Surface plasmon modes of metal granules chains

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Parameters

$a\,,\ l$ - granule size;

δ - gap between granules;

$\varepsilon(\omega) = \varepsilon' + i\varepsilon''$ - metal permittivity;

$\varepsilon' \gg \varepsilon'';\ \varepsilon' < 0;\ |\varepsilon'| \gg 1$

For 620 nm wavelength the constants are:

Gold – $\varepsilon' = -10.66; \quad \varepsilon'' = 1.37$

Silver - $\varepsilon' = -17.24; \quad \varepsilon'' = 0.5$

Equations

We assume that: $\sqrt{\varepsilon}ka \ll 1;\ k = \omega / c$

Thus, electric field is potential:

$$\mathbf{E} = -\nabla\phi; \quad \nabla^2\phi = 0$$

$$\text{Boundary conditions:}$$

$$\phi_{in} = \phi_{out};\ \varepsilon\frac{\partial\phi_{in}}{\partial n} = \frac{\partial\phi_{out}}{\partial n}$$

Modes of single granule

Single spherical granule:

$$\phi^{in} = r^n Y_{nm}; \quad \phi^{out} = r^{-(n+1)} \tilde{Y}_{nm}$$

$$\varepsilon_n^{res} = -\frac{n+1}{n}$$

Single cylinder: $\varepsilon_n^{res} = -1$

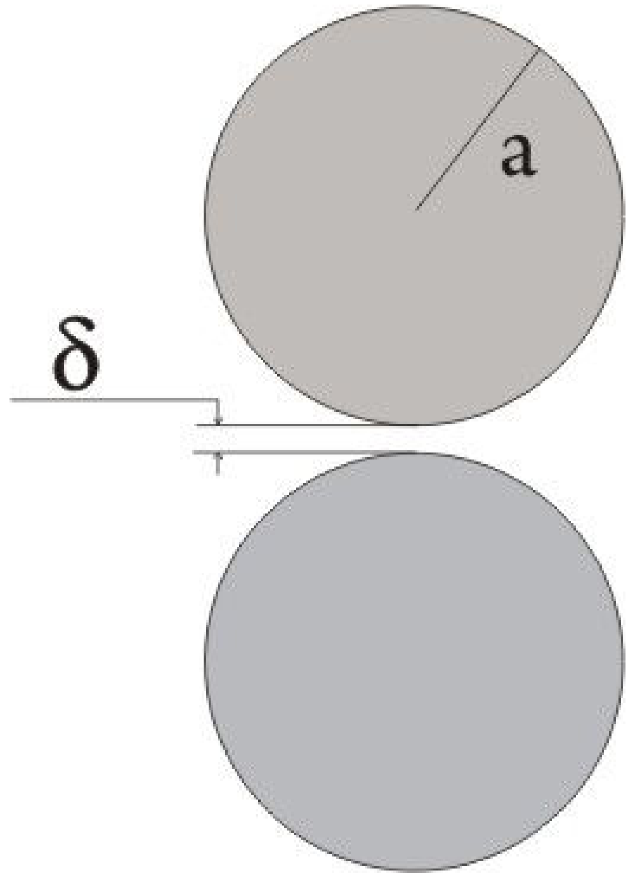
Modes of pares of granules

Gap width: $\sim \delta$

Gap length: $\sim \sqrt{a\delta}$

$$a \gg \delta$$

$$\varepsilon_n^{res} \sim -\frac{1}{n}\frac{\sqrt{a\delta}}{\delta} \sim -\frac{1}{n}\sqrt{\frac{a}{\delta}};$$

