec{k}} |^2$$

$u_{\vec{k}}$ — Fourier mode of $u(x, y)$

If $\sigma < 0$, then modes $u_{\vec{k}}$ with

$$|\vec{k}| < \frac{|\sigma|}{\kappa}$$

becomes unstable

**Buckling instability**

# Weak excitation of high order harmonics

**Vesicle volume**

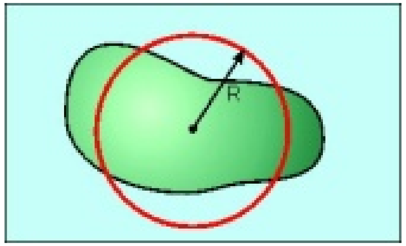
$$\mathcal{V} = \frac{4\pi}{3} R^3,$$

**Surface area of the membrane**

$$\mathcal{A} = (4\pi + \Delta) R^2$$

**Excess area**

$$\Delta = \Delta(\Delta_2 + \Delta_3 + \ldots)$$




**Vesicle form is almost smooth**

$$\Delta_3 = 1 - \Delta_2 << 1$$

$$S^{-1}\tau_* \, \partial_t \Theta = -\frac{\sin\Theta \sin(2\Phi)}{\sqrt{\Delta_2}} - \frac{1}{S}\cos(3\Theta) + \mathrm{DF}_\Theta + \xi_\Theta$$

$$S^{-1}\tau_* \, \partial_t \Phi = \frac{1}{2}\left( \frac{\cos\Phi}{\sqrt{\Delta_2}\ \cos\Theta} - \Lambda \right) + \xi_\Phi$$

$$S^{-1}\tau_* \frac{\partial_t (1-\Delta_2)}{2} = *(1-\Delta_2)\left( \cos\Theta \sin(2\Phi) - \frac{*}{S\sqrt{\Delta}} \right) + \mathrm{DF}_3 + \xi_3$$

**Thermal noise**

**(Negative) surface tension**

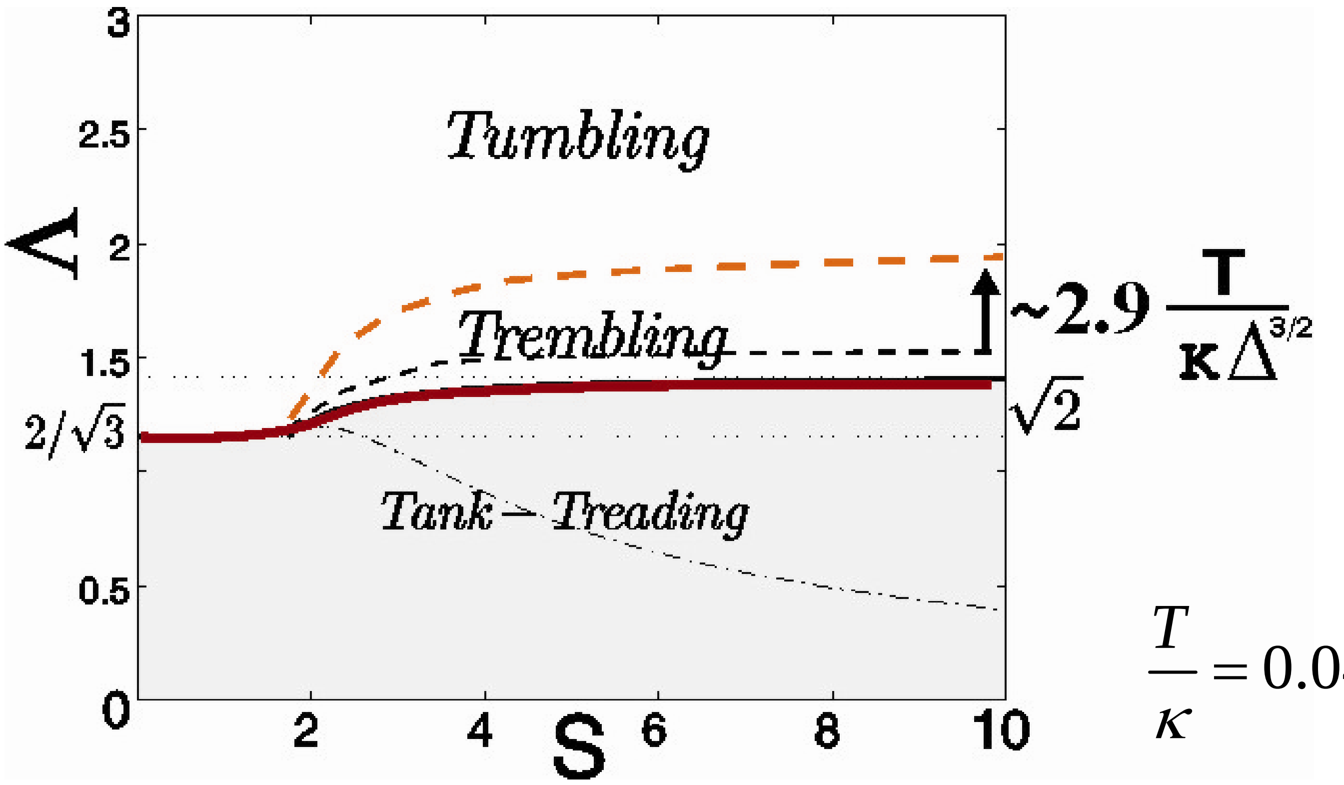
**Damping due to membrane potential forces**