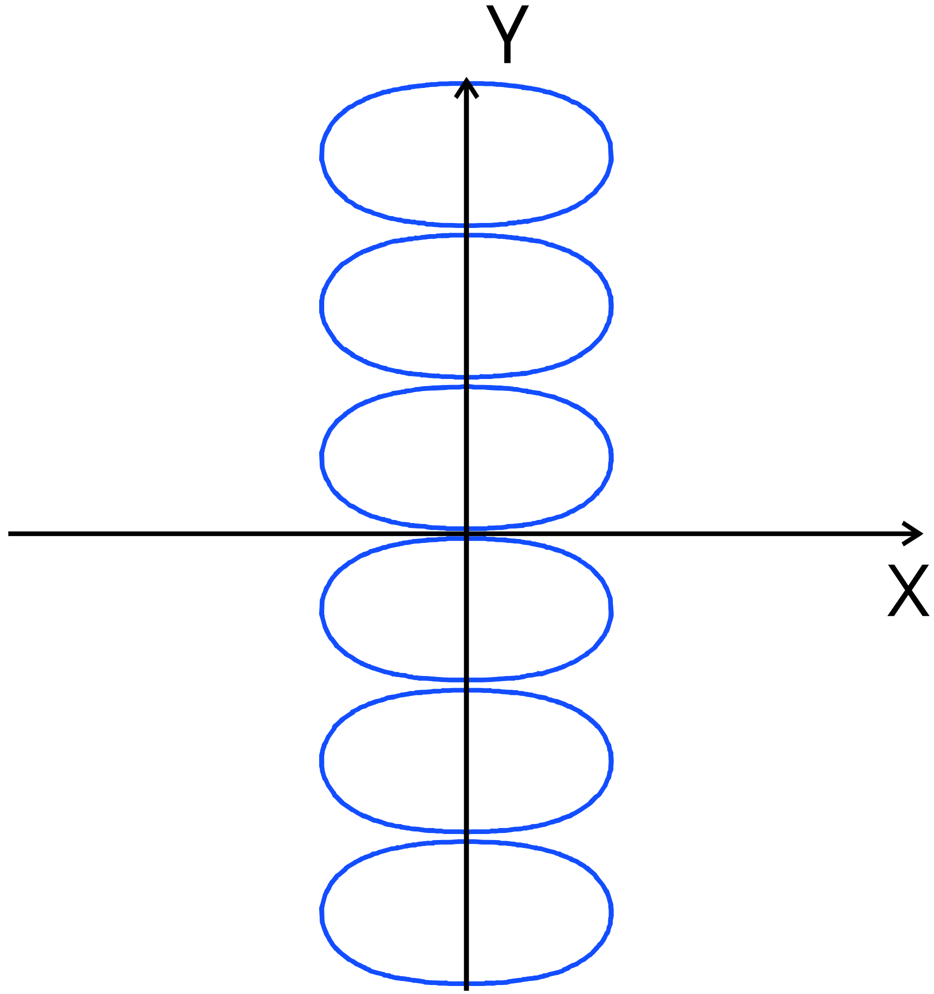


Conformal mapping

$$\xi + i\eta = \ln\left[tg\left\{ \frac{\pi(1/2 - iz/l)}{2} \right\} \right]$$

$$w = \xi + i\eta$$

Surfaces $\xi = \xi_0$



Parameters

$$\xi_0 \ll 1; \qquad \xi_0 = \frac{\pi\delta}{2l}; \qquad R = \frac{\pi^2}{2}\frac{l^2}{\delta}$$

- R – curvature radius of the granule surface at the gap center
- Gap width - $\sim \delta$,
- Gap length - $\sim \sqrt{R\delta} \sim l$

Quasi-momentum

$$\Phi(x, y+l) = e^{iql}\Phi(x, y);$$

$$\Phi(\xi, \eta) = e^{\pm iql}\Phi(-\xi, \pi - \eta);$$

$$\Phi(\xi, \eta) = e^{\mp iql}\Phi(-\xi, -\pi - \eta)$$

Quasi-momentum q=0

$$\Phi = sh\left[(2n+1)\xi\right]\cos\left[(2n+1)\eta\right]$$

$$\varepsilon_n^{res} = -cth\left[(2n+1)\xi_0\right]$$

$$\varepsilon_0^{res} = -\frac{1}{\xi_0} = -\frac{2l}{\pi\delta}$$

Quasi-momentum q=π / l

$$\Phi = sh\left[2(n+1)\xi\right]\cos\left[2(n+1)\eta\right]$$

$$\varepsilon_n^{res} = -cth\left[2(n+1)\xi_0\right]$$

$$\varepsilon_0^{res} = -\frac{1}{2\xi_0} = -\frac{l}{\pi\delta}$$

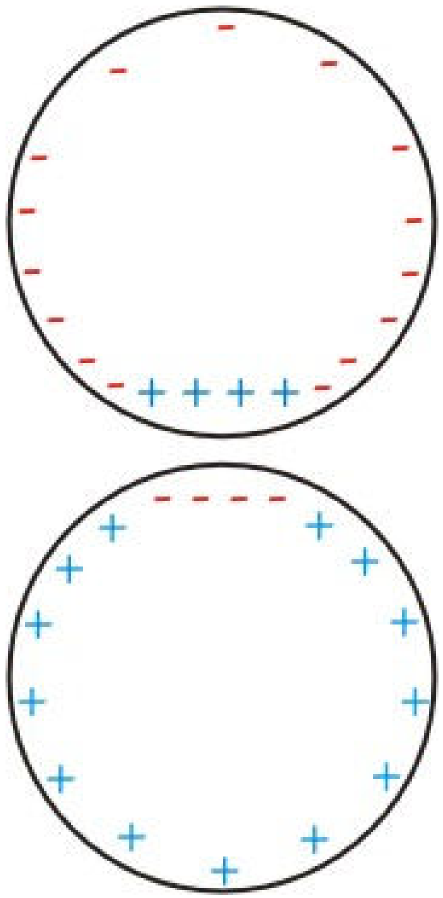
Resonance zones

$$\Delta\varepsilon_n^{res} = cth\left[(2n+1)\xi_0\right] - cth\left[2(n+1)\xi_0\right];$$

$$\Delta\varepsilon_0^{res} = \frac{2l}{\pi\delta} - \frac{l}{\pi\delta} = \frac{l}{\pi\delta}$$