$$\langle \xi_3(t)\ \xi_3(t') \rangle = «\ldots» \frac{1}{S^2} \frac{T}{\kappa\ \Delta^{3/2}}\ \tau_* \delta(t - t')$$

**and the same for** $\xi_\Theta,\ \xi_\Phi$

$$DF_\Theta,\ DF_\Phi = «\ldots» \frac{1}{S} \frac{T}{\kappa\ \Delta^{3/2}}$$

## The phase diagram of the vesicle dynamical regimes in presence of thermal noise




$$\frac{T}{\kappa} = 0.04, \quad \Delta = 0.5$$

$$\sqrt{1-\Delta_2} \approx 0.3$$

# Extremely large shears

**High order angular harmonic dynamics (number $l$)**

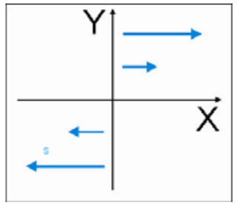
$$S^{-1}\tau_* \frac{\partial_t \Delta_l}{2} = {*l}\,\Delta_l\left(\cos\Theta\sin(2\Phi) - \frac{{*l^2}}{S\sqrt{\Delta}}\right) + \mathrm{DF}_l + \xi_l$$

$$1-\Delta_2 \sim 1, \qquad S >> \left[\ln\left(\frac{S^{3/5}\kappa}{T}\right)\right]^{5/2}$$

Acceleration of dynamics for high order angular harmonics

(Negative) surface tension

Damping due to membrane potential forcesΛ

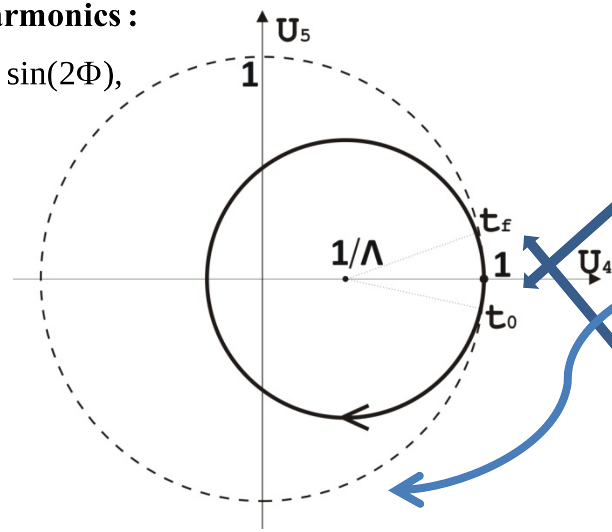


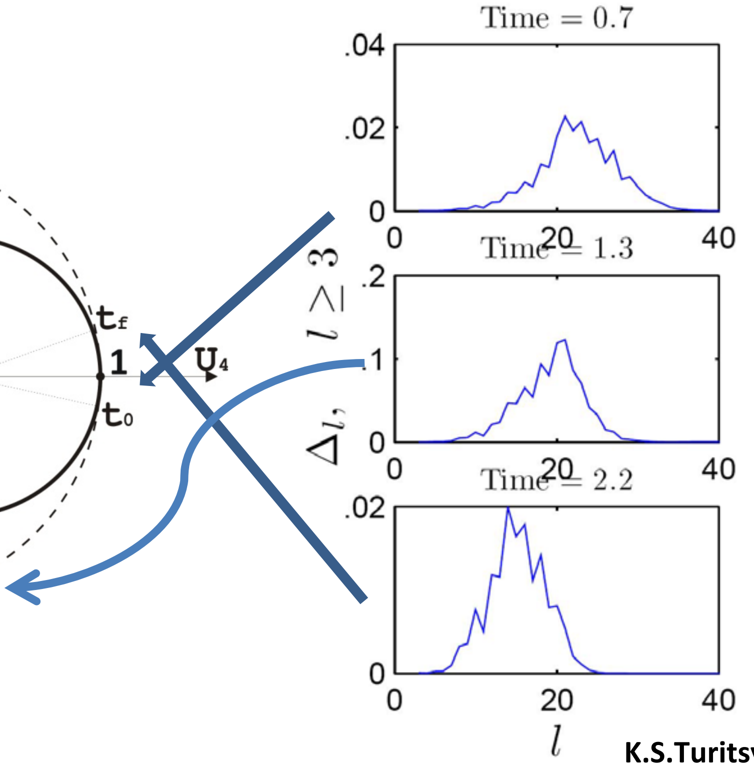

**Dynamics of second order harmonics:**

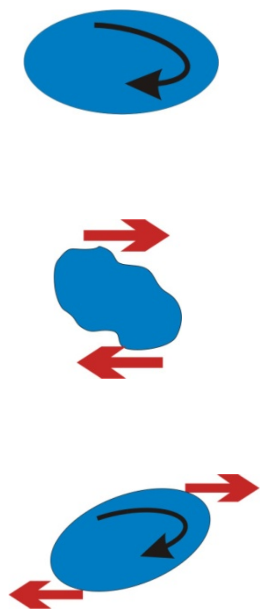
$U_4 = \sqrt{\Delta_2}\cos(2\Phi), \quad U_5 = \sqrt{\Delta_2}\sin(2\Phi),$

**(In particular case $\Theta = 0$)**

**Only advection by flow, without influence of membrane surface tension**



**High order harmonics: universal dynamics**

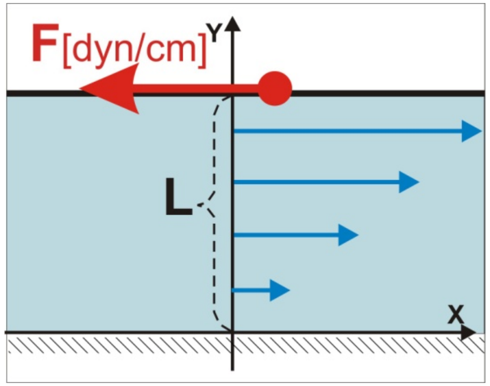
$$\bar{l} = l_0\left(\frac{t}{t_0}\right)^{-1/3}, \qquad l^* \sim S^{1/3}$$

**Surface tension**

$$\sigma = \sigma(t_0)\left(\frac{t}{t_0}\right)^{-2/3}, \quad t_0 = \frac{T}{S^{1/5}}, \quad l_0 \sim S^{2/5}$$

K.S.Turitsyn & S.S.Vergeles, PRL 100, 028103 (2008)

# Viscosity of a suspension




**Special geometry: shear flow** $V_x = \dot{\gamma} y$

**(Shear) viscosity** $\eta_s = \dfrac{F}{\dot{\gamma} L}$

$W$ – **applied external power, with suspension**

$W^{(0)}$ – **applied external power, with pure solvent**

**Flow is slow, thus is described by Stokes equation**

$$\eta \, \Delta \mathbf{v} = \nabla p - \boldsymbol{f}_{\text{ext}}, \qquad \text{div}\, \mathbf{v} = 0$$