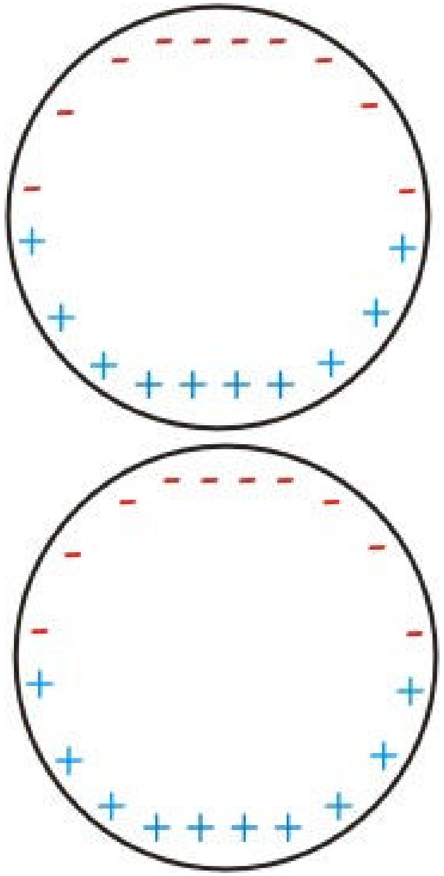
$$\Delta\varepsilon_n^{res} \sim |\,\varepsilon_n^{res}\,|$$

Charge distribution.
Pair of granules.

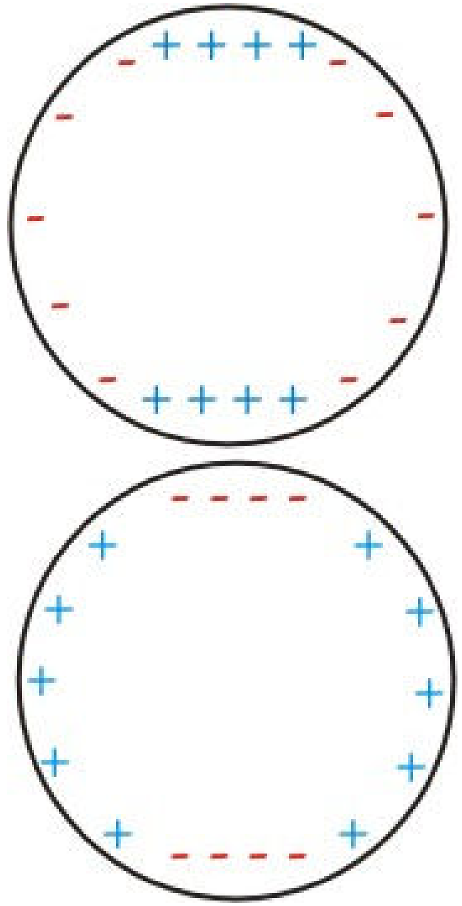


Charge distribution. Chain of granules.

q=0



q= π / l



General expression for potential.

$$\Phi = \tilde{\Phi}(\xi + i\eta) + \tilde{\Phi}(\xi - i\eta)$$

$$\Phi = \left[th(w/2 - i\pi/4)\right]^{iql/\pi} \sum B_m(\cos\{im(w - i\pi/2)\} - (-1)^m)$$

$$\Phi^{in} = \sum A_m e^{-m\xi} \cos(m\eta)$$

Mode dispersion

For small values of quasi-momentum: $ql \ll 1$

$$\varepsilon_n^{res} = -\frac{1}{\xi_0} + \frac{2ql}{\pi {\xi_0}^2}$$

External field enhancement

Round cylinders:

$$d \sim E_c a^{3/2} \delta^{1/2}; E_c / E_0 = \frac{1}{\varepsilon''} \left(\frac{a}{\delta} \right)^{3/2}$$

Oblate cylinders:

$$d \sim E_c l^2;\ \ E_c / E_0 = \frac{1}{\varepsilon''} \left(\frac{l}{\delta} \right)^2$$

Reflection from thin plate.