**Viscosity of the suspension**

$$\eta = \frac{W}{W^{(0)}} \eta_{\text{solvent}}$$

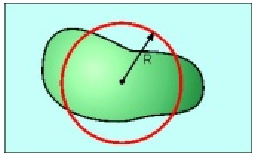
# Viscosity of a suspension

**Applied external power for suspension**

$$W = \int_\Gamma dS^i\, \Pi^{ik}\, v^k_{(0)} = W^{(0)} + \sum_a W^a$$

$$\eta = \eta_{\text{solvent}} \left[ 1 + \frac{1}{W^{(0)}} \sum_a W^a \right]$$

**Velocity of solvent near a particle**




$$\boldsymbol{v} = \boldsymbol{v}_{(0)} + \delta \boldsymbol{v}$$

**Power, wich is dissipated on the particle**

$$W^a = -\frac{5\eta \mathcal{V}}{2} \int \frac{do}{4\pi}\, \partial_i v^k_{(0)}\, n^i n^k \left[ \frac{12\, \delta v^r}{R} + 3 \partial_r \delta v^r \right]_{r=R}$$

**for limit when volume fraction** $\varphi << 1$

**Rigid balls with radius** $R$ (Einstein)

$$W^a = \frac{5\eta \mathcal{V}}{2}\, \partial_i v^k_{(0)} \left( \partial_i v^k_{(0)} + \partial_k v^i_{(0)} \right), \quad \mathcal{V} - \text{volume of the particle}$$

**Viscosity of the suspension**

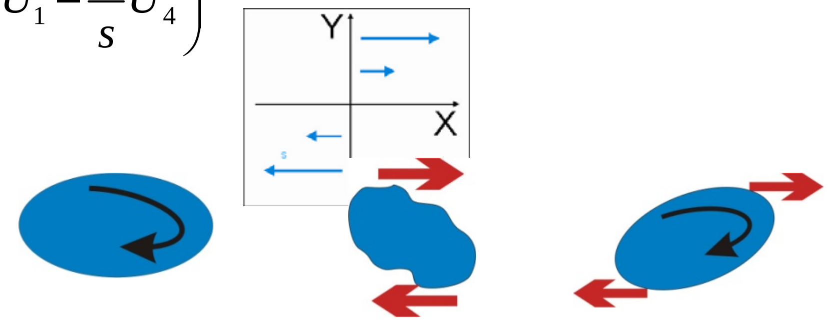
$$\eta = \eta_{\text{solvent}} \left( 1 + \frac{5}{2} \varphi \right)$$

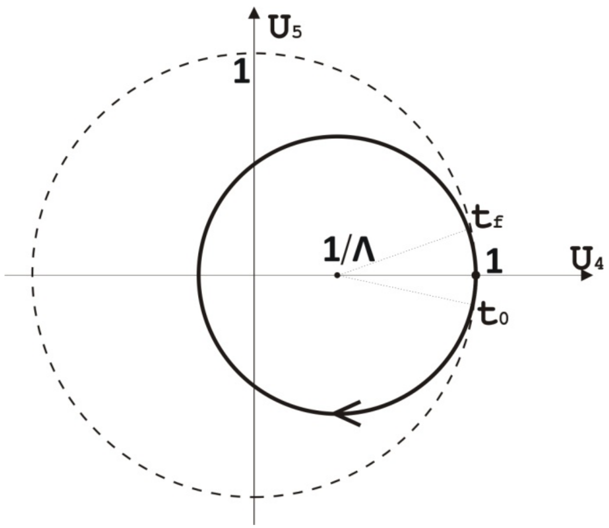
# Viscosity of vesicle suspension

**Viscosity of the vesicle suspension**

**($\varphi$ – volume fraction)**

$$\frac{\eta - \eta_{\text{solvent}}}{\varphi\ \eta_{\text{solvent}}} = \frac{5}{2} + \left\langle \sqrt{\Delta}\ Q \right\rangle_{\text{vesicles,time}}, \qquad Q = *\left( *U_1 - \frac{\omega}{s} U_4 \right)$$




**For random distribution of vesicle smooth shapes**

**1)Without wrinkling:**

$$\langle Q \rangle = 0$$

**2)With wrinkling:**

$$\langle Q \rangle = -0.77 \frac{\omega}{s} \frac{(2\Lambda - 1)(\Lambda - 1)}{3\Lambda^4} < 0$$

# Conclusions

- **Influence of short wave excitations on the vesicle membrane during motion in stationary flow was considered**
- **Limit of small influence from short wave excitation on the vesicle motion was investigated: correction to regime diagram was obtained**
- **Limit of extremely strong flows was considered: membrane wrinkling was established and power law for typical wave vector was found**
- **Rheology properties of the vesicle suspension was discussed, in particular for limit of extremely strong flows.**