a - plate thickness, $\sqrt{\varepsilon} ka \ll 1;$

If $\varepsilon ka \ll 1$ - reflection coefficient $R \ll 1$

If $\varepsilon ka \gg 1$ - reflection coefficient $R \sim 1$

Reflection coefficient

$$E = E_{inc}(e^{ikx} + Be^{-ikx})$$

B – reflection amplitude.

$$B = -\frac{ik}{2L}\int \frac{D_y}{E_{inc}} ds;$$

$$D_y = GE_0; \;\; E_0 = E_{inc}(1 + B)$$

Reflection coefficient. Estimations

Round cylinders:

$$B \sim \frac{ka^{3/2}}{\varepsilon''\delta^{1/2} + ka^{3/2}}; \quad \varepsilon''\delta^{1/2} \le ka^{3/2} \rightarrow B \sim 1$$

Oblate cylinders:

$$B \sim \frac{kl^{3}}{\varepsilon''\delta^{2} + kl^{3}}; \quad \varepsilon''\delta^{2} \le kl^{3} \rightarrow B \sim 1$$

Reflection coefficient. Analytic solution for oblate cylinders.

$$\int \frac{D_y}{l} ds = E_0 l \sum \frac{\alpha_n}{\varepsilon + cth\left[(2n+1)\xi_0\right]}$$

$$\alpha_n = \frac{2(\varepsilon - 1)^2 \exp[-2(2n+1)\xi_0]}{\pi(2n+1)}$$

Reflection coefficient. Analytic solution for oblate cylinders.

$$\varepsilon = -cth\left[\xi_0\right];$$

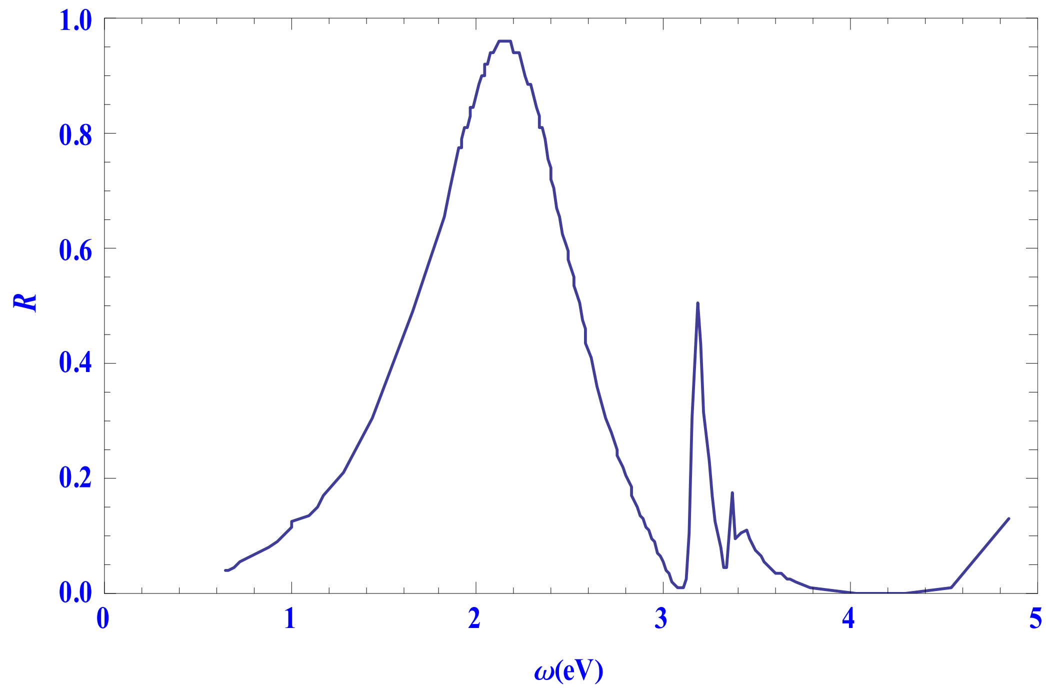
$$B = \frac{4kl^3}{\pi^2 \varepsilon'' \delta^2 + 4kl^3}$$

$$-cth\left[\xi_0\right] < \varepsilon < -cth\left[3\xi_0\right] \rightarrow \; B = 0 \quad (\varepsilon'' = 0);$$

Conclusions

- In chains of metal granules plasmonic modes can exist in broad ranges of metal permittivity values – zones.
- The width of each zone is of the order of permittivity value inside the zone.
- Plasmonic resonance can lead to strong reflection of electromagnetic wave from the chain of granules compared to the metal plate.

Reflection coefficient. Numerical results.



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Silver granules. First peak: $\lambda \approx 570nm,\quad \varepsilon' \approx -12;\quad \varepsilon'' \approx 0.5